mathematics education in the margins

Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia

Edited by Margaret Marshman, Vince Geiger & Anne Bennison

Mathematics Education in the Margins
Proceedings of the 38th Annual Conference of the
Mathematics Education Research Group of Australasia

ISBN 978-1-920846-40-4

Published by
The Mathematics Education Research Group of Australasia
GPO Box 2747
Adelaide SA 5001
www.merga.net.au

Local organising committee:
Margaret Marshman, University of the Sunshine Coast – Conference Convenor
Anne Bennison, The University of Queensland
Shelley Dole, University of the Sunshine Coast
Vince Geiger, Australian Catholic University
Merrilyn Goos, The University of Queensland
Peter Grootenboer, Griffith University
Michael Jennings, The University of Queensland
Janeen Lamb, Australian Catholic University
Glorianne Porter, University of the Sunshine Coast

Conference sponsors:
Faculty of Science, Health, Education and Engineering, University of the Sunshine Coast
University of the Sunshine Coast

Preface

This is a record of the Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia (MERGA), held at the University of the Sunshine Coast. The Proceedings were published on the MERGA website at www.merga.edu.au, as well as being made available to conference delegates on a USB.

The theme of the conference was *Mathematics education in the margins*. There were several reasons for choosing this theme. First, the University of the Sunshine Coast is a regional university seen by many as being in the “margins”. The conference was therefore an opportunity to share the endeavours of the many people who teach, research, and reach out to those students who feel that their mathematical experiences are in the margins. It was also an opportunity to question the role of mathematics education in helping students come out from the margins. To that end, the keynote presentations addressed this theme with Professor Peter Sullivan presenting his work on engaging students by posing challenging mathematics tasks to prompt learning through problem solving and reasoning. He discussed how these tasks could be differentiated so that all students could achieve whilst developing persistence. Professor Tom Lowrie challenged us by stating “It is also important that our research empowers people, and that our recommendations and implications improve systems, especially for the disadvantaged.” Professor Jill Adler discussed her work with some of the world’s most marginalised teachers and students, in *schools for the very poor* in South Africa.

Presentations at the conference comprised research papers, round tables, and short communications that covered a wide variety of topics relevant to mathematics education across all countries, with a particular focus on the Australasian region. Research into mathematics education in early childhood settings, primary and secondary schools, or in tertiary institutions was presented and discussed. In accordance with established MERGA procedures, all research papers were blind peer reviewed by panels of mathematics educators with appropriate expertise in the field. Papers could be accepted for presentation only, or for both presentation and publication in the Proceedings. All papers initially accepted for presentation only were reconsidered by two members of an additional panel of independent expert reviewers. Only those research papers finally accepted for presentation and publication are published in these Proceedings. The abstracts for short communications and round tables were also blind peer reviewed. The published Proceedings include the keynote papers, research papers, and abstracts for round tables and short communications.

The Editorial Team would like to acknowledge and thank Review Panel Chairs and all reviewers for their efforts in reading and providing constructive feedback in a short timeframe. Ensuring the published papers meet high academic standards is an important and shared responsibility of the MERGA community. We would also like to thank the authors for taking the time and necessary care to use the MERGA conference paper template and guidelines before submitting their papers.

The Conference Organising Committee welcomed participants from all states and territories of Australia, as well as from many countries including Canada, Germany, Japan, New Zealand, Nigeria, and Singapore. We hope you enjoyed your time at the conference, making new connections and new friends, and that you had a chance to enjoy the Sunshine Coast and the many wonderful places to visit. We would also like to thank our Queensland-based colleagues who helped us to welcome you and show you around. We certainly enjoyed meeting you and hosting this conference.

Margaret Marshman
Chair, Conference Organising Committee and Chief Editor
Vince Geiger and Anne Bennison (Editors)

MERGA38 Reviewers

Megan Anakin
Judy Anderson *
Glenda Anthony
Stephen Arnold
Catherine Attard
Robin Averill
Lynda Ball*
Monica Baker
Bill Barton
Anne Bennison
Brenda Bicknell *
Trevor Bills
Susan Blackley
Janette Bobis *
Kathy Brady
Fiona Budgen
Raymond Brown
Nigel Calder
Scott Cameron
Michael Cavanagh
Eric Chun Ming Chan
Lu Pien Cheng
Barbara Clarke
Julie Clark *
Phil Clarkson
Audrey Cooke
Lorraine Day
Shelley Dole
Michael Drake
Raewyn Eden
Trish Forrester
Kym Fry
Vince Geiger
Rosemay Golds
Merrilyn Goos
Peter Grootenboer

Ngarewa Hawera
Sandra Herbert
Joanna Higgins *
Kate Highfield
Kai Fai Ho
Weng Kin Ho
Kath Holmes
Jodie Hunter *
Roberta Hunter
Derek Hurrell
Chris Hurst *
Lorraine Jacob
Dan Jazby
Jyoti Jhagaroo
Harry Kanasa
Berinderjeet Kaur *
Virginia Kinnear
Barry Kissane
Mun Yee Lai
Janeen Lamb
Kevin Larkin
Generosa Leach
Gilah Leder
Gail Ledger
Christina Lee
Ngan Hoe Lee
Yew Hoong Leong
Karen Major
Katie Makar
Margaret Marshman
Sandi McCutcheon
Paula Mildenhall
Judith Mills
Judy Mousley
Joanne Mulligan
Carol Murphy

Olivia Naylor
Dawn Kit Ee Ng
Marina Papic
Pam Perger
Tim Perkins
Anne Prescott
Elena Prieto
Maria Quigley
Felipe Almuna Salgado
Yvette Semler
Pep Serow
Sashi Sharma
Kaye Stacey
Vicki Steinle
Eng Guan Tay
Merilyn Taylor
Zain Thompson
Steve Thornton
Pee Choon Toh
Tin Lam Toh
Wee Tiong Seah
Colleen Vale
Jana Visnovska *
Margaret Walshaw
Jane Watson
Jenni Way
John West
Bruce White
Paul White
Karina Wilkie
Tiffany Winn
Monica Wong
Andy Yeh
Joseph Boon Wooi Yeo

* Panel chair

Contents

Preface 3

MERGA38 Reviewers 4

KEYNOTES

Preamble to the Annual Clements/Foyster Lecture 13

Mathematics Education as a Field of Research: Have We Become Too Comfortable? 14
Tom Lowrie

Researching and Doing Professional Development Using a Shared Discursive Resource and an Analytic Tool 25
Jill Adler

Exploring a Structure for Mathematics Lessons that Foster Problem Solving and Reasoning 41
Peter Sullivan, Nadia Walker, Chris Borcek & Mick Rennie

Practical Implications Award

Teacher Actions to Facilitate Early Algebraic Reasoning 58
Jodie Hunter

Research Papers

The challenge of supporting a beginning teacher to plan in primary mathematics 69
Judy Bailey

Contemplating symbolic literacy of first year mathematics students 77
Caroline Bardini, Robyn Pierce & Jill Vincent

Problematising Mathematics Education 85
Andy Begg

Identity as an Embedder-of-Numeracy: Identifying ways to support teachers to embed numeracy across the curriculum 93
Anne Bennison

Young Children's Number Line Placements and Place-Value Understanding 101
Brenda Bicknell & Jenny Young-Loveridge

The Role of Cultural Capital in Creating Equity for Pāsifika Learners in Mathematics 109
Trevor Bills, Roberta Hunter

The importance of praxis in financial literacy education: An Indigenous perspective 117
Levon Blue, Peter Grootenboer & Mark Brimble

Coming to do Mathematics in the Margins 125
Raymond Brown & Trevor Redmond

"You play on them. They're active." Enhancing the mathematics learning of reluctant teenage students 133
Nigel Calder & Anthony Campbell

CAS or Pen-and-paper: Factors that Influence Students' Choices 141
Scott Cameron & Lynda Ball

The Language Used to Articulate Content as an Aspect of Pedagogical Content Knowledge 149
Helen Chick

Specialised Content Knowledge: Evidence of Pre-service teachers' Appraisal of Student Errors in Proportional Reasoning 157
Mohan Chinnappan & Bruce White

Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-designed Experimental Mathematics Lessons 165
Doug Clarke, David Clarke, Anne Roche & Man Ching Esther Chan

Inclusive Practices in the Teaching of Mathematics: Supporting the Work of effective primary teachers 173
Barbara Clarke & Rhonda Faragher

Supporting Students to Reason About the Relative Size of Proper and Improper Fractions 181
Jose Luis Cortina & Jana Visnovska

Proportional Reasoning as Essential Numeracy 189
Shelley Dole & Annette Hilton

A Case Study of the Pedagogical Tensions in Teacher's Questioning Practices When Implementing Reform-Based Mathematics Curriculum in China 197
Lianchun Dong, Wee Tiong Seah & David Clarke

Improving the Effectiveness of Mathematics Teaching through Active Reflection 205
Kerryn Driscoll

Promoting Teacher Growth through Lesson Study: A Culturally Embedded Approach 213
Marlon Ebaeguin

The Self-Efficacy of students with Borderline, Mild and Moderate Intellectual Disabilities and their Achievements in Mathematics 221
Agbon Enoma & John Malone

Identifying Core Elements of Argument-Based Inquiry in Primary Mathematics Learning 229
Jill Fielding-Wells

STEM Education: What Does Mathematics Have To Offer? 237
Noleine Fitzallen

The Challenge for Non-first-language-English Academic Publishing in English Language Research Outlets 245
Vince Geiger & Rudolf Straesser

The Impact of Let's Count on Children's Mathematics Learning 253
Ann Gervasoni, Bob Perry & Linda Parish

Comparing the Development of Australian and German 7-Year-Old and 8-Year-Old's Counting and Whole Number Learning 261
Ann Gervasoni & Andrea Peter-Koop

Learning at the Boundaries 269
Merrilyn Goos

The Practice of 'Middle Leading' in Mathematics Education 277
Peter Grootenboer, Christine Edwards-Groves & Karin Rönnerman

Teaching Computation in Primary School without Traditional Written Algorithms 285
Judy Hartnett

Calculating for probability: "He koretake te rima" (Five is useless) 293
Ngārewa Hāwera & Merilyn Taylor

Students’ Relationships with Mathematics: Affect and Identity 301
Naomi Ingram

Using Alternative Multiplication Algorithms to ‘Offload’ Cognition 309
Dan Jazby & Cath Pearn

Successful Mathematics Lessons in Remote Communities: A Case Study of Balargo 317
Robyn Jorgensen

Differentiated Success: Combining Theories to Explain Learning 325
Robyn Jorgensen & Kevin Larkin

The Mathematics Instructional Leader: What a Difference Crucial Conversations Make........ 333
Janeen Lamb, Carmel Diezmann & Jillian Fox

The Search for Fidelity in Geometry Apps: An Exercise in Futility? 341
Kevin Larkin

Pre-service teachers and numeracy in and beyond the classroom 349
Gilah C Leder, Helen J Forgasz, Natalie Kalkhoven & Vince Geiger

Gender Differences in Mathematics Attitudes in Coeducational and Single Sex Secondary Education 357
Kester Lee & Judy Anderson

Developing a Theoretical Framework to Assess Taiwanese Primary Students’ Geometric Argumentation 365
Tsu-Nan Lee

Starting a Conversation about Open Data in Mathematics Education Research........ 373
Tracy Logan

A Snapshot of Young Children’s Mathematical Competencies: Results from the Longitudinal Study of Australian Children 381
Amy MacDonald & Colin Carmichael

Examining PCK in a Senior Secondary Mathematics Lesson........ 389
Nicole Maher, Tracey Muir & Helen Chick

Teacher’s Scaffolding over the Year to Develop Norms of Mathematical Inquiry in a Primary Classroom 397
Katie Makar, Arthur Bakker & Dani Ben-Zvi

Middle Years Students Influencing Local Policy........ 405
Margaret Marshman

Early Years Teachers’ Perspectives on Teaching through Multiple Metaphors and Multimodality 413
Paula Mildenhall

Young Indigenous Students’ Engagement with Growing Pattern Tasks: A Semiotic Perspective421
Jodie Miller

Professional Knowledge Required when Teaching Mathematics for Numeracy in the Multiplicative Domain........ 429
Judith Mills

Determining a Student’s Optimal Learning Zone in Light of the Swiss Cheese Model 437
Patricia Morley & Simone Zmood

Student and Parent Perspectives on Fipping the Mathematics Classroom 445
Tracey Muir

Authority and Agency in Young Children's Early Number Work: A Functional Linguistic Perspective 453
Carol Murphy

Examples in the Teaching of Mathematics: Teachers' Perceptions 461
Lay Keow Ng & Jaguthsing Dindyal

How Inquiry Pedagogy Enables Teachers to Facilitate Growth Mindsets in Mathematics Classrooms 469
Mia O'Brien, Katie Makar, Jill Fielding-Wells & Jude Hillman

Challenging the Mindset of Sammy: A Case Study of a Grade 3 Mathematically Highly Capable Student 477
Linda Parish

Facebook as a Learning Space: An Analysis from a Community of Practice Perspective 485
Sitti Maesuri Patahuddin & Tracy Logan

Strategies for Solving Fraction Tasks and Their Link to Algebraic Thinking 493
Catherine Pearn & Max Stephens

Mentoring to Alleviate Anxiety in Pre-Service primary mathematics Teachers: an orientation towards improvement rather than evaluation 501
Timothy Perkins

Spatial Visualisation and Cognitive Style: How Do Gender Differences Play Out? 508
Ajay Ramful & Tom Lowrie

The Practice of Teacher Aides in Tasmanian Primary Mathematics Classrooms 516
Robyn Reaburn

Qualitative Facets of Prospective Elementary Teachers' Diagnostic Proceeding: Collecting and Interpreting in One-on-one Interviews 524
Simone Reinhold

Describing the nature and effect of teacher interactions with students during seat work on challenging tasks 532
Anne Roche & Doug Clarke

Teachers' talk about Robotics: Where is the Mathematics? 540
Annie Savard & Kate Highfield

Teaching Statistics in Middle School Mathematics classrooms: Making Links with Mathematics but Avoiding Statistical Reasoning 547
Annie Savard & Dominic Manuel

Context counts: The potential of realistic problems to expose and extend social and mathematical understandings 555
Carly Sawatzki

Theorising about Mathematics Teachers' Professional Knowledge: The Content, Form, Nature, and Course of Teachers' Knowledge 563
Thorsten Scheiner

Understanding Geometric Ideas: Pre-service Primary Teachers' Knowledge as a Basis for Teaching 571
Rebecca Seah

Mathematical Language Development and Talk Types in Computer Supported Collaborative Learning Environments 579
Duncan Symons & Robyn Pierce

The Individual Basic Facts Assessment Tool 587
Sandi Tait-McCutcheon & Michael Drake

Affording and Constraining Local Moral Orders in Teacher-Led Ability-Based Mathematics Groups ... 595
Sandi Tait-McCutcheon, Joanna Higgins, Mary Jane Shuker & Judith Loveridge

Exploring relationship between scientific reasoning skills and mathematics problem solving ... 603
Nor'ain Mohd Tajudin & Mohan Chinnappan

Developing Adaptive Expertise with Pasifika Learners in an Inquiry Classroom ... 611
Zain Thompson & Jodie Hunter

Getting out of Bed: Students' Beliefs ... 619
Jane Watson & Rosemary Callingham

Improving Student Motivation and Engagement in Mathematics Through One-to-one Interactions ... 627
Jennifer Way, Amelia Reece, Janette Bobis, Judy Anderson & Andrew Martin

A Cross-cultural Comparison of Parental Expectations for the Mathematics Achievement of their Secondary School Students ... 635
Daya Weerasinghe & Debra Panizzon

"I was in year 5 and I failed maths": Identifying the Range and Causes of Maths Anxiety in first year Pre-service Teachers ... 643
Sue Wilson

Enhancing Mathematics (STEM) Teacher Education in Regional Australia: Pedagogical Interactions and Affect ... 651
Geoff Woolcott & Tony Yeigh

Mathematics, Programming, and STEM ... 659
Andy Yeh & Vinesh Chandra

RESEARCH PRESENTATION ABSTRACTS

Laying the Foundation for Proportional Reasoning ... 668
Ann Downton

The Development and Evaluation of an Individualised Learning Tool for Mathematics students with Intellectual Disability: IMPELS ... 668
Agbon Enoma & John Malone

Capturing Mathematical Learning in an Inquiry Context: There are Some Things Not Easily Measured ... 669
Kym Fry

Teacher Professional Growth through using a Critical Mass Mentoring System: Effective Whole School Teacher Professional Development ... 669
Judy Hartnett & Jim Midgley

Anatomy of a Mathscast ... 670
Carola Hobohm & Linda Galligan

An Exploration of Strategies That Teachers Use When Teaching Beginning Algebra ... 670
Christina Lee & Christine Ormond

Factors Influencing Social Process of Statistics Learning within an IT Environment ... 671
Ken W. Li & Merrilyn Goos

Identifying categories of Pre-service Teachers' Mathematical Content Knowledge ... 671
Sharyn Livy

Language and Mathematics: Exploring a New Model to Teach in Bi/Multilingual Mathematics Classroom 672
Charly Muke

Exploring the Influence of Early Numeracy Understanding Prior to School on Mathematics Achievement at the End of Grade 2........ 673
Andrea Peter-Koop & Sebastian Kollhoff

An Irish Response to an International Concern: Challenges to Mathematics Teaching 673
Lisa O'Keeffe, Olivia Fitzmaurice & Patrick Johnson

An Analysis of Modelling Process based on McLuhan's Media Theory: Focus on Constructions by Media in Cases of Using Geoboard 674
Hiro Ozasa

The Knowledge Dimension of Revised Bloom's Taxonomy for Integration 674
Farzad Radmehr, Robin Averill & Michael Drake

Developing an analysing tool for dynamic mathematics-related student interaction regarding affect, cognition and participation 675
Laura Tuohilampi

Thinking Strategies Used by 7th-Grade Students in Solving Number Sense Problems 675
Palanisamy Veloo & Parmjit Singh

ROUND TABLE DISCUSSION ABSTRACTS

Working Across Disciplinary Boundaries in Pre-service Teacher Education........ 677
Merrilyn Goos, Judy Anderson, Jo Balatti, Kim Beswick, Tricia Forrester & Jenni Way

Promoting Positive Emotional Engagement in Mathematics of Prospective Primary Teachers.. 678
Joanna Higgins & Janette Bobis

Senior Secondary Students' Pre-calculus and Calculus Understanding........ 679
Michael Jennings & Peter Adams

Investigating Mathematical Inquiry........ 680
Katie Makar, Jill Fielding-Wells, Kym Fry, Sue Allmond & Jude Hillman

SHORT COMMUNICATION ABSTRACTS

A Problem Solving Lesson: Pre-service Teachers Initiation to Lesson Study 682
Jaguthsing Dindyal

Teachers' Beliefs about Knowledge of Content and Students and its Effect on their Practice.... 682
Vesife Hatısaru

Exploring Students' Views on using iPads in Mathematics........ 683
Janelle Hill

Mapping school students' aspirations for STEM careers 683
Kathryn Holmes, Adam Lloyd, Jenny Gore & Max Smith

Breaking down Barriers........ 684
Peter Howley

Building upon the Language Model of Mathematics 684
Harry Kanasa & Kevin Larkin

The Australian Mathematics Competition: What's the Score? 685
Andrew Kepert & Mike Clapper

A Focus Question Approach to the Teaching of Mathematics 685
John Ley

Promoting the Development of Foundation Content Knowledge in all Primary Pre-service Teachers 686
Chris Linsell, Naomi Ingram & Megan Anakin

Paternal influence on school students' aspirations for STEM careers 686
Adam Lloyd, Jenny Gore & Max Smith

Understanding Mathematics: Teacher Knowledge, Task Design and Evaluating Students' Mathematical Reasoning 687
Christine Mae, Janette Bobis & Jenni Way

The Pattern and Structure of the Australian Curriculum—Mathematics 687
Catherine McCluskey, Joanne Mulligan & Michael Mitchelmore

Mathematical Thinking in a Context of 'General Thinking': Implications for Mathematics Education 688
Corinne Miller, Geoff Woolcott & Christos Markopoulos

Conceptual Connectivity in Mathematics 688
Joanne Mulligan & Geoff Woolcott

Primary-Middle Pre-Service Teachers reported use of the Mathematics Textbook 689
Lisa O'Keeffe

Examining a Students' Resource for Reconstructing the Limit Concept at Need: A Structural Abstraction Perspective 689
Thorsten Scheiner & Márcia M. F. Pinto

Pre-service Teachers' Views on Mathematics Homework Practices 690
Sven Trenholm & Mohan Chinnappan

Teaching out-of-field: Meanings, representations and silences 690
Colleen Vale, Linda Hobbs, Christopher Speldewinde & Zahra Parvanehnezhadshirazian

Promoting Financial Literacy in Pre-service Teacher Education through On-line Modules 691
Leigh Wood, Joanne Mulligan, Carmel Coady, Michael Cavanagh & Damian Bridge

Keynotes

The Beginnings of MERGA

Preamble to the Annual Clements/Foyster Lecture

In the middle of 1976 John Foyster, who was then based at the Australian Council for Educational Research (ACER), came to see me at Monash University, where I was in charge of the Mathematics Education program. John talked about how the Australian Science Education Research Association (ASERA) had recently been established, with Professor Richard Tisher (then of Monash University) as the prime mover. John wondered whether the time was ripe for a similar national group interested in mathematics education research to be established, and asked whether he and I might take steps to establish such a group.

My immediate reaction was yes, we should do it. Then came the doubts and reservations. How would the Australian Association of Mathematics Teachers (AAMT) react to such an initiative? After all, AAMT already had a "Research Committee." In any case, would there be enough mathematics educators in Australia, interested in such a group to make it a viable proposition? Who would provide the funds likely to be needed for the establishment of such a group?

It was John's and my opinion that the AAMT Research Committee had not reached out to embrace most of the people lecturing in mathematics education in Australia at teachers colleges or in universities at that time. Intuitively, I thought Australia needed a group like the one John was proposing. My intuition told me that AAMT was not the organisation to move towards the establishment of such a group.

John assured me that he would put up any funds needed to get the group going (and, hopefully, any group that was established would be able to pay him back within a few years). Hence we decided to proceed with the idea of establishing the group and to strike while the iron was hot, so to speak, by conducting a national conference at Monash University in the middle of 1977. I came up with the name "Mathematics Education Research Group of Australia" which John liked because of the acronym MERGA, which suggested a "merging together." We sent out notices of our intention to form MERGA late in 1976. Neither of us knew many of the people who might be interested in joining such a group, so the notices were addressed to the "Mathematics Lecturers at…"

Soon after we had decided to go ahead, I heard of the existence of a group, based in New South Wales, called the Mathematics Education Lecturers' Association (MELA). John and I talked about whether MERGA and MELA might become one from the outset, but we decided that the aims of MELA seemed to be sufficiently different from those that we envisaged for MERGA, focused far more on research than lecturing, that we should proceed with the MERGA idea.

And so it came to be that in May 1977, the first of what was to become the annual conference of MERGA took place. About 100 people attended, with papers frenetically being read from 9 am to about 10 pm, for three days, in a Rotunda Theatre at Monash University. Professor Richard Tisher was present at the start of the Conference, and talked of his experiences in establishing ASERA. Frank Lester, of Indiana University, was among those present. In the event, two volumes of papers read at the Conference were produced (the first volume being available on the first day of the Conference, and the second several months later).

At a post-Conference meeting it was decided that, yes, MERGA should be formed, that the second meeting would be at Macquarie University in May 1978, and that an annual conferences should be held each year at a different academic institution. At that second conference it was decided by those present that MERGA should continue and a constitution and election of offices would be decided on at the third conference to be held at the then Brisbane College of Education. And so MERGA was born.

Ken Clements

Mathematics Education as a Field of Research: Have We Become Too Comfortable?

Tom Lowrie
University of Canberra
<thomas.lowrie@canberra.edu.au>

Mathematics education is highly regarded as a research field within our region, especially when compared to other fields within the broader education discipline. The field has been relatively cohesive, well organised and internationally influential in a universally strong field. Mathematics education research has developed and evolved in challenging times—when other fields have become fragmented and lost vision—have we more to offer? This keynote paper considers the challenges we face as a field of research as we navigate our theoretical underpinnings and pedagogical practices, within both the mathematical sciences and broader education disciplines.

The International and Regional Strength of Mathematics Education

As a field of research, mathematics education has developed a reputation of scientific strength within a relatively short timeframe. In terms of research intensity, the international reputation of mathematics education is one of considerable productivity and engagement. The field is internationally connected via the regular production of international handbooks, collaborative manuscripts, well-regarded journals, and a number of international conferences that attract participants from a diverse number of countries. These international conferences often have specific strength within particular aspects of mathematics education: including psychology (e.g., Psychology of Mathematics Education [PME]) and sociology (e.g., Mathematics Education and Society [MES]). Such is the magnitude of the mathematics education field.

More localised strength within the field usually comprises strong regional organisations like MERGA, well-organised conferences like the four-yearly ICMI-East Asia Regional Conference of Mathematics Education (EARCOME), or special interest groups within larger education conferences (e.g., mathematics education is the largest special interest group of the American Educational Research Association [AERA]). Such strength and connectivity is certainly not unique; nevertheless the national and international strength is somewhat privileged within the education discipline.

In terms of professional development and engagement, mathematics education researchers in Australia and New Zealand are somewhat advantaged. Our geographical "remoteness" and relative small populations have shaped our research-based organisations in dramatic ways. We have few discipline-based competitors and our scale ensures that we know each other well. Most members attend the annual MERGA conference on a regular basis—providing a connected network of scholars that develops both collective capacity and identity formation. Such remoteness also tends to develop a "collective" mentality, moving collectively to international conventions across the globe. Our members are regularly among the highest proportion of attendees outside that of the host country at international conferences like the PME. With perhaps the exception of science education, no other discipline-based field within education offers such strength and direction. Most disciplines in Australia and New Zealand rely on general national education communities (i.e., Australian Association of Research in Education [AARE] and New Zealand Association for Research in Education [NZARE]) to harness research strength. It is of no

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 14–24. Sunshine Coast: MERGA.

surprise that mathematics education does not have a special interest group at AARE—which is in stark contrast to that in the United States and AERA. Not only do we have strength in numbers, but our propensity to engage with colleagues from all over the world makes our international presence and reputation palpable.

Most, if not all, education-based discipline fields conduct research and professional practice that are abreast of both general-education and specific-discipline paradigms. For example, both the English and education fields influence literacy education researchers, while in human movement and education fields those researchers who concentrate on physical education. To this point, most discipline-based fields within education are concerned with the promotion of the discipline content and the pedagogical constructs that support the learning of the discipline. At least from a regional perspective, it would be fair to say that literacy and numeracy attract most of the attention from the collective education community—including politicians, policy makers, assessment experts, school communities and the general public. As mathematics education researchers, such community attention provides us with scope, capacity and opportunity not afforded to other fields of research within education—even strong fields such as science education. Although this may well be as good as it gets, mathematics education is a field with high credibility and sustained influence within the broader parameters of education research.

Enhancing Our Regional Reputation: Respectful Acknowledgement via Tradition or Ground-Breaking Innovation?

As Galbraith (2014) maintained, mathematics educators within our region have made a substantial contribution to the field internationally in terms of theoretical development and practical applications. This success can be gauged across various measures of impact and contribution. New Zealand has a past President of the International Congress on Mathematical Instruction (ICMI); and a winner of the Felix Klein Medal and the current Editor of *Educational Studies of Mathematics* (ESM) are from Australia.[1] Our members have been editors of several international handbooks and have contributed substantially to invited papers and keynote presentations at PME and the International Congress on Mathematics Education (ICME)[2]. In fact, our attendances at the annual PME conferences or as contributors to *International Handbooks of Mathematics Education* are higher than that of any other country, apart from the United States. For countries of relatively small populations, such international contributions are considerable (Singh & Ellerton, 2012). Indeed, they point to the strength of our research-based community. From its inception in the early twentieth century, mathematics education has been a field that has been dominated by European and North American mathematicians and mathematics educators (Singh & Ellerton, 2012). Yet MERGA members, in particular, have been able to establish enough credibility and presence to make an impact within such restricted structures. It is noteworthy that the only ICME conference to be held outside of the Northern Hemisphere was in Adelaide (1984), with Sydney contesting to hold the event in 2020—which would, if successful, be the second such occasion. Within the field we are certainly influential,

---

[1] Professor Bill Barton was President of ICMI from 2008–2012); Professor Gilah Leder was awarded the Felix Klein Medal for research excellence from ICMI in 2009; Professor Merrilyn Goos was appointed Editor of ESM in 2014.

[2] In the four ICME conferences since 2000, MERGA members have presented four plenary activities and 16 regular lectures.

despite our relatively small populations. Perhaps it is our size and isolation that shapes such successful practices?

Such data highlight the fact our members have been able to engage (and certainly contribute) internationally in an environment that has been focused on European and North American traditions. To some degree, these traditions remain self-absorbed and somewhat conservative. It may be the case that such attributes lend themselves well to mathematics education since "mathematics" tends to flourish on traditional approaches and foundation principles. The rise and rise of Asian countries in terms of prosperity and influence within the world's increasingly networked society (Castells, 2010) has begun to dramatically shift the education focus from Europe and North America. From a political perspective, attention first shifted toward Asia when it became apparent that Confucian-heritage nations consistently performed better than students from North America and Europe. Interestingly, the current fixation on comparing student performance across countries—along with the obvious performance advantage these Asian countries exhibit on the Programme for International Student Assessment and Trends in International Mathematics and Science Study—reinforces the notion that traditional and structured approaches to teaching mathematics are most effective.

In his keynote presentation in Singapore, Clements (2012) argued that a more inclusive MERGA could become a power block to rival those in North America and Western Europe. Both regionally and contextually, we are well placed to engage deeply with colleagues from Asia—indeed, many of us are doing so already. Anecdotally, I would suggest that at least half of our MERGA's current membership has sustained research relationships with colleagues and/or countries in Asia, and especially in southeast Asia. Respectfully, we have at least as much to learn from, and engage with, our colleagues in this region than we do from the traditional two blocks.

Mathematics Education and Mathematics

Mathematicians made most policy, curriculum and pedagogical decisions concerning mathematics education, as recent as thirty years ago (Clements, 2012). There were few *mathematics education specialists*—and those folk who possessed such skills were not influential. This landscape has changed dramatically, both regionally and internationally. As Fried (2014) commented:

> …over the last quarter century or so, and for better or for worse, this simple notion of where the core of mathematics education lies has been offset by goals and interests allying it, as an academic field, more closely with psychology of learning, cultural differences, and social justice, among others, than with mathematics itself. Thus, while the first two-thirds of the twentieth century could boast of great mathematicians such as Felix Klein, Jacques Hadamard, George Pólya, and Hans Freudenthal making contributions to mathematics education, today, not only are such figures rare in the field, they have also been to an extent alienated by it. (p. 12)

It is also the case that those innovative and highly capable mathematicians established our field. I am convinced that some of our colleagues today are just as creative and innovative, however the field is much larger—consequently, "big fish in a big pond". It would also be fair to suggest that our society is more complex and interactive than it was thirty years ago—demanding that we consider psychology, cultural differences and social justice dimensions with as much rigor and attention as mathematics content and processes.

For some time now, most of our mathematics education academics completed their doctorates in Australia or New Zealand—initially guided by a handful of our community's most respected researchers. The vast majority of these new doctoral scholars emerged from a secondary teaching background. Only a handful of our new early career researchers moved into universities from primary or early childhood backgrounds—a pattern that has changed considerably in the past ten years. It could be argued that our discipline has stronger foundations within *education* than it does in *mathematics*, at least from the orientation of our community. In both Australia and New Zealand, mathematics educators are typically in Faculties and Departments that include other education experts—in contrast to many North American and European universities where mathematical science and mathematics educators belong to the same department. I seldom engaged with teacher educators in my first two sabbaticals to North America, yet was surrounded by mathematics colleagues. It is certainly the case that the discipline profile of MERGA members will look very different in ten years' time, with fewer people having mathematics as their major postgraduate qualification.

There has been a concerted effort to find common ground with our mathematics colleagues in recent years. This has especially been the case with MERGA under the leadership of Merrilyn Goos, with strong collaborative support from the Australian Association of Mathematics Teachers (AAMT). From a political perspective, this has included a determination to have a common voice with the Australian Mathematical Sciences Institute (AMSI), the Australian Mathematical Society (AustMS) and the Statistical Society of Australia (SSAI) on a range of issues. The cohesive and collaborative nature of our work (and common aspirations) was no more evident than in the concerted effort to host ICME-14 in Sydney in July 2020. The ICMI delegation were both surprised and overjoyed with the evident goodwill, common ground and working relationships that existed among our organisations—collectively and individually commenting that this was rarely seen elsewhere in the world.[3]

From a research perspective, however, this connectivity is less apparent. It may be the case that our philosophical lenses and ways of knowing are too dissimilar. As Brown (2010) argued:

> Mathematicians who see mathematics as an entirely abstract domain are a different breed to those attentive to its historical evolution and hence its potential immersion within the social sciences. To move from one domain to another requires a major switch in modes of thinking, from one conception of life to another. (p. 341)

Fried and Dreyfus (2014) produced a manuscript that encouraged mathematicians and mathematics educators to consider the common ground among their fields of research. They suggested that mathematicians were primarily concerned with content and ideas, and approaches for ensuring ideas could be presented as fluently as possible. By contrast, mathematics educators were concerned with students' thinking and how understanding is embedded in culture and everyday experiences. Any research nexus between the discipline fields seems to be closely associated with teachers of mathematics in the classroom. In mathematics education, this research tends to be associated with classroom teachers' content knowledge (CK) and pedagogical content knowledge (PCK). In mathematics, the research is associated with developing more informative assessment practices and the necessity to produce good quality teachers of mathematics.

Mathematics researchers have re-invented themselves in part, at least in our region, by necessity. Although mathematics education has become increasingly concerned with the mathematics knowledge our teachers possess when enrolling in education degrees, our

[3] The ICMI delegation were similarly impressed with how closely connected our Australian and New Zealand members were to our colleagues in Asia—as we presented a case for our regional cohesion.

mathematics colleagues are concerned with the decrease in the number of students wanting to undertake degrees with a mathematics specialisation. We may have too many students; by contrast they have too few. One way that mathematicians have responded to this loss of capacity is to become more broad or balanced in their scope. The term *mathematical sciences* has emerged, commonly defined in our region as "encompassing mathematics, statistics and the range of mathematics-based disciplines including the teaching of mathematics and teacher education". The first part of the statement is unsurprising, and the identification of mathematics-based disciplines increasingly necessary. For example, mathematics knowledge and tools have become critical to commerce and industry in an increasingly technological age. However, it is noteworthy that such statements about components that encompass mathematical sciences would mention teacher education. This may be a strategic decision that is politically astute to ensure the discipline remains vital and influential. After all, it must be difficult to "compete" for exposure and relevance in a science-dominated landscape (especially in terms of physics, chemistry and medical science). Although there must be some advantages of being considered a "hard science", there are challenges when your field is required to share the same research space.

Some of my mathematics colleagues lament at the challenge of demonstrating impact when the most prestigious journal in their field, *Annals of Mathematics*, has an impact factor of 2.8. They quote journals in engineering and general science with impact factors substantially higher than their gold standard. This is also the case in our field. *Educational Studies in Mathematics* has a far lower impact factor than the most well-regarded teacher education and general education journals (see Table 1). Science-based journals seem to be more widely read and quoted. The science-based journal equivalent to ESM would be *Journal of Research in Science Teaching*, which has an impact factor of 3.02.

Table 1
Impact Factors of Well-Respected Journals by Discipline Field

Journal	Discipline Field	Impact Factor
Educational Studies in Mathematics	Maths Education	0.6
Journal of Teacher Education	Teacher Education	2.2
Review of Educational Research	Education	5.0
Annals of Mathematics	Mathematics	2.8
International Journal of Civil Engineering & Technology	Engineering	9.1
Science	Science	31.4

In an environment of heavily reduced research funding opportunities, such cross-disciplinary comparison becomes relevant and potentially debilitating. Most large-scale and long-term research projects are funded within cross-disciplinary panels and assessment committees. No mathematics education consortium has been awarded an ARC Centre of Excellence or Cooperative Research Centre (CRC). In fact, no education-led consortia have ever been awarded such sustained funding to work on complex research questions. By contrast, our mathematics colleagues have recently been awarded an ARC Centre of Excellence in "Mathematical and statistical frontiers of big data, big models, new insights". The Centre is led by a statistician, Professor Peter Hall.

What our mathematical science colleagues have been able to achieve is commendable. They have been able to show how their discipline-based research can be applied to, and have impact on, the broader community. For the ARC Centre of Excellence, they were able to demonstrate that the mathematical models they were going to develop would be vital to the Centre's collaborative domains, namely: healthy people, sustainable environments and prosperous societies. In order to receive such funding opportunities, we have some way to go—nevertheless, there are some positive signs concerning how our work is regarded on a national stage.[4]

Mathematics Education within the Education Discipline

In an Australian context, the Australian Research Council (ARC) Discovery Grants are often regarded as the gold standard. These grants allow researchers to frame research projects from a position of personal strength and focus. Unlike many funding schemes, the open nature of the funding rules has no set agenda—apart from the need to demonstrate national significance and innovation. Over the past ten years, mathematics educators have received a high proportion of funding from this scheme, relative to other disciplines within the field of education. In fact, of all grants awarded in the past ten years, mathematics educators have been awarded 20% of the grants—all of whom are members of MERGA. The data are more compelling when considering all grants awarded within the curriculum and pedagogy component of the education discipline—where almost all mathematics education grants are assigned. Within this categorisation, our members have been awarded 40% of the grants awarded by the ARC. Typically, success rates for this Discovery scheme are less than 20%, from a pool of the education disciplines' most highly regarded researchers. Although it is difficult to ascertain what the success rate for mathematics educators would be within this funding scheme, in some years it would be more than 50%. As a community of scholars, we must be doing something right! Table 2 provides data on the number of grants awarded by the ARC in the Discovery Scheme within the curriculum and pedagogy discipline, by content specialisation.

[4] The ARC awarded Special Research Initiative funding to a Science of Learning Research Centre in 2012. Although the Centre is led by neuroscientists and cognitive psychologists from a brain centre, pleasingly, a number of MERGA members are Chief Investigators in this Centre—raising the possibilities for mathematics education researchers.

Table 2
Grants Awarded in the Discovery Scheme (2005–2014) Curriculum and Pedagogy (4-Digit Code) by Year and Field Specialisation

Discipline Field	Total Awarded	Proportion (%)
Mathematics education	23	40%
Science education	17	30%
English/literacy education	5	9%
Technologies	3	5%
Physical education/health	2	3.5%
Curriculum/national	2	3.5%
Social justice	1	<2%
Democracy	1	<2%
Learning cycles	1	<2%
Assessment	1	<2%
Integration	1	<2%
Total	*57*	

There are a number of plausible explanations for this high proportion of success, relative to other sub-disciplines within the field. These grants are typically awarded to investigators with very strong research profiles, with 40% of the assessment criteria afforded to the research team's record of research productivity. A further 30% is awarded to the grant's contribution to national priorities and strength of the team's research environment. Mathematics, and specifically numeracy, is considered to be of critical importance to the nation's prosperity and capacity to remain competitive in global markets. To this point, our field is well placed to take advantage of the fact that numeracy (along with literacy) is afforded more attention politically than other areas of learning in schools. By contrast, the fields of arts education, human society or physical education rarely gain such community-based attention. Consequently, the general view that mathematics is necessary for the development of the next generation of global citizens, combined with the international reputations of the research team, ensures higher-than-average levels of success.

The "Education" That Surrounds Mathematics Education

One of the central criticisms of teacher education is that both research approaches and the implementation of practice(s) revolve around "cottage industries" that cyclically repeat and reinvent similar initiatives (McKernan, 2008). Elsewhere, I have argued that new frameworks need to be developed, trialled and implemented across different contexts and countries to provide research and practice opportunities which not only value add to previous initiatives, but reflect sophisticated research designs (Lowrie, 2014). A lack of sustained long-term research funding, and the challenge of meeting education jurisdictions' restrictive timeframes make such aspirations challenging. Moreover, education jurisdictions within countries generally require new approaches and innovations to be tailored to their specific cultural and political circumstances. As a result, research studies are difficult to replicate across jurisdictions. It is also the case that education research is complex. As Berliner (2002) maintained:

> In education, broad theories and ecological generalizations often fail because they cannot incorporate the enormous number or determine the power of the contexts within which human beings find themselves…. The participants in those networks have variable power to affect each other from day to day, and the ordinary events of life (a sick child, a messy divorce, a passionate love affair, migraine headaches, hot flashes, a birthday party, alcohol abuse, a new principal, a new child in the classroom, rain that keeps the children from a recess outside the school building) all affect doing science in school settings by limiting the generalizability of educational research findings. (pp. 18–19)

Context is critical to, and in, educational research. It could be argued that it is more influential in replicating findings than anything else. Even in studies of more than 200 participants, it is often difficult to replicate findings because the within-group variance is typically larger than the between-group variance. So many variables are at play, even for well-defined and structured treatment programs or multivariate analyses that provide a battery of instruments to "control" for variables. At the same time, student behaviour could be interacting with a teacher's mathematics knowledge, beliefs about pedagogy or even assessment practices—not to mention the socioeconomic status of the students of the community. Most large-scale studies are drawn from a participant base that is familiar to, or in close proximity of, the researcher's own context. This is also the case in educational psychology research. This might include undergraduate students drawn from the researcher's own university, or schools in their own district. Increasingly, it is difficult for "outsiders" to get into different and new jurisdictions within their own country, let alone another country.

In teacher education, in particular, small-scale qualitative research dominates (Adler, Ball, Krainer, Lin, & Novotna, 2005). As Adler et al. (2005) suggested, such findings are unsurprising since theory-practice relationships can be explored in authentic ways via teacher voice. It may also be the case that context and cultural aspects of the investigation are not generalisable until theorising and modelling can be established. Nevertheless, the criticism that teacher education is concerned predominately with a cottage industry is understandable—where most research is focused on what is taking place nearby and repeated in multiple sites across the world. Such perceptions are especially salient when most small-scale research is conducted with teachers with whom the researcher knows and has worked with in the past. In fact, as much as 80% of all investigations are conducted with relationships already formed (Adler et al., 2005). Given the competitiveness of securing external funding, and the challenges of securing ethics clearance from out-of-region jurisdictions, it is hard to imagine this changing in the foreseeable future.

Most of the research conducted in education research is situated within familiar contexts, irrespective of paradigm used to collect and analyse data. To some degree, our mathematics foundations provide opportunities for our work to be more varied than the descriptions presented above. Perhaps our well-designed cognitive models and theoretical frameworks provide some opportunities to cross boundaries more so than other fields within the education discipline? It may also be the case that the well-connected international community we belong to enhances such prospects. Brown (2010) has argued that this is not the case, since our education origins dominate our practice(s).

> …mathematics education research rests on supposed cognitive models in which the human being is understood in particular ways with pedagogical models/apparatus shaped accordingly. Yet, learning can be productively viewed as an experience through time where there are changes in both the human subject and the objects they apprehend… [since] the prominence of Piaget and Vygotsky in our research has overly restricted analytical opportunities. (Brown, 2010, p. 342)

Without cognitive models or theoretical frameworks to help us explain learning within socio-cultural contexts it is difficult to imagine moving beyond small-scale studies that (seemingly) have little impact on policy makers and practitioners alike. This may not be a bad thing, in and of itself, since research within the social sciences is complex and integrated—and to some extent the most sophisticated multivariate design becomes a single case study. However, as Galbraith (2014) pointed out, there are numerous theoretical frameworks at our disposal with which we can embed our research programs. Too much choice, it would seem?

Beyond Comfortable: Barricades, Warning Signs and New Opportunities

As a field of research, mathematics education would certainly be regarded as a success story—especially when compared to most fields within the education discipline. For the past thirty years our field has become increasingly diverse in terms of what research questions we pose, the theoretical underpinnings and lenses we adopt, and the methodological frameworks we construct in order to gather data. Perhaps some of our greatest achievements have been associated with issues of social justice, cultural diversity and affective dimensions of mathematics. To some degree, these contributions to teaching and learning show how essential our work is to *mathematics* and *education*.

At the same time I worry that our field has stagnated, at least in relation to some important mathematics topics. By way of example, the most highly regarded mathematics education researcher in my area of specialisation has had two articles published in the last twelve months that suggest nothing new has happened in the specific field for quite some time. The first manuscript had 34 references with no new worked cited over the past ten years, aside from the person's own work and cross-referencing from that very issue of the journal. The second manuscript is the entry from the *Encyclopedia of Mathematics Education.* It has 24 references, however none of the works cited in the topic area have been published in the last twenty years, aside from this person's own work. The citing of one's own work is understandable, especially someone so highly regarded. What I find most extraordinary about this is how much this specific field (topic) has changed in the last twenty years, in part due to technological advances. This topic has seen major contributions from neuroscience, cognitive psychology, educational psychology, and practical applications from chemistry in the past ten years; yet such research seems to have not shaped this sub-field (in mathematics education) at all! I suspect and worry that this might not be an isolated case. In his Forster/Clements keynote presentation at the 2014 MERGA conference, Peter Galbraith (2014) challenged us to consider any theory or practice in mathematics education that has outraged us—I find such practices bewildering and outrageous. As much for the fact that such highly regarded mathematics educators are able to get away with such practices. Perhaps this is why they are able to get away with it?

If I took a more restrained and considered approach, I would need to form a conclusion that the contributions from other disciplines do not add sufficient value to expand our field? Or perhaps there is nothing new or innovative to have come out in the past twenty years? Perhaps, what I think is new or innovative might just be replicating the seminal works of the past?[5] Notwithstanding these questions, it would be problematic (and perhaps

[5] This reminds me of my perception of popular music. Sam Smith is one of the most creative new talents in music. His hit single, *Stay with me*, sold more than 5 million copies and was a Number 1 hit in seven countries in 2014. Within six months of release, Smith was required to give co-writer credits to Tom Petty

reckless) to assume that our field can be barricaded by research in mathematics and mathematics education alone. As Lerman (2014) reminds us:

> Mathematics educators have traditionally drawn on psychology, but nowadays draw also on sociology, anthropology, philosophy, ethics and other fields. The central focus is, of course, the teaching and learning of mathematics, and thus the nature of mathematical activity and thinking are a crucial focus for study in the field.... (p. 65)

It is essential that we maintain our focus on the teaching and learning of mathematics, to flourish in such a competitive research environment. We will only remain relevant if we remain true to our core principles. It is also important that our research empowers people, and that our recommendations and implications improve systems, especially for the disadvantaged. Our research should also be fun! Nevertheless, it is conceivable that many other fields will shape our work into the future. Societies and communities are changing rapidly, which can only mean our work becomes more complex and integrated. In addition, we might well be simultaneously focusing on practice *and* theory, rather than one or the other. The theoretical frameworks and learning models we develop will need to have the flexibility to be applied in various practice-based contexts.

Over the next five years or so, many of our most influential mathematics education researchers will be transitioning into retirement, or have started that journey already. Many of the research leaders who have been most influential on my work are closer to 70 (or beyond) than they are to 60. Then again, I remembering thinking the same thing five years ago—perhaps they and their work are more enduring than I had first thought. They have strengthened our field considerably, especially in relation to establishing research programs that would be sustaining in a post-mathematician dominated landscape. That is, strengthening our field with sociology, anthropology and philosophy (Lerman, 2014), as well as considering the foundations of the mathematics, psychology and general education disciplines. The alienation of mathematicians from our field (as prescribed by Fried, 2014) seems to me to be more about generational change than anything else, in terms of both people and societies. We are nearing such a point in time again. We need our transitioning research leaders as much as ever; however, we also need new opportunities for our early career researchers. It will be interesting to see what the *profile* of the next wave of professors and research leaders will be. Will the majority of these folk come from school teacher and early childhood backgrounds? How many will have degrees in pure or applied mathematics? How many will have doctorates from outside of the Australian and New Zealand university systems? What proportion of these professors would have studied under our MERGA leaders currently transitioning into retirement?

Despite the dramatic generational change that will occur in the coming years, the core principles of *what MERGA is about* will remain relatively constant—such is the influence of those foundational MERGA leaders. However, our next phase of development might need to go beyond the professional support and camaraderie we all experience from our Association. The intentional support and connectivity we offer one another is atypical in education organisations, where policy, research and practice goals are often disparate and fragmented. The "cottage industry" raises its ugly head, not due to selfishness or ignorance but rather an embracing of idiosyncratic ways. Organisationally, MERGA needs to become more strategic by establishing a common voice on issues that really matter to us. This will not be easy, given our support for one another and of MERGA is always "in kind" support.

---

and Jeff Lynne. It was revealed that the melody line of the song was astonishingly similar to their 1989 hit; ironically titled *I won't back down.*

It appears that our expertise and influence will move much further along the education spectrum, from a point that was much closer to the mathematics end of the spectrum when MERGA began. This might require different forms of engagement, whilst remaining true to our mathematics discipline ways. As Jorgensen (2014) indicated, we may need to build a transformative knowledge-making paradigm, which completing disrupts current (and past) pedagogical and classroom-based practices. Such dramatic shifts in how mathematics is taught would take time, both politically and organisationally. Nevertheless, such aspirational goals would position us to have more influence within education and school contexts. Our influence should be both discipline and practice based.

References

Adler, J., Ball, D., Krainer, K., Lin, F., & Novotna, J. (2005). Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics, 60,* 359–381.

Berliner, D. (2002). Educational research: The hardest science of all. *Educational Researcher, 31*(8), 18–20.

Brown, T. (2010). Truth and the renewal of knowledge: The case of mathematics education. *Educational Studies in Mathematics, 75,* 329–343.

Castells, M. (2010). *The rise of the networked society (2nd Ed).* (2010). United Kingdom: Wiley-Blackwell Publications.

Clements, M. A. (2012). MERGA 2012: Where we've been, where we are, and where we're going. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons* (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, pp. 4–19). Singapore: MERGA.

Fried, M. (2014). Mathematics and mathematics education: Searching for common ground. In M. Fried & T. Dreyfus (Eds.), *Mathematics and mathematics education: Searching for common ground.* Dordrecht, The Netherlands: Springer.

Fried, M., & Dreyfus, T. (Eds.) (2014). *Mathematics and mathematics education: Searching for common ground.* Dordrecht, The Netherlands: Springer.

Galbraith, P. (2014). Custodians of quality: Mathematics education in Australasia where from? Where at? Where to? In J. Anderson., M. Cavanagh, & A. Prescott. (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp.38–53). Sydney: MERGA.

Jorgensen, R. (2014). Social theories of learning: A need for a new paradigm in mathematics education. In J. Anderson., M. Cavanagh, & A. Prescott. (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp.311–318). Sydney: MERGA.

Lerman, S. (2014). Expectations according to a mathematics educator. In M. Fried & T. Dreyfus (Eds.), *Mathematics and mathematics education: Searching for common ground.* Dordrecht, The Netherlands: Springer.

Lowrie, T. (2014). An educational practices framework: The potential for empowerment of the teaching profession. *Journal of Teacher Education, 40*(1), 34–46.

McKernan, J. (2008). *Curriculum and imagination: Process theory, pedagogy and action research.* New York: Routledge.

Singh, P., & Ellerton, N. F. (2012). International collaborative studies in mathematics education. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 806–839). Dordrecht, The Netherlands: Springer.

Researching and Doing Professional Development Using a Shared Discursive Resource and an Analytic Tool

Jill Adler
University of the Witwatersrand
jill.adler@wits.ac.za

Linked research and development forms the central pillar of the 5-year Wits Maths Connect Secondary Project in South Africa. Our empirical data emphasised the need for teaching that mediates towards mathematics viewed as a network of scientific concepts, and the development of the notion of ‘mathematical discourse in instruction’ (MDI), as an analytic tool and discursive resource for working on research and professional development. This paper describes and reflects on MDI, its emergence in a particular education context, and what this discursive resource offers more generally as it works across different discourses and practices.

Introduction

It is well known that poverty is strongly associated with poor educational outcomes, and that inequitable socio-economic conditions are the most significant factor in inequitable educational outcomes (OECD, 2013). We also know in our field of mathematics education, that despite building expertise over many years in doing and researching professional development, links between investments in such activity, the quality of teaching school mathematics, and equitable educational outcomes remain tenuous. In the light of these claims, a question must be asked as to whether, and then if so how, it is possible to impact mathematics teaching and learning in conditions of deep inequality and high levels of poverty through professional development. What might be appropriate and meaningful goals for improving learner attainment in low-income communities, or in the context of this conference, in the margins? What role, if any, is there for discursive resources in realising such goals?

The first question remains the driving force for the Wits Maths Connect Secondary Project (WMCS), a research and development project working with mathematics teachers in ten disadvantaged secondary schools in one district in South Africa. WMCS has worked in its first five-year phase (2010-2014) with a key goal of strengthening teaching and learning of mathematics through professional development of teachers in these ten schools. It is a complex project with multiple additional, and at times competing goals: goals for advancing knowledge and research on related questions and problems in mathematics education, for building research capacity through linked doctoral studies; and for developing and investigating sustainable models of professional development. These professional goals are complemented by a social justice goal in our field, where 90% of the research we do and thus much of the knowledge we build, takes place in, or in relation to, adequately resourced and functioning schools (Skovsmose, 2011). WMCS has been inspired by the challenge of investigating the research-development nexus in mathematics education in poorly resourced conditions – and so the learning and teaching of mathematics in *schools for the poor* (Shalem & Hoadley, 2009). We have learned a great deal over the past five years, and have reported results on the impact of our professional development intervention on student attainment (Pournara et al, forthcoming), and the workings of the overall project (Adler, 2014). In this paper and presentation I focus in on one key aspect of our work, and that is the discursive resource and analytic tool developed

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 25–40. Sunshine Coast: MERGA.

to support our professional development work and our research, to engage the second question posed above. The more we learned with and from teachers and learners in their classrooms, the more we were able to sharpen our core research questions, and to construct a framework – called Mathematics Discourse in Instruction (MDI) – to support deliberate movement between the discourses of research, professional development and teaching, and so between the overlapping communities of practice (Wenger, 1998) in which the overall project was participating.

The MDI framework characterises the teaching of a mathematics lesson as a sequence of examples together with the tasks they are embedded in, and the accompanying explanatory talk, two commonplaces of mathematics teaching (and thus high-leverage practices (Grossman et al, 2009)), that occur within particular patterns of interaction in the classroom, and towards a particular goal or what we refer to as an 'object of learning' (Marton & Tsui, 2004). As intimated above, MDI has developed over time. In previous research work across WMCS and a similar project in primary schools, we conceptualised MDI to examine coherence within a task, and so between the stated problem or task, its exemplification or representation, and the accompanying explanations (Venkat & Adler, 2012); and more recently to examine coherence across a sequence of tasks/examples and accompanying explanatory talk within a lesson, and in relation to the intended object of learning (Adler & Venkat, 2014; Adler &Ronda, 2014). It was our empirical data that emphasised the need for coherence, and teaching that mediates towards mathematics viewed as a network of scientific concepts (Vygotsky, 1986), and towards generality (Watson & Mason, 2006). More recently we have used an expanded MDI analytic framework, illustrated in Figure 1 below, to examine shifts in exemplification and explanatory talk in classroom discourse, and have described our methodology in some detail (Adler & Ronda, forthcoming).

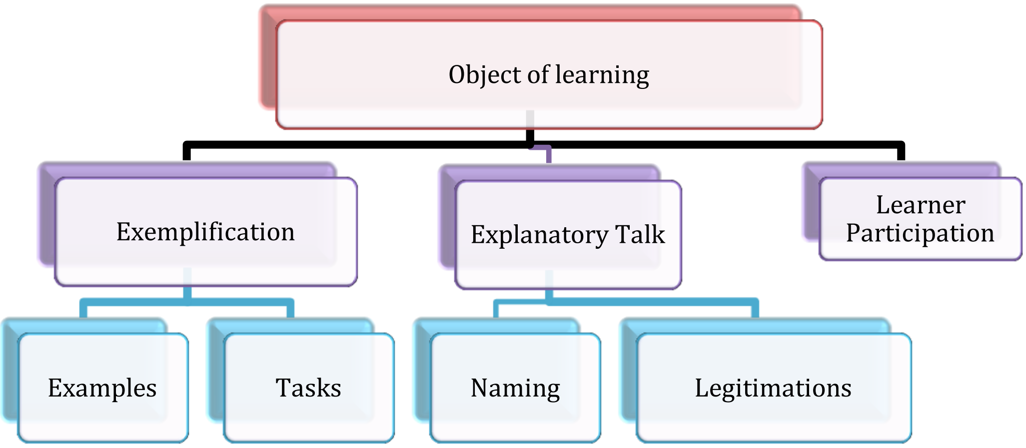


Figure 1: The MDI analytic framework (in Adler & Ronda, forthcoming)

Amidst this research work, we reported on our understanding of MDI as a *boundary object* as we were simultaneously using a form of it in our professional development work with teachers (Venkat & Adler, 2013). Drawing on Star & Griesemer's (1989) notion of *boundary objects* we viewed MDI illustrated above as "plastic enough to adapt to local needs and constraints" of our different practices and discourses, but "robust enough to maintain a common identity across sites" (p. 393). We were particularly concerned with an instrument that resonated with teachers, connecting with their practices in ways that enabled us to engage in the joint and shared enterprise of working on teaching to improve opportunities for mathematics learning in their schools and classrooms. We developed and refined MDI informally first and then through trialing across *mathematical knowledge for teaching* courses – 20 day content knowledge for teaching courses that are offered within our respective projects.

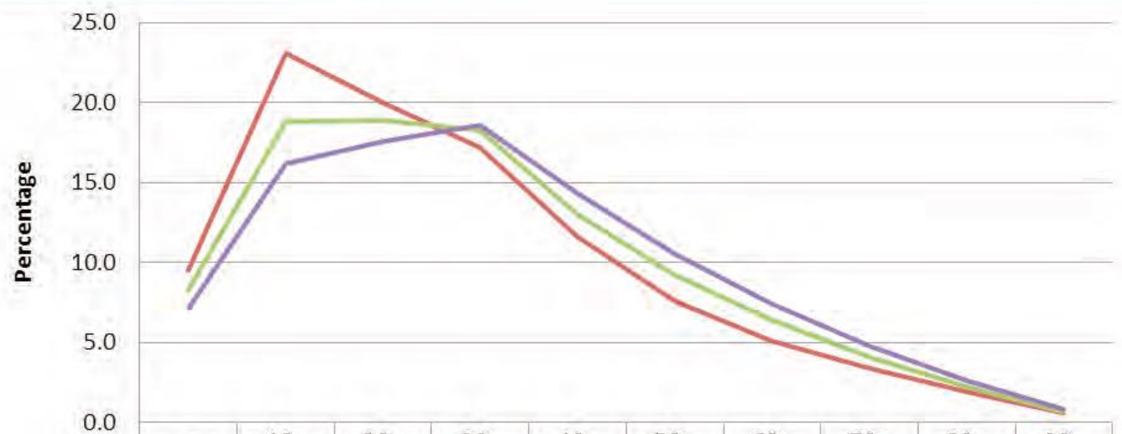
I draw from all the above work and its interconnectedness as I describe how and why MDI emerged in this form and reflect on what it does in relation to the goal of impacting the teaching and learning of mathematics. It goes without saying that the emergence of MDI is a function of its context, specifically mathematics education in post-apartheid South Africa, and the interaction of this 'ground' with discourses in the field of mathematics education and my own previous research. It is also a function of the desire early on in the project to produce a resource – an overarching frame - that could move across our overlapping communities and differing discourses. I thus begin with a brief account of the mathematics education terrain in South Africa, and the conditions of teachers' work in *schools for the poor*; followed by a brief detailing of some of the 'realities' indicated by research findings early on in our project, that further illuminate common mathematics teaching practices in South Africa, and provide the impetus for the MDI framing above. I link these with literature and research in mathematics education and so too the elaboration of MDI before moving on to illustrate how we bent MDI towards the needs and design of our professional development work, and describe how we extended and operationalised it for research. This background, I hope, will enable appreciation of the WMCS in its location, and at the same time, connect with mathematics education on the margins elsewhere. I conclude with some reflections, what MDI illuminates and obscures, and with work therefore that lies ahead.

The South African Mathematics Education Context

Broad Patterns of Performance and Conditions of Teachers' Work

We are twenty years into our new and still rather young democracy. It is deeply troubling that education in post-apartheid South Africa is described, in research and in public debate, as being in a state of 'crisis' (Spaull 2013; Taylor, Van Der Berg & Mabogoane, 20013). Research over the last decade has established that problems of low educational outcomes for a majority of learners is apparent in South Africa as early as the end of the Foundation Phase or third grade. Whilst this is the pattern across education, the problems of performance in mathematics are deeper, with Mathematics showing consistently lower levels of performance at Grade 12 level than most other subjects (South African Institute of Race Relations, 2012).

The graphs in Figure 1 below show the performance distribution curves for Mathematics (2011 - 2013), as presented in the National Senior Certificate Diagnostic report in South Africa (DBE, 2013, p. 126). While improvements in the system as a whole are visible, with failures decreasing and more obtaining better scores, the evidence is stark: the South African education system, and mathematics within this, is failing most of its learners. The performance curves in 2009 and 2010 in the WMCS schools had a similar shape, though more exaggerated, as all are relatively poorer performing schools. The challenge for the project was whether a research informed professional development project could work with teachers to shift this curve in and across schools, to reduce the large failure rate and very low performance of the majority, and increase learners obtaining scores over 60% and so with possibilities for tertiary study in the mathematical sciences.



	0 - 9.9	10 - 19.9	20 - 29.9	30 - 39.9	40 - 49.9	50 - 59.9	60 - 69.9	70 - 79.9	80 - 89.9	90 - 100
2011	9.5	23.1	20.0	17.2	11.6	7.6	5.1	3.4	1.9	0.6
2012	8.3	18.8	18.9	18.3	13.0	9.2	6.4	4.1	2.2	0.7
2013	7.1	16.2	17.6	18.6	14.3	10.5	7.4	4.8	2.6	0.8

Figure 1: Performance distribution curves Mathematics (DBE, 2013, p. 126)

Two additional contextual issues in South Africa are important to highlight here, as they are typically not foregrounded in the research on professional development, and both relate to the conditions of teachers' work. We learned very early on in the project, that whatever the desired intervention might be, it would interact with and thus need to be deeply cognisant of the conditions of teachers' work. We were guided in this, firstly by time spent becoming familiar with the schools in the first year, but also by an insightful analysis of the dual economy of schooling in South Arica, and the impact of this on teachers work. Shalem & Hoadley (2009) studied the relationship between inequality, teacher morale and their conditions of work, and identified four factors that impact on this work. They argue that:

> [t]eachers experience inequalities in terms of their access to:
>
> - learners who are cognitively well-prepared for schooling, are physically healthy and whose homes function as a second site of acquisition;
> - meaningful learning opportunities in the past and in the present and a reservoir of cognitive resources at the level of the school;
> - a well-specified and guiding curriculum; and
> - functional school management that mediates the bureaucratic demands on teacher time. (p.127)

The relevance of this study to our work was that it revealed resources (the authors refer to these as assets) in teaching that are less visible, but resources non-the-less. We can divide South African teachers into three analytic categories based on this understanding of assets. In one category are (roughly 20%) teachers whose experiences are mediated by the presence of all the resources listed above. They work in schools for the rich, produce good student achievement and are associated with the provision of quality education. At the other end, also roughly 20%, are teachers who work in schools for the poor and whose work and experience is shaped by the absence of all these assets. In between, and also with relatively low educational outcomes are the majority - 60–70% - of teachers in South Africa, whose work is mediated by some but certainly not all of these assets. The teachers in the schools in our project are in this last category, facing a situation where many learners in their classes are not academically prepared for the grade level they are in, and so an ongoing tension between meeting curriculum requirements for the specific grade, and at the same time meeting many learners where they are, mathematically speaking. Collectively, teachers in this schooling band, while qualified, have had poorer disciplinary and professional learning opportunities, and their schools are on lower scales of functionality. As Shalem & Hoadly argue, teachers here have to "… expend much more effort to develop their learners and the task is insurmountable given the property and organisational assets available to them" (2009, p.128). Six of the schools in the WMCS project were termed *priority* schools, which meant they were subject to significant levels of bureaucratic control. The mathematics teachers have to follow a specified term by term, week by week, teaching schedule and learners write common assessments set by officials in the district offices who also then check on the school and teachers for compliance. In broad terms, increased time pressures bear down on teachers who are subjected to high levels of bureaucratic demands that aggravate already low morale.

Linked to this, and the second area of impact on teachers' work, there is increasing curriculum prescription and an assessment regime that impacts teaching and learning - a condition shared in some countries that do not have extremes of poverty and inequality (e.g. the United Kingdom). In South Africa, we currently have Annual National Assessments in Grades 3, 6 and 9, and while these are meant to be for diagnostic and systemic purposes, they have become an additional pressure on teachers and schools. The effect of these processes in secondary schools in particular, in addition to broad low morale, is on teaching/learning time. The space for exploring and building, for example, more exploratory mathematical work and dialogic classroom interaction valued in the field is highly constrained, and markedly so in priority schools where the bureaucracy bears down heavily, expecting and monitoring teachers' compliance with official decrees.

How does or can a professional development (PD) intervention meet these conditions on the ground, where the shared goal with teachers and schools of improving opportunities to learn come up against low morale and this key tension in PD work – teachers' time? PD is premised on the availability of time; however this might be organised, for the teacher to engage in life-long learning in their work. The irony here that I have tried to make visible, is that while time constraints and pressures for improved mathematics performance affect PD everywhere, and this is well documented (see discussion of tensions in Adler (2013)), these are acute in schools with low educational outcomes – and generally then in schools *for the poor*.

Performance and Practice in our Schools and Further Rooting MDI

The first year of WMCS is best described as a time of 'getting to know' the schools, and mathematics teaching and learning in them. We piloted a diagnostic test in algebra, with Grade 8 and 10 learners. The results of these tests, and a rerun of this in Grade 9 and 11 the following year, were distressing. Not only did errors proliferate across items, but within an item errors were too diffuse to formulate clear categories to organise and enable discussion of the range of responses offered. As we shared these results with teachers, we were able to use this data to open up conversation about the absence of both skill and meaning with respect to algebraic symbolic forms for the majority of learners, even learners who had selected mathematics as a subject of study in Grade 10; and thus open discussion on the daily challenges they faced given the under-preparedness of many of their learners.

Our observations of many lessons provided us opportunity to consider how teaching, and more specifically MDI, was implicated in the apparent incoherence in learner productions in the tests. We observed teachers explain some examples for the announced focus of a session, often with poor levels of coherence between the example and its elaboration, and/or across a sequence of examples. By way of brief example, in one lesson on the products of expressions, three different sets of rules were provided: multiplying expressions with exponents ("if the base is the same we add the exponents"); multiplying a monomial and binomial ("you multiply everything inside the bracket by the term outside the bracket"; multiplying two binomials ("we use the distributive law, and multiply first, inner, outer and last terms [FOIL]). Aside from the instructional talk being focused on the 'how to' steps of procedures, devoid of explanations that provided rationales for these steps (Adler & Venkat, 2014), there was no narrative related to operation of multiplication of different expressions that could have connected the lesson parts and reduced the inevitable result of learners having to rely on multiple visual cues and memory if they were to reproduce such products independently themselves. Compounding such practices was the ways in which teachers used words to name what they were talking about – we observed extensive use of ambiguous referents in teacher talk.

Most of the lessons we observed proceeded with examples and explanations of what was stated as the focus of the lesson, but, as illustrated above, mathematical goals or *objects of learning* were out of focus. We identified two key areas of issue within pedagogy to focus on in our professional development work: Mediating mathematical ideas (this point takes in findings related to ambiguity within teacher talk, and the lack of explanations that establish rationales for action in teachers' handling of specific examples); Progressing understandings towards ideas that build generality, effectiveness and efficiency (this point incorporates the selection and sequencing of examples and ongoing promulgation and acceptance of rule-based strategies that relied on visual cues, memory or imitation). Much of reform based mathematics teacher education engages these pedagogic issues of mediation and progression towards generality through *rich* tasks where mathematical exploration becomes possible through orchestrated dialogic teaching. These practices are viewed as providing possibilities for deepening mathematical knowledge for teaching, and advocacy of such task based or problem based teaching in teachers' classrooms.

Whether the underlying or source of the issues is in pedagogic practice or mathematical knowledge for teaching (and later I discuss the intervention and its focus on the latter), both construct the teacher and the teaching as in deficit, as wholly problematic. We believed strongly that a reform-based orientation would not be an appropriate route to take for WMCS. So we focused our attention on the object of learning being out of focus and how this might be pursued through the themes of *exemplification* (selection and sequencing of examples and related tasks) and teachers' mediation of these through *explanatory talk* taking cognizance of learners' current understandings – and so with resonances with their deeply interwoven cultural practices in their classrooms. Hence, the initial and first layer of elements of the MDI framework in Figure 1 above.

Interestingly, within mathematics education, significant bodies of literature underlie both aspects, and I turn briefly to those studies dealing with examples and talk/explanations in ways that are particularly salient to the issues we have raised above as well as to my own prior research in the field.

Linking with Mathematics Education Research

Focus on examples

The ubiquity of examples within the terrain of mathematics teaching and learning has been acknowledged (Bills, et al., 2006). This follows from a basic maxim that initial experiences of mathematical concepts and procedures, given their abstract nature, will be through some exemplification: through examples and the tasks in which they are embedded. Watson & Mason (2006), for example, have noted the importance of carefully structured example sequences that draw attention towards generality whilst working with particulars:

> the learning of particular interest to us here is conceptual development. This means to us that the learner experiences a shift between attending to relationships within and between elements of current experience (e.g., the doing of individual questions) and perceiving relationships as properties that might be applicable in other situations (p. 92)

Rowland (2008) has also emphasized the need for careful selections and sequencing of example for practice, noting that learners should also experience the range of examples that a procedure can be applied to, to have a sense of the breadth of the 'example space', and to build not just fluency, but also efficiency across the procedures one is practising.

Both Rowland, and Watson & Mason discuss the importance of variation amidst invariance in the teaching and learning of mathematics, referring to theoretical work on variation theory (e.g. Marton & Tsui, op cit; Runesson, 2006) that has come to figure in the literature in mathematics education and exemplification. Variation theory rests on the underlying notion that learning something depends on access to distinguishing variation in the thing to be learned. The form of example sequences 'fits' this model of learning well, with traditional example sets in mathematics often being set in graded forms that lend themselves to analysis through the lens of variation.

Focus on mediating talk/explanations

The ubiquity of 'explanation' as a form of pedagogic talk in mathematics classrooms has also been acknowledged. Andrews (2009) for example, noted the need for teacher explanations to be 'relevant, coherent, complete and accurate'. In previous research work (e.g. Adler & Davis, 2006), we operationalised such explanatory talk through Bernstein's (2000) key insight that pedagogy proceeds through evaluation, and through what was legitimated as knowledge in pedagogic practice. We developed tools for analysing the criteria transmitted as to what was valued in school mathematics or in teacher education, finding this productive and illuminating of what was constituted as mathematics in these pedagogic sites. We have included this in MDI as part of explanatory talk, and as a means for observing whether and how explanations in school mathematics classrooms are coherent and accurate.

In addition we also drew on previous research that foregrounded the importance of how words are used in multilingual mathematics classrooms (Adler, 2001). Mathematical objects come to life not only through activity on tasks and selected examples, but also in how they are named, and thus the importance of movement between informal or colloquial talk and more formal and literate use of mathematical words in school mathematics. In the context of WMCS work, ambiguous use of referents, and so not naming mathematical signifiers appropriately can obstruct learner participation in mathematical discourse. Hence, our specific and additional attention to naming within explanatory talk in MDI.

This brief foray into the literature illuminates the third row of boxes in Figure 1 above, and so the expanding out of two key elements of MDI (exemplification and explanatory talk) to include examples, tasks, naming and legitimating criteria.

As suggested but not explicitly stated, our observation of teaching across classrooms in the schools in which we work is that there is a dominance of more traditional teacher-led whole class working rather than the more dialogic interactional forms described in the international literature. Thus, the focus on teacher's selections and use of examples and explanations 'fits' with the prevalence of more traditional pedagogies. A critique of this twin focus relates to the relative *absence* of the learner in this frame. Linked therefore to the earlier mention that the goal of pedagogy is to improve mathematical learning, we added in a focus on *participation* alongside the other two categories in MDI, guided by the need to explore mediation and progression of mathematical ideas across these features. We have used this discursive resource, underlain by the lenses gained from the more local, and broader research findings, as a tool for analysing videotaped lessons, and as a boundary object for developing pedagogy for mathematics learning. In the remaining sections of the paper, I turn now to discuss our PD work, and related research. There are constraints on space here, and so I only provide illustration of our work in relation to MDI.

MDI in WMCS Professional Development Work

Earlier, I mentioned the "20-day" mathematics for teaching courses in our PD intervention. These are the major components of our work. We have two courses: Transition Maths 1 (TM1), which is aimed at the transition from Grades 9 to 10 (in our system between what are referred to as General and Further Education); and Transition Maths 2 (TM2) aimed at the transition from Grades 11 and 12 into tertiary study. As we got to know and appreciate the diverse knowledge and experience of the range of mathematics teachers across the ten schools, so it became necessary to organise our mathematics focused PD at different levels. The TM courses were not part of our original plan, but became the form in which we could meet teachers mathematically, so to speak, as well as practically. Teachers come to the University for 16 full days in eight 2-day sessions spread over the academic year. We negotiated with the district and schools for teachers to be released from school on 10 of those days, with 6 days then committed from teachers' own time (on Saturdays or in school vacation time). The additional 4 days of the course were allocated for in-school work. This arrangement dealt with the practicality of time for PD work for teachers. Mathematically, we realised that it would be of most value if teachers had adequate opportunity to engage with mathematics in their PD time, hence the two-day sessions; but also that they would have time in between sessions for working on their own mathematics with their colleagues, independently from course tutors. Between each of the two-day sessions, teachers had mathematics assignments that included work on strengthening their fluency and conceptual understanding, as well as a teaching assignment to try out in their classroom or with some learners.

The bulk of each course, 75%, was on mathematics, a function of our developing understanding that an underlying difficulty for many teachers was articulating what it was, mathematically, they wanted learners to know and be able to do. Our starting point then for strengthening this was to provide opportunity for teachers to strengthen their own relationship with mathematics in the first instance.

The remaining 25% of time in the courses focused on MDI and its elements (exemplification, explanation, and learner participation, all in relation to an object of learning) and we called this a Mathematics Teaching Framework in the PD. We worked on each element separately and then together in various sessions in the courses, structured by the discursive resources in Figures 3 and 4 below. For example, in the first day of a course we would have a two-hour session where we worked on selecting and sequencing examples, typically for a lesson related to content being dealt with in the mathematics sessions earlier that day. Teachers examined textbooks, and other teaching materials for what were exemplified, and how, in a particular topic; whether these were *good* examples, and well sequenced. This opened space for discussion of what made examples, and sets of examples, *good*, or *coherent with the object of learning*, and we shared with teachers, key tenets of variation theory, of seeing similarity and difference, as a means for doing this work. At some point following, we would introduce the framework, and so our boundary object recast for work in the PD. In following sessions we then dealt with each of the columns in Figure 3, elaborating these, as illustrated in Figure 4 for explanation.

In the latter half of the course we have a *lab* lesson during one of the course days, where a class of learners from one of the schools comes to the University (this was typically arranged for a Saturday session). The course leader and teachers planned the lesson together in a session on the previous day. They used the framework to bring attention to the mathematical goal, and how the selected examples and tasks, their sequencing and their mediation in talk through naming and justifying, supported the intended learning object. Attention was then also turned to learner participation – to what learners would be asked to do, say, write and how this would enable their learning. The course leader then taught the lesson, teachers observed, and made notes, using the framework, on an empty version of the table in Figure 3. After the lesson, the course session would be a reflection on the lesson, again using the framework to guide discussion. This adapted version, drawing from both Lesson Study (with resonances with the Japanese model) and Learning Study (the Swedish model), is also then carried out in schools. Teachers from neighbouring schools come to one school once a week in the afternoon for three consecutive weeks once a term, to work in a similar fashion as described above. Planning takes place in week 1, the lesson is taught by one of the teachers in week 2 with one class of learners, and revised, and the revised version is taught by a different teacher with another class in week 3. While the project assists with co-ordination and planning, teachers themselves teach the lessons, and collaborate on its design, reflection, redesign and so on. One WMCS team member works with each group of teachers. Here too, the framework is used as a discursive resource to guide planning and reflection.

This in-school work, while occurring after school hours, provided an opportunity for teachers to collaborate on and study their teaching with their own learners (or those of a colleague), and on an agreed and shared problem. Questions like: “What do we want learners to know and be able to do?” “How do the examples and tasks selected support this?” “Where the examples well sequenced?” “What of the talk? How did it move between every day or informal and then mathematical talk”. “How full were explanations that evolved?” “Were learners participating and how?” “Ultimately, did learners learn what we intended? How can we know?”

The tables in Figures 3 and 4 below are examples of the resources that structure this working on practice together, using MDI in its practice-based version.

Object of learning : teaching x to y		
Examples and tasks	**Explanations and talk**	**Learner activity**

What examples are used?
• To start off the lesson
• To develop the lesson
(these may be "examples of")
  • To introduce a concept
  • To ask questions
  • To explain further
• For learners to practise/ consolidate (these are "examples for")
 	
What are the associated tasks?
• What are learners required to do with the example/s?
 	
➤ How do these combine to build key concepts and skills?
	What kinds of explanations are offered?
	• What (and why)
	• How (and why)
	
	• What representations are used?
	
	➤ How do these help to build the key concepts and skills?
		What work do learners do?
		
		e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class
		
		➤ How does their activity help to build key concepts and skills?

Coherence: Are there **coherent connections** between the object of learning, examples, tasks and explanations?		

Figure 3: The Mathematics Teaching Framework

Object of learning				
Examples and tasks	**Explanation** What does the teacher say and do to help learners make sense of the mathematics beyond the current lesson?			**Learner activity**
	What is written? What does the teacher write (publicly) regarding the mathematical object?	**What is said?** How does the teacher talk about the mathematical object?	**How is the maths justified?** How does the teacher justify the mathematics?	
	Words, phrases, sentences Terminology and expressions Graphs, illustrations, figures Definitions Procedures Solutions Proofs	**Colloquial language** Everyday language e.g. "taking x to the other side" Ambiguous referents for objects e.g. this, that, thing	**Non-mathematical cues** Visual cues, mnemonics e.g. smiley parabola Metaphor related to features of real objects e.g. This is how it "looks", "sounds", "how you remember"	
		Some mathematical language to name object, component e.g. factor, parabola, derivative Reading a string of symbols e.g. "x into x plus 2",	**Local mathematical** Specific/single cases e.g. triangles in standard position, expressions with only positive terms Established short-cuts and conventions e.g. FOIL, SOHCAHTOA	
		Extended and appropriate mathematical language to name mathematical objects and procedures e.g. "the product of two binomials", "subtracting the additive inverse"	**General mathematical** equivalent representations, definitions, properties, principles, structures, previously established generalizations Note: A general mathematical justification could be partial/incomplete/full.	

Figure 4: The Mathematics Teaching *Framework*, with elaboration of explanation.

As anyone working with Lesson Study would know, building and sustaining such communities is not trivial work, nor is the functioning of the study group. It is beyond the scope here to elaborate our trials and tribulations in this work in detail – I will talk to this in the presentation. I focus some discussion, however, on the salience of the framework and the discursive resources that support it and teachers working with it.

We know from our research study (see below) of videos of lessons of teachers prior to taking the TM1 course and then some time after completing it, that the selection and sequencing of examples improved – with respect to criteria we established - across many of the teachers in the research sample (Adler, 2014; Adler & Ronda, 2014; forthcoming). This research result confirms our experience with the lesson/learning studies we have done in 2014 that planning, reflecting on and critiquing the example sets in the lesson is the part of the framework and tool that teachers engage with most easily. There were also shifts in our research data on attention to word use, and working between informal and formal mathematical talk across teachers over time. Here too, and this is not a surprise in a multilingual setting, teachers noticed learners' use of words, and the particular words or phrases that they found difficult – and were aware of their own challenges in navigating and revoicing these. Teachers who taught the lessons in our after school work often raised these language issues as the first things to discuss in the lesson reflection.

At the same time, these shifts in parts of exemplification and explanatory talk were not supported by tasks that required more than simple known procedures by learners, and where learners had more opportunity to enter the discourse both through what they did and what they were able to communicate. Alongside this, the kinds of explanations for both procedures and concepts did not seem to move from justifications asserted by the teacher, stated rather than derived, or single case examples and so without connections and moves to greater generality and mathematical power. These difficulties were visible in the research data and remain a challenge in our lesson/learning study work. The discursive resources do not function at this point, as a means for thinking about and talking about this key aspect of teaching and so MDI in the classroom. Task demands and how justifications are built are linked with learner participation, and how teachers connect learners with mathematics. The entrenched cultural forms of pedagogy in these classrooms remain difficult to shift. Where these shifts are visible, and across our data are numerous attempts by teachers to invite learners into more complex tasks, and to agree, disagree with what is being offered, their mediation has tended to reduce the task demands. In managing discussion where disagreements were voiced, the mathematical substance of these remained largely hidden or implicit.

Evident in this discussion of using MDI in our professional development work with teachers, the particular form it has taken and what has taken place is deeply interwoven with our research work and insights from analysis of video lessons. As these happen coincidently, each has informed and shaped the other. I now move on to discuss how we have used MDI in research.

An Analytic Framework for MDI

Table 1 in Figure 5 below presents the framework. I briefly elaborate each of the analytic resources, and our analytic categories, derived from the research literature mentioned earlier and in interaction with our empirical data. Visible in the categories is our interest in scientific concepts and increasing generality in examples; increasing complexity in tasks; colloquial and formal mathematical talk and mathematical justifications for what counts in the discourse. With respect to participation, we are interested increasing the opportunity for learners talk mathematically, and teachers' increasing the use of learners' ideas.

Our unit of study is a lesson, and units of analysis within this, an event. The first analytic task is to divide a lesson into events, distinguished by a shift in content focus, and within an event then to record the sequences of examples presented. Each new example becomes a sub-event. Our interest here is whether and how this presentation of examples within and across events brings the object of learning into focus, and for this we recruit constructs from variation theory (Marton & Pang, 2006). The underlying phenomenology here is that we can discern a feature of an object if we can see similarities and differences through what varies and what is kept invariant. Variation through similarity is when a feature to be discerned is varied (or kept invariant), while others are kept invariant (or made to vary), with possibilities then for seeing generality; contrast is when there is opportunity to see what is not the object, e.g. when an example is contrasted with a non-example; when there is simultaneous discernment of aspects of the object is possible, further generalisation is possible. These three forms of variation (similarity, contrast and simultaneity) can operate separately or together, with consequences for what is possible to discern – and so, in more general terms, what is made available to learn. In WMCS we are interested in analysing the teacher's selection and sequencing of examples within an event and then across events in a lesson, and then whether and how, over time, teachers expand the set of examples and the sequencing constructed in a lesson.

Object of learning				
Exemplification		**Explanatory talk**		**Learner Participation**
Examples	**Tasks**	**Naming**	**Legitimating criteria**	
Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of *similarity* **(S),** *contrast* **(C),** *simultaneity* **(U)**	Across the lesson, learners are required to: *Carry out known operations and procedures* **(K)** e.g. multiply, factorise, solve; *Apply known skills, and/or decide on operation and /or procedure to use* **(A)** e.g. Compare/ classify/ match representations; *Use multiple concepts and make multiple connections.* **(C/PS)** e.g. Solve problems in different ways; use multiple representations; pose problems; prove; reason.etc	Within and across events word use is: *Colloquial* **(NM)** *e.g.* everyday language and/or ambiguous referents such as this, that, thing, to refer to signifiers *Math words used as name only* **(Ms)** e.g. to read string of symbols *Mathematical language used appropriately* ***(Ma)*** to refer to signifiers and procedures	Legitimating criteria: *Non mathematical* ***(NM)*** *Visual* **(V)** – e.g. cues are iconic or mnemonic *Positional* ***(P)*** – e.g. *a* statement or assertion, typically by the teacher, as if 'fact'. *Everyday* ***(E)*** *Mathematical criteria:* *Local* **(L)** e.g. a specific or single case (real-life or math), established shortcut, or convention *General* **(G)** equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial **(GP)** or 'full' **(GF)**	Learners answer: *yes/no questions or offer single words* to the teacher's unfinished sentence **Y/N** Learners answer (what/ how) questions in phrases/ sentences **(P/S)** Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions (D)

Figure 5: Analytic framework for mathematical discourse in instruction.

Of course, examples do not speak for themselves. There is always a task associated with an example, and accompanying talk. With respect to tasks, we are interested in cognitive demand in terms of the extent of connections between concepts and procedures. Hence, in column two we examine whether tasks within and across events require learners to carry out a known operation or procedure, and/or whether they are required to decide on steps to carry out, and some application, and/or whether the demand is for multiple connections and problem solving. These categories bear some resemblance to Stein et al's (2000) distinctions between lower and higher demand tasks.

With respect to how explanation unfolds through talk, and again the levels and distinctions have been empirically derived through examination of video data, we distinguish firstly between naming and legitimating, between how the teachers refer to mathematical objects and processes on the one hand, and how they legitimate what counts as mathematics on the other. For the latter, we have drawn from and built on the earlier research discussed above. Specifically, we are interested in whether the criteria teachers transmit as explanation for what counts is or is not mathematical, is particular or localised, or more general, and then if the explanation is grounded in rules, conventions, procedures, definitions, theorems, and their level of generality. With regard to naming, we have paid attention to teacher's discourse shifts between colloquial and mathematical word use.

Finally, all of the above mediational means (examples, tasks, word use, legitimating criteria) occur in a context of interaction between the teacher and learners, with learning a function of their participation. Thus, in addition to task demand, we are concerned with what learners are invited to say i.e. whether and how learners have opportunity to use mathematical language, and engage in mathematical reasoning, and the teacher's engagement with learner productions.

Illustration of the use of this framework first on one selected lesson appeared in Adler & Ronda (2014). Further extension of the framework and its use in comparing lessons and so shifts in MDE over time can be found in Adler & Ronda (forthcoming), where categorising events over time accumulate into levels based on our privileging of development towards scientific concepts and generality in the discourse. I do not reproduce these here and refer to the full research papers. Nevertheless, in Figures 6 and 7 below, taken from Adler & Ronda (forthcoming), is the coding of events and how these accumulate into levels for one teacher's lessons in 2012 and then 2013. I present these here, despite the analysis on which they are based not appearing here, so as to enable the discussion following.

Events	Exs	Tasks	Naming	Leg Criteria	Lr Partic
1 – Review Exponent laws	NA	K	Ms, Ma	NA	P/S
2 – Application numerical bases	U, S	A, K	Ms, Ma	L, GP	P/S
3 – Application – literal bases	U, S	A, K	Ms, Ma	L, GP	Y/N
Cumulative level	L1	L2-L1	$L2^{-}$	L2	L2

Figure 6. Summary codes and analysis of Lesson of Teacher X in 2012, in Adler & Ronda (forthcoming)

Events	Exs	Tasks	Naming	Leg Criteria	Lr Partic
1 – Meaning of a Term	S, C, U	K	NM, Ms, Ma	G	Y/N
2 – Meaning of common factor	NA	K	Ms, Ma	G	Y/N, P/S
3 – Simplify algebraic fraction	S, C, U	A - K	NM, Ms	NM, L	Y/N
4 -Divide algebraic fractions (+)	S, U	A - K	NM, Ms	NM, L, G	Y/N
5 – Extension to (-) coefficients	S, U	A - K	Nm, Ms	L	Y/N
Cumulative Code	L3	L2- L1	L2	L2	L1

Figure 7. Summary codes and analysis of Lesson of Teacher X in 2013, in Adler & Ronda (forthcoming)

Discussion

Our MDI framework allows for an attenuated description of practice, prising apart parts of a lesson that in practice are inextricably interconnected, and how each of these contribute overall to what is made available to learn. It co-ordinates *various variables, situations and circumstances* of teacher activity (Ponte & Chapman, op cit). There is much room for this teacher to work on learner participation patterns, as well as task demand (and these are inevitably inter-related); at the same time her example space evidenced awareness of and skill in producing a sequence of examples that bring the object of learning into focus, hence the value of this specific aspect of MDI. Contrasting levels in earlier observation of this teacher indicates an expanded example space and more movement in her talk between colloquial and mathematical discourse. The MDI framework is thus helpful in directing work with the teacher (practice), and in illuminating take up of aspects of MDI within and across teachers (research). As noted, our analysis across teachers suggests that take-up with respect to developing generality of explanations is more difficult.

The MDI framework provides for responsive and responsible description. It does not produce a description of the teacher uniformly as in deficit, as is the case in most literature that works with a reform ideology, so positioning the teacher in relation to researchers' desires (Ponte & Chapman, op cit). We are nevertheless aware that we have illustrated MDI on what many would refer to as a *traditional* pedagogy. We have *tested* it out on lessons structured by more open tasks, but this requires more systematic study on varying classroom practices.

Conclusion

I have written this paper to capture the work of the WMCS project and the development and use of MDI. It is a descriptive paper, as the more directed research is reported elsewhere. Through this I hope to have shared some of the *in betweenness* in our work, as many of us are simultaneously practitioners in mathematics teacher education and in research – and thus boundary crossers in our work. As a keynote paper and without the boundaries imposed by research practices on the one hand, or development descriptions on the other, I have been able to share how we worked within and between these. I hope too that by setting WMCS explicitly in its location, and linking with research literature, I have enabled connections between this work with mathematics education on the margins elsewhere.

As we reflect back and look ahead we are aware and it is important to explicitly acknowledge this here, that there is the learning progression in professional development implicit in the WMCS model as it has developed. The courses and their greater focus on content knowledge of teaching, and teachers' own relationship with mathematics in contrast to attention to pedagogy indicate that we view this as primary. We hold strongly to this view but understand at the same time, that the lesser focus on pedagogy, and further how we have done this with MDI is implicated in that it is teachers have taken it up and what are clearly more challenging aspects of teaching and related opportunities for learning, specifically setting up and maintaining more demanding tasks, and orchestrating greater learner participation in classroom discourse. At this point we do not see this as necessarily as a weakness in the programme, but more an indicator of how learning progresses over time. As we move into Phase 2 of the project, our plan in the first instance is to develop MDI further, where we bring learner participation and the nature of tasks into focus with the teachers, and research what is entailed in this work. An additional focus for our future work is that while we have evidence of the impact of the courses on teachers' knowledge and their learners' performance, we are aware of the time invested as we developed the courses, of our own learning and developing expertise as these were implemented and refined. What then is entailed in making the materials and rationales for the course available for others to use and so more teachers to have such opportunities? What happens as these are taken up and expanded out – to the mathematical experiences offered in the courses on the one hand, and to the interweaving of MDI within and across the sessions? There will be recontextualisation! But of what, how and with what consequences? There is much work to do going forward.

Acknowledgements

This work is based on research of the Wits Mathematics Connect Project, University of the Witwatersrand, supported by Department of Science and Technology, and FirstRand Foundation, and administered by the National Research Foundation. Any opinion, findings, conclusions, recommendations are those of the authors. The authors also acknowledge the WMCS project team all of whom have contributed to MDI.

References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht ; Boston: Kluwer Academic Publishers.

Adler, J. (2013). *The research and development learning curve continued: Report of the Wits FRF Mathematics Education Chair and the Wits Maths Connect Secondary Project, 2013*. University of the Witwatersrand. Johannesburg.

Adler, J. (2014). *The research and development learning curve continued: Report of the Wits FRF Mathematics Education Chair and the Wits Maths Connect Secondary Project, 2014*. University of the Witwatersrand. Johannesburg.

Adler, J., & Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education*, 37(4), 270-296.

Adler, J. & Ronda, E. (2014) An analytic framework for describing teachers' mathematics discourse in instruction. In C. Nichol, P. Liljedahl, S. Oesterle, & D. Allan (Eds.), *Proceedings of the of the 38th Conference of the International Group for the Psychology in Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol 2, pp.9-16). Vancouver, Canada: PME.

Adler, J. & Ronda, E (forthcoming) A framework for mathematics discourse in instruction: Documenting shifts in practice. *African Journal for Research in Mathematics, Science and Technology Education.*

Adler, J., & Venkat, H. (2014). Teachers' mathematical discourse in instruction: focus on examples and explanations. In M. Rollnick, H. Venkat, J. Loughran, & M. Askew (Eds.), *Exploring content knowledge for teaching science and mathematics* (pp. 132-146). London: Routledge.

Andrews, P. (2009). Mathematics teachers' didactic strategies: Examining the comparative potential of low inference generic descriptors. *Comparative Education Review, 53*(4), 559-582.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, Research, Critique*. Maryland: Rowman & Littlefield.

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in Mathematics Education. Prague, Czech Republic: PME. In J. Novotna (Ed.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*. (Vol 1, pp. 126-154) Prague: PME.

DBE. (2013). *National Senior Certificate Examination: National Diagnostic Report*. Pretoria.

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers & Teaching, 15*(2), 273-289. doi: 10.1080/13540600902875340 2012

Marton, F., & Pang, M. F. (2006). On some necessary conditions of learning. *The Journal of the Learning Sciences, 15*(2), 193–220.

Marton, F., & Tsui, A. (2004). *Classroom discourse and the space of learning*. New Jersey: Lawrence Erlbaum Associates.

OECD. (2013). *OECD Economic Surveys*: South Africa 2013.

Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 461-494). Rotterdam: Sense

Pournara, C., Hodgen, J., Adler, J. & Pillay, V. (forthcoming) *South African Journal of Education.*

Rowland, T. (2008). The purpose, design and use of examples in the teaching of elementary mathematics. *Educational Studies in Mathematics, 69*, 149-163.

Shalem, Y., & Hoadley, U. (2009). The dual economy of schooling and teacher morale in South Africa. *International Studies in Sociology of Education, 19*(2), 119-134.

Skovsmose, O. (2011) *An Invitation to Critical Mathematics Education*. Springer.

South African Institute of Race Relations. (2012). *2010/2011 South Africa Survey*: SAIRR.

Spaull, N. (2013). *South Africa's education crisis: The quality of education in South Africa 1994-2011*. Johannesburg, CDE Centre for Development and Enterprise.

Star, S. L., & Griesemer, J. (1989). Institutional ecology, 'translations' and boundary objects: amateurs and professionals in Berkeley's museum of vertebrate zoology, 1907 - 1939. *Social Studies of Science, 19*, 387-420.

Stein, M., Smith, M., Henningsen, M., & Silver, E. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. Reston.

Sullivan, P. (Ed.). (2008). *Knowledge and beliefs in mathematics teaching and teacher development (Vol. 1).* Rotterdam: Sense Publishers.

Taylor, N., Van der Berg, S., & Mabogoane, T. (Eds.). (2013). *Creating Effecitve Schools*. Cape Town: Pearson.

Venkat, H., & Adler, J. (2012). Coherence and connections in teachers' mathematical discourses in instruction. *Pythagoras, 33*(3). doi: http://dx.doi.org/10.4102/pythagoras.v33i3.188

Venkat, H., & Adler, J. (2013) From research to teacher development in mathematics education: Creating boundary objects. In *Gouws, E. & Wolhuter,C. (Eds). Proceedings of the SAERA Conference, 2013*. Oxford University Press.

Vygotsky, L. S. (1978). *Mind in society : the development of higher psychological processes*. Cambridge: Harvard University Press.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning, 8*(2), 91-111.

Exploring a Structure for Mathematics Lessons that Foster Problem Solving and Reasoning

Peter Sullivan
Monash University
<peter.sullivan@monash.edu>

Nadia Walker
Monash University
<Nadia.walker@monash.edu>

Chris Borcek
St Kilda Primary School
<borcek.christine.c@edumail.vic.gov.au>

Mick Rennie
St Kilda Primary School
<rennie.michael.a1@edumail.vic.gov.au>

While there is widespread agreement on the importance of incorporating problem solving and reasoning into mathematics classrooms, there is limited specific advice on how this can best happen. This is a report of an aspect of a project that is examining the opportunities and constraints in initiating learning by posing challenging mathematics tasks intended to prompt problem solving and reasoning to students, not only to activate their thinking but also to develop an orientation to persistence. The results indicate that such learning is facilitated by a particular lesson structure. This article reports research on the implementation of this lesson structure and also on the finding that students' responses to the lessons can be used to inform subsequent learning experiences.

Introduction

Teachers commonly report experiencing difficulties in incorporating problem solving and reasoning into their mathematics classrooms while at the same time catering for students with a wide range of prior experiences. Rather than the common approach of starting learning sequences with simple tasks intending to move to more challenging tasks subsequently, we are exploring an approach based on initiating learning through a challenging task — described as activating cognition. In particular, we describe the implementation of a particular lesson structure designed to initiate learning through an appropriate challenge, effectively differentiating that challenge for particular student needs, and consolidating the learning through task variations.

The data reported below are from one aspect of a larger project[1] that is exploring the proposition that students learn mathematics best when they engage in building connections between mathematical ideas for themselves (prior to instruction from the teacher) at the start of a sequence of learning rather than at the end. The larger project is studying the type of tasks that can be used to prompt this learning and ways that those tasks can be optimally used, one aspect of which is communicating to students that this type of learning requires persistence on their part. Essentially the notion is for teachers to present tasks that the students do not yet know how to answer and to support them in coming to find a solution for themselves.

There are many scholars who have argued that the choice of task is fundamental to opportunities for student problem solving and reasoning. Anthony and Walshaw (2009), for example, in a research synthesis, concluded that "in the mathematics classroom, it is through tasks, more than in any other way, that opportunities to learn are made available to

---

[1] The Encouraging Persistence Maintaining Challenge project was funded through an Australian Research Council Discovery Project (DP110101027) and was a collaboration between the Monash University and Australian Catholic University. The views expressed are those of the authors. The generous participation of project schools is acknowledged.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 41–56. Sunshine Coast: MERGA.

the students" (p. 96). Similar comments have been made by Ruthven, Laborde, Leach, and Tiberghien (2009) and Sullivan, Clarke, and Clarke (2013).

There are also scholars who have proposed that those tasks should be appropriately challenging. Christiansen and Walther (1986), for example, argued that non-routine tasks, because they build connections between different aspects of learning, provide optimal conditions for thinking in which new knowledge is constructed and earlier knowledge is activated. Similarly, Kilpatrick, Swafford, and Findell (2001) suggested that teachers who seek to engage students in developing adaptive reasoning and strategic competence, or problem solving, should provide them with tasks that are designed to foster those actions. Such tasks clearly need to be challenging and the solutions needs to be developed by the learners. This notion of appropriate challenge also aligns with the *Zone of Proximal Development* (ZPD) (Vygotsky, 1978). Similarly, the National Council of Teachers of Mathematics (NCTM) (2014) noted:

> Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (p. 17)

This approach was described in *PISA in Focus* (Organisation for Economic Co-operation and Development (OECD) (2014) as follows:

> Teachers' use of cognitive-activation strategies, such as giving students problems that require them to think for an extended time, presenting problems for which there is no immediately obvious way of arriving at a solution, and helping students to learn from their mistakes, is associated with students' drive. (p. 1)

The OECD (2014) explicitly connected student drive, which we associate with persistence, with higher achievement.

There are many research findings that elaborate how such advice can be implemented in classrooms, some of which is reviewed below. This report seeks to extend this advice in three significant ways: first, by investigating a specific lesson structure and particular tasks; second, by suggesting how such tasks can be adapted to accommodate differences in students' prior experiences; and third, by considering how the learning from the challenging tasks can be consolidated.

The Connection Between the Research Framework and the Structuring of Lessons

The data reported below are informed by a framework as shown in Figure 1, adapted from Clark and Peterson (1986), that proposes that teachers' intentions to act are informed by their knowledge, their disposition, and the constraints they anticipate experiencing. The particular focus in this article is the ways that each of these factors connect to the structuring of lessons.

One node of this framework presents decisions on lesson structure as being informed by the knowledge of the teacher. The different aspects of such knowledge, specifically teachers' knowledge of mathematics, of pedagogy and of students, are represented schematically by Hill, Ball, and Schilling (2008).

The inference is that it is more likely that teachers will intend to use challenging tasks if they understand the mathematics and its potential, are aware of approaches to implementing the tasks in classrooms and can anticipate student responses.

Another node in the framework suggests that teachers' planning intentions are informed by their dispositions including their beliefs about how students learn (Zan, Brown, Evans, & Hannula, 2006), the ways that challenge can activate cognition (Middleton, 1995), and perspectives on self-goals, a growth mindset and the importance of student persistence (Dweck, 2000).

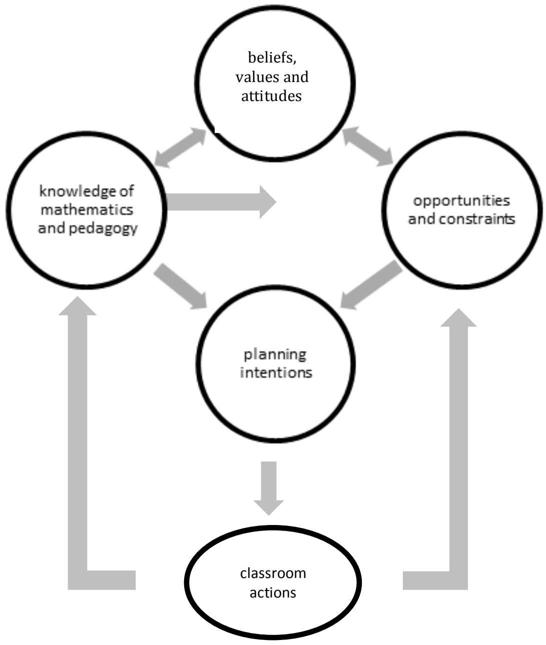


Figure 1: The framework informing the research.

A third node proposes that the ways teachers plan are influenced by constraints that they anticipate they might experience. For example, teachers may be more likely to enact lessons based on challenging tasks if they do not fear negative reactions from students (see Desforges & Cockburn, 1987).

These three nodes interact with each other and together they inform teachers' planning intentions which in turn influence the classroom actions.

An analogous framework that similarly connects teachers' knowledge, beliefs and perceptions was presented in a diagram by Stein, Grover, and Henningsen (1996), which, when converted to text, proposes that the features of the mathematical task when set up in the classroom, as well as the cognitive demands it makes of students, are informed by the mathematical task. These features are, in turn, influenced by the teacher's goals, subject-matter knowledge, and their knowledge of their students. This then informs the mathematical task as experienced by students which creates the potential for their learning.

The particular lesson structure being explored addresses four aspects arising from consideration of both frameworks, specifically:

- The ways the tasks are posed in the introductory phase that is described by Lappan, Fey, Fitzgerald, Friel, and Phillips (2006) as the Launch, and by Inoue (2010), in outlining the structure of Japanese lessons as Hatsumon, meaning the initial problem, and Kizuki which is what it is intended that the students will learn;
- Actions taken to differentiate the task for particular students that occur during what Lappan et al. call Explore, and what Inoue describes as Kikanjyuski which is the individual or group work on the problem. Note that Inoue uses the term Kikan shido to suggest that the teacher actions during this aspect include thoughtfully walking around the desks;
- Ways that the student activity on the task is reviewed, described by Lappan et al. as Summary; and which includes both what Inoue calls Neriage which is carefully

managed whole class discussion seeking the students' insights, and Matome which is the teacher summary of the key ideas; and
- Subsequent teacher actions to pose additional experiences that consolidate the learning activated by engaging with the initial task.

The four aspects are elaborated below. In each of the aspects, teachers' actions connect directly to their knowledge of the mathematics involved in the task, their beliefs about what students can do and their anticipation of any constraints they may experience.

Posing the Task

A key aspect of the structuring of a lesson is the information provided to students as part of the introduction. If the teacher is working on the proposition that the students can be offered the opportunity to explore the problem and associated mathematics for themselves, then the introductory phase of the lesson becomes critical. Jackson, Garrison, Wilson, Gibbons, and Shahan (2013), for example, argued that there are two key issues for teachers to consider in the *set up* of the task. The first is that a common language can be established not only for students to interpret the task appropriately but also so they can contribute to the subsequent discussion. Second, it is productive if teachers consciously maintain the cognitive demand of the task. It can be assumed that decisions teachers make in maintaining the challenge are directly connected to their knowledge and beliefs about mathematics and pedagogy. Also connected to the maintenance of the challenge is whether teachers anticipate negative student reactions. Interestingly, in an earlier iteration of the project, Sullivan, Askew, Cheeseman, Clarke, Mornane, Roche, and Walker (2014) found that the majority of students do not fear challenges: they welcome them. Further, rather than preferring teachers to instruct them on solution methods, many students reported that they prefer to work out solutions and representations for themselves or with a partner.

Differentiating the Task

A second aspect of structuring lessons is anticipating ways that different students within the class might respond to the challenge, noting that this is important whether the students are grouped by their achievement or not. Sullivan, Mousley, and Jorgensen (2009) described two key actions in differentiating the experience:

- The provision of enabling prompts, which involve reducing the number of steps, simplifying the complexity of the numbers, and varying the forms of representation for those students who cannot proceed with the task with the explicit intention that they work on the initial task subsequently; and
- Offering extending prompts to students who complete the original task quickly which ideally elicit abstraction and generalisation of the solutions.

These differentiated experiences are offered after students have engaged with the original task for some time and have the same characteristics as the original task, meaning that students engage with the task for themselves as distinct from being told what to do.

Reviewing Student Activity on the Task

A further key aspect of the structuring of lessons is the review of students' solutions and strategies on the challenging task. The key elements of such lesson reviews were described by Smith and Stein (2011) as:

- Selecting particular responses for presentation to the class and giving those students some advance notice that they will be asked to explain what they have done;
- Sequencing those responses so that the reporting is cumulative; and
- Connecting the various strategies together.

Consolidating the Learning

So far, the lesson structure has facilitated the activation of cognition. The next phase is to provide opportunities for students to do what Dooley (2012) describes as *consolidating* the learning. This may involve posing a task similar in structure and complexity to the original challenge that helps to reinforce or extend the learning prompted by engagement with the original task.

Variation Theory offers a process that can guide the planning of these consolidating tasks. Kullberg, Runesson, and Mårtensson (2013), for example, described a study that used variation theory to plan lessons subsequent to an initial lesson on division of decimals. Their intention was that such task variations would prompt students to interpret the concepts in a different way from what they had seen previously. Kullberg et al. (2013) argued:

> In order to understand or see a phenomenon or a situation in a particular way one must discern all the critical aspects of the object in question simultaneously. Since an aspect *is noticeable only if it varies against a back-ground in invariance* (emphasis in original), the experience of variation is a necessary condition for learning something in a specific way. (p. 611)

In the application of Variation Theory to the creation of tasks intended to consolidate the learning prompted by the initial task, the intent is that some elements of the original task remain invariant, and other aspects vary so that the learner can focus on the concept and not be misled by over-generalisation from solutions to a single example. It is possible that this aspect is underemphasised in much commentary on student centred approaches.

In summary, the teachers' intentions include identifying the mathematical potential within a task; planning the elements of lessons that engage learners in creating their own solutions to problems including deliberately maintaining the challenge of the task; anticipating the need to differentiate the task for some students; effectively reviewing students' reporting on their activity on the task; and consolidating that learning through similar tasks thoughtfully varied. The overall project is continuing to explore all of these aspects.

The results below are intended to offer insights into the following research questions:

(a) To what extent is the proposed lesson structure manageable by teachers and to what extent does it support student engagement with the challenging tasks?

(b) How does the lesson structure connect student learning with subsequent teacher actions?

The Context of the EPMC Project and Processes of Data Collection

The data reported below were sought from teachers of students in Years 3/4 (ages 8 to 9) in schools serving communities across a variety of socio economic backgrounds. The project adopted a design research approach which "attempts to support arguments constructed around the results of active innovation and intervention in classrooms" (Kelly, 2003, p. 3). The key elements are an *intervention* by the researchers to propose (possibly) different pedagogies from those used normally, the approach is *iterative* in that subsequent interventions are based on previous ones, and the intent is that findings address issues of *practice*, in this case the structuring of lessons.

The first step in this iteration was a full-day professional learning session in which the teachers worked through a set of 10 tasks and lessons that focused on aspects of addition and subtraction. The 30 teachers were from 15 primary schools serving students from diverse socio economic backgrounds, mainly in metropolitan Melbourne. The teachers were a mix of age and experience, although skewed toward being more experienced. The teachers nearly all claimed to be confident that they know both the relevant mathematics and ways of teaching it.

The professional learning included teachers solving the task for themselves, discussing various solutions and considering pedagogies associated with each task and lesson. The importance of anticipating the student experience by exploring possible solutions and variations in advance was emphasised. Even though not part of this report, various strategies to elicit student motivation and persistence were suggested to the teachers.

After each lesson teachers completed a proforma, gathering data on the implementation of lessons using scaled and open responses. While there are advantages in observing lessons to examine the nature of implementation, such observations create interventions of their own and can make the data less representative of natural teaching. In the following, the data on the lessons are from self-report but it is stressed that the teachers were responding to a specific proforma immediately after having taught each lesson, offering readers confidence in the authenticity of the teachers' self-reports.

Teachers also completed additional summative surveys. The Likert-style items on the surveys were descriptive in form, and representative responses are reported below. The qualitative responses were read through and themes identified, especially where the responses aligned with aspects of the research questions.

The students completed an online instrument that included pre-assessments of content and some survey items. Similar questions were asked on a post-test. The main analysis of the test responses was through quantifying the types of student responses and comparing and considering changes in the profile of responses from pre-test to post-test.

The specifics of one lesson constitute the thrust of the data presented below. Even though this runs the risk of overgeneralising from a subset, the focus on this report is on the specifics of the structure of lessons and the details of one lesson elaborate the structure. Data on some other lessons are included for comparative purposes and to establish claims of wider applicability of the structure.

The lesson reported in detail below, titled *Making Both Sides Equal*, included the initial task, termed *learning task*, which was posed as follows:

> Work out some numbers that make both sides of these equations equal
>
> 898 + ? = 900 + ?
>
> 95 - ? = ? - 10
>
> Give a range of responses for each.

The main learning focus of this task is on equivalence; although there are aspects of pattern identification, partitioning and regrouping that might emerge. It is noted that equivalence is important mathematically. To emphasise this point, the 2013 examiners' report for the top level mathematics in Year 12 (17 year old students) (Victorian Curriculum and Assessment Authority, 2014) included the following statement:

> Equals signs should be placed between quantities that are equal—the working should not appear to be a number of disjointed statements. If there are logical inconsistencies in the student's working,

> full marks will not be awarded. For example, if an equals (sic) sign is placed between quantities that are not equal, full marks will not be awarded. (p. 1)

It is stressed that this is part of the first statement in the report from the examiners of the subject taken mainly by the very best students. Clearly it is important that students come to experience the notion of equivalence and there is no reason why students aged 8–9 years should not start to learn this.

It may not be obvious in what ways this task is challenging. Readers are invited to describe not only the relationship between the unknowns, especially in the subtraction example, but also the reasons that the relationship exists. It is such dimensions of the task that justify the categorisation of "challenging".

Through working on the task, it is hoped that, having found a number of solutions to the task, the patterns associated with creating the equal statements emerge. As with most of the other lessons, there is potential for multiple solutions. This has four benefits:

- It allows a low "floor" for the task in that all students can find at least one solution readily;
- There is an expectation that students will determine their own strategy for answering the questions and it is this opportunity for decision making that is engaging for the students;
- There is a high "ceiling" in which students who complete the learning task can seek to propose a generalisation; and
- Having found their own solution strategy, the openness means that students can make unique contributions to the class discussions.

The lesson documentation offered to the teachers also included a rationale for the lesson, the relevant extract from the *Australian Curriculum: Mathematics*, suggestions for a possible introduction to the lesson, and an indicative statement of the goals for student learning.

Enabling prompts (for students experiencing difficulty), which are intended to be posed separately, were suggested as follows:

> In your head, work out the number that would make this equation true:
>
> 9 + 6 = 10 + ?
>
> 9 – 5 = 7 - ?

Note that these use a similar structure to the learning task but with only one unknown and smaller numbers. If some students experience difficulty with the learning task, the teacher would present those students with one or both of these prompts after waiting an appropriate length of time. The intention is that, having completed the prompt(s), those students then proceed with the original learning task.

An extending prompt (for those who find solutions quickly) was suggested as follows:

> Describe the pattern that summarises all of your answers to the question.

One of the intentions of such prompts is to encourage students towards making a generalisation, in this case by finding a clear and concise way to describe the pattern of responses.

The "consolidating" task to follow up the initial learning was suggested as follows:

> What might be the missing numbers? Give at least 10 possibilities.
>
> 224 + ? = ? + 10

? – 10 = 100 – ?

Again, readers are invited to describe the relationship between the unknowns in the subtraction example, noting the ways that the relationship is both similar to and different from the earlier example. Readers are also reminded that there is no initial instruction other than clarifying relevant language.

Figure 2 presents the titles and learning tasks of a selection of four of the other nine lessons that are most similar in form and focus to *Making both sides equal*. Some data of teachers' responses to these lessons are presented below for comparison purposes. It is noted that the information to the teachers on the structuring of these lessons was similar to the lesson described above.

Lesson title	Learning Task
Addition Shortcuts	Work out the answer to 3 + 4 + 5 + 35 + 37 + 36 in your head. What advice would you give to a friend about how to work out answers to questions like these in their head?
Ways to Add in your Head	Work out how to add 298 + 35 in your head. What advice would you give to someone on how to work out answers to questions like this in their head?
Missing Number Subtraction	I did a subtraction question correctly for homework, but my printer ran out of ink. I remember it looked like 8 ☐ - 2 ☐ = ☐ 2 What might be the digits that did not print? (Give as many sets of answers as you can)
Two purchases	I bought a new pair of shoes and a new pair of sandals. The total cost was $87. I know that the shoes cost at least $50 more than the sandals. How much might the shoes cost?

Figure . The learning tasks of some other lessons in this iteration of the project.

Note that this final task is more complex that the others and requires different pieces of information to be processed simultaneously.

Results

The results are presented in two sections: teachers' reports of the implementation of different elements of the lesson structure; and students' responses to both a pre-test and post-test, including a follow-up discussion with teachers from a school with high improvement.

Reports of the Implementation of the Lesson Structure

The following presents the reactions of the teachers to the teaching of the lessons, seeking to answer the first of the research questions. It should be noted that the following represents a substantial data collection exercise in that around 30 teachers responded to a proforma immediately after teaching each of the 10 lessons (around 300 lessons in all).

Table 1 presents the profile of responses to general prompts about the *Making Both Sides Equal* lesson, rating the propositions from strongly disagree (SD) to strong agreement (SA). Note that the numbers of SD, disagree (D), and Unsure (U) responses were small so they have been aggregated.

Table 1
Teachers' Ratings of Aspects of the 'Making Both Sides Equal' Lesson Immediately After its Teaching (n = 30)

Prompts about the *Making both sides equal* lesson	SD, D, U	A	SA
The level of challenge was about right	4	17	9
I would use this lesson again even if I adapt it a little	1	13	16
Most students learned the main mathematical ideas	4	15	11
The contribution of the students to the discussion was good	4	14	12

The teachers endorsed these propositions (87% or more indicating agree or strongly agree). The most positive response was to the prompt about using the lesson again, which is a strong indication of the productivity and potential of the lesson, especially since the teachers had just taught it. The teachers were only slightly less positive about the students' learning. Note that it was not possible to differentiate teachers' responses based on background factors since those data were gathered anonymously. Overall the teachers gave very positive reactions to the lesson and the responses of the students.

Such positive responses were also evident in their responses to the other lessons. Table 2 presents summaries of responses to the comparison lessons. For ease of presentation, and recognising the potential of such analysis to be reductionist, responses of strongly disagree were allocated a score of 1, disagree 2, etc., and then those scores were averaged.

Table 2
Teachers' Ratings of Aspects of Other Lessons Immediately Their Teaching (n = 30)

	Addition Shortcuts	*Ways to Add in Head*	*Missing Number Subtraction*	*Two Purchases*
The level of challenge was about right	4.2	4.2	3.9	3.3
I would use this lesson again even if I adapt it a little	4.7	4.6	4.4	3.7
Most students learned the main mathematical ideas	4.0	4.0	3.7	3.4
The contribution of the students to the discussion was good	4.1	4.3	3.9	3.7

Overall these are very positive reports by the teachers to the lessons and the reactions of students, indicating that the responses to the *Making Both Sides Equal* lessons are representative of these other lessons as well. Even the responses to the more challenging *Two Purchases* lesson were very positive. The inference is that the teachers considered that the students engaged with the challenge, learned the mathematics and made productive contributions to discussions.

The teachers were also given the opportunity to provide written reactions to various open response prompts, some representative responses of which are presented below. In the post lesson proforma, some teachers commented on the engagement of the students during the *Making Both Sides Equal* lesson, especially with regard to the sharing of the learning:

> It was great to see every child have a go and once we came together and shared ideas the number of kids that were successful increased.
>
> I really enjoyed this lesson. I found it interesting and children were engaged.
>
> Children enjoyed the challenge and discussion was good.
>
> That they enjoyed having a go to equal both sides. … kids learnt from one another and were eager to go and fix their mistakes.
>
> Children enjoyed the lesson and were totally engaged. Although they found the concept bewildering at the start, they were still interested enough to persevere and complete the task, cross checking and evaluating as they went.

Many teachers also commented on the experience of the students with the concept of equivalence, such as:

> Great. It highlighted students' misconceptions of what = means
>
> I found this lesson valuable to show that the equal sign means the same as.

There were also teachers who reported on aspects of the challenge. For example:

> It was more difficult for students than I predicted. Again they generally used patterning well but did not always check it was accurate. The subtraction was more difficult.
>
> The difficulty they experienced with the concept—they tend to write e.g. 6+4=10+4 then want to do another problem. It was surprisingly hard to explain how each side needed to balance. After a while most got the idea and were then able to use pattern.

Of course, it does not matter that students find a task difficult—and indeed that is the intention—but it is critical that teachers are aware of student difficulties and take action to resolve them. This is addressed further below.

To explore ways that teachers implemented the various lesson elements, the post-lesson proforma sought an indication of the number of minutes teachers spent on each. Table 3 presents the mean in minutes for each element in each of the five comparison lessons.

Table 3
The Mean of the Duration in Minutes of the Lesson Elements (n = 28)

	Making Both Sides Equal	*Addition Shortcuts*	*Ways to Add in Head*	*Missing Number Subtraction*	*Two Purchases*
The introduction to the learning task	6.0	6.4	6.5	6.6	6.3
Students working on the learning task	16.0	15.2	12.1	15.8	17.0
Whole class review of the learning task	10.6	10.4	10.7	9.7	12.3
Introduction to the consolidating task	5.9	5.0	5.4	5.7	7.2
Students working on the consolidating task	16.9	14.7	17.4	15.9	16.7
Whole class review of the consolidating task	8.9	9.8	9.6	8.5	10.1

The most striking aspect of this is the similarity across lessons. Noting that this table presents summary data from 140 lessons, it seems that the lessons took around one hour (derived by adding up the mean times of the lesson elements), the teachers spent about 6 minutes introducing each of the tasks, the students spent around 15 minutes working on each of the tasks, and the teacher spent 10 further minutes on the whole class reviews of each task. Given the brief introductions, the extended time for students to work on a single task, even if differentiated, and time allocated to the review of their work, the inference is that teachers implemented the various lesson elements in the way that was recommended.

Another key aspect of the implementation of the lesson structure was the extent to which teachers reported using the prompts. As part of the post lesson proforma, the teachers noted the number of students who were given an enabling prompt, and the time they waited before giving the prompts. Table 4 presents of the mean and median of the number of students over the 28 lessons, the fewest and greatest number of students in any lesson given the prompts, and the average time that the teachers waited before giving out any prompts.

Table 4
The use of the enabling prompts over the 28 implementation of each of the lessons

Lesson title	Mean number of prompts given per lesson	Median number of prompts per lesson	Low number of prompts given in a single lesson	High number of prompts in a given lesson	Time until prompts given
Making Both Sides Equal	6.3	4	0	23	6.3
Addition Shortcuts	6.7	4	1	25	6.8
Finding Ways To Add In Your Head	6.2	4	0	20	6.6
Missing Number Subtraction	5.7	5	0	18	6.6
Two Purchases	10.9	10	1	23	7.0

To elaborate these data for the *Making Both Sides Equal* lesson, the teachers reported the enabling prompts were given to between two and six students, with the mode number being 4. There was one teacher who gave no prompts and three who gave prompts to more than 20 students. Nearly all teachers waited between five and 10 minutes before doing so. The intention is that all students first have opportunity to engage with the learning tasks and are only offered prompts after this opportunity. The data suggest that the prompts were implemented by teachers in ways compatible with the proposed lesson structure.

Table 5 presents similar data for the extending prompts.

Table 5
The use of the extending prompts over the 28 implementation of each of the lessons

Lesson title	Mean number of prompts given per lesson	Median number of prompts per lesson	Low number of prompts given in a single lesson	High number of prompts in a given lesson
Making Both Sides Equal	6.9	5	0	20
Addition Shortcuts	7.3	6	0	22
Finding Ways To Add In Your Head	7.9	6.5	0	22
Missing Number Subtraction	7.4	6.5	0	20
Two Purchases	3.3	1	0	20

In the *Making Both Sides Equal* lesson, only four teachers did not give out the extending prompt indicating that in most classes there were students for whom the learning task was not challenging. Most of the teachers gave the extending prompt to between one and ten students. There were three teachers who gave the extending prompt to more than 20 students. Noting the variability across the classes, overall this also suggests that such prompts were used judiciously.

Across all of the lessons, it seems that teachers made active and deliberate use of the prompts depending on the responses of the class. No teacher reported a negative response to the prompts which seem to be a useful device to differentiate learning opportunities while maintaining not only the challenge of the task but also a sense of the class as a learning community. This data in the table suggest that this aspect of the recommended lesson structure was implemented by the teachers.

Overall, this is compelling evidence, based on the teachers' reactions, that the ways they implemented the lesson structure aligned with the advice they were offered both as part of the professional learning and in the lesson documentation. This lesson structure is feasible and manageable and may have potential for transfer to other types of lessons as well.

Pre- and Post-Assessment of Student Learning

To gain a different indication of the implementation of the lesson, and to seek insights into whether participation in this and the other lessons improved the chances that students would answer associated assessment items correctly, students completed an online assessment before and after the set of 10 lessons. Three of the items sought responses to questions presented in a similar format to the *Making Both Sides Equal* tasks. Table 6 presents the overall results from the items. The prompt for each of the items was "What should be the "?" ?". The items were open response, which gives more confidence in the responses than had the items been multiple choice. Only responses from students in classes who completed both assessments are included. Even though the number range and placement of the unknown varies, it is arguable that the items are assessing similar mathematical knowledge to the *Making Both Sides Equal* lesson.

Table 6
Number (%) of Student Correct Responses to the Three Equivalence Items Pre and Post Implementation

	Pre-test n = 1050	Post-test n = 1080
100 + 56 = ? + 53	215 (20.3%)	497 (45.8%)
19 + 22 = 20 + ?	254 (25.0%)	487 (45.3%)
95 - ? = 75 - 10	180 (17.7%)	399 (37.2%)

The improvement is similar in all three items, with around 20 to 25% more of the group answering correctly after the intervention in comparison to before. In other words, about one quarter of the group improved overall. To put this another way, one third of those who could not respond correctly before the lessons could do so after the lessons.

Considering the responses to the three items together, in the post-test, 40% of the students got none of the three items correct, meaning 60% of the students got one or more correct, representing improvement compared to the individual items from the 32% of students who answered one or more correctly on the pre-test. That is, nearly half of the students who could not answer an item previously could now respond correctly at least once. Twenty five percent of the students answered all these items correctly on the post-test, an increase from 8% from the pre-test.

In short, a significant minority of students were better able to respond to the items after the lessons than they were before. Tests of proportions on the items are highly significant statistically ($p = .000$), but the issue is whether this constitutes a meaningful educational improvement. Indeed, it might have been expected that the improvement would be greater, given that the students had completed an apparently successful lesson specifically on the particular concept of equivalence and other lessons on related topics.

One possible interpretation is that these gains are impressive but that this type of learning takes longer than one lesson for many students and learning gains overall take time. This is exemplified by the modest gains on comparable items on the national numeracy assessment between Year 3 and Year 5, for example. Sullivan and Davidson (2014) noticed a comparable apparently limited gain on particular assessment items in a previous iteration of the project. They followed up with a delayed assessment using a pencil and paper format and also examined students' worksheets. From these, the new knowledge demonstrated on the assessments of the students was substantially greater than was revealed by the on line pre/post comparisons.

An interesting aspect of the results was that, in comparing results of school cohorts, it was noticed that there were quite wide variations in the extent of student improvement between the pre- and post-test. To explore this further, some teachers of a school who were particularly successful in terms of improvement in students' responses to the items, were asked whether they could explain the special results of their classes. These teachers' responses indicated that they:

- Allowed students the time to consolidate their learning;
- Specifically addressed the issue of student persistence;
- Worked through the tasks prior to the lessons to enhance their chances of anticipating student responses; and

- Used the same structure, incorporating each of the lesson elements, for each of the lessons.

The teachers also commented on ways they used the students' responses. For example, the summary phase after the learning task was described as follows:

> Discussion always ended with the learning task on the smart board and we allowed questions for clarification. Students worked independently. When it came to sharing, we made sure we had a range of strategies from least to most efficient which were all presented in different ways. All strategies were celebrated. Our main goal was to promote the most efficient strategies, but to try and show them in a range of different ways. Students learnt to articulate themselves clearly by listening to each other explain their thinking. It also validated their thinking, by listening to others.

Perhaps more critically, though, is the assessment information that they gathered:

> The learning tasks acted as Rich Assessment Tasks where we encouraged students to try their best because their strategies and attempts would help us to plan the follow up lessons. We used the results to plan the next sequence of lessons.

It is those words "we used the results to plan the next sequence of lessons" that may well be critical in consolidating the learning. While not directly connected to lesson structure, it does indicate that the cognition activated by the challenging task may need to be followed up by subsequent further challenges and explanations. Noting that none of the teachers in schools with less than average improvement were interviewed and so it is not clear how their responses might differ, it seems that the purposeful actions by the teachers of these classes with high gains have contributed to those gains.

Summary

The research reported above intended to explore the implementation of a particular lesson structure to exemplify the common advice on mathematics teaching, to offer teachers strategies for differentiating learning opportunities and to propose experiences to consolidate learning for the students.

The lesson that was the focus of this article was based on a challenging task intended to activate the thinking of students around the concept of equivalence and patterns. The lesson information offered to teachers included prompts for students who experienced difficulty and those who finished quickly and a consolidating task for all students. The teachers reported that the lesson was successful in terms of student learning and contribution to whole class discussion. The teachers reports of the time they spent on each of the lesson elements and the overall data suggests that they implemented the various elements as recommended. The reported frequency of use of the enabling and extending prompts indicated that this strategy was manageable and that teachers made active decisions on which and when students should be offered the prompts. There was satisfactory overall improvement in the students' responses presented. The comparative data from other lessons indicate that the responses were not idiosyncratic to the focus lesson but were comparable across the other lessons presented. It seems that the lesson structure is useful to teachers and may act as a guide in further teacher professional learning.

As indicated in the framework used to guide the research, it seems that teachers do make implementation decisions based on their knowledge about the mathematics (which was perhaps gained from the teacher professional learning day and other pre-lesson planning), about the pedagogy (which is mainly built into the lesson structure and associated advice), and about the students (which is partially gained from observing students closely while they interact with the tasks). Teachers do need to anticipate the constraints they might experience, such as negative student reactions and plan to address them. Teachers beliefs that students can solve problems for themselves are presumably reflected in the time allocated to the lesson elements and especially the time for students to work on the tasks.

The teachers of classes with impressive improvement between the pre- and post-test reported a series of actions that seemed productive. This suggests that having suitable tasks and lessons is necessary but not sufficient to ensure learning. Given the current interest in schools on improving students' responses to external assessments, further research on how to consolidate learning activated through challenging tasks would be useful.

References

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. *Educational Series—19.* Brussels: International Bureau of Education, Geneva.

Christiansen, B., & Walther, G. (1986). Task and activity. In B. Christiansen, A. G. Howson, & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 243–307). Holland: Reidel.

Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.) *Handbook of research on teaching* (pp. 255–296). New York: Macmillan.

Desforges, C., & Cockburn, A. (1987). *Understanding the mathematics teacher: A study of practice in first schools*. London: The Palmer Press.

Dooley, T. (2012). Constructing and consolidating mathematical entities in the context of whole class discussion. In J. Dindyal, L. P. Cheng, & S.F. Ng (Eds.). *Mathematics education: Expanding horizons* (Proceedings of the 35th conference of the Mathematics Education Group of Australasia, Singapore. pp. 234–241). Singapore: MERGA.

Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development.* Philadelphia, PA: Psychology Press.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Inoue, N. (2010). Zen and the art of *neriage*: Facilitating consensus building in mathematics inquiry lessons through lesson study. *Journal of Mathematics Teacher Education*, Retrieved online from http://www.springerlink.com/content/g33022h2k1384461/.polt

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction**.** *Journal for Research in Mathematics Education***,** *44*(4), 646–682.

Kelly, A. (2003). Research as design. *Educational Researcher, 32*(1), 3–4.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Kullberg, A., Runesson, U., & Mårtensson, P. (2013). The same task? Different learning possibilities. In C. Margolinas (Ed.), Task design in mathematics education. In *Proceedings of the International Commission on Mathematical Instruction (Study 22)* (pp. 609–616). Oxford, UK: ICMI.

Lappan, G., Fey, T., Fitzgerald, W. M., Friel, S., & Phillips, E. D. (2006). *Connected Mathematics 2: Implementing and teaching guide.* Boston, MA: Pearson, Prentice Hall.

Middleton, J. A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal construct approach. *Journal for Research in Mathematics Education, 26*(3), 254–279.

National Council of Teachers of Mathematics. (2014). *Principles to action: Ensuring mathematics success for all.* Reston, VA: NCTM.

Organisation for Economic Co-operation and Development (2014). *Do students have the drive to succeed? PISA in Focus.* Paris: OECD Publishing.

Ruthven, K., Laborde, C., Leach, J., & Tiberghien, A. (2009). Design tools in didactical research: Instrumenting the epistemological and cognitive aspects of the design of teaching sequences. *Educational Researcher*, *38*(5), 329–342.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematical discussions.* Reston, VA: National Council of Teacher of Mathematics.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*. DOI 10.1007/s10857-014-9279-2.

Sullivan, P., Clarke, D. M., & Clarke, B. (2013). *Teaching with tasks for effective mathematics learning.* New York: Springer.

Sullivan, P., & Davidson, A. (2014). The role of challenging mathematical tasks in creating opportunities for student reasoning. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice*. (Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia, Sydney, pp. 605–612). Sydney: MERGA.

Sullivan, P., Mousley, J., & Jorgensen, R. (2009). Tasks and pedagogies that facilitate mathematical problem solving. In B. Kaur, M. Kapur, & K. Berinderjeet, (Eds.), *Yearbook of the Association of Mathematics Educators* (pp. 17–42). London: AME and World Scientific Publishing.

Victorian Curriculum and Assessment Authority (2014). *2013 Examination report: Specialist Mathematics 1GA2.* Melbourne: VCAA.

Vygotsky, L. S. (1930–1934/1978). Mind in society: The development of higher psychological processes (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Cambridge, MA: Harvard University Press.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.

Practical Implications Award

The Beth Southwell
Practical Implications Award

The Beth Southwell Practical Implications Award was initiated and sponsored by the National Key Centre for Teaching and Research in School Science and Mathematics, Curtin University, Perth, Western Australia. Curtin sponsored the “Practical Implications Award”, as it was then known, for the first ten years. The Award is now sponsored by the Australian Association of Mathematics Teachers (AAMT). In 2008, MERGA was honoured to be able to rename the PIA as the Beth Southwell Practical Implications Award, in honour of MERGA’s and AAMT’s esteemed late member, Beth Southwell.

The award is designed to stimulate the writing of papers on research related to mathematics teaching or learning or mathematics curricula. Application for the award is open to all members of MERGA who are registered for the conference.

Applications for the PIA are judged against specific criteria by a panel consisting of two members of MERGA, two from AAMT, and chaired by the MERGA Vice President (Development).

Teacher Actions to Facilitate Early Algebraic Reasoning

Jodie Hunter
Massey University
<j.hunter1@massey.ac.nz>

In recent years there has been an increased emphasis on integrating the teaching of arithmetic and algebra in primary school classrooms. This requires teachers to develop links between arithmetic and algebra and use pedagogical actions that facilitate algebraic reasoning. Drawing on findings from a classroom-based study, this paper provides an exemplar of one teacher's journey in shifting her practice to integrate early algebra into her everyday mathematics lessons. The findings highlight the importance of addressing different areas including algebraic content, task development and enactment, and the classroom and mathematical practices to facilitate algebraic reasoning.

Introduction

Significant changes have been proposed for Western mathematics classrooms of the 21st century in order to meet the needs of a *knowledge society*. One key focus has been on the learning of algebra as an essential type of thinking for "participation in a democratic society" (Mason, 2008, p. 79). Algebra takes an important role in ensuring access to both potential educational and employment opportunities (Knuth, Stephens, McNeil, & Alibabi, 2006). Given this position, there has been a growing consensus in both research (e.g., Bastable & Schifter, 2008; Blanton & Kaput, 2005; Carpenter, Franke, & Levi, 2003) and policy documents (e.g., Department for Education and Employment, 1999; Ministry of Education, 2007) that algebra be introduced at a much younger age with a focus on the integration of arithmetic and algebra as a unified curricula strand.

To ensure links to early algebra are developed and maintained, teachers have a key role in developing and enacting tasks that integrate arithmetic and algebra and reforming classroom practice. However, many primary teachers have not had experience in how to develop links between arithmetic and algebra or in using pedagogical actions that facilitate algebraic reasoning (Blanton & Kaput, 2005). To meet the need for models of how early algebra can be integrated into the primary classrooms, this paper provides an exemplar of one teacher's journey in shifting her practice to integrate early algebra into her everyday mathematics lessons.

Many studies (e.g., Bastable & Schifter, 2008; Blanton & Kaput, 2005; Carpenter, Franke, & Levi, 2003) illustrate how teachers can develop aspects of algebraic reasoning in their classrooms. Key findings of these studies include the importance of content areas within the existing curriculum with which early algebra has connections, a focus on student thinking and reasoning, and the use of task design and implementation to promote algebraic reasoning. There are also many studies (e.g., Fosnot & Jacob, 2009; McCrone, 2005; Monaghan, 2005) that address productive classroom and mathematical practices in the mathematics classroom. However, there are few studies that specifically attend to algebraic content, task development and enactment, and the classroom and mathematical practices that facilitate primary students to engage in early algebraic reasoning. The present paper aims to address this gap in the literature by presenting a framework of teacher actions to facilitate early algebraic reasoning that addresses algebraic content, task development and enactment, and the classroom and mathematical practices.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 58–67. Sunshine Coast: MERGA.

The theoretical framing of this paper draws on a socio-cultural perspective. In this view, individuals participate in the everyday activities within a classroom community of practice (Lave & Wenger, 1991) and through this participation learn the ways of thinking and acting that are valued by the community. Social participation facilitates the development both of a sense of what it means to be a member of a specific community and a sense of self in relation to the community.

Method

This paper reports on episodes drawn from a larger study (Hunter, 2014) that involved a year-long continuing professional development (PD) classroom-based intervention focused on developing early algebraic reasoning. The participants included two separate groups of primary teachers (from England and the British Isles) from schools that used the Mathematics Enhancement Programme (MEP) curriculum. The focus in this paper is on one teacher who was an experienced teacher interested in strengthening her ability to develop early algebraic reasoning within her classroom. Her class consisted of 25 Year Three students from a semi-rural primary school in the British Isles. The students were from predominantly middle socio-economic home environments and represented a range of ethnic backgrounds.

The model for PD used during the intervention initially drew on research literature. As the intervention progressed, the re-design of the PD drew on a range of sources including researcher observations from the classrooms, study group meetings, teacher interviews and discussions. The focus for professional learning comprised four separate but related components; understanding of early algebraic concepts; task development, modification, and enactment; classroom practices; and mathematical practices. Key aspects of the PD included the use of research articles to extend teachers' understanding of early algebra, to provide models of classrooms that would support early algebraic reasoning, and to promote reflection on current practice. Also central was a focus on the selection, design, and enactment of tasks. This included the teachers completing algebraic tasks themselves, analysing tasks from the MEP material to identify opportunities for algebraic reasoning and investigating ways of modifying existing tasks. In addition, the teachers engaged in activities where they both predicted and analysed student responses to algebraic tasks. A final key element of the PD was facilitating reflection on practice, including developing tools and skills for noticing relevant aspects of their own practice. To support this, the teachers were provided with an adapted framework from Hunter (2009) and also engaged in a series of lesson study cycles.

Data gathering included classroom observations prior to the beginning of the professional development and throughout the school year, video records of professional development meetings, audio recorded interviews with the teachers and students, detailed field notes, and classroom artefacts. On-going data analysis supported the revision of the model for professional development. Retrospective data analysis used NVivo 10 qualitative software programme (2012). The initial codes were developed from a variety of sources including research literature, the initial viewing of the video records, and the observational and reflective field notes. Repeated viewing of the videos and re-reading of the transcripts and field notes confirmed or refuted the initial hypotheses and codes and other hypotheses and codes were developed as necessary.

Results and Discussion

The results and discussion will present the *Framework of Teacher Actions to Facilitate Algebraic Reasoning.* This framework integrates four separate, interlinked components that the study identifies as key to the development of early algebraic reasoning. An analysis will be undertaken of the shifts across the three phases of the study.

Teacher Awareness of and a Purposeful Focus on Algebraic Concepts

Prior to the PD, the teacher demonstrated some awareness of the links between arithmetic and algebra. Instantiations of types of early algebra such as the commutative property, equivalence, inverse relationships were evident during the observed lessons. However, there was no explicit identification or examination of the properties of numbers or operations during lessons. This meant that for students, the properties remained implicit and they were not provided with opportunities to develop deep generalised understanding as advocated by researchers (e.g., Bastable & Schifter, 2008; Carpenter et al., 2003).

Central to each phase was a purposeful focus on algebraic concepts. This is not intended as an exhaustive list but consists of algebraic concepts that are identified as relevant to primary classrooms. The following sections of the findings and discussion will show the teacher's growing propensity to focus on these concepts and integrate exploration of these into her everyday mathematics lessons.

Table 1
Teacher awareness of and a purposeful focus on algebraic concepts

Phase One to Three	Address the following concepts: understand the equal sign as representing equivalence; relational reasoning including whole numbers and rational numbers; commutative property; inverse relationships; odd and even numbers; identity elements; distributive property; associative property; properties of rational numbers; using and solving equations; function

Teacher Actions to Develop and Modify Tasks and Enact Them in Ways That Facilitate Algebraic Reasoning

Prior to the initial PD, the teacher used tasks from the MEP curriculum and carefully guided students through the steps required to complete the task with an emphasis on a fast pace. Her questioning focused attention on computational approaches and was characterised as leading or funnelling students towards correct responses or teacher chosen solution strategies.

Developing new methods of task implementation was an important pedagogical strategy to facilitate algebraic reasoning. In the first phase, an immediate change involved the implementation of tasks as problem-solving opportunities. This included emphasising student effort to approach and complete cognitively challenging tasks. Enabling prompts such as described by Sullivan, Mousley, and Zevenbergen (2006) were used to scaffold all students to access the tasks, without lowering the cognitive demand. Another key change in the second phase related to task implementation involved shifting attention away from recording answers to focusing on patterns and relationships. Teacher questioning oriented students to use a structural focus. For example, in one lesson the teacher introduced a task involving a series of number sentences (100 – 10 =, 90 – 9 =, 80 – 8 = …). She said: "Look at those questions and see if there is a pattern, don't work out the answers yet, just look at it." She then drew attention to the patterns in the answers by asking: "As there is a pattern in the questions, do you think there might be a pattern in the answers?" Many researchers (e.g., Carpenter et al., 2003; Fosnot & Jacob, 2009) argue that the development of structural perspectives is an important aspect of algebraic reasoning.

Changes to lesson planning were important in integrating algebraic reasoning into the everyday mathematics lessons. In the first phase, the teacher began by examining the MEP lesson plans and selecting parts of tasks that focused attention on an algebraic concept. At this point, this did not extend to engagement in a deeper investigation of algebraic concepts. For example, one task involved an array and two number sentences with missing parts ($3 \times \_ = 6$, $6 \div \_ = 2$). Initially teacher questioning focused attention on the general relationship between multiplication and division:

> Three times two equals six and six divided by three equals two. With your partner, what do you notice about those please? A student responded: They're just the other way around… because the three is in the middle and the six is at the beginning and at the end.

After this response, the teacher shifted to ask students to examine related equations where the position of the numerals had changed. This limited opportunities for students to further explore the relationship between multiplication and division as the focus moved to specific equations.

Through the second phase, there was growth in the teacher's understanding of different types of algebraic reasoning. This meant that she was able to more readily modify tasks to include early algebra. It also led to her noticing when students provided responses related to algebraic reasoning. Later during this phase the teacher began to recognise and use spontaneous opportunities for algebra as tasks were enacted. In this phase, the shift in teacher actions also extended to structuring tasks to address misconceptions. For example, in one lesson, students were asked to solve $36 – 6 = \_\_ + 20$. Some students responded by writing 30. The teacher used this as an opportunity to engage the class in prolonged discussion focused on the equal sign.

In the final phase, the teacher consistently planned classroom activities in a way that focused on opportunities for early algebra. She described herself thinking as she planned about how to: "Draw out the commutative law from this one, or this could be a great discussion point for timesing by one, or dividing by zero, get them to come out with conjectures." Another point of difference in this phase was the teacher's propensity to engage in anticipating the outcomes of the task enactment. This supported her to develop her use of monitoring, noticing and sequencing student responses that could be used to spontaneously investigate algebraic concepts. For example during one lesson, the teacher asked her students to think about an efficient method to solve $26 – 8 =$. A student suggested breaking the eight into six and two. The teacher then used this as a spontaneous opportunity to investigate how numbers could be partitioned to solve subtraction tasks: "If you were doing 34 take away seven, with your partner can you just talk about how Misty would tackle that?" Blanton and Kaput (2005) note that spontaneously integrating algebraic reasoning opportunities into lessons is key to developing a classroom context that emphasises algebraic reasoning.

These changes resulted in a clear focus on algebraic reasoning integrated into lessons and included coverage of a broad range of algebraic concepts. In summary, the teacher actions are illustrated in Table Two.

Table 2
Teacher Actions to Develop and Modify Tasks and Enact them in Ways that Facilitate Algebraic Reasoning

Phase One	Implement tasks as problem-solving opportunities
	Emphasise student effort to approach and complete cognitively challenging tasks
	Extend or enact tasks to include opportunities for generalisation
	Interrogate tasks for opportunities to highlight structure and relationships
Phase Two	Adapt tasks to highlight structure and relationships. This includes changing numbers or extending to multiple solutions
	Structure tasks to address potential misconceptions
	Use enabling prompts to facilitate all students to access tasks
	Implement tasks by focusing attention on patterns and structure
	Use spontaneous opportunities for algebraic reasoning during task enactment
Phase Three	Recognise and use links to algebra in tasks across mathematical areas
	Implement tasks as open-ended problems
	Anticipate student responses that could provide opportunities for algebra
	Use spontaneous opportunities for algebraic reasoning from student responses

Teacher Actions to Develop Classroom Practices That Provide Opportunities for Engagement in Algebraic Reasoning

Prior to the initial PD, paired work was a feature of the classroom but rather than complete tasks collaboratively, the partnerships were used as a support mechanism when students were stuck. The discourse patterns in the classroom were dominated by the teacher. Students frequently gave answers with no mathematical reasoning and the teacher provided the majority of mathematical explanations.

In the first phase, to support student engagement in algebraic reasoning it was necessary to address the ways in which students worked collaboratively and the forms of talk used in the classroom. The teacher explicitly discussed with her students how to successfully talk together and facilitated them to generate rules for productive talk similar to what is described by Monaghan (2005). A key expectation was that students developed a shared understanding of a jointly constructed solution strategy. The teacher drew on student models to develop understanding of the new expectations and to affirm productive shared discourse norms. For example after observing small group work she said to the class: "Zanthe said to everybody 'do you get it?' And everyone nodded, but you didn't get it, did you? How did you know that Calvin hadn't got it?" This was followed by asking Zanthe to share with the class how she had known her group member, Calvin, was unsure by asking him to explain the jointly constructed solution strategy.

In the second phase, to advance all students' opportunities to engage in algebraic reasoning it was important to extend collaboration to whole class discussions. The teacher positioned students to listen actively to their peers' reasoning and explanations and make sense of these. During whole class discussions she intervened to provide space for other students to question or modelled how to ask a question herself. For example, in one lesson she asked the students to generate different two factor equations using the digits two, three and five. A student provided her group's solution strategy: "We think we should work out two times two first, then two times three and two times five." At this point the teacher provided a space for questions that led to a student question focused on clarification and justification: "If you were to do that, how would you be able to know whether you'd done the two and five, or two and three, or two and two, how would you know?"

In the final phase, a consistent expectation was established that students would work as a collaborative community. When students explained their strategy solutions during whole class discussions, the teacher emphasised that their partners or group needed to listen carefully and support them when necessary. She made the speaker aware of peer support and facilitated the rest of the class to listen to the explanation and make sense of it while supporting everyone in the class to understand it. This was similar to the pedagogical actions described by McCrone (2005). Although an emphasis was placed on developing a collaborative community, teacher continued to use pedagogical actions to ensure that students did not view this as always needing to agree with their peers. She emphasised mathematical argumentation when working with partners: "I was really impressed with the discussion that was going on when you didn't agree with your partner." This focus led to students attending both to their own thinking and the thinking of others and using mathematical reasoning to agree or disagree.

In summary, the teacher actions are illustrated in Table Three.

Table 3
Teacher Actions to Develop Classroom Practices that Provide Opportunities for Engagement in Algebraic Reasoning

Phase One	Lead explicit discussion about classroom and discourse practices Ask students to apply their own reasoning to the reasoning of someone else Require students working in pairs or small groups to develop a collaborative solution strategy that all can explain
Phase Two	Require that students indicate agreement/disagreement with part of an explanation or a whole explanation and provide mathematical reasons for this Lead explicit discussions about ways of reasoning Provide space for students to ask questions for clarification Request students to add on to a previous contribution Ask students to repeat previous contributions Use student reasoning as the basis of the lesson Facilitate students to examine solution strategies for similarities or differences
Phase Three	Lead explicit discussion about mathematical practices Sequence solution strategies to advance mathematical thinking and reasoning Provide space for students to question for justification

Teacher Actions to Develop Mathematical Practices That Support the Development of Algebraic Reasoning

Prior to the PD, key mathematical practices such as making conjectures, developing generalisations, justification and proof were not established within the classroom. The introduction of key mathematical practices associated with algebraic reasoning was important aspects to support student engagement with algebraic reasoning. In the first phase this included the new expectation that students would explain and clarify their ideas and reasoning. In the second phase of the study, a key shift for the teacher was her emphasis on facilitating student development of mathematical explanations rather than continuing to provide the majority of explanations herself. To achieve this, the teacher trialled the use of prompts such as: "I want you to think because I'm sitting here and I'm dead confused, how you could explain it to us. So I'm not just interested in your answer, I'm interested in you explaining it."

The introduction of the mathematical practice of using representations was an important aspect in the second phase of the study. This included facilitating students' use of representations as a key way for them to support their own reasoning and to access the structure of tasks and develop understanding. The teacher also promoted the use of different representations (e.g., verbal, concrete materials and written) as a way of developing the clarity of explanations and to link tasks and representational forms. In the final phase, the teacher continued to encourage use of multiple representations. But more than just using a selected representation, she now developed an expectation that the students would translate between different representations. This included asking students to draw on multiple representations in relation to a task and to listen to explanations by their peers and then to use an alternative representation for the explanation.

In the second and third phase of the study, the teacher introduced her students to the mathematical practices of generalisation, justification, and proof. She began by purposefully planning an investigation of identity elements similar to the approach advocated by Carpenter et al., (2003). This familiarised students with the processes of making conjectures and finding examples to illustrate these. The teacher initiated a growing expectation that generalisations would be expressed and treated as conjectures. In doing this, she facilitated a 'conjecturing atmosphere' such as described by Bastable & Schifter, (2008) and Mason (2008) where students readily expressed conjectures. This meant that the teacher was able to draw on the conjectures and then use these to engage students in the mathematical practices of generalisation, justification and proof. Also in the third phase, representations were introduced as a powerful form of concrete justification. With further classroom experiences focused on justification, students more readily drew on material to prove reasoning.

In summary, the teacher actions are illustrated in Table 4.

Table 4
Teacher Actions to Develop Mathematical Practices that Support the Development of Algebraic Reasoning

Phase One	Require students to explain their reasoning
Phase Two	Require students to develop mathematical explanations that refer to the task and context. Facilitate students to use representations to develop understanding of algebraic concepts. Ask students to develop connections between tasks and representations. Provide opportunities for students to formulate conjectures and generalisations in natural language. Lead students in examining and refining conjectures and generalisations. Listen for conjectures during discussions. Facilitates examination of these. Require use of different representations to develop the clarity of explanations. Model and support the use of questions that lead to generalisations; Does it always work? Can you see any patterns? Would that work with all numbers?
Phase Three	Listen for implicit use of number or operational properties. Uses these as a platform for students to make conjectures and generalise. Facilitate students to represent conjectures and generalisations in number sentences using symbols. Ask students to consider if the rule or solution strategy they have used will work for other numbers or for a general case. Promote use of concrete forms of justification. Require students to translate between different representations.

Conclusions and Implications

This study sought to illustrate the pathway that a teacher took in shifting her practice to integrate algebra into her everyday mathematics lessons. Similar to the findings of other researchers (Bastable & Schifter, 2008; Blanton & Kaput, 2005), it was evident that it is the teacher who makes the integration of algebraic reasoning into the learning community possible. The findings highlight the important role that the teacher takes in implementing and leading change within the classroom. In the first phase of the study, although the teacher began to consciously plan to integrate algebra into lessons, some of the existing classroom practices limited opportunities for engagement with algebra. Through the second and third phase, the teacher continued to extend her planning for algebraic reasoning and also began to notice and respond to spontaneous opportunities during lessons. Increasingly, the classroom practices and mathematical practices supported the students to engage with algebraic reasoning. These changes meant that the students became engaged in the key mathematical practices linked with algebra.

Overall, this study illustrates that the integration of early algebraic reasoning requires more than the introduction of algebraic concepts. It was necessary for the teacher to reflect on both the planning and implementation of tasks. Also of importance was attending to the development of the classroom community and facilitating the growth of classroom practices and mathematical practices that supported collective student participation and engagement with algebraic reasoning.

Practical Implications

A challenge for teachers in recent years has been to develop classroom contexts that integrate arithmetic and algebra and facilitate learners to shift from arithmetical to algebraic reasoning. The results of this study provide some important practical implications for thinking about ways in which early algebraic reasoning can be integrated into primary mathematics classrooms. A clear contribution is seen in the broad perspective of algebra that is taken to include both areas of content and classroom and mathematical practices that support student engagement in algebraic reasoning.

The *Framework of Teacher Actions to Facilitate Algebraic Reasoning* that is outlined in the paper is offered as a contribution to the field. Importantly this framework integrates four separate, interlinked components that the study identifies as key to the development of early algebraic reasoning. These include:

- Teacher awareness of and a purposeful focus on algebraic concepts
- Teacher actions to develop and modify tasks and enact them in ways that facilitate algebraic reasoning
- Teacher actions to develop classroom practices that provide opportunities for engagement in algebraic reasoning
- Teacher actions to develop mathematical practices that support the development of algebraic reasoning.

Each of the four key aspects integrated within the framework has been linked with specific supportive teacher actions. Based on evidence of 'what works' in terms of teacher practice, this is an important contribution to enhance professional learning and development opportunities to build capacity to enact reforms in early algebra teaching and learning. This framework can be used both by teachers to investigate and develop their own practice and as a productive model for researchers and designers of professional development to use while working with teachers.

This study illustrates the complexity and challenges of teacher change and enactment of changes within the classroom. The integration of algebraic reasoning into classroom mathematical activity was a gradual process. It required a focus on developing teacher understanding of algebraic concepts and involved changes to task implementation and design, shifts in pedagogical actions and the facilitation of new classroom and mathematical practices. It is important that teachers view algebra as encompassing classroom culture. This means that both pedagogical content knowledge of algebra and a focus on classroom and mathematical practices that facilitate algebraic reasoning opportunities needs to be incorporated into professional learning and development.

Of importance is the need for teachers to develop understanding of algebra beyond their schooling experiences. Initially the teacher in this study held understandings of algebra that were grounded in her own schooling experiences. This involved more traditional approaches where computational arithmetic was taught in primary school followed by the introduction of abstract algebra in secondary school. In her own words, she described her previous view of algebra as: *the missing number and shoving in an X here*. An important factor in the shift in the teacher's understanding and practice was the re-conceptualisation of her understanding of algebra.

Planning for algebraic opportunities was a key element in the teacher's development. However, an important implication for both teachers and teacher educators is that simply planning and developing algebraic tasks is insufficient to ensure that early algebra is integrated into mathematics lessons and learners shift from arithmetical to algebraic reasoning. Attention also needs to be focused on how tasks are implemented and enacted in the classroom. Enacting a task successfully requires teachers to identify the focus of the task, the purpose of any adaptation, and anticipate the possibilities that may happen in the task enactment. The framework provides some key teacher actions that relate to task implementation and enactment. It highlights the importance of implementing tasks in ways that focus on structural and relational aspects as well as drawing on spontaneous opportunities arising from both task enactment and student responses to engage all students in algebraic investigation.

Also evident from the findings of this study is that there are a number of key pedagogical strategies and classroom and mathematical practices that support student engagement in algebraic reasoning. Understanding of the classroom and mathematical practices that link to the development of algebraic reasoning are a further key aspect of teachers developing classrooms that integrate algebra into everyday mathematics lessons. The teacher in this study progressively introduced new classroom practices. There was an increased expectation on students to talk and work collaboratively. This collaborative work included developing shared understanding of a jointly constructed solution strategy. Another key emphasis was on student development of mathematical explanations. Also illuminated in this study is the importance of teacher understanding of mathematical practices such as generalising and justifying. An initial lack of understanding of these mathematical practices resulted in the teacher shifting student focus from general cases to specific examples. Developing understanding in this area enabled the teacher to draw on student generated conjectures and use these to engage students in justifying and generalising.

In summary, the important implication of this study for both teachers and teacher educators is that if we want to develop classroom contexts in which early algebra is a focus and students engage in algebraic reasoning, we must take a multi-faceted approach that addresses not only algebraic concepts but also task design and implementation as well as classroom and mathematical practices.

References

Bastable, V., & Schifter, D. (2008). Classroom stories: Examples of elementary students engaged in early algebra. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 165 - 184). Mahwah, NJ: Lawrence Erlbaum.

Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36*, 412-446.

Carpenter, T., Franke, M., & Levi L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth: Heinemann.

Department for Education and Skills. (1999). *The national curriculum for England and Wales*. London: DfES.

Fosnot, C. T., & Jacob, B. (2009). Young mathematicians at work: The role of contexts and models in the emergence of proof. In D. A. Stylianou, M. L. Blanton, & E. J. Knuth (Eds.), *Teaching and learning proof across the grades: A K-16 perspective* (pp. 102-119). New York: Routledge.

Hunter, J. (2014). *Developing early algebraic reasoning in a mathematical community of inquiry*. Unpublished doctoral thesis, Plymouth University, Plymouth.

Hunter, R. (2009). *Teachers developing communities of mathematical inquiry*. Auckland: Massey University.

Knuth, E., Stephens, A., McNeil, N., & Alibali, M. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*(4), 297-312.

Lave, J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation, New York: Cambridge University Press.

Mason, J. (2008). Making use of children’s powers. In J. Kaput, D. Carraher, & M. Blanton (Eds.), Algebra in the early grades (pp. 57-94). Mahwah, NJ: Lawrence Erlbaum.

McCrone, S. (2005). The development of mathematical discussions: An investigation of a fifth-grade classroom. Mathematical Thinking and Learning, 7(2), 111-133.

Ministry of Education. (2007). The New Zealand Curriculum. Wellington: Learning Media.

Monaghan, F. (2005). Don’t think it in your head, think aloud: ICT and exploratory talk in the primary mathematics classroom. Research in Mathematics Education, 7, 83-100.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Teacher actions to maximize mathematics learning in heterogeneous classrooms. International Journal of Science and Mathematics Education, 4, 117-143.

Research Papers

The challenge of supporting a beginning teacher to plan in primary mathematics

Judy Bailey
The University of Waikato
<jlbailey@waikato.ac.nz>

Effective lesson planning is a real challenge for many beginning teachers. This paper presents a case study of one such teacher, and the author's efforts to support her in the planning process. Results show supporting the beginning teacher's planning by (a) providing access to resources such as web-sites and teaching handbooks, (b) modelling, and (c) providing an explication of planning were insufficient to create substantive and necessary changes in the teacher's planning during the period of research. Implications for supporting beginning teachers are considered.

The New Zealand primary classroom is a multi-faceted, complex context in which beginning teachers are required to learn how to teach mathematics, a subject the New Zealand government places particular emphasis on (see Ministry of Education, 2004, 2007 & 2009). Not only is the teaching and learning of mathematics just one of eight learning areas, curriculum expectations also require beginning teachers to learn how to (a) embed values such as excellence and inquiry; (b) develop key competencies such as thinking and relating to others; and (c) embody principles such as cultural diversity and inclusion within their day-to-day teaching practice (Ministry of Education, 2007). The everyday demands of classroom organisation and management are yet another important focus for the beginning teacher. It is thus understandable that identifying ways to support the learning of beginning teachers is seen as vital (Desimone, Hochberg, Porter, Polikoff, Schwartz & Johnson, 2014).

Planning and preparation are considered to have a central role in teacher practice (Neill, Fisher & Dingle, 2010; Roche, Clarke, Clarke & Sullivan, 2014). Planning is concerned with knowing what and how to teach (such as sequencing content), while preparation involves organisational elements including the getting and/or designing of materials (Fernandez & Cannon, 2005, as cited in Roche, Clarke, Clarke & Sullivan, 2014). Roche, Clarke, Clarke and Sullivan (2014) suggest that, "… it is difficult to imagine that teachers of mathematics can perform their role without substantial planning" (p. 854). These researchers have proposed a theoretical framework for teacher planning. The framework begins with four 'elements'. The first two of these interconnected elements include teachers checking (a) web and text resources, and (b) school and curriculum documents. The second set of two interconnected elements relates to teachers drawing on (c) their own and colleagues' experience, and (d) assessment data. All of the information thus generated is used to establish specific learning goals, which in turn inform the selecting and sequencing of tasks and finally planning the teaching and assessment, including differentiating tasks for particular students (Roche, Clarke, Clarke & Sullivan, 2014).

There is great variation in the way teachers plan (John, 2006; Roche, Clarke, Clarke & Sullivan, 2014) reflecting teachers' (and teacher educators') varied perspectives about learning, teaching, curriculum and education. For example, John (2006) outlines how rationalistic, technical curriculum planning has been the dominant model underpinning lesson-planning in teacher education for many years. Within this model planning begins

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 69–76. Sunshine Coast: MERGA.

with the setting of objectives, and then follows a set order finishing with lesson evaluation. It exemplifies a focus on outcomes-based education (John, 2006). Given that such planning does not take into account the context or contingencies of teaching, John (2006) offers an alternative dialogical model where constructing the plan (as a product) is seen as secondary to the planning process (although the end product of a plan is not ignored). Roche, Clarke, Clarke and Sullivan (2014) also refer to the process of creating a plan as key, rather than the plan as a product. John furthers justifies his alternative planning model on the basis of learners being agents in their own learning. In his words, "…the negotiated nature of learning needs to be added to the planning equation if spontaneity and improvisation are to be allowed" (John, 2006, p. 487). The main core of the alternative model is fixed by the aims, objectives and goals of the plan, and around this are a large number of 'nodes' such as subject content, national curriculum, classroom control, and tasks and activities. Each of these in turn is subdivided; for example, factors relating to subject content include a consideration of conceptual understanding, representations, depth and breadth and schemes of work. Unlike the rationalistic model, John's model does not privilege a fixed order for the process of planning, and recognises that the planning process will change as teachers become more experienced.

Lesson planning is regarded as difficult for teachers to learn, with a problematic range of outcomes (John, 2006; Mutton, Hagger & Burn, 2011; Steketee & McNaught, 2007). John (2006), for instance, found that once novice teachers are planning on their own, their responses range from creativity to bewilderment and anxiety. More experienced teachers' planning is likely to involve a concurrent consideration of a wide number of elements, rather than a linear progression of decision-making (John, 2006). However, a teacher's level of experience is only one factor influencing a teacher's planning. Others include depth of subject knowledge and pedagogical knowledge, teaching style, and perceptions and knowledge of pupils (Roche, Clarke, Clarke & Sullivan, 2014). Novice teachers are likely to engage in short-term planning, and generally describe planning as time-consuming and complex (John, 2006; Mutton, Hagger & Burn, 2011). Once exposed to teaching, novices begin to realise that planning and preparation are concepts associated with unpredictability, flexibility and creativity (John, 2006).

Research literature on effective mathematics teachers is mainly centred on teaching practices and tends not to emphasise planning (Roche, Clarke, Clarke & Sullivan, 2014). A recent publication by the New Zealand Education Review Office (2013) on developing a responsive curriculum for priority learners in mathematics also focuses on learning tasks and teaching strategies, referring to the planning aspect of teaching only briefly. Although it is argued that the described practices of effective teaching are likely to be underpinned by sound planning (Roche, Clarke, Clarke & Sullivan, 2014, p. 854) learning how to plan is critical to the development of teaching expertise (Mutton, Hagger & Burn, 2011).

Teaching is a profession that involves continual learning by teachers and children alike (Gorodetsky & Barak, 2009), and it is recognised that pre-service teacher education provides just a beginning in learning to teach (Feiman-Nemser, 2012; Mutton, Hagger & Burn, 2011). There has been little research that explores how beginning teachers are best supported in the development of their planning expertise (Mutton, Hagger & Burn, 2011). Considering the complex demands made of beginning teachers and the importance of supporting their ongoing learning, the small study reported in this paper was designed to explore the research question: what form of support enables a beginning teacher to plan effectively in primary mathematics?

Methodology

This small study occurred within a wider two-year project focused on raising school wide achievement in mathematics in a relatively large urban primary school (catering for children aged 5 – 11 years) within a middle socio-economic city suburb. This paper reports on data relating to the author, a university mathematics educator and researcher, working alongside a beginning teacher. The beginning teacher, Rebecca (a pseudonym), had completed a three-year Bachelor of Teaching degree that included a range of professional practice and curriculum papers. Within the three professional practice papers (one in each year of the degree) planning is discussed with a focus on theoretical aspects, for example, why planning is important. A range of models and formats are encountered during three practicum placements (one in each year of the degree) drawing on associate teacher's expertise with planning. In curriculum papers key aspects such as learning intentions, progression of lessons, and activities are discussed. Within the one and a half mathematics education papers, two (of five) assignments included a planning requirement, one on lesson planning and the other on unit planning. Additional mathematics education assignments explored and assessed content knowledge, pedagogical content knowledge, and the use of worthwhile teaching activities for supporting learning in mathematics.

The class Rebecca was teaching at the time of this study was a co-educational composite class of year three-four (seven and eight year-old) children. Rebecca was in her second year of teaching but it was her first year working with children this age. A group of nine children in the class were achieving below expected levels and regarded by the teacher as a concern.

Over a period of two terms (terms three and four of the second year of the school-wide mathematics development project) Rebecca and the author met to discuss how Rebecca could provide effective support within her daily mathematics programme for the nine lower-achieving children. Informal discussions between Rebecca and the author took place in the classroom, usually after school, on nine occasions. One of these discussions was audio-taped; and field notes were recorded for all meetings. Rebecca also invited the author to observe her teaching, and during one lesson she asked the researcher to teach the class so she could observe a more experienced teacher in action. This led to a short series of lessons (over a two-week period) where both Rebecca and the author took turns in teaching, with each observing the other. The planning for these lessons was initially led by the author but later, ideas for planning and teaching were shared and discussed. Communication was also maintained via e-mails. Some of these were organisational, others extended face-to-face discussions and provided a forum for the sharing of ideas, and the asking and answering of questions.

For four weeks at the beginning of the following year (an informal continuation of the two year project) the author and Rebecca kept in touch via e-mail sharing ideas about how the learning of another group of year three-four children not achieving at expected levels could be supported in a small group environment, but this time outside of the normal mathematics programme. The research was curtailed when ill-health led to Rebecca leaving teaching for the remainder of the year.

Data include e-mail communications and field notes of oral discussions; the author's planning for the lessons she taught (within a two-week number unit); planning shared by the teacher; and field notes of all taught and observed teaching sessions. An additional electronic journal recording the author's thinking was kept throughout the research period, and also maintained as data were analysed. This process aligns with the ideas of St. Pierre (2011) who states data are collected during thinking and writing and suggests, "if we don't read the theoretical and philosophical literature, we have nothing much to think with during analysis except normalised discourses that seldom explain the way things are" (St. Pierre, 2011, p. 614).

Data analysis has occurred in the reading, re-reading, listening to audio-tapes, some transcribing of the audio-tapes, chronological organising of data, and the author's ongoing thinking and writing, and reading of literature (St. Pierre, 2011). An emergent analytical approach (Borko, Liston and Whitcomb, 2007) was also employed. As data were read and re-read, and audio-tapes listened to, the author made notes about issues and themes that emerged from the data. One of these was 'planning'. As this issue emerged, all data were re-read to explicitly search for all references made to planning by the researcher and teacher, and analyse these against the useful framework for teacher planning proposed by Roche, Clarke, Clarke and Sullivan (2014). Thus evidence was sought of the beginning teacher: (a) checking the web and texts, and (b) school and curriculum documents as planning resources; (c) drawing on the teacher's own and colleagues' experiences; (d) drawing on assessment data; (e) establishing specific learning goals; (f) selecting and sequencing tasks; and (g) planning the teaching and assessment, including differentiating tasks.

Results and Discussion

Checking School or Web Documents, Teacher Resources and/or Student Texts

The resource Rebecca most relied on for her planning was a unit plan, consisting of a list of topics and associated activity sheets, provided by another teacher within her syndicate. She explained that as a beginning teacher she would be given activities for teaching and the colleague responsible for planning the unit would find these. In Rebecca's words, the teacher "who plans the unit finds all the resources with them". Rebecca also said that available text-books were not helpful because they were written to align with a nation-wide mathematics project that was not followed in her school. She mentioned that, "the text books which aren't very helpful… not very helpful… because these pretty much align with the … project, but of course we don't go near there, and I struggle to match them all up again". This comment suggests it was difficult for Rebecca to reconcile the activities in the textbooks with the learning needs of the children in her class. She did, however, refer to using the 'Figure It Out" series (a Ministry of Education publication of approximately 80-90 separate titles for supporting mathematics teaching and learning from levels 2-5 of the New Zealand Curriculum) and also explained that she usually "forgets" about the teacher resource website, nzmaths.co.nz for planning support. A teacher resource she did find helpful was a handbook that listed and briefly outlined ideas children at each level of the curriculum are expected to learn (see Biddulph, 2011). She said,

> It's all off the check-list … . By the end of year 4… because we know the year 4s are going to be there. And the year 3s will have got a good grounding and really have it drilled in next year.

Of the planning resources referred to in Roche, Clarke, Clarke and Sullivan's (2014) framework, Rebecca accessed only some of these, namely, school documents in the form of the syndicate unit plan, and some teacher resources. Web documents and student textbooks were not consulted on a regular basis or were regarded as unhelpful.

Examining Curriculum Content Descriptions to Identify the Important Idea(s)

Rebecca did not make any references to curriculum expectations within the recorded conversations, or in any of the written planning she shared during the research period. She seemed unaware that the handbook she found useful was a detailed clarification of curriculum requirements. Thus, there was no evident link in the teacher's planning (oral or written) to the framework element, "examining curriculum content descriptions to identify the important ideas" (Roche, Clarke, Clarke and Sullivan, 2014, p.862).

Drawing on Experience (Self and Colleagues)

Teachers drawing on their own and others' experience is another aspect of the planning framework proposed by Roche, Clarke, Clarke and Sullivan (2014). As a beginning teacher Rebecca clearly had limited experience on which to draw. She recognised this, and was also aware of the possibilities of drawing on collegial support. She explained that support, "would be helpful cos I've really never gone back that far. Last year I had senior kids…". Rebecca was open and keen to learn all she could to more effectively cater for all of the children's learning needs in her class. She frequently asked questions such as, "How long would you spend on …?", and her willingness to learn and receive guidance from colleagues was exemplified by her comment, "I'm just really wanting to know where to go from here". Rebecca was appreciative of working alongside more experienced colleagues. In one conversation, she stated, "I found it very beneficial watching you today so I would love it if you would like to teach tomorrow… . Would it be ok if you took the whole lesson then I can see the sequence that you go with?". She referred to a similar process with her more experienced syndicate colleagues as being a supportive part of her learning to teach.

Drawing on Assessment of Student Readiness

Rebecca had assessed and identified children who were not achieving at expected levels. One-to-one interviews conducted by the author during the period of research verified Rebecca's previously determined assessment of all nine children. Assessment tools used by Rebecca included the standardised 'Progressive Achievement Tests' conducted at the beginning of the school year; and her own ongoing overall teacher judgments of the children's learning. These were based on informal observations of the children's learning, and children's more formal written assessments.

Establishing Specific Learning Goals

Rebecca appeared to find it difficult to establish specific learning goals. In one conversation she said, "It will be fine once I get a clear idea of what… I think I need a check-list of basically what they need to know… basically teach to the test". Rebecca actually already had access to a check-list of what children need to learn at years 3-4, and made reference to this resource a little later in the same conversation. While the list outlined concepts and ideas to be taught, this on its own did not appear to be enough to support Rebecca in determining the finer details of planning and teaching. Six months later there was still a similar state of uncertainty about what to teach, and how to go about it. She wrote in an e-mail:

> I would appreciate any help possible really. I feel like im (sic) kind of doing this blind. I have assessed the children's thinking ….. My year 3's (5 of them) need the following help….. My year 4's (5 of them) need the following help…Now I have this information I am stuck on what order to do it in?

These challenges in establishing specific learning goals when planning lessons were also evident in the observed taught lessons, with ideas being introduced that were not closely connected to what appeared to be the main idea of the lesson. For example, in a lesson about the number of tens in two-digit numbers (eg. there are 9 tens in 93) Rebecca began listing different combinations of coins to make a particular amount ($4), and also noted the colours of different dollar bills. While she recognised and verbally acknowledged to the children that she had lost focus, there remained an overall lack of clarity or purpose within that particular lesson.

A similar lack of clarity about specific learning goals was evident in Rebecca's oral and written communications. Typical of the challenge in articulating the ideas being taught is this comment, "I think that last group has definitely grasped the concept of working with under $100 and I think the next step would be working towards the numbers in the hundreds". While there is evidence of Rebecca learning to sequence ideas the actual idea being taught is not clearly expressed, and often, she was not able to move beyond re-stating an idea from the list of ideas being taught to the children.

Selecting and Sequencing Tasks including adapting them for your Students

The next aspect of the framework proposed by Roche, Clarke, Clarke and Sullivan (2014) focuses on the selecting, sequencing and adapting of tasks. Rebecca found it challenging to do this beyond following the list of activities and worksheets that were provided with the syndicate plan. Some progress began to be made with sequencing ideas but this was not secure six months after the beginning of the research. For example, after assessing the second group of children achieving below expected levels (at the beginning of the second year of the research) she wrote, "now I have this information I am stuck on what order to do it in? I have started the year 3's on counting in 2's which will lead to doubles and odd and even numbers".

Some progress was also made with selecting tasks. Two months into the research period she wrote, "Tomorrow I plan to carry on with doubles to 20 and I have found some activities on nz.maths [a web-site] to support this" indicating some move towards being able to independently locate tasks for teaching and learning. However, this was not secure, as indicated by her writing at four months, "I am after as much advice as possible in regards to activities and equipment that I can use". Later on, at about six months, similar comments and requests were being made. For example, "I am still working on making 10 with the year four group so I can move on to addition and subtraction but they can not understand the concept! Do you have any ideas on efficient ways of teaching this?".

Planning the Teaching and Assessment including Differentiating for Particular Students:

Rebecca was aware early on in the research period that differentiating tasks would be one way of supporting this group of children's particular learning needs. She initially wondered about having to plan separate programmes saying,

> I'm going to have to go right back, aren't I with them? So, do I carry on with my normal programme with the majority but have this as completely separate? Not touch on the whole syndicate's plan, and not even touch those on them…..

After discussions and observing the author's planning and teaching of the whole class followed by the use of differentiated tasks, Rebecca was keen to trial this way of catering for the learning needs of the whole class. She later commented that it appeared to be a manageable way of catering for the diversity in learning needs.

During the research the author shared her written planning with Rebecca, and during the audio-recorded discussion, and later on in email conversations, explications and modelling of the planning process was provided. During these the author outlined key elements of what might be helpful to consider when planning including identifying the key idea(s) that children could learn, thinking about the sequencing of ideas, planning key questions that could be asked of the children to support their learning, carefully choosing appropriate numbers for equations, as well as considering what equipment could be used. Tasks and the differentiating of these to cater for diversity in learning needs were also discussed. On one occasion Rebecca shared her teaching 'notes' with the author. These notes included an explanation of an activity, modelled on some of the author's previous planning, and were annotated with the children's learning over a period of two days. All other written communications listed the ideas Rebecca wanted the children to learn, but beyond this she did not appear to formalise or extend the planning provided in the unit. Several respectful requests asking for Rebecca's planning, with the hope of it informing and guiding discussions, were made during the research period, but nothing further was offered. It must be acknowledged that much teacher planning is done mentally (Roche, Clarke, Clarke and Sullivan, 2014), and perhaps this was the case for Rebecca. Learning to make pre-existing plans and schemes for teaching 'one's own' is also an important aspect of learning to plan (Mutton, Hagger and Burn, 2011), and it appears this is an aspect that Rebecca could be supported to develop.

Conclusion and recommendations:

When analysing Rebecca's planning practice against the framework proposed by Roche, Clarke, Clarke and Sullivan (2014) it is evident that some aspects were present in her planning. She was able to draw on assessment data of student readiness; used the unit plan written by another teacher, and was aware of and consulted an appropriate teacher handbook outlining the lists of concepts/ideas to be taught. Remaining aspects of the framework proposed by Roche et. al. were absent. Neither the provision of numerous resources (by the school; and during discussions with the author) including handbooks, web-sites and various text books nor an explication and sharing of the planning process were enough to support Rebecca, within the six-month research period, to confidently and consistently address the questions she had about what to teach, and how to sequence the ideas the children needed to learn. Rebecca's focus on activities rather than identifying mathematical learning goals or objectives is consistent with findings by Roche et.al (2014) who determined that teachers did not rate 'establishing specific learning goals' as a high priority.

Given the importance of planning on what happens in the classroom (Roche, Clarke, Clarke & Sullivan, 2014), and the contention that "it is through planning that teachers are able to learn about teaching" (Mutton, Hagger & Burn, 2011, p. 399) it is possible that engaging in more planning and/or exploring alternative models of planning such as that proposed by John (2006), may have enhanced Rebecca's learning about meeting the needs of all children in her class. Bearing in mind the clear limitations of drawing conclusions from a small and truncated case study (due to the teacher's ill health), it appears the provision of resources such as text-books and web-sites on their own were not sufficient to support a beginning teacher's planning. It seems that at least some beginning teachers need more specialised and longer-term support to establish the wider understanding and expertise needed to plan, including establishing specific learning goals. This is consistent with the findings of Desimone, Hochberg, Porter, Polikoff, Schwartz and Johnson (2014) who point towards the need for the support of beginning teachers to focus on deeper understandings of the teaching process rather than simply being provided with resources. The framework proposed by Roche et. al. (2014) could be a useful starting point to guide planning, with a particular focus on encouraging beginning teachers to check school, curriculum and web documents and other relevant teacher resources in order to establish specific learning goals, select and sequence tasks and plan for teaching and assessment including suitable differentiation. This is a complex task, particularly for the beginning teacher, unfamiliar with each and every mathematics unit they teach.

References:

Biddulph, F. (2011). *New Zealand Mathematics Education Handbook: Primary and Intermediate School Years.* Hamilton, New Zealand: Biddulph Group.

Borko, H., Liston, D., & Whitcomb, J. (2007). Genres of empirical research in teacher education. *Journal of Teacher Education, 58*(1), 3-11.

Desimone, L., Hochberg, E., Porter, A., Polikoff, M., Schwartz, R., & Johnson, L. (2014). Formal and informal mentoring: Complementary, compensatory, or consistent? *Journal of Teacher Education 65*(2), 88-110.

Education Review Office (2013). *Mathematics in years 4 to 8: Developing a responsive curriculum*. Wellington: Education Evaluation Reports.

Feiman-Nemser, S. (2012). *Teachers as learners*. Cambridge, Massachusetts: Harvard Education.

Gorodetsky, M. & Barak, J. (2009). Back to schooling: Challenging implicit routines and change. *Professional Development in Education, 35*(4), 585-600.

John, P.D. (2006). Lesson planning and the student teacher: re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498.

Ministry of Education (2004). *The National Education Goals*. Retrieved 6 January, 2014 from http://www.minedu.govt.nz/TheMinistry/EducationInNewZealand/EducationLegislation/TheNationalEducationGoalsNEGs.aspx

Ministry of Education. (2007). *The New Zealand Curriculum*. Wellington: Learning Media.

Ministry of Education (2008). *Numeracy Professional Development Project Booklets*. Wellington: Learning Media.

Ministry of Education. (2009). *The New Zealand Curriculum: Mathematics Standards for years 1 – 8*. Wellington: Learning Media.

Ministry of Education. (various). *Figure It Out* (series of booklets). Wellington: author.

Mutton, T., Hagger, H., & Burn, K. (2011). Learning to plan, planning to learn: The developing expertise of beginning teachers. *Teachers and Teaching: theory and practice, 17*(4), 399-416.

Neill, A., Fisher, J., & Dingle, R. (2010). *Exploring mathematics interventions: Exploratory evaluation of the 'Accelerating Learning in Mathematics' pilot study: Report to the Ministry of Education*. Wellington: NZCER.

Roche, A., Clarke, D. M., Clarke, D. J., & Sullivan, P. (2014). Primary teachers' written unit plans in mathematics and their perceptions of essential elements of these. *Mathematics Education Research Journal, 26*(4), 853-870.

Steketee, C., & McNaught, K. (2007). The complexities for new graduates planning mathematics based on student need. *Mathematics: Essential Research, Essential Practice. (*Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 671-677).

St. Pierre, E. A. (2011). Post qualitative research: the critique and the coming after. In N. K. Denzin & Y. S. Lincoln. (Eds.), *The Sage handbook of qualitative research* (pp. 611-625). Thousand Oaks: Sage.

Contemplating symbolic literacy of first year mathematics students

Caroline Bardini
The University of Melbourne
<cbardini@unimelb.edu.au>

Robyn Pierce
The University of Melbourne
<r.pierce@unimelb.edu.au>

Jill Vincent
The University of Melbourne
<jlvinc@unimelb.edu.au>

Analysis of mathematical notations must consider both syntactical aspects of symbols and the underpinning mathematical concept(s) conveyed. We argue that the construct of *syntax template* provides a theoretical framework to analyse undergraduate mathematics students' written solutions, where we have identified several types of symbol-related errors. A focus on syntax templates may address the under-developed symbol sense of many tertiary mathematics students, resulting in greater mathematics success, and with the potential to improve retention rates in mathematics.

Introduction

Mathematics derives much of its power from the use of symbols (Arcavi, 2005), but research at secondary level has shown that their conciseness and abstraction can be a barrier to learning (Pierce, Stacey, & Bardini, 2010; MacGregor & Stacey, 1997). Since symbols form the basis of mathematical language, mathematical fluency, like fluency in any language, requires proficiency with symbols, which we call *symbolic literacy*. Under the notion of symbolic literacy lies the notion of *symbol sense* described by Arcavi (1994, 2005), which includes among other components the ability to manipulate, *read through* symbolic expressions, realise that symbols can play different roles in different contexts (this will be emphasised throughout this paper), and develop an intuitive feel for those differences. We have privileged the term *literacy* in order to convey the idea of mathematics as a language of discourse (Usiskin, 2012) that can take place in oral or written form.

> Mathematics is, among its many other attributes, a language of discourse. It is both a written language and a spoken language, for – particularly in school mathematics–we have words for virtually all the symbols. Familiarity with this language is a precursor to all understanding. (Usiskin, 2012, p. 4)

The notion of *symbolic literacy* encompasses the understanding of what we believe to be one major feature of mathematical development (see also Usiskin, 1996; Rubenstein and Thompson, 2001) and is at the core of our current studies (e.g., Bardini & Pierce, 2015). However, for the purpose of this paper we will focus on its written aspects since this feature is the nature of our data.

Quinnell and Carter (2012) note that while inaccuracies in spelling and word usage in everyday English text usually do not prevent the reader from understanding the text, even small errors in the use of mathematical symbols may have a major impact on making meaning of the written mathematics. Take, at a very basic level, the common error of omission or misuse of parentheses. Students do not always recognise, for example, that $(-1)^2$ and -1^2 have different meanings or that $\left[2+6\times\sqrt{4}\right]^2$ and $\left[(2+6)\times\sqrt{4}\right]^2$ do not mean the same and do not have the same value.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 77–84. Sunshine Coast: MERGA.

At university, not only does mathematics become much more symbolic, but its writing is more subtle and requires increased *flexibility* from the reader; we anticipate that many students may have difficulty with the new and more intense ways in which symbols are used at university, referred to as *symbol load* in our previous work (Bardini & Pierce, 2015). In a study involving first year university physics students (Torigoe & Gladding, 2007), it was found that students' performance is highly correlated to their understanding of symbols. We anticipate that similar outcomes apply to other mathematical sciences at university, with the consequence that students may not understand the mathematical content as well as they did at school, potentially leading to a decrease in positive affect, which in turn might discourage enrolment in further mathematical subjects.

As a first step towards investigating these larger questions our aim is to provide tools that enable us to better examine students' understanding and use of mathematical symbols and therefore gain a better comprehension of students' symbolic literacy. In the following sections we will present the frameworks underlying the construct of such tools and show how these enable us to gain a fine-grained description of students' understanding of symbols, in particular through their writings.

Theoretical Framework

Skemp (1982) identified two levels of language, distinguishing between the surface structures (syntax) of mathematical symbol-systems and the deep structures that embody the meaning of a mathematical communication—the mathematical ideas themselves, and their relationships.

Serfati (2005) also provides us with an epistemological approach to mathematical notations that takes into account both the syntactical aspect of a symbol and the underpinning mathematical concept(s) conveyed. Note that we will use the term *symbol* throughout this paper, but in this particular instance the term *sign* could be thought to be more appropriate (the limitations of this paper do not allow us to fully discuss this).

Following Serfati's work we can analyse symbolic expressions by considering each of their components and distinguishing three features:

- the *materiality*. The materiality of a symbol focuses on its *physical* attributes (what it looks like), including the category the symbol belongs to (a letter, a numeral, a specific shape, etc.).
- the *syntax*. The syntax of a symbol relates to the rules it must obey in the symbolic writing. This includes the number of operands for symbols standing for operators but also the *legitimacy* of a symbol being juxtaposed to adjacent symbols.
- the *meaning*. The meaning of the symbol is the concept being conveyed (the representation of an unknown, of a given operation, etc.). Meaning for Serfati is that commonly agreed by the community of mathematicians and it does not refer to a person's individual understanding.

To work with a mathematical symbol, one not only has to recognise it in the text (i.e., through its materiality), but to select the right meaning and appropriate syntax in that context, which sometimes has to be interpreted very locally (e.g.,, the symbol '–' in front of a number, between matrices).

Since we are considering students' symbolic literacy from a writing perspective, the syntactical aspect of mathematical expressions plays a substantial role. Sherin (1996) provides an alternative yet closely related framework to Serfati's notion of *syntax* (originally called *combinatorial syntax* in Serfati 2005) for the syntactical aspect of mathematical expressions. In a study with third semester engineering students, Sherin asserted that particular arrangements of symbols in physics equations express particular meanings for students, allowing them to understand the equations in a relatively deep manner. He introduces the concept of *symbol patterns*, which can be understood as templates for the arrangement of symbols. As the students developed physics expertise, they acquired knowledge elements that Sherin (1996) refers to as *symbolic forms* consisting of two components: a symbol template, for example □ = □, and a conceptual schema. The schema is the idea to be expressed and the symbol template specifies how that idea is written in symbols, so that students learn to associate meaning with certain mathematical structures. Sherin's *symbolic forms* bear resemblance to Tall's (2001) *procepts*.

Methodology

The research described in this paper formed part of a preliminary study of the extent to which first year university mathematics students experience symbol overload due both to increased symbol intensity and their lack of familiarity with the symbols themselves. This preliminary study led to a current three-year project on this matter funded by the Australian Research Council.

The participants (21 in total) were a tutorial class of first semester undergraduate students enrolled in Calculus 1 in a major Australian university. Data was collected during normal weekly tutorials in which students completed worksheet exercises and problems based on their current lecture topics. It was the normal practice in these tutorials for students to work, standing in pairs or groups, writing their solutions on whiteboards. The tutor moved around the tutorial room, checking students' progress, pointing out errors in the students' solutions and suggesting appropriate methods when students were unsure how to proceed. As observers, the authors of this paper were able to photograph students' solutions but were not able to converse with them as this could disturb the progress of the students' work. These photographs constituted the data. The students' written solutions captured in these photographs were analysed in order to look for evidence of facets of their symbolic literacy through identified errors in particular. This paper focuses on students' solutions to some exercise questions during one of two tutorials relating to complex numbers (tutorials 7 and 8, end of April 2014).

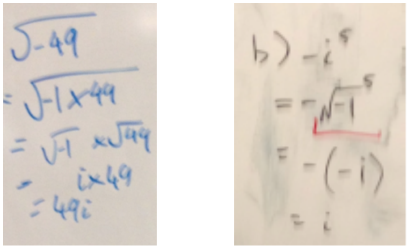
Results and Discussion

The student solutions included below have been selected as representative illustrations of typical errors made by the students. These will be analysed by both considering Serfati's (2005) notions of materiality, syntax, and meaning and by incorporating the idea of symbol template (Sherin 1996) that we will rather call *syntax template* so to ensure coherence with Serfati's framework. For most of these students, the week of tutorial 7, which had included two lectures on the topic, was their first encounter with complex numbers. The materiality, that is, the *shapes* of the symbols and their combination with other symbols, were all familiar from school algebra but some of the syntax and meaning were not. For example while students were already familiar with Latin letters standing for unknowns, variables, etc., the letter i in a complex number takes a very precise and new meaning. Also, while square roots were so far applicable to positive numbers, here the syntax of *square root* is expanded to include negative numbers.

It was clear that every example in these practice exercises involved complex numbers so students were focusing on applying their new learning. In these circumstances it seems that errors in their established templates for syntax were exposed. Illustrations of such errors come from students' responses to questions in tutorial 7 and are detailed in what follows.

Illustration 1

Question 1 of tutorial 7 asked: "Simplify the following, expressing your answers in Cartesian form $a + ib$ where a and b are real numbers. (a) $\sqrt{-49}$; (b) $-i^5$". Figure 1 shows the solution to those items given by two groups of students.



a b

Figure 1. Answers to Question 1a and 1b.

Figure 1a shows the solution to Question 1a, where students have omitted to take the square root of 49, resulting in an incorrect answer of $49i$ instead of $7i$. We conjecture that this is not a mere case of *having forgotten* (a common response from students and, we believe, a likely reply from these students had we had the opportunity to query them). We believe that one potential source for this error lies in the difference in meanings that a same materiality of a symbol (here '$\sqrt{\ }$') conveys. So far, students have always *decoded* '$\sqrt{\ }$' as meaning the process *take the square root of* along with its specific properties (the same that apply for exponents). With the introduction of the imaginary unit i with the property $i^2 = -1$, '$\sqrt{-1}$' is no longer considered as a *square root of* or, in other words, that its syntax template is of the form $\sqrt{\quad}$, rather it has to be considered as one *block* $\sqrt{\square}$, and perceived as the symbolic representation of i. Figure 1a shows that the students did this successfully, moving from $\sqrt{-1}$ (third line) to i (fourth line). However, it seems that the students at the same time see the whole sentence '$\sqrt{-1}\times\sqrt{49}$' with the syntax template $\sqrt{\quad} \times \sqrt{\quad}$ and apply (wrongly) the properties for square roots, in particular the one that says that if you multiply two square roots (provided the arguments are the same) then they *cancel out*.

In Figure 1b, the students have incorrectly evaluated $\sqrt{-1}^5$ as $-i$ instead of i. Similarly to students' response shown in 1a, they have correctly *translated* the symbol i into the *symbol block* $\sqrt{-1}$, but this seems to be what causes them to move incorrectly from the second line to the third. Having considered $\sqrt{-1}$ as one element, this might have led students to now view $-\sqrt{-1}^5$ with the syntax template *negative to an odd power is negative* and too quickly applying this rule to what the *block* $\sqrt{-1}$ means (this thinking is apparent from the usage of brackets in '$(-i)$'), leading to the incorrect intermediary result '$-(-i)$'.

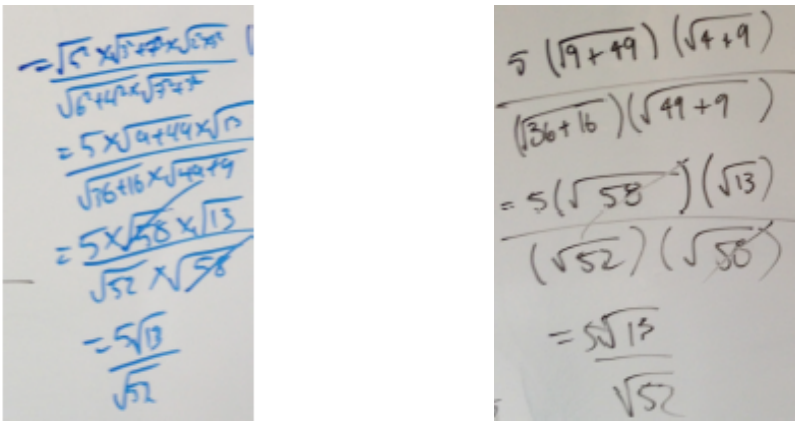
Illustration 2

Equally interesting to looking at students' answers is analysing the questions themselves, since being symbolically literate also means, in some sense, to appropriately read and make meaning of what is asked, including having to sometimes decode *hidden messages* in the stimulus.

In Question 4b of the tutorial, students were asked: "Find the modulus of the following complex numbers without multiplying into Cartesian form: $\frac{-5i(3-7i)(2+3i)}{(6+4i)(7+3i)}$,,

Question 1, for the tutorial, required students to flexibly navigate between different meanings of a symbol with the same materiality ($\sqrt{\ }$); that is, to easily translate square roots in terms of imaginary units as well as to use the fact that $i = \sqrt{-1}$. In order to successfully answer Question 4, students must, on the contrary, *lock* the meaning of *i* as a symbol standing for the imaginary unit, without further considering its intrinsic property. Should the students replace *i* by $\sqrt{-1}$, that would indeed lead them to the numerical dead end $\frac{-135\sqrt{-1}-25}{46\sqrt{-1}+30}$. In fact (and as a consequence), the whole sentence, for example, $3–7i$ is now to be seen as a whole. This is reinforced by the prompt in the stimulus *without multiplying into Cartesian form*. Because *i* has the same syntax as any other letter, one might be tempted to apply the distributive law to $(3–7i)(2+3i)$. Whilst applying the distributive law eventually leads to the expected answer (5/2), underlying the question is the need to work with properties of the modulus of complex numbers (the modulus of the product of complex numbers). The need to *see the sentence as a whole* goes beyond the syntactical interpretation just described (i.e., to not apply algebraic manipulations as one would for syntactically similar expressions). This specific item required going (or at least was intended to go) beyond the syntax template '□ – □*i*' and rather view it as a complex number. It is the context (complex numbers) and certainly the mathematical conventions (except if we are in electricity or electronics courses where *j* stands for the imaginary unit) that guide the interpretation of the syntax. More importantly, it is the context that will signal an efficient approach to finding the appropriate answer. This will be discussed below.

Figure 2 shows the approach taken by two groups of students in Question 4. First of all, let us note that students have indeed recognised each element of the expression as a given complex number as they then immediately start by (correctly) applying the definition of the modulus of complex numbers and their properties. They then carry out the correct mathematical procedures to finally provide numerical answers. The students have certainly failed to notice that 13 is a factor of 52, hence not recognising that the fraction 13/52 is equivalent to ¼, yet their answer is mathematically correct. So where is the problem (if any)?



a b

Figure 2. Unsimplified numerical answers.

At a basic level, we expected students to question their approach: is it reasonable, at this stage of their mathematical experience that the question posed is meant to test the ability of manipulating square roots? Also, students seem to blindly manipulate mathematical expressions, without ever questioning their meaning in context (certainly a magnitude of a complex number can take any numerical positive value, but we expected that students would have used the meaning of the original expression–the modulus of the complex number–to try and make sense of their final answer and, therefore, prompt them to simplify the result). But the issue is less about students providing a mathematically valid answer than it is about them having not fully unravelled the subtleties of the question, including reading *beyond the mere syntax* of the mathematical expression provided. In fact, a successful and more efficient solution to the problem requires interpreting the modulus of complex numbers without necessarily having recourse to the Pythagorean formula, and to rather interpret the meaning of, for example, $|3-7i|$ (and all other expressions) in the geometrical sense. Having done so, students would have been able to *cancel out* pairs of moduli (e.g., $|3-7i|$ and $|7+3i|$) and come up with a very much more efficient solution. We see in this example to the complexity of being able to navigate between meanings of expressions *with same materiality* and we anticipate this is even more problematic if students are too often exposed to drill types of exercises, as these students' responses seem to suggest.

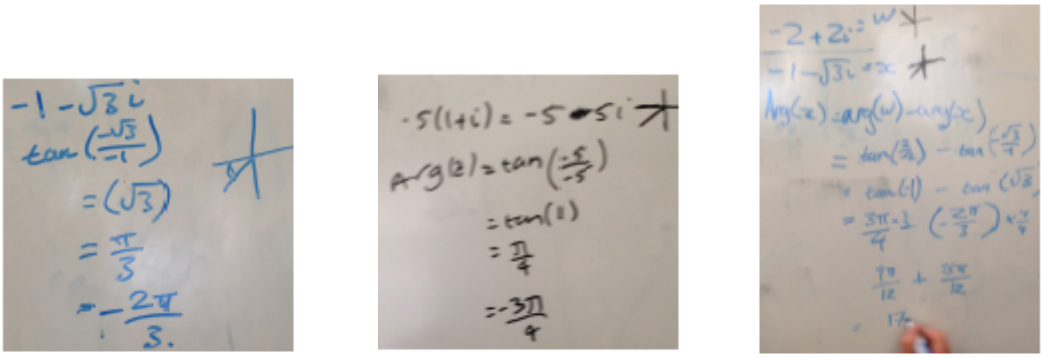
Illustration 3

Question 5 of the tutorial asked:

"Find an argument θ, where $-\pi < \theta \leq \pi$, for the following complex numbers. For part (iii), use facts about the argument of a product or quotient, rather that simplifying the expression.

(a) (i) -5	(ii) $1+i$	(iii) $-5(1+i)$
(b) (i) $-2+2i$	(ii) $-1-\sqrt{3}i$	(iii) $\frac{-2+2i}{-1-\sqrt{3}i}$"

Taking a generic complex number, $a+bi$, the appropriate symbolic form for the argument θ is $\theta = \tan^{-1}\frac{b}{a}$ (or $\theta = \arctan\frac{b}{a}$), taking into account, of course, the signs of a and b to determine the appropriate angle. The students whose solutions are shown in Figures 3a, 3b, and 3c have each obtained the correct values for the arguments but all three show flaws in their written responses.



a b c

Figure 3. Incorrect symbol template and disregard for meaning of equals sign.

First is the confusion between the tangent and the inverse operation, leading to an inappropriate use of the syntax template for the tangent of an angle. In fact, as tautological as it may seem, one has to note that when considering a syntax template, not only are we considering it as a template (much as equation editors in document processing software) but also the syntactical rules that apply for each of its elements (precisely what Serfati 2005 called *combinatorial syntax*). It is almost as if each of the *empty boxes* of the template come with a precise *domain* (in the functional sense). So, for example, the symbol $\sqrt{}$ has the template of the form $\sqrt{\quad}$, where the empty box has to be filled by a number (given or unknown). Interestingly enough, some of these *domains* evolve or change depending on the mathematical context where they are used. In the case of $\sqrt{}$, we have seen that, while we remain within the set of real numbers $\mathbb{R}$, only positive numbers can fill the empty box. Once we incorporate the set of complex numbers $\mathbb{C}$, this restriction is no longer valid and the *template for the same materiality'* (loosely described) then gains an extended *domain*. Students' responses in Figure 3 suggest that students do not consider the syntax of expressions when it comes to the *domain* of the template for tan , not realising that *tan* prompts for its argument to be an angle. It would seem that students should be encouraged to verbalise their symbolic expressions, stating orally that the argument is equal to *the angle whose tangent is* (see Figure 1b) and linking this with the appropriate syntax template.

The students' syntax, if read aloud, does not make sense. They seem to be working out *the answer* without expecting that the symbols they are writing convey a meaning to the reader. Their responses suggest they are using '=' to say "and then I did something (the reader must guess what that was) and the result is*"*. This and *the result is* meaning of the '=' sign dates from primary school and is deeply set in students' thinking. The notion of expecting symbols to have meaning and a habit of checking the meaning of the symbols used is an aspect of working mathematically that needs to be cultured at all levels: primary, seconday, and tertiary. The work shown in Figures 3a, b, and c suggests that students have thought about the meaning of the symbols, indicating the size and position of the angle locating the complex number on the Argand plane, but have only taken this into consideration once they had finished their calculations.

Conclusions and Implications

The examples that we have chosen illustrate the value of following Serfati's (2005) approach to analysing mathematical notation that takes into account both the syntactical aspect of a symbol and also the underpinning mathematical concept(s) conveyed.

First, careful consideration of materiality is important for both teachers and students. The choice of letters and the form of the symbol act as a cue to the student in making choices about efficient solution methods (Illustration 2). Teachers need to help their students learn to recognise such cues and students need to take a moment to consider the makeup of each symbol rather than relying on unthinking recognition of syntax templates.

Secondly, in the examples shown above it is clear that the students' focus is on the new aspects of working with complex numbers. We can see them trying to employ new syntax templates but either failing to look at familiar materiality in a new way or, in a combination of new and old, misapplying old syntax templates. The notion of syntax templates can help teachers identify likely causes for students' errors and provides a way of talking about the structure and meaning of symbols where in one context students need to recognise a symbol as indicating a process but in another identifying a combination signifying a concept (Illustration 1)(Tall et al., 2001).

Thirdly, Illustration 3 highlights what happens when students do not expect mathematics to be read with logical meaning. Here the lack of conventional templates, where '=' indicates that the expressions prior and following are equal, leave the reader guessing as to the meaning intended.

Mathematical literacy (Usiskin, 2012) may be promoted through contemplation of syntax templates by both teachers and students.

References

Arcavi, A. (2005). Developing and using symbol sense in mathematics. *For the Learning of Mathematics, 25*(2), 42–48.

Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics*, *14*(3), 24–35.

Bardini, C., & Pierce, R. (2015). Assumed knowledge: the challenge of symbols. *International Journal of Innovation in Science and Mathematics Education*, *23*(1), 1–9.

Gray, E., & Tall, D. (2001). Relationships between embodied objects and symbolic procepts: an explanatory theory of success and failure in mathematics. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education*, pp. 65–72. Utrecht: PME.

Pierce, R., Stacey, K., & Bardini, C. (2010) Linear functions: Teaching strategies and students' conceptions associated with y=mx+c. *Pedagogies: An International Journal, 5*(3), 202-205.

Pimm, D. (1991). Communicating mathematically. In K. Durkin, & B. Shire (Eds.), *Language in Mathematical Education: Research and Practice* (pp. 17–23). Philadelphia: Open University Press.

Quinnell, L. & Carter, M. (2012). Greek or not: the use of symbols and abbreviations in mathematics. *Australian Mathematics Teacher*, *68*(2), 34–40.

Rubenstein, R. N. (2008). Mathematical symbolization: Challenges across levels. Paper presented at ICME 11, *TSG 31: Language and communication in mathematics education*. Retrieved 06 March 2014 from www.tsg.icme11.org/document/get/853

Rubenstein, R. N., & Thompson, D. R. (2001). Learning mathematical symbolism: Challenges and instructional strategies. *Mathematics Teacher, 94*(4), 265–271.

Serfati, M. (2005). *La révolution symbolique. La constitution de l'écriture symbolique mathématique.* Paris: Pétra.

Sherin, B. L. (2001). How students understand physics equations. *Cognition and Instruction, 19*(4), 479–541.

Skemp, R. (1982). Communicating mathematics: surface structures and deep structure, *Visible Language*, *16*(3), 281–288.

Tall, D., Gray, E., Bin Ali, M., Crowley, L., DeMarois, P.,McGowen, M., Pitta, D., Pinto, M., Thomas, M. & Yusof, Y. (2001). Symbols and the bifurcation between procedural and conceptual thinking. *Canadian Journal of Science, Mathematics and Technology Education, 1*(1), 81–104.

Usiskin, Z. (2012). What does it mean to understand some mathematics? In *Proceedings of the 12th International Congress on Mathematical Education*. Seoul, Korea: ICME.

Problematising Mathematics Education

Andy Begg
Auckland University of Technology
<andy.begg@aut.ac.nz>

We assume many things when considering our practice, but our assumptions limit what we do. In this theoretical/philosophical paper I consider some assumptions that relate to our work. My purpose is to stimulate a debate, a search for alternatives, and to help us improve mathematics education by influencing our future curriculum documents and practice.

Many assumptions are made about mathematics education practice at all levels of education; these need to be identified and questioned as some are no longer appropriate. They relate to our aims, and our conceptions of mathematics, curriculum, teaching and learning, thinking, and assessment; and problematising these is the basis of this paper.

Educational Aims

The taken-for-granted aim of many who teach mathematics is to follow a prescribed curriculum knowing that learners will be assessed summatively. This may seem cynical, but it is also the view of many students, parents, and future employers. It assumes that summative assessment is more important than diagnostic, formative, or self-assessment. While many able students often find such assessment tasks trivial, less-able students learn from such assessment that they are not mathematically capable.

But, putting assessments to the side for the moment, what are our general educational aims and are these relevant for mathematics? Early in my teaching my ideas were influenced by a set of aims published by our teacher organisation (Munro, 1969, p. 1), these were "the urge to enquire, concern for others, and desire for self respect."

These hardly changed when our new curriculum (Ministry of Education, 2007, p. 12) identified five key competencies, "thinking, using language, symbols and text, managing self, relating to others, participating and contributing," which I interpreted as aims. For me both sets of aims are similar and relate to mathematics and to other subjects. For me, thinking includes caring thinking which implies: concern for and relating to other people and living things, and caring for and respecting self. Accepting and interpreting these aims, our task is not to prepare students for the future by teaching for assessment, but: to foster student enquiry, thinking, and self-management. And, as enquiry involves creative and critical thinking, and self-management involves metacognitive thinking, these aims become one key aim, thinking, but it is rarely implemented.

Defining Mathematics

Everyone knows what mathematics is, but it is difficult to define without words such as arithmetic, geometry, algebra, statistics, … This is evident with dictionary definitions, for example: "Mathematics is a group of related sciences including algebra, geometry, and calculus, which use a specialised notation to study number, quantity, shape and space." But, is mathematics a subject formed by a partitioning of knowledge into disciplines, subjects, topics and sub-topics, or, is knowledge holistic? For me all knowledge is interrelated, which implies that when teaching a subject we should emphasise links with other subjects. I see mathematics as the study of relations, relations are sets of ordered

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 85–92. Sunshine Coast: MERGA.

pairs, and all operations are relations where the first element of the ordered pair is itself another ordered pair, e.g., $+ = \{((2, 3), 5), ((1, 4), 5), \ldots\}$. This definition from the 'new maths' of the 1960s as presented by Papy and Papy (1963/68, 1971) unifies mathematics; and I believe we need to integrate mathematics with other subjects whenever possible.

Curriculum

Rather than using thinking (or educational aims) as a basis for planning, teachers often begin with the curriculum which has been defined as 'all that is planned for the classroom', where the classroom may be at any level of formal or informal education, or in the classroom of life where learning is determined by and unique to the learner. Within each level of formal education curriculum documents have been written, these include:

- the regional level (national/state/district curriculum; or assessment syllabus);
- the institutional level (school scheme/institution curriculum);
- the individual teacher level (reinterpretation of curriculum to 'suit' the students)
- the learner level (student-constructed learnt curriculum).

The traditional model for curriculum was derived from the 'tree of knowledge' in the Bible (Genesis, 2:17). This metaphor implies a knowledge structure with the trunk as the fundamental ideas, branches as the disciplines, and on to small branches, twigs and leaves.

An alternative is a rhizome metaphor from Deleuze and Guattari (2004/1980). This is based on the notion from botany—a rhizome being a plant with roots that grow underground that sends up shoots all over the place. These shoots seem disconnected, but are linked. What we learn seems like this; we learn little bits first and the connections come later.

Both metaphors imply organising curriculum by structuring knowledge. The tree implies a formal organisation, while the rhizome is less rigid, each shoot represents a topic and only after numerous topics have been explored will the connected structure of our subject emerge. With both metaphors knowledge is the goal—the 'content' rather than the 'context' for learning. With these metaphors the assumption is that some specific knowledge needs to be learnt by everyone regardless of beliefs, backgrounds, and interest.

Considering curriculum more radically involves thinking of knowledge and learning as a complex/living/emerging system. Every aspect is connected and interacts with every other aspect and complexity implies that all connections and interactions are unpredictable. This is based on the work of the Maturana and Varela (1987) who saw living and learning as a complex system, and said 'to live is to learn'; it also explains how other living things learn (e.g., Anathaswamy, 2014; Birkhead, 2012; Chamovitz, 2012). From this perspective all learning is connected, knowing (epistemology) is inseparable from being (ontology); we are always learning, and our task as educators is to enrich the living-learning process. However, what is learnt differs for individuals because of differing degrees of awareness, ability to 'be' in the learning moment, prior knowledge, attentiveness, and ability to make sense of what is said; and learning occurs consciously and subconsciously with each learner developing a unique web of understanding that grows in complexity over time.

Teaching and Learning

From this 'living is learning' perspective, teachers are merely catalysts for learning; but powerful catalysts who influence the direction, speed, and depth of learning. Traditionally teachers based their work on learning theories; and theories like theorems are based on assumptions that are not made explicit, so practitioners are often not aware of what they are assuming. I have a list of over 160 theories, though none start with *X*; they cannot all be true as they contain elements that are contradictory; Table 1 is an abbreviated version of it:

Table 1
Theories related to learning; what is the X-factor?

Theories related to learning		
Abstraction theory	**J**ob-based learning	**S**elf-directed learning
Behaviourism	**K**inaesthetic education	**T**rial and error
Communities of practice	**L**ecturing	**U**nconscious learning
Drill and practice	**M**astery learning	**V**ocational-based learning
Enactivism	**N**arrative pedagogy	**W**omen's ways of knowing
Friere's critical education	**O**bservation-based learning	**X** ????
Goal-based learning	**P**rogrammed instruction	**Y**in-yang learning
Holistic learning	**Q**uestion-based learning	**Z**one of proximal development
Imitative learning	**R**adical constructivism	

It is useful to think about the learning theory that influences one's work, and wonder what the originator of the theory assumed, and how the theory is interpreted today. I am drawn to theories E and Q in the table—Q is self-explanatory, and E, enactivism, best fits with the view of learning I described when I wrote (Begg, 2013, pp. 81–82) the essence of enactivism is, "learning is living, living is learning, and this is true for all living organisms." From this perspective, I see we and the world as inseparable; we co-emerge—cognition (learning) cannot be separated from being (living). Knowledge is the domain of possibilities that emerges as we respond to and cause changes within our world.

As teachers we know our task is to teach. For me teaching is 'stimulating enquiry by asking questions', not 'telling'; and this is possible. The best mathematics lessons I have seen was in Japan—during the 50-minutes the teacher only asked questions, "What do you think? What do you others think? …" Accepting the cultural concern regarding individualism, the teacher ensured that group work dominated so responses given by individuals were group ones and no 'loss of face' occurred. This epitomised 'teaching as asking, not telling.' Related to teachers 'asking' is learners 'researching and thinking'; thus our task as teachers is to provide researching/thinking activities, but that is not always easy. As Heidegger (2004/1954, p. 15) puts it, "Teaching is more difficult than learning because what teaching calls for is this: to let learn" and I would add: 'and to let think!'

Thinking

Mathematical thinking is often considered as being logical (or critical) thinking; but all the other forms of thinking also seem to me to be relevant within mathematics education. There are many possible classifications of the forms of thinking; my own classification divides thinking into nine slightly overlapping forms, namely: empirical, critical, creative, meta-cognitive, caring, contemplative, subconscious, cultural, and systems thinking.

Empirical Thinking

Empirical or sense-based thinking occurs when we are aware through our senses—seeing, hearing, feeling, tasting, or smelling. It is the dominant form of thinking of young children and the starting point for most conscious thinking. It seems valued by both western and non-western people; it is important both in its own right and as the basis for other forms of thinking. Being aware through one's senses and remembering is nearly automatic—though by improving one's noticing skills (Mason, 2002) or by becoming more aware (Depraz, Varela, & Vermersch, 2003), the process can become richer.

Empirical thinking involves sensation followed by perception (Restak, 2012); sensation involves detection of information (awareness) using sense organs, and perception is the interpretation/analysis of that information so that it can be remembered and used for some purpose. Interpretation involves constructing meaning, thus empirical thinking is not direct knowing as interpretation is based on prior experience. Sometimes, before a sensation has been interpreted, our body has already reacted unconsciously but intelligently to it, e.g., one cuts one's finger and the body's cells immediately begin to 'intelligently' repair the cut before the brain receives and interprets the cutting sensation.

In mathematics education the main forms of sense-based thinking are visual thinking (interpreting and imagining 2 and 3-dimensional diagrams; using Venn diagrams, arrow graphs, flow charts, Cartesian and statistical graphs, symbols, signs, and gestures; picturing, modelling ideas; noticing (Mason, 2002)); and aural/oral thinking (involving: making sense/interpreting what one hears, and saying what one means).

Critical Thinking:

Critical (rational or logical) thinking is fundamental to mathematics; it involves logic, (which depends on a 'logic' system and initial assumptions). Usually western logic is taken for granted and initial assumptions are made without considering alternatives. Absolute proof is not possible with critical thinking as it depends on assumptions made and the logic system used. One can gather evidence to support a hypothesis; and if all the assumptions are made explicit then a 'relative' proof may be useful—but one counter-example disproves a hypothesis. Words (or symbols) are usually used in critical thinking, but diagrams can also be used (e.g., Venn diagrams in set theory)—proofs are not always possible with diagrams, but diagrams are useful when exploring a problem; though they can mislead (e.g., 'are two straight line that never intersect parallel?' One approach is to draw many examples and conclude that that is true; but it is not true in 3-dimensions).

Western critical thinking has dominated western thinking and resulted in 'advances' in many subjects, but the underpinning assumptions seem often not to be made explicit. This has resulted in 'solutions' to problems without consideration of the consequences (e.g., science problems have been solved without considering the environment; western economics has been based on having more, not having enough; and western philosophy has been concerned with individual rights, not community good).

Creative Thinking

Creative thinking occurs in art, music, literature, but also in other aspects of life when we consider alternatives and ask "what if …?" Our ideas of self, of others, and of things we learnt at home and school depend on assumptions and one can be creative by making these explicit, questioning them, and considering alternative assumptions that other people make. Creative thinking can involve making connections within contexts, finding alternative connections, and finding different solutions to problems in different contexts, or with different initial assumptions (and often assumptions are culturally specific), imagining possibilities, visualising options, conjecturing, modelling reality, designing things, making and seeing patterns, generalising and specialising, and using analogies. It is important in mathematics at all levels as many problems can be solved in different ways.

Metacognitive Thinking

Metacognition is monitoring one's thinking; it involves: learning to learn, thinking about thinking, reflecting, and self-assessing. It occurs consciously, unconsciously, and automatically. The more one attends to this consciously the more one feels in control. Typical questions one might ask oneself are: Have I done enough? Should I do more? What else could I do? What have I assumed, and could I make a different assumption? Am I happy with this, or do I need to improve it, and how might I improve it?

Caring Thinking

Lipman (2003) wrote about caring thinking, and his ideas fit with aims related to self, family, others, living things, the environment, and culture. Caring is influenced by values, and activities for clarifying values help learners become more aware of (and strengthen) their values. One value is respect, including respect for others with different values. Caring depends on cultural beliefs about 'being', and one may ask, are we all separate; could we exist without other people, other living things, and our planet? Caring thinking involves ethical thinking, emotional thinking and critical thinking. It relates to caring for self, for others, and for the community (local, national and international), and for other living things. In education caring is involved when someone is stuck with a problem—when should one intervene? One person steps in at once to help so the person is not frustrated; another allows the person time to consider alternatives—both reflect caring thinking.

Contemplative Thinking

Contemplative thinking can involve having hunches (intuition), noticing, being still, meditating, and developing awareness. It is associated with religious contemplation and is evident in Shamanic, Vedic, Buddhist, Christian, Islamic (Sufism), and Jewish (Cabalistic) traditions; and in the ways of knowing of numerous indigenous cultures (Abram, 1997; Buhner, 2014; Davis, 2007; Kharitidi, 1996; Wolff, 2001). Contemplation is not emphasised as it was in the past because we emphasise science and critical thinking, but numerous scientists, mathematicians, and philosophers acknowledge its importance (Buhner, 2014). Contemplative thinking builds on empirical thinking and complements critical thinking, thus developing contemplative thinking (or awareness) means developing noticing skills (Mason, 2002), sensory awareness, and openness using analogical thinking (Buhner, 2014). Teachers want students to be reflective, but when asking students to reflect on something they often mean 'think critically about it'. Reflecting from a contemplative perspective means holding an idea in one's mind without processing it.

Subconscious Thinking

Subconscious, unconscious, or bodily thinking is important. Mlodinow (2012) wrote how we are only aware of 5% of what goes on in our brains; our brains unconsciously handle the other 95%. This means our subconscious thinking shapes our empirical (sense-based) thinking, our ever-changing memories, our social interactions, our logic, and our cultural beliefs; how we think about self, others, the world around us; and the assumptions that influence our conscious thinking. According to Davis, Sumara and Luce-Kapler (2008, p. 24) our sense organs register about 10 million bits of information each second but we are only consciously aware of about 20 of these bits; our subconscious 'thinking' or bodily knowing occurs within the cells of our bodies (and within the cells of all living things) and these cells 'know' what must occur for life—but we are not consciously aware of this knowing. Intuition involves the subconscious becoming conscious. One example of this emerged when a mathematics professor was asked, 'how do you go about solving these difficult problems?' He replied, 'I read the question carefully before going to sleep, then when I wake up I write out the solution.' Thus, mathematics not only involves logical/critical thinking, it also involves contemplative (or unconscious) thinking.

Cultural Thinking

Cultural thinking includes communal/collective and global thinking; and differences arise with people from different cultures. Nisbett (2003) wrote about the different ways that Asians and Westerners think, and indigenous peoples think differently in other ways. These ways are not right or wrong, just different—different starting assumptions, different experiences, different vocabulary, different beliefs and philosophies, different logic systems, different emphasis on nouns and verbs, and so on. An example of this (Nisbett, 2003, p. 141) is when given three pictures—some grass, a hen, and a cow—and asked what goes with the cow? Westerners used an animal/vegetable division; while easterners used a thematic relationship cows eat grass. Two other examples of cultural thinking are:

- from economics—maximising profit is the basis of decisions in some countries, but in other countries environmental considerations are more important;
- regarding self-image—my western-enculturated brain believes I am a self-sufficient individual, yet I cannot exist without the world, the air to breath, and the life forms that provide food; so, am I an individual, or a part of a bigger organism? (And that raises the question, how do people from other cultures see themselves?)

Systems Thinking

Systems thinking is based on notions of complex (rather than simple or complicated) systems. Simple systems are mechanistic, based on cause and effect relationships (A causes B); complicated systems are also predictable though not always obviously (A causes B which causes C which causes … which causes Y which causes Z). Contrasting with these are complex systems; complexity assumes a web of interrelationships with ideas emerging that are not predictable (A, B, C, … all interact but the result is unpredictable as the result emerges from the complexity of the interactions). Systems thinking explains how small catalytic events that are separated by distance and time can cause significant changes in complex systems. Systems thinking techniques are used to study physical, biological, social, scientific, engineered, human, and conceptual systems; and it explains how students who have attended the same class come away with different learning because of slightly different initial ideas.

Thinking in Education

These nine forms of thinking seem to imply a partitioning of thinking into categories, but these overlap and merge. When focusing on a task one does not limit oneself to one form of thinking, one moves smoothly from form to form as the task progresses.

For example:

firstly noticing, sensing, or perceiving a situation (empirical thinking),
then analysing it using logic (critical thinking),
pausing and reflecting (contemplative thinking),
deciding to stop and reconsider (metacognition),
asking oneself, might another assumption be made (creative thinking),
thinking of possible undesirable implications (caring thinking),
being influenced by notions we are unaware of (unconscious thinking),
and so on …

Within education, every topic in every subject at every level is a context for thinking. Thinking in mathematics education is more than listening and remembering, and is enhanced by activities involving: communicating, connecting, problem solving (and problem posing), applying knowledge, using tools (including IT), and reflecting (which links with metacognitive thinking and self-assessment). Ideally such activities need to be included when designing thinking/research focused classroom activities (such as literature reviews, projects, creative activities, discussions, and free-writing). For me research simply means 'enquiry'; and research tasks come in many sizes and in all subjects, and can involve independent or group learning, problem-solving, project work, and tutorials where groups of students with their teacher discuss their plans and progress with projects and receive feedback.

Assessment

The dominant types of assessment are internal (in-school) and external (for awards), but traditionally there are three forms:

- diagnostic before learning to find what students need/want to know;
- formative during learning to find how students are forming ideas/ coming to know;
- summative after learning to find what students know (and understood).

For me, changing our emphasis from knowing to thinking shifts the focus of assessment from summative to formative and to an emphasis on metacognition (thinking about thinking) and life-long learning. The responsibility for all forms of assessment shifts from teacher to learner and becomes self-assessment; which fits with preparing our students for life-long learning. Additionally, when the mode of learning involves research projects then the assessment is unlikely to be whole class, but rather, project based.

Conclusion—Making Changes

I see virtually everything we are doing in mathematics classrooms as needing to be changed! Our efforts to change in the past have been like 'shifting the deck chairs on the Titanic.' What should we do to implement our aims? How can we encourage thinking? Can we change our teaching to 'let learn' and shift from telling to asking? How might we reduce the subject silo effect? What forms of assessment are appropriate? Should we encourage learning to learn rather than learning what is taught? Will learning to learn prepare students for a lifetime of learning?

In the past educational authorities sought 'top-down change' by legislation with new curriculum or assessment policies, but the desired changes were never fully implemented. The alternative is 'bottom-up-change' with small groups of teachers taking professional responsibility and making numerous small changes. In this situation the role of mathematics educators is to model the ideal changes, discuss them, and encourage and support practitioners in their efforts to change. My aim is that we re-conceptualise:

- teaching as asking, not telling;
- learning as researching and thinking, not memorising;
- assessment as formative and self-assessment, not summative;
- mathematics as being integrated with other subjects, not separated from them: and
- making changes as our personal responsibility, not that of external authorities.

References

Abram, D. (1997). *The spell of the sensuous: perception and language in a more-than-human world.* New York, NY: Vintage Books.

Anathaswamy, A. (2014, Dec. 6). Roots of consciousness. *New Scientist 2998*, 34–37.

Begg, A. (2013). Interpreting enactivism for learning and teaching. Invited paper: *Education Sciences and Society 4*(1), 81–96.

Birkhead, T. (2012). *Bird sense: what it's like to be a bird.* New York, NY: Walker & Company.

Buhner, S. (2014). *Plant intelligence and the imaginal realm: beyond the doors of perception into the dreaming of earth.* Rochester, VT: Bear & Co/Inner Traditions.

Chamovitz, D. (2012). *What a plant knows: a field guide to the senses.* New York, NY: Scientific American/ Farrar, Strauss and Giroux.

Davis, B., Sumara, D., & Luce-Kapler, R. (2008). *Engaging minds: changing teaching in complex times.* (Second and revised edition). New York, NY: Routledge.

Davis, W. (2007). *Light at the edge of the world: a holistic vision for our time.* Creskill, NJ: Hampton Press.

Deleuze, G., & Guattari, F. (2004/1980). *A Thousand Plateaus.* (Translated by Brian Massumi.) London, England & New York, NY: Continuum Books (Translation of: *Capitalism and Schizophrenia,* Vol.2. Mille Plateaux. Paris, France: Les Editions de Minuit.)

Depraz, N., Varela, F., & Vermersch, P. (2003). *On Becoming Aware: a Pragmatics of Experiencing.* Amsterdam, The Netherlands: John Benjamins.

Heidegger, M. (2004). What is called thinking? New York, NY: Harper Perennial. (First published in 1954 as *Was Heisst Denken?* By Max Niemeyer Verlag, Tuebingen, Germany, Translated by J. Glenn Gray.)

Kharitidi, O. (1996). *Entering the Circle: Ancient Secrets of Siberian Wisdom Discovered by a Russian Psychiatrist.* London, England: Thorsons.

Lipman, M. (2003). *Thinking in Education.* (Second edition). New York, NY: Cambridge University Press.

Mason, J. (2002). *Researching your own Practice: the Discipline of Noticing.* London, England: Routledge-Falmer.

Maturana, H., & Varela, F. (1987). *The Tree of Knowledge: the biological roots of human understanding.* Boston, MA: Shambhala Press.

Ministry of Education. (2007). *The New Zealand Curriculum for English-medium teaching and learning in years 1–13*. Wellington, NZ: Ministry of Education.

Mlodinow, L. (2012). *Subliminal: the new unconscious and what it teaches us.* London, England: Allen Lane; (& 2014, London, England: Penguin Books).

Munro, R. [Chairman]. (1969). *Education in change: report of the curriculum review group of the New Zealand Post-Primary Teachers' Association.* Auckland, NZ: Longman Paul/NZPPTA.

Nisbett, R. (2003). *The geography of thought: How Asians and Westerners think differently ... and why.* London, England: Nicholas Brealey Publishing.

Papy, G., with Papy, F. (1963/68) *Mathématique Moderne 1*, Brussels, Belgium & Paris, France.: Marcel Didier. [English version: Papy (1968) *Modern Mathematics 1*, London. England: Collier/Macmillan Ltd, Translated by F. Gorner].

Papy, F. (1971) *Mathematics and the child.* Montreal, Canada: Algonquin. (First published in 1968 as: *Les enfants et la mathématique.* Brussels, Belgium: Marcel Didier. Originally presented as the first thesis requirement for her doctoral degree at Brussels University).

Restak, R. (2012*). Mind: the big questions.* London, England: Quercus Editions.

Wolff, R. (2001). *Original wisdom: stories of an ancient way of knowing.* Rochester, VT: Inner Traditions.

Identity as an Embedder-of-Numeracy: Identifying ways to support teachers to embed numeracy across the curriculum

Anne Bennison
The University of Queensland
<a.bennison@uq.edu.au>

The context in which mathematics is used is an important aspect of numeracy. Therefore, students' numeracy capabilities need to be developed in subjects across the curriculum. The case study of a secondary school history teacher is presented to demonstrate how a framework for identity as an embedder-of-numeracy can be used to identify ways that this teacher might be supported to embed numeracy into the history curriculum. While the framework was generally effective for this purpose, a potential limitation was identified.

Introduction

The pressure on schools to demonstrate improved outcomes on the National Assessment Plan - Literacy and Numeracy (NAPLAN) influences school organisation, curriculum, and pedagogy (Hardy, 2014). This includes the use of practice tests and teaching to the test with some resultant focus on a definition of numeracy as the mathematical skills required to successfully answer NAPLAN questions. However, numeracy encompasses much more than just mathematics. The Organisation for Economic Co-operation and Development (OECD) defines mathematical literacy (the term used in some international contexts) as:

> an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals in recognising the role that mathematics plays in the world and to make well-founded judgments and decisions needed by constructive, concerned and reflective citizens (OECD 2013, p. 25).

This widely accepted definition of numeracy recognises that the context in which mathematics is used is as an important aspect of numeracy. In fact, Steen (2001) argued that context is what distinguishes numeracy from mathematics, and if students are to develop the capabilities needed to become numerate, they need to be provided with opportunities to use mathematics in a range of contexts; in other words, in subjects across the curriculum. While the need for this type of approach has also been recognised in Australia for some time (DEETYA, 1997), it is only recently with the introduction of the Australian Curriculum (ACARA, 2014), where numeracy was seen as a general capability to be developed in all subjects, that there has been a national approach to this. However, for this approach to be successful, teachers from all disciplines need to exploit numeracy learning opportunities that exist in the subjects they teach. Therefore, there is a need to investigate how teachers can be supported to develop this capacity. One way of doing this is to use teacher identity as the analytic lens.

This paper builds on previous research in which a framework for identity as an embedder-of-numeracy was developed (Bennison, 2015, hereafter referred to as *EoN Identity*). Specifically, the purpose of this paper is to investigate the efficacy of the *EoN Identity framewor*k to answer the following research question: In what ways can the EoN Identity framework be utilised to identify ways to support a teacher to develop the capacity to embed numeracy into the subjects she/he teaches?

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 93–100. Sunshine Coast: MERGA.

A Framework for EoN Identity

An individual's identity develops as they negotiate meaning through individual cognition and social interactions within their environment; it is dynamic and context dependent (Wenger, 1998). These attributes make teacher identity a useful construct for investigating how teachers can be supported to exploit numeracy learning opportunities across the curriculum. However, it is difficult to design empirical studies that capture the complexity of teacher identity but are still viable practically (Enyedy, Goldberg, & Welsh, 2005). To overcome this limitation, a framework for EoN Identity was developed that identifies characteristics that are most likely to have greatest impact on a teacher's capacity to embed numeracy into the subjects they teach (Bennison, 2015). The two understandings that underpin the EoN Identity framework are that:

1. being numerate involves having the dispositions that support the critical use of mathematical knowledge and appropriate tools in a range of contexts: these five dimensions of numeracy are encapsulated in the numeracy model developed by Goos, Geiger, and Dole (2014); and
2. the belief that the best way for teachers to support numeracy learning is to embed numeracy into the subjects they teach in order to enhance discipline learning.

In Goos et al.'s (2014) numeracy model, numeracy requires *dispositions* (i.e., confidence and willingness) to use *mathematical knowledge* (concepts and skills, problem solving, and estimation) and representational, physical, and digital *tools* (e.g., graphs, measuring instruments, and calculators, respectively) in a range of *contexts* (both within school and beyond school settings). These four dimensions are set within a *critical orientation* that enables decisions and judgments about mathematical information. (See pp.83-85 for further elaboration). This model of numeracy was used to underpin the EoN Identity framework because each dimension of numeracy was made explicit and the model provided an effective means of describing a teacher's personal conception of numeracy and classroom activities. The second of the understandings that underpinned the EoN Identity framework stemmed from recognition of the important role numeracy has for conceptual understanding in disciplines across the curriculum. For example, an understanding of chronological conventions was seen as essential to conceptual understanding of history (Blow, Lee, & Shemilt, 2012), whereas lack of well-developed numeracy skills was identified as a barrier to learning science (Quinnell, Thomson, & LeBard, 2013).

The EoN Identity framework (summarised in Table 1) was arranged around five *domains of influence*: knowledge, affective, social, life history, and context. Within each of these domains were characteristics that impact on a teacher's capacity to support numeracy learning. For example, mathematical content knowledge (MCK), pedagogical content knowledge (PCK) and curriculum knowledge (CK) were the aspects of the knowledge domain. Only these types of knowledge, instead of all of the types of knowledge that Shulman (1987) suggested were needed for teaching, were included because, in order to support numeracy learning, a teacher needs to be competent in the mathematics inherent in the discipline (MCK), be able to use curriculum documents to identify where numeracy would support discipline learning (CK), and be able to design tasks that utilise some or all of the dimensions of numeracy in Goos et al.'s (2014) numeracy model (PCK). These nuanced meanings of MCK, PCK and CK focus on the knowledge needed to support students' numeracy learning. (See Bennison, 2015 for further details of how the framework was developed).

Table 1
Framework for identity as an embedder-of-numeracy (adapted from Bennison, 2015, p.15)

Domains of influence	Characteristics
Life History	Past experiences of mathematics
	Pre-service program
	Initial teaching experiences
Context	School policies
	Resources
Knowledge	Mathematics content knowledge (MCK)
	Pedagogical content knowledge (PCK)
	Curriculum knowledge (CK)
Affective	Personal conception of numeracy
	Attitudes towards mathematics
	Perceived preparation to embed numeracy
Social	School communities
	Professional communities

Research Design and Methods

The data presented in this paper were drawn from a two-year study (2013-2014) that was conducted in two schools in Queensland: one metropolitan and one regional. The study employed case study methodology (Stake, 2003) with Kylie (pseudonym), the teacher who is the focus of this paper, being one of eight teachers who were recruited for the study. These teachers had different disciplinary backgrounds and levels of experience and were recruited because they had previously agreed to participate in a larger study (hereafter referred to as the *Numeracy Project*). Thus, they had indicated an interest in developing their capacity to support numeracy learning across a range of disciplines (English, mathematics, science, and history) and provided an opportunity to learn about how teachers develop an EoN Identity.

The main sources of data for the study were semi-structured interviews with the teachers and lesson observations. Kylie was visited six times during the study. On each occasion, at least one lesson was observed and she was interviewed after the lesson(s) about the tasks she had used as well as student and teacher learning. She participated in two additional interviews: a scoping interview that was conducted near the beginning of the study (but after Kylie had participated in two workshops for the Numeracy Project) and a final interview that was conducted during the last school visit. The first of these interviews was about her background, beliefs about numeracy, and school context, whereas the final interview asked Kylie about her experiences during this study and provided an opportunity to get clarification of comments she had made during earlier interviews. Lesson observations and post lesson interviews provided data for this study and the Numeracy Project, whereas the scoping interview and final interview, the sources of data for this paper, were conducted solely for this study.

Data analysis involved coding the transcripts of Kylie's scoping and final interviews to identify comments that were related to aspects within each of the domains of influence. For example, comments she made about her studies of mathematics at school were included in both her knowledge and affective domains because they gave some indication of her MCK and her attitudes towards mathematics, respectively. Judgments were made about her level of MCK, PCK, and CK based on her comments during interviews.

Kylie's EoN Identity

Life History Domain

While at school, Kylie focussed on humanities subjects and reported that:

> [M]aths was something I kind of endured … Like, I did Maths A [a subject taken in the final two years of school by students who do not require a knowledge of calculus], and I did quite well in Maths A, but I took it because it was the easy one (Final interview).

At university, Kylie completed a Bachelor of Arts degree, majoring in Ancient History and English Literature. Although her studies at university did not require any further formal mathematics subjects, she reported that she used mathematical knowledge, especially statistics, in some of her history courses. After travelling and working overseas, Kylie returned to Australia and completed a Graduate Diploma in Education with teaching areas in English and History. Although she could not remember much emphasis being placed on literacy and numeracy during her pre-service teacher education program, she did remember having to comment on both in an assessment task for a course she completed during her final semester. However, Kylie reported that numeracy was not a focus for her in this task.

When this study commenced, Kylie was at the beginning of her teaching career in a secondary school in a regional city. During the study, she taught mainly junior classes (Years 8 and 9), which she took for English and history. However, the focus in this paper is on her EoN Identity in the discipline of history.

Context Domain

There were three aspects to Kylie's context domain: the Australian Curriculum, the school where she teaches, and the Numeracy Project.

Australian Curriculum. The Australian Curriculum: History (ACARA, 2014) was implemented in Kylie's school in the first year of the study. Although the numeracy demands in history were identified with icons and online filters in curriculum documents, Kylie felt that "the numeracy that's outlined isn't particularly in depth or challenging" (Final interview). Despite this lack of guidance from curriculum documents, over the course of the study Kylie identified a number of learning opportunities that existed within the history curriculum.

The implementation of the new curriculum presented Kylie with some challenges. She felt pressure to cover the content: "We don't have time at the moment and that's what I am particularly concerned about" (Scoping interview), and would like to see reduced content to allow greater focus on historical skills such as reading maps and constructing graphs. Access to appropriate resources was also an issue for Kylie, who described some of the difficulties she experienced with a research project on the medieval period.

> [W]e just didn't quite have enough resources … we only have one free computer lab … four classes on each line … we've had one lesson on computers and the rest has been from books. Obviously we are building up books, [but it] will probably take a couple of years before we have enough books (Scoping interview).

Kylie's school. The school where Kylie taught was located in a regional city where the main industry was mining. The school was classified as being in an average socioeconomic area and had around 1,000 students who came from both metropolitan and rural areas. School NAPLAN results for numeracy had been close to the Australian schools' average but those for some aspects of literacy, although close to the Australian schools' average, had declined over the last couple of years. This had resulted in "such a focus in English to prepare students for NAPLAN" (Scoping interview).

In the final year of the study, the school had set up a number of committees and each teacher was asked to join one. As Kylie had participated in the Numeracy Project and had taken on the role of Literacy Coach, she joined the Literacy and Numeracy Committee. The task for this committee was to track implementation of literacy and numeracy strategies in the school in order to evaluate their impact on NAPLAN data. In light of a decline in NAPLAN results for literacy, the focus of the committee had been on implementing a whole school approach to literacy. A similar approach was not considered necessary for numeracy: "The numeracy people, the Maths department, feels that on their level they've achieved this prize for what they are doing" (Final interview).

Numeracy Project. Kylie and three other teachers from her school participated in the Numeracy Project. This project investigated the potential of professional development based on Goos et al.'s (2014) numeracy model for supporting teachers to promote numeracy learning across the curriculum. Teachers across a range of disciplines had been recruited from primary and secondary schools in Queensland and Victoria. During the project (2012-2014), the teachers participated in cycles of professional development workshops followed by visits to the school by researchers who observed lessons where teachers implemented activities to support numeracy learning and interviewed the teachers.

Knowledge Domain

Although Kylie felt that she "probably need[ed] a refresher for a lot of [the mathematical knowledge]" (Scoping interview) required for embedding numeracy in history, her mathematics background (as outlined earlier in the section on her Life History domain) had probably given her the requisite MCK. With a major in Ancient History, Kylie had a strong discipline background. However, as the history curriculum was still relatively new, it may take time for her to develop the CK needed to identify where numeracy can be used to support discipline learning in history.

Prior to her participation in the Numeracy Project, Kylie had not had any opportunities to learn about embedding numeracy in history. Early in this study, Kylie thought that she needed to learn how to "adequately incorporate numeracy without losing the focus on historical issues" (Scoping interview). However, by the end of this study, she had demonstrated some PCK through her classroom practice. For example, she described a lesson where she utilised a scaled timeline to assist students to understand that historical events had duration and could be concurrent.

> I was trying to get them to think about timelines because my students, like, we have been trying timelines since we started this unit at the end of last term and we have done two or three and they are just [pause] having trouble with the times themselves, like, they're having troubles conceptually understanding when things happened. So we did a whole world timeline about all the ideas we could think of. Like, 'When was the relationship between the Black Plague and the Age of Discovery?' And, 'When was Captain Cook and the Age of Slavery?' And, 'When, you know, what was happening in Australia at this time?' But they're just [pause]; to them 1400 to 1600 is just this blurry blob in the middle of nowhere. They just have no idea what's going on (Final interview).

Affective Domain

Kylie reported that before her participation in the Numeracy Project, she did not "have an awareness of what numeracy was … [and was] probably one of those teachers who was like, 'Numeracy, well I'm sure they'll cover that in maths'" (Scoping interview). However, even in the early stages of the Numeracy Project, Kylie felt that her ideas about numeracy were changing as numeracy became more obvious to her. Initially, she thought that numeracy was important, although not as important as literacy, but had come to believe that "if you are innumerate, that's on a level with not being able to read" (Final interview). Kylie felt that embedding numeracy in subjects across the curriculum required explicit attention to numeracy within subjects and breaking down the view, held in Kylie's opinion by many teachers and students, that each subject has its own knowledge and practices that are only applicable within that subject.

Kylie thought that there were many opportunities to use numeracy to develop students' conceptual understanding of history. For example, she thought that using representational tools enabled students to gain a better understanding of some of the data that was encountered in history.

> We spend a lot of time discussing the concepts, like, 'What percentage here? What percentage there?' … We don't spend a lot of time transferring that into easy to look at information … We don't particularly follow through with those kinds of tools like turning it into a pie chart, into a graph (Scoping interview).

Kylie also described how she had used numeracy to a The purpose of the paper is to test the existing framework for identity as an embedder-of-numeracy with empirical data. ssist students to understand what it was like in medieval times.

> [W]e look at the Black Plague and how many were affected and if, what percentage of people in the world today. Like, we did how many people in the world would have been killed, one to two thirds, one to two thirds of the world, of Europe and then we looked at the school and then we looked at the classroom and decided who gets killed by the Black Plague. They all re-enacted it with disgusting accuracy and so it's much simpler. They just needed to understand how bad the Black Plague was. So it was a very easy concept to apply numeracy to … we said it was devastating and I think the problem was that they didn't understand, like, they have a lot of difficulty identifying the concepts in the medieval world … it was trying to build their understanding (Scoping interview).

Kylie was reasonably confident that she could deal explicitly with the mathematical content in history lessons: "Once I look at it I can probably do it as long as it's not too complicated" (Scoping interview).

Social Domain

As teachers at Kylie's school were allocated to staffrooms based on their discipline, Kylie shared her staffroom with other English teachers as well as with Business and Information Technology teachers. She reported that some of her English teaching colleagues found it strange that she had participated in a research project on numeracy and that the general feeling in her staff room was that there was no difference between numeracy and mathematics. Although there were three other teachers at Kylie's school who participated in the Numeracy Project, opportunities to work with these teachers had been limited because they were located in other staffrooms. Kylie expressed a desire for more internal professional learning within her school community and more integrated planning across disciplines, which she felt, could be achieved by having:

> a general meeting at the beginning [of the year] and just that awareness and that way it would open those communication lines … breaking down those kid’s ideas … separate ideas, separate subjects, separate skills. When it’s not. It’s one subject, one skill (Final interview).

Apart from her interactions with researchers and teachers from other schools who were also participants in the Numeracy Project, opportunities for Kylie to network with others outside her school community were limited. She was not involved in any professional associations for history and, although she had seen advertisements for a small number of professional development workshops about numeracy, the majority of these were offered in the state’s capital city or another regional city, both about 500 kilometres from her school. Kylie had discussed her plans for embedding numeracy in history with a mathematics teacher from another school whom she lives with. She had found these discussions useful, even though were limited to whether students in a particular year level could be expected to understand the mathematics needed for the tasks she was planning.

Discussion

Kylie’s EoN Identity included affordances and constraints on her capacity to embed numeracy into the discipline of history. Kylie’s life history domain has contributed to the current state of her knowledge and beliefs. Within her knowledge domain, she seemed to have the appropriate MCK and her CK is likely to develop over time as she becomes familiar with the new curriculum. However, the absence of a focus on numeracy across the curriculum during her pre-service teacher education program suggests that she may need support to develop the appropriate PCK. Kylie’s affective domain was supportive of an across the curriculum approach to numeracy. Although her initial understanding of numeracy was focused on mathematics, only one of the five dimensions in Goos et al.’s (2014) numeracy model, her personal conception of numeracy appeared to be broadening as a result of her participation in the Numeracy Project. She provided evidence that she saw a need for numeracy in supporting students’ learning in the discipline of history, in a similar way to that described by Blow et al. (2012), and expressed confidence that she had the mathematics ability to support this. Within Kylie’s context domain, the new history curriculum and her participation in the Numeracy Project provided support for an across the curriculum approach to numeracy. However, Kylie must overcome the challenges that implementation of the new curriculum brings in a school environment where the focus is primarily on improving students’ NAPLAN performance in literacy: a focus that can influence pedagogy (Hardy, 2014). Kylie’s social domain, apart from her interactions with those associated with the Numeracy Project, although not negative, did not actively promote an across the curriculum approach to numeracy.

This analysis suggests that there are several ways that Kylie could be supported to strengthen her EoN Identity. Within her knowledge domain, her main need appeared to be increasing her PCK to enable her to design tasks that support numeracy learning while at the same time enhancing conceptual understanding in history. However, there are also changes that could be made within her context and social domains to facilitate this learning. For example, within her context domain the Literacy and Numeracy Committee could put greater emphasis on a whole of school approach numeracy or within her social domain, the expertise of the mathematics department could be utilised to assist teachers of other disciplines in their planning for numeracy.

Concluding Remarks

In this paper, the EoN Identity framework (Bennison, 2015) has been used to describe Kylie's EoN Identity and identify some ways to support her to embed numeracy into the discipline of history. However, as identity is dynamic (Wenger, 1988), the EoN Identity framework provides a snapshot Kylie's EoN Identity at one point in time and her current needs to support embedding numeracy into history. At a different time in her career, and for other teachers, the aspects of the domains of influence will be different: resulting in different EoN Identities and needs. One of the strengths of the EoN Identity framework is that it accommodates the temporal nature of identity because the domains of influence overlap and are continually changing.

The purpose of the empirical phase of the study reported on in this paper was to test the framework for EoN Identity that was developed from a theoretical perspective (see Bennison, 2015). Therefore, extensive data collection was warranted. However, a potential practical limitation of the EoN Identity framework is the time required to collect sufficient information to describe a teacher's EoN Identity. Therefore, further research in needed to fully test the EoN Identity framework with the case studies of the other teachers participating in the study and to develop a streamlined means of collecting adequate information for describing a teacher's EoN Identity.

References

Australian Curriculum Assessment and Reporting Authority [ACARA]. (2014). Australian Curriculum (Version 7.0). Retrieved 6 August 2014, from http://www.australiancurriculum.edu.au/Download/F10

Bennison, A. (2015). Developing an analytic lens for investigating identity as an embedder-of-numeracy. *Mathematics Education Research Journal*, *27*, 1-19. doi:10.1007/s13394-014-0129-4

Blow, F., Lee, P., & Shemilt, D. (2012). Time and chronology: Conjoined twins or distant cousins? *Teaching History, 147*, 26-34.

Department of Employment Education Training and Youth Affairs [DEETYA]. (1997). Numeracy = everyone's business. The report of the Numeracy Education Strategy Development Conference. Adelaide: Australian Association of Mathematics Teachers.

Enyedy, N., Goldberg, J., & Welsh, K. M. (2005). Complex dilemmas of identity and practice. *Science Education, 90*, 68-93. doi:10.1002/sce.20096.

Goos, M., Geiger, V., & Dole, S. (2014). Transforming professional practice in numeracy teaching. In Y. Li, E. Silver, & S. Li (Eds.), *Transforming mathematics instruction: Multiple approaches and practices* (pp. 81-102). New York: Springer.

Hardy, I. (2014). A logic of enumeration: The nature and effects of national literacy and numeracy testing in *Australia. Journal of Educational Policy*. doi: 10.1080/02680939.2014.945964

Organisation for Economic Co-operation and Development [OECD]. (2013). PISA 2012 Assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy. Retrieved 6 August 2014, from http://dx.doi.org/10.1787/9789264190511-en

Quinnell, R., Thompson, R., LeBard, R. J. (2013). It's not maths; it's science: Exploring thinking dispositions, learning thresholds and mindfulness in science learning. *International Journal of Mathematical Education in Science and Technology*, *44*(6), 808-816. doi: 10.1080/0020739X.2013.800598

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-21.

Stake, R. E. (2003). Case studies. In N. K. Denzin, & Y. S. Lincoln (Eds.), *Stategies of qualitative inquiry* (2nd ed., pp. 134-164). Thousand Oaks, CA: Sage.

Steen, L. A. (2001). The case for quantitative literacy. In L. A. Steen (Ed.), *Mathematics and democracy: The case for quantitative literacy* (pp. 1-22). Princeton, N.J.: National Council on Education and the Disciplines.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.

Young Children's Number Line Placements and Place-Value Understanding

Brenda Bicknell
The University of Waikato
<bicknell@waikato.ac.nz>

Jenny Young-Loveridge
The University of Waikato
<educ2233@waikato.ac.nz>

In this paper we report on two assessment tasks extracted from a larger study. The tasks involved number-line placements on two different number lines (0-to-10 and 0-to-20) and place-value understanding. Participants were 119 children from four different classes (Years 1-3). Children's placements were more accurate on the 0-to-20 than the 0-to-10 number line but many found midpoint placements difficult. Children with good place-value understanding were better than their peers at making accurate number-line placements. The findings have implications for practitioners in making more explicit the connections between number and space.

Background

The representation of numerical quantity is a complex multi-dimensional domain. Dehaene, Piazza, Pinel, and Cohen (2003) propose three systems that contribute to the processing of number, each involving activation of different parts of the brain. The verbal system represents numbers as words and focuses particularly on the memorisation and recall of number facts. The other two systems are nonverbal, including a "visual system" that encodes numbers in terms of a mental number line running from left to right, and a "quantity system" that represents the size and distance relations between numbers. One type of magnitude estimation comprises translation from one non-numerical magnitude into another form of non-numerical magnitude, such as estimating a quantity by indicating it as a position on a number line. The other type of magnitude estimation is numerical, such as assigning line lengths to numbers.

The nature of the cognitive systems associated with magnitude estimation are strongly debated in the literature (Moeller, Pixner, Kaufmann, & Nuerk, 2009; Núñez, 2011; Núñez, Cooperrider, & Wassmann, 2012). Number seems to be initially coded *logarithmically* where the distances between adjacent numbers on the mental number line decrease as their magnitudes increase. It has been argued that formal schooling and other cultural practices lead to changes in coding from logarithmic to linear (Booth & Siegler, 2008; Siegler & Booth, 2004; Dehaene, Izard, Spelke, & Pica, 2008; Núñez, et al., 2012), and this is correlated positively with mathematics achievement. For example, learning to integrate tens and ones in the place-value system could help to explain the apparent transition from logarithmic to linear representations with age (Moeller et al., 2009).

Older children are better at magnitude estimation than younger children, and smaller numbers are represented more accurately than larger numbers (e.g., Barth & Paladino, 2011; Praet & Desoete, 2014; Rouder & Geary, 2014). Children's ability to place numbers on a number line is strongly related to their understanding of proportional reasoning and overall mathematical achievement (Rouder & Geary, 2014). Anchor points at the beginning and end of the line are used to help place numbers by children as young as six years old. Older children (7- to 10-year-olds) are able to make use of a third anchor point (the midpoint) to place numbers more accurately (Slusser, Santiago, & Barth, 2013). Understanding geometric ideas such as the axis of symmetry also helps children to use the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 101–108. Sunshine Coast: MERGA.

midpoint in making placements on a number line (Mulligan & Mitchelmore, 2013; Spence & Krizel, 1994).

The research on numerical magnitude and number-line representation links to the distinction made by Yackel (2001) between counting-based and collections-based approaches to working with numbers. Both approaches are important for developing a deep and connected understanding of number. There is an 'inherent contradiction' in the way that Western children are initially encouraged to count by ones (unitary *counting-based* concepts), but then are expected to reorganise these into *collections-based* concepts involving units consisting of tens and ones when place-value instruction begins (Yang & Cobb, 1995).

Research on children's awareness of mathematical pattern and structure (AMPS) shows the importance of students developing an awareness of structural relationships in mathematics (e.g., Mulligan, 2011). Low levels of AMPS seem to be associated with having poor visual and working memory. Mulligan found that students with low AMPS tended to "rely on superficial unitary counting by ones" (p. 36), and did not develop efficient and flexible strategies for solving problems. AMPS also appears to impact on the development of measurement concepts and proportional reasoning. Mulligan's work on promoting awareness of pattern and structure is consistent with other research on the importance of helping children develop knowledge of place-value structure (Cobb, 2000; Fuson, Smith, & Cicero, 1997; Thomas, Mulligan, & Goldin, 2002).

The development of place-value understanding requires children to be familiar with the concept of unit, and appreciate the difference between units of ten and units of one. Children need to be part-whole thinkers in order to partition numbers into tens and ones (Fuson, Smith, & Cicero, 1997; Ross, 1989). A key feature of place-value development is the shift from a unitary (by ones) way of thinking about numbers to a multi-unit conception (e.g., tens & ones). Place-value knowledge has four major properties: positional, base-ten, multiplicative, and additive (Ross, 1989). Because place-value understanding is inherently multiplicative, it is more complex than additive thinking (Clark & Kamii, 1996; Vergnaud; 1994). Multi-digit arithmetic requires not only an understanding of the place-value system for the Arabic number system but also an understanding of the magnitude of numbers (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011).

The theoretical perspective taken in this paper was informed primarily by the extensive work of Mulligan and colleagues on the importance of pattern and structure for mathematical thinking (e.g., Mulligan, 2010, 2011; Mulligan, Mitchelmore, English, & Crevensten, 2013) and the literature in the multiplicative conceptual field (e.g., Clark & Kamii, 1996; Vergnaud, 1994). These two fields of research led to the research question focussing on the relationship between young children's number line placements and place-value understanding. We explored how accurately young children mapped one- and two-digit numbers on number lines, their understanding of two-digit numbers, and the relationship between these constructs. This research was part of a larger study that focused on developing children's part-whole thinking through the use of multiplication and division problem-solving contexts. Selected baseline data from the study was analysed to answer the research question.

The Study

This exploratory study was set in an urban school (medium socioeconomic status) in New Zealand. The participants were 119 five- to seven-year-olds (59 girls and 60 boys) from Years 1 to 3 (average age at each year level: 5.5, 6.5, 7.3 years). There were 42 Year

1, 34 Year 2, and 43 Year 3 children from four different classes. The children were from a diverse range of ethnic backgrounds, with approximately one third Māori (the indigenous people of New Zealand), one third European, one fifth Asian, and the remainder from other ethnicities including African and Pasifika (Pacific Islands people). Approximately one quarter of the children were English Language Learners. The children were assessed using an individual diagnostic task-based interview designed to explore number knowledge and problem-solving strategies. The assessment tasks included: subitising, addition, subtraction, multiplication, division, basic facts, incrementing in tens, counting sequences, number-line placement, and place value. The two tasks reported in this paper focused on the latter two categories.

In the first task (number-line placements), children were shown a number line with 0 and 10 marked on it (see Figure 1). The interviewer said: "This number line goes from zero to ten. Where does five belong on this number line?" The children then indicated the estimated position on the number line, which was recorded by the interviewer. This was followed by questions about the placements of two and one. Children were then shown another number line with 0 and 20 marked on it (See Figure 1). The same process was used for the placement of 19, 10, and one. Later, the researcher measured and recorded the distance in millimetres between zero and the child's placement of the target numbers on the number lines. The number line placements were coded from 0 to 3 based on the accuracy of the position. Placements within 10 per cent of the target position were coded 3, 11 to 20 per cent were coded 2, 21 to 50% were coded 1, and the others were coded 0.

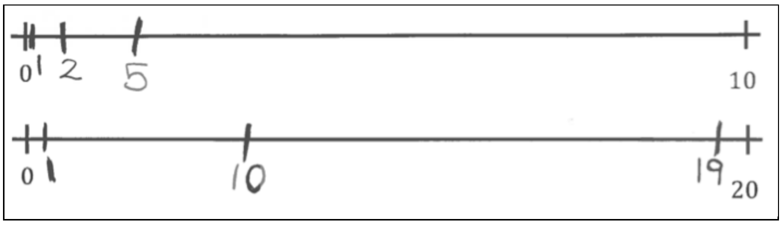


Figure 1. Record of one child's responses to number-line placements

In the second task (place value), children were shown a picture of two ten-sticks (each ten stick represented by a row of five grey boxes joined to a row of five white boxes) and four singleton boxes. The children were asked to find the total number of boxes (their strategy was recorded) and then to write this number above the picture. The interviewer circled the digit "4" in "24" and asked: "which boxes might the four mean?" The collection indicated by the child was circled and a line drawn connecting the boxes with the digit "4." The interviewer circled the digit "2" in "24" and asked: "Which boxes does the '2' in '24' mean?" This was recorded in the same way as the "ones" digit. Finally, the interviewer asked: "So, what is the '2' in '24' telling you?" The interviewer recorded how the children determined the total number of boxes, and whether they linked the "4" to four boxes, and "2" to 20 boxes. Figure 2 shows a record of one child's correct responses to the task.

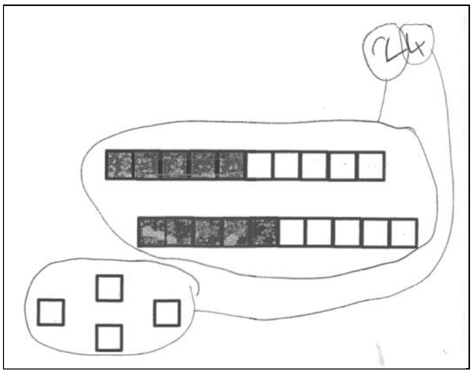


Figure 2. Record of one child's responses to questions about the meaning of 4 and 2 in 24

Results

The first task required children to estimate the placement of numbers on the two different number lines, 0-to-10 and 0-to-20. Table 1 shows the median number-line placements for 5, 2, and 1 on the 0-to-10 number line, and 19, 10, and 1 on the 0-to-20 number line (measured in millimetres) by year group. The table also shows the discrepancy between the median and correct position in brackets, and the minimum and maximum values (range).

Table 1
Median Number-Line Placement in mm, (Discrepancy), and Range for Each Year Level

	Correct Place	Y1 (n=42)	Y2 (n=34)	Y3 (n=43)
0-to-10 Line				
Place "five"	80	40 (40)	38 (42)	67 (13)
Range		1 to 169	15 to 100	21 to 85
Place "two"	32	12 (20)	12 (20)	14 (18)
Range		3 to 157	4 to 25	6 to 31
Place "one"	16	5 (11)	3 (13)	5 (11)
Range		1 to 170	1 to 12	1 to 19
0-to-20 Line				
Place "nineteen"	152	142 (10)	147 (5)	148 (4)
Range		2 to 169	71 to 155	79 to 156
Place "ten"	80	84 (-4)	74 (6)	78 (2)
Range		-2 to 162	29 to 132	33 to 130
Place "one"	8	7 (1)	5 (3)	6 (2)
Range		-2 to 145	1 to 11	1 to 15

Year 3 children were, on average, far more accurate with their placements than the other year groups. These children were also most accurate in placing 10 and 1 on the 0-to-20 number line, with a median discrepancy of only 2 mm short of the correct position. Their accuracy was greater on the 0-to-20 number line than on the 0-to-10 number line, and they were least accurate in placing 2 on the 0-to-10 number line, with a median discrepancy of 18 mm short of the correct position.

Year 1 and Year 2 children were most accurate in placing 1 on the 0-to-20 number line, with the median placements being less than 5 mm short of the correct position. The next most accurate placement for these children was 10 on the 0-to-20 number line, with the median position 4 mm beyond the actual position for Year 1 (shown as a negative value in Table 3), and for Year 2, the median placement was 6 mm to the left of the position (a positive value).

The second task (place-value) was given to all children who could successfully complete several ten-structured tasks such as subitising a ten-frame and finding half of 20. This reduced the sample size for the place-value tasks to 12 Year 2 and 35 Year 3 children (n = 47). Table 2 shows the strategies used by the 43 out of 47 children who correctly determined that there were 24 boxes in total (see Figure 2). These strategies included counting by ones, fives, and tens. Approximately half of the children counted by tens to determine the number of boxes. Almost one-quarter counted by ones (n = 12), while seven children counted by fives.

Table 2
Strategies Used to Count 24 Boxes and Make the Links between Digits and Quantity

Year	By ones	By fives	By tens	Links “4” to 4 Boxes	Links “2” to 20 boxes
2	6	1	4	8	3
3	6	6	20	32	20

When asked to link digits with quantities, most (85%) of these 47 children were able to link the “4” in “24” to four single boxes (see Table 2). Twenty-three children (49%) made the correct place-value link (the “2” in 24 to two tens or to 20). The children who could count 24 boxes by tens were not necessarily the same children who could link the “2” in 24 to 20 boxes. Individual profile data showed that one Year 2 and 17 Year 3 children counted by tens and made this place-value link. The 23 children who were able to connect the “2” in “24” to two tens or 20 were selected for further analysis to explore the relationship between place-value understanding and number-line knowledge. The accuracy of their number-line placements is shown in Table 3.

Table 3
Accuracy of Number-line Placements for the 23 Children who Correctly Linked 2 in 24 with Two Tens or 20

	0-to-10 number line			0-to-20 number line		
Number	"five"	"two"	"one"	"nineteen"	"ten"	"one"
Actual length	80 mm	32 mm	16 mm	152 mm	80 mm	8 mm
≤ 10%	10	2	1	22	13	8
11-20%	6	1	4	1	4	2
21-50%	3	7	5	0	5	9
> 50%	4	13	13	0	1	4

Placing 5 on the 0-to-10 number line (the midpoint) was easier for these children than placing 2 or 1 (43% vs. 9% and 4%, respectively). All but one child accurately placed 19 on the 0-to-20 number line (96%). They were not so accurate in placing either 10 or 1 on the 0-to-20 number line (57% and 35% within 10% of the target, respectively). The placement of 10 (the midpoint) was more accurate than the placement of 1, showing children did not notice that 19 and 1 are equidistant from the two endpoints. These children were more accurate in placing numbers on the 0-to-20 number line than the 0-to-10 number line. When the 23 children who made accurate place-value links were compared to the 43 Year 3 children (the oldest year group), the 23 children were slightly better at making number-line placements, with a greater proportion making the most accurate placements (within 10% of the target) for 5 and 10 (43% vs. 35%; 57% vs. 49%).

Discussion

The assessment tasks reported in this paper were designed to explore young children's number-line knowledge reflected in the placement of one- and two-digit numbers on number lines, and their understanding of two-digit numbers as represented by ten sticks (composed of two groups of five) and singleton boxes. The 119 children were more accurate in placing numbers on the 0-to-20 number line than 0-to-10. This could be explained by the fact that the 0-to-10 line (two anchor points) was presented first to help the children to become familiar with the task. The first placement question for the 0-to-10 line was 5, but most children did not recognise 5 (the midpoint) as a third anchor point that could help in making a more accurate placement (Rouder & Geary, 2014). This recognition of the midpoint relates to understanding about an axis of symmetry and proportional reasoning, which was evident in responses from older children in other studies (e.g., Mulligan & Mitchelmore, 2013; Spence & Krizel, 1994).

Overall, children were more accurate in placing 19 on the 0-to-20 line than 1, failing to recognise that both these placements were equidistant from the anchor points at each end. These young children had not yet established number-to-space connections that could have supported their number-line placements (Núñez, et al., 2012). A few children made negative placements (i.e., to the left of zero), lacking awareness that all whole numbers are to the right of zero. Other children placed 10 and 19 to the right of 20, suggesting that they had some weaknesses in their knowledge of number sequences.

Many of the children in this study did not appear to have developed a sense of the midpoint as the third anchor point because of little or no experience with number-line placement. This finding is consistent with researchers who have found that the use of the midpoint to make number line placements appears about the third year of school (e.g., Barth & Paladino, 2011; Slusser et al, 2013). This could be explained by the focus in many New Zealand schools on teaching number in isolation from the other domains within the mathematics curriculum. This practice does not help children to build the connections highlighted by the research on spatial structuring and number. For example, experiences with the axis of symmetry in the context of geometry, as well as halving quantities and shapes could help children build a deeper more connected understanding of the relationships among numbers (Mulligan & Mitchelmore, 2013).

In the place-value task, children used a range of strategies to determine that there were 24 boxes in the picture. Half of the children recognised that two groups of ten (as represented by ten-sticks) made 20 in total, and quickly determined that there were 24 boxes altogether. This is consistent with the work of Fuson and colleagues (1997) on the developmental trajectory from unitary to ten-structured thinking, and then progression to multi-unit conceptions of number. A few children (n = 7) took advantage of the quinary structure of the ten-sticks and counted by fives to 20 (Mulligan, 2010). One quarter of the children counted the ten-sticks by ones (unitary counting), and this included both Year 2 and Year 3 children. This could be explained by the continued emphasis in the early years of school on counting by ones, as reflected in curriculum documents such as the Mathematics Standards (Ministry of Education, 2009) which expect children after one year at school to add by counting all, and after two years at school, to add by counting on.

Only 18 children used groups of ten to determine that there were 24 boxes, and correctly linked the “2” in “24” to 20 boxes. Despite having the beginnings of place-value understanding (as reflected in linking digits to quantities), five of the 23 children had used counting by ones to determine the total, ignoring the groups of five and ten clearly evident in the picture. These results reflect the complexity of part-whole understanding and ten-structured thinking (Fuson et al, 1997; Ross, 1989).

The 23 children, mostly from Year 3, who successfully made place-value links between digits and quantities, were also reasonably competent with the number-line placement task. This provides evidence that their recognition of the linear aspect of number lines is consistent with research showing that older children perform better on magnitude estimation (Praet & Desoete, 2014; Rouder & Geary, 2014). However, these children were more accurate in placing 19 and 10 than 1 on the 0-to-20 number line, and placing 5 than 2 or 1 on the 0-to-10 number line. Perhaps it was easier for them to place the 5 and 10 because they used the midpoint as a third anchor point to make the placement. They also may have used their awareness of the axis of symmetry (Mulligan & Mitchelmore, 2013).

The findings reported here could be useful for classroom teachers in emphasising the importance of helping children make connections between different representations of two-digit numbers. Multiple representations for two-digit numbers, including using materials such as Unifix cubes, ten-frames, Slavonic abacus, numeral cards, and number lines, could help children strengthen connections between visual and non-visual systems. Links between measurement concepts, proportional reasoning, and numerical magnitude could be made as children learn to divide a distance on a number line in order to estimate more accurately the placement of numbers (e.g., by folding a number line in half). By varying the anchor point at the right-hand end of the number line (e.g., 0-10, 0-20, 0-100), an appreciation of proportionality could be further developed (e.g., half of 10 is 5, half of 100 is 50). This could strengthen understanding of relationships among numbers and enable children to make more accurate number-line placements.

Acknowledgements

This project was made possible by funding from the Teaching and Learning Research Initiative [TLRI] through the New Zealand Council for Educational Research and the interest and support of the teachers and children involved in the project.

References

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science, 14*, 125-135.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79*, 1016-1031.

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education*, 27, 41-51.

Cobb, P. (2000). Constructivism in social context. In L. P. Steffe & P. W. Thompson (Eds.), *Radical constructivism in action: Building on the pioneering work of Ernest von Glasersfeld* (pp. 152-178). London, UK: Routledge/Falmer.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*, 487-506.

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science, 320*, 1217-1220.

Fuson, K. C., Smith, S. T., & Cicero, A. M. L. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*, 738-766.

Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H-C. (2009). Children's early mental number line: Logarithmic or decomposed linear? *Journal of Experimental Child Psychology, 103*, 503-515.

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance: A longitudinal study on numerical development. *Research in Developmental Disabilities, 32*(5), 1837-1851. http://dx.doi.org/10.1016/j.ridd.2011.03.012

Ministry of Education (2009). *The New Zealand curriculum mathematics standards for years 1-8*. Wellington, NZ: Author.

Mulligan, J. T. (2010). Reconceptualising early mathematics learning. *Teaching mathematics? Make it count: What research tells us about effective teaching and learning of mathematics* (Proceedings of the Australian Council for Educational Research Conference) (pp. 47-52). Camberwell, Vic: ACER.

Mulligan, J. (2011). Towards understanding the origins of children's difficulties in mathematics learning. *Australian Journal of Learning Disabilities, 16*, 19-39.

Mulligan, J. T., & Mitchelmore, M. C. (2013). Early awareness of mathematical pattern and structure. In L. D. English and J. T. Mulligan (Eds.), *Reconceptualizing early mathematics learning* (pp. 29-46). New York, NY: Springer.

Mulligan, J. T., Mitchelmore, M. C., English, L. D., & Crevensten, N. (2013). Reconceptualising early mathematics learning: The fundamental role of pattern and structure. In L. D. English and J. T. Mulligan (Eds.), *Reconceptualizing early mathematics learning* (pp. 47-66). New York, NY: Springer.

Núñez, R. E. (2011). No innate number line in the human brain. *Journal of Cross-Cultural Psychology, 42*, 651-668.

Núñez, R., Cooperrider, K., & Wassmann, J. (2012). Number concepts without number lines in an indigenous group of Papua New Guinea. *PloS one, 7*(4), e35662, 1-8.

Praet, M., & Desoete, A. (2014). Number line estimation from kindergarten to grade 2: A longitudinal study. *Learning and Instruction, 33*, 19-28.

Ross, S. H. (1989). Parts, wholes, and place value: A developmental view. *Arithmetic Teacher, 36* (6), 47-51.

Rouder, J. N., & Geary, D. C. (2014). Children's cognitive representation of the mathematical number line. *Developmental Science, 17,* 525-536.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*, 428-444.

Slusser, E. B., Santiago, R. T., & Barth, H. C. (2013). Developmental change in numerical estimation. *Journal of Experimental Psychology: General, 142*, 193-208.

Spence, I., & Krizel, P. (1994). Children's perception of proportion in graphs. *Child Development, 65*, 1193-1213.

Thomas, N. D., Mulligan, J. T., & Goldin, G. A. (2002). Children's representation and structural development of the counting sequence 1–100. *The Journal of Mathematical Behavior, 2*1, 117-133.

Vergnaud, G. (1994). Multiplicative conceptual field: What and why? In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 41–59). Albany, NY: State University of New York Press.

Yackel, E. (2001). Perspectives on arithmetic from classroom-based research in the United States of America. In J. Anghileri (Ed). *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom*. (pp. 15-31). Buckingham, UK: Open University Press.

Yang, M. T. & Cobb, P. (1995). A cross-cultural investigation into the development of place-value concepts of children in Taiwan and the United States. *Educational Studies in Mathematics, 28*, 1-33.

The Role of Cultural Capital in Creating Equity for Pāsifika Learners in Mathematics

Trevor Bills
Massey University
<t.d.bills@massey.ac.nz>

Roberta Hunter
Massey University
<r.hunter@massey.ac.nz>

Despite the Ministry of Education Statement of Intent 2014 – 2018 that the performance of the education system for priority students – Māori, Pāsifika, students with special education needs and students from low socio-economic areas needs to improve rapidly these groups remain a concern in the New Zealand Education System. This article explores what happens to a group of Pāsifika students and their teachers when the teachers draw on Pāsifika focused culturally responsive teaching in the mathematics classroom. Changes to the identity and mathematical disposition of the Pāsifika students are illustrated when their ethos becomes the cultural capital valued in the classroom using teacher and student voice.

Introduction

In New Zealand a disproportionate number of Pāsifika students perform below their European and Asian counterparts (New Zealand Qualifications Authority, 2013). In order to address significant disparities in numeracy and literacy achievement the Ministry of Education affords priority to this specific group of learners (Ministry of Education, 2014). The goal for priority Pāsifika learners is placed on ensuring high quality and inclusive teaching that incorporates aspects of the students' language, identity, and culture. Educationalists are charged with the responsibility to place them, their parents, fānau (families), and communities central to changes aimed to increase Pāsifika capability and competence. Central to the changes is the goal to draw on knowledge and understandings of Pāsifika culture and its use in Pāsifika focused pedagogy – a goal which may hold challenges for many teachers. Developing appropriate pedagogy situated within the known world of their Pāsifika students is difficult given that the cultural experiences of Pāsifika learners may be different in both obvious and subtle ways from those commonly experienced by many New Zealand teachers. This paper explores what happens when teachers explicitly explore ways they can engage with the language, culture, and identity of their Pāsifika students to structure the mathematical activity in the classroom. The key question we examined was: What is the effect on Pāsifika students' relationship with mathematics when teachers use Pāsifika focused culturally responsive pedagogy?

Literature Review

Pāsifika students enter New Zealand schools with a rich background of experiences. However, the lived reality in the school life of many of them can be significantly different from their home life experiences. As a result researchers (e.g., Anae, Coxon, Mara, Wendt-Samu, & Finau, 2001; Barton, 1995; Hunter & Anthony, 2011) argue that this is a contributing factor in their underachievement and disengagement with their New Zealand schooling. As Bartolomé (1996) explains, unless educational methods are situated in students' cultural world they will continue to show difficulty in mastering content area. This is because the learning is not only alien to their reality, but may also be antagonistic to their culture and lived experiences – that is their cultural capital. The term cultural capital used by Bourdieu and Passeron (1973) is defined by McLaren (1994) as being the general

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 109–116. Sunshine Coast: MERGA.

cultural background, knowledge, disposition, and skills that are passed on from one generation to another. Cultural capital represents "ways of talking, acting, and socialising, as well as language practices, values, and types of dress and behaviour" (McLaren, 1994, p. 219).

The cultural capital valued in many New Zealand mathematics classrooms reflects the cultural capital of the dominant New Zealand cultural groups. For example in recent professional development reforms (Ministry of Education, 2004), the grouping system promoted and used in most mathematics classrooms drew on the dominant group of Western origins beliefs and values. The use of streamed groups encouraged competitiveness and placed importance on individual success. An emphasis on the individual in the NZ Numeracy project contrasts directly with Pāsifika notions of the value of collectivism. Within a Pāsifika view the success of group members is measured by the success of the collective as a whole. This may well suggest a reason for why Pāsifika students were not as successful in this project as the Asian and European cohort. However, recent New Zealand studies (Hunter, 2013; Hunter & Anthony, 2011) illustrate that when teachers draw on the cultural background of their Pāsifika students and use them to shape the interactions in the mathematics classroom both their mathematical achievement results and engagement in communicating their mathematical reasoning increase. These studies provide persuasive evidence that when teachers draw on the cultural context and values of their Pāsifika students and use them to engage and connect them to mathematics their mathematical learning is accelerated and their mathematical disposition enhanced.

To engage and connect Pāsifika students to mathematics not only do teachers need to consider the cultural context and values of the students they also need to take into account Pāsifika languages, culture and identity. This is in line with the seminal work of Paulo Freire (2000). He argues the need for education to transform oppressive structures by engaging people who have been marginalized and dehumanized and drawing on what they already know. Other researchers (e.g., Frankenstein, 2010; Stinson, Bidwell, & Powell, 2012) building on this epistemology through ethnomathematics argue the need for educators to learn about how cultural practices – daily practice, language, power, and ideology constitute people's views of mathematics and their ways of thinking mathematically. Moreover, Freire (2000) emphasises the importance of dialogical education built on respect with people working together in reciprocal relationships. Freire (2000) argues the importance of egalitarian dialogue, based on the validity of the argument and not on power imbalances in the teacher and student relationship. He also contends that praxis or informed action embedded in values is also important as is situating the educational activity in the lived experience of the learners. This is illustrated in Boaler's (2002) study which showed that in classrooms where teachers hold the balance of control over which methods and procedures are used students have less opportunity to interact with the mathematics. They also perceived that they were secondary to the teacher and became passive recipients of knowledge. In contrast in classrooms where power was more evenly distributed students developed positive dispositions which led to deeper mathematical understandings.

In a recent MERGA paper, Jorgensen (2014) presented a challenge to mathematics researchers. She argued that a new paradigm in mathematics education was needed because despite the many social theories of learning there remains a "significant problem with the outcomes in indigenous education in particular and equity target groups in general" (p. 311). She questions whether the inequitable outcomes may be structural and not something random. She suggests that schools and education are structured in ways that reproduce inequality. In response, this paper draws on a critical pedagogical lens in order to understand what the relationship is between learning and social change (Giroux, 1983). Giroux (1983) explains that within this frame, students are listened to and provided with a voice and role in their own learning. In turn, teachers not only educate students but also learn from them in a reciprocal relationship. As such critical pedagogy is a humanizing pedagogy that values students' (and teachers') background knowledge, culture, and lived experiences (Bartolomé, 1996).

Methodology

This research is part of a large project which involves 240 teachers in twenty New Zealand urban full primary schools. The students come from very low socio-economic home environments and are of predominantly Pācific Nations groupings. Many of them speak English as a second language and some are new immigrants with little English. The project looks at changing the cultural capital of the mathematics classroom so that it better matches that of Pāsifika students. Complex and challenging mathematics problems are devised around aspects of Pāsifika culture and the students' daily lives.

Core Pāsifika values are explicitly used to underpin the social and sociomathematical norms constructed within the mathematical inquiry community. These values include reciprocity, respect, service, inclusion, family, relationships, spirituality, leadership, collectivism, love, and belonging. The students are placed in mixed ability groups which runs counter to common New Zealand practices and the Pāsifika concept of collectivism is used to structure small group activity. This builds on the notion of success of the group as a collective, rather than as individual members in the group. Another example of the use of Pāsifika values is the use of reciprocity and service. The students are structured to be responsible to ask and respond to questions and use mathematical argumentation in ways considered respectful as a Pāsifika person.

In this project students were also encouraged and supported in using their first language when discussing, explaining and justifying their mathematical understandings. This recognised that Pāsifika students often have to deal with various language difficulties when learning mathematics including at times their first language not having an equivalent word for the concept that they are learning about. In addition, in order to achieve success in mathematics the students need to be able to read and understand the mathematical problems set for them. This can prove difficult for Pāsifika students who do not have English as their first language. During the course of the research these issues were minimised by encouraging the students to switch between their first language and English in order to develop deeper understandings. One of the Samoan teachers also regularly switched from English to Samoan to help individual students clarify the problems and to get them to explain their reasoning.

In contrast, prior to involvement in the larger project the teachers taught according to the New Zealand Numeracy Project (Ministry of Education, 2004). The students experienced mathematical activity within ability groups and tasks were drawn from the Numeracy Project which was English language based and tends to better match the beliefs and values within the frame of the more dominant Western participants. This paper reports on three teachers (2 Pāsifika and 1 Māori) and their students who ranged in age from 10 – 12 years. Pseudonyms are used for the three teachers. In their previous mathematics teaching all three teachers described themselves as the disseminator of knowledge and the main power holder in the classroom.

For this paper only one section of the data was used. This involved a set of open ended questions which allowed for multiple responses. Analysis of the data consisted of comparing and contrasting responses from the different teachers and students. Emerging themes and patterns were determined and analysed.

Results and Discussion

The initial interview question explored how addressing the cultural identity and the core Pāsifika value of reciprocity changed how the students considered agency and power in the mathematical activity. In response Hone one of the classroom teachers explains:

> Hone: What I've noticed over the time I have been part of the programme is the importance of having the student voice as opposed to being teacher directed. When it's all teacher directed you can share till the cows come home but if it doesn't make sense to them it's not going to. If you give them a problem and say go away and have a look at that, come back to me, what do you think is happening? They start sharing their ideas and they feel valued because they are being listened to.

Student ownership of the mathematics had become shared within an expectation of mutual responsibility. In the changed roles students could be teachers while the teachers could learn from the students as Eti another teacher explains:

> Eti: There has been a shift in the percentage of student voice. The locus of control is not so much us, but what the students are discussing and sharing and that's really powerful seeing that shift from us having all the power to a shared power…and it's a shared responsibility and we become the facilitator and the students become teachers to each other and that's really good to see. Because there is more student voice and they have more control you can also see what the students know and how they think, they can explain their reasoning more. We can also learn more about them, like they come out with strategies to solve some of the problems that I haven't thought of and it's like…. yeah that works.

The students also described the way in which they now shared the responsibility for their own learning and the learning of others. They described how the power was shared between the teacher and students and how this changed the classroom mathematical talk. As the students explained:

> Sione: Yeah, It's not just one person answering the question; we get others' opinions so everyone understands. It's our responsibility as a team to work together. And it's good because you can talk to people. Before you just had your book and you got a growl if you talked or asked someone a question but now you can.

> Luana: In this maths we have more power. He (teacher) gives us the problem but the problem is about us …. our reality and we have to figure it out, we are responsible for our own learning and others' learning too, we have control.

A number of students also described how the teachers through listening to them learnt new ways of thinking mathematically. They also identified how the teachers were able to respond and progress their reasoning 'in the moment'. For example Siale stated:

> Siale: He can learn what we think and how to help us improve.

Clearly, within the more balanced power relationships what Freire (2000) described as dialogical education was evident. In these classrooms the mutual respect of all classroom members supported the development of reciprocal relationships.

The second interview question examined how integration of core Pāsifika values into the social and sociomathematical norms in the mathematical activity assisted the students with their learning and identity. The teachers described metaphorically how they were a Pāsifika family and how this frame shaped the interactions:

> Hone: Family is big, it’s everything. The way our classes are set up now everyone has a chance to share ideas, and like a family everyone helps out, and nobody is left out because everybody has a job to do and that’s the Pāsifika way and the Māori way. We talk about that a lot as a class, like if you are doing the housework everybody helps or if you are making an umu or hangi (earth oven) everybody has a job to do. It might be dig the hole or peel the spuds but you have a job… and like with a vaka (canoe) everybody has got to paddle in the same direction, in time if you are going to move and the kids can relate to that because that’s their world.

They described how respect shaped the way the students engaged in mathematical argumentation:

> Eti: Respect is a big part of being a family as well. Everyone shows respect because we are a family and they know they all have a say and have a chance to listen to what other people say and if things are not clear to them they can generate friendly arguments and say… hey where did you get that from because my understanding is this and everyone comes away with a broader understanding.

Taking risks with mathematical reasoning and making mistakes were presented as learning contexts within the context of the Pāsifika values:

> Sina: We talk about the value of respect and about accepting others, and that’s really strong. We encourage them to participate and take a risk but it’s safe because there is respect and inclusion and love so nobody is going to put you down if you get it wrong because we learn from our mistakes. We are all learners and we have the right to learn and the responsibility to listen to others because we are family.

The students also described how the different Pāsifika values shaped the way they engaged in mathematics. They described how respect for each other shaped how they engaged in mathematical argumentation:

> Sione: Respect is real important. When you have respect you can have friendly arguments and you argue about the maths so it’s not nah you’re wrong or you’re dumb eh, it’s like I don’t agree with your maths and this is what I think or you have to convince me.

> Grace: You can have friendly debates about maths……and then you have to justify your answer. And then if you made a mistake you learn from your mistake.

Reciprocity and collectivism also shaped their interactions as they took responsibility for their own mathematical reasoning and the reasoning of others’:

> Luana: It is not fair if you have the answer and nobody else knows what you are talking about. So you have to explain step by step to help them get to their answer so they understand and not just go I got an answer so…….You have to help each other figure it out. Everyone has to be included and contribute to the work. You have to encourage them to get their own answer though, not just give them the answer.

> Josef: Respect is important because they may have an answer that’s wrong, but don’t judge them. If you don’t show respect then how are we a team? How can we work together and take our ideas and put them together.

With the Pāsifika values incorporated into the ethos of the classroom there was no longer disconnect between the home life of the students and the school. Family, sharing and collectivism had become the cultural capital promoted in the classrooms rather than individualism and competition. Mutuality and respect were integral to the development of the mathematical inquiry community.

The next question explored how the use of problems which drew on the context of the students' every-day lives and were culturally relevant affected their learning. The teachers recognised how the different contexts lent themselves to learning across the mathematical strands. They also described how the students became more deeply engaged when the problems connected to their own reality:

Eti: Things around family…even if you were talking about food….how many corned beef cans… you can also link the strands in…measurement, volume, capacity, how many plates will you need…and the students can relate to it. Cultural contexts might be looking at tapa patterns and the types of maths in the patterns and they get to share back their experience and they can compare what they call it; because in Samoa it's Siapo and Tonga it's Ngatu and Fiji it's called Masi because if you just said tapa lots of the kids wouldn't know what you were talking about but they have these discussions and they all have a concept of the patterns but they hadn't seen the maths in that. You can then pose a problem and if they have a grasp you have a foundation for the discussion and we can then expand the maths….it's not until they make those connections they realise the real life situations they are involved in like a hair cutting ceremony or making ula lole (lolly necklaces) that maths links in so when we highlight that they are like WOW, there is maths here. Until we started to bring these types of problems they didn't make those links and they saw maths as something they did at school that was not relevant. This has been one of the most powerful parts of the maths project. The biggest concept for our kids to know is maths is everywhere……it is not just for maths time. That's the hook in…. we practise that with our family… we practise this in our church and in our community so when they make those links and can tie it into…. well maths is everywhere. Maths is part of my culture… the value of maths changes and the idea that maths is hard or alien or random changes as well.

The teachers recognised how the problems connected the students' home and school mathematical lives. The students also affirmed the importance of the use of relevant contexts and seeing themselves in the mathematics problems:

Josef: The maths is about us, about the community. The problems relate to our cultures and celebrations which makes it more understandable.

Luana: It makes it easier for us to learn…like the ula lole (lolly necklace) problem because most of us have made it before and we can see it and have a picture in our minds so we can see how it's proportions and ratios like one chocolate to three fruit burst or minties.

Grace: When the problems are about us you can see that maths is real and it's useful…...not just something random you do at school.

Importantly, not only did the problems connect their home and school mathematical life but it also normalised them as citizens within their own culture.

Sione: When the maths is about us and our culture it makes me feel normal, and my culture is normal.

Luana: Yeah like it is normal to be Samoan or Tongan.

The students were provided with opportunities to see mathematics in their 'lived life' while at the same time their 'lived life' was affirmed through ensuring that their cultural capital was reflected in both mathematics problems and activity.

The final question explored how the use of the first language of the different students supported them in engaging and learning mathematics. The teachers described how all class members supported each other:

Sina: My ESOL girl had only been in the country for a week so some of my Samoan girls helped her by speaking Samoan to her. They would read the question to her in Samoan so she could understand and she is able to talk back to them and explain what she is doing. I am Samoan so I understand what they are saying as well but if they were Cook Island I would just get some of the Cook Islanders to talk in their language and translate for me or represent in a different way so I would get them to draw it and I would understand what they are drawing so it doesn't matter what nationality they are. So it's just using different ways because she wasn't getting it but when they were able to switch and talk to her in her own language she was able to make connections and go okay now I know what they are talking about.

The teachers also described the use of different Pāsifika languages to deepen understanding and ensure that all members of the community could interact:

John: It's really powerful if they can use their own language because sometimes it might just be that they don't understand the question or even the ones that speak English there might not be a word in English that represents what they are talking about or they might be more confident speaking Samoan or Tongan and then others can translate. Without that, like in the past those kids didn't have a voice and you would just think they couldn't do it. It really helps transfer the power as well, as I don't always understand and they have to translate for me and their understanding really improves when they do this.

The students also realised the value of using different languages:

Sione: Sometimes it helps to explain things in Tongan because some of the Tongans in our class are new and their English isn't that good but they can understand the maths in Tongan which is cool because before you didn't really speak Tongan in class.

The encouragement of the use of students' first language in mathematical activity not only helped with deepening the conceptual understanding it also improved their self-esteem and disposition towards mathematics as language was no longer a barrier to success.

Conclusion and Implications

Jorgenson (2014) challenged the mathematics community to find other ways to address the inequitable outcomes which are evident when we look at who achieves in mathematics. This paper suggests some ways of beginning to address her challenge. The findings illustrate that when teachers do seriously consider the cultural capital of the students they carefully consider its influence in their teaching and so they begin to balance more equitable outcomes for their learners in mathematics. The lived reality of their students became what guided the ethos of the classroom. This supports Bartolomé's (1996) contention that when educational activity does not match the world of the learner they are precluded from achieving.

The responses from teachers and students reinforced what Hunter and Anthony (2011) had suggested regarding the importance of culturally relevant values when working with Pāsifika students. In this study evidence is provided of how students learnt how to interact with each other in respectful ways and appreciate the cultural differences. Issues of cultural differences and perceived abilities also lessened as the students began to perceive what their roles were as users and doers of mathematics. Boaler (2006) viewed this as gaining relational equity. Illustrated here in these findings is how she argued that students develop respect for different cultural groups when they are provided with opportunities to learn through culturally relevant examples and actions.

In the findings in this paper we have presented data that suggests not only a shift in identity but also a shift in mathematical disposition. Through culturally responsive teaching and the teachers' actions of drawing on the cultural capital of the students it allowed them access to the mathematical problems and also supported them to engage with each other mathematically in culturally appropriate ways. The teachers too were inducted into a world where they could develop a vision of culturally Pāsifika responsive actions which they could use in their mathematics teaching. Closely aligned was the way in which the views of the students had shifted towards seeing mathematics as part of the world they inhabited and something they needed within that world.

Clearly evident in the findings of this paper is the way in which culture and language shape the identity of students. These Pāsifika learners were empowered by the actions of their teachers as they explicitly drew on and used the cultural capital of the students in ways that supported them as mathematical learners. The results were transformative in that not only did the teachers bridge the gaps maintained by what is taken as normal or status quo – they also were able to bridge the same gaps for their students and empower them as learners. The implications of this research link directly to the need for more research in this field. The problem society is creating in allowing groups of underachieving students to continue needs to be addressed. This study suggests that we do have some strategies which directly illustrate the effect of culturally responsive teaching but we need to explore this on a larger scale.

References

Anae, M., Coxon, E., Mara, D., Wendt-Samu, T., & Finau, C. (2001). *Pasifika education research guidelines*. Ministry of Education: Wellington. New Zealand.

Bartolomé, L. I. (1996). Beyond the methods fetish: Toward a humanizing pedagogy. In P. Leistyna, A. Woodrum, & S. A. Sherblom (Eds.), *Breaking free: The transformative power of critical pedagogy* (pp. 229–252). Cambridge, MA: Harvard Education Press.

Barton, B. (1995). Cultural issues in NZ mathematics education. In J. Neyland (Ed.), *Mathematics education: A handbook for teachers,* Vol. 2 (pp. 150-164). Wellington: Wellington College of Education.

Boaler, J. (2002). *Experiencing school mathematics: traditional and reform approaches to teaching and their impact on student learning*. Mahwah, NJ: Lawrence Erlbaum Association.

Bourdieu, P., & Passeron, J. C. (1973). Cultural reproduction and social reproduction. In R. K. Brown (Ed.), *Knowledge, education and cultural change*. London: Tavistock

Frankenstein, M. (2010). Critical mathematics education: An application of Paulo Freire's epistemology with new afterword. *Journal of Education, 165*(4), 315-339.

Freire, P. (2000). *Pedagogy of the oppressed.* New York: Continuum.

Giroux, H. (1983). *Theory and resistance in education: A pedagogy for the opposition*. South Hadley, MA: Bergin and Garvey.

Hunter, R., (2013). Developing equitable opportunities for Pasifika students to engage in mathematical practices. In A. M. Lindmeier & A. Heinze (Eds.). *Proceedings of the 37th International Group for the Psychology of Mathematics Education*. Vol 3. pp. 397-406. Kiel, Germany: PME

Hunter, R. K., & Anthony, G. (2011). Forging mathematical relationships in inquiry-based classrooms with Pasifika students. *Journal of Urban Mathematics Education*, 4(1), 98-119.

Jorgensen, R, (2014). Social Theories of Learning: The need for a new paradigm in Mathematics Education. In J. Anderson, M. Cavanagh,, & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 311–318). Sydney: MERGA.

McLaren, P. (1994). *Life in schools: an introduction to critical pedagogy in the foundations of education.* New York, Longman.

Ministry of Education. (2004). *Book 3: Getting Started*. Wellington: Learning Media.

Ministry of Education. (2014). *Statement of intent 2014 – 2018*. Wellington, New Zealand: Ministry of Education.

The importance of *praxis* in financial literacy education: An Indigenous perspective

Levon Blue
Griffith University
<levon.blue@griffithuni.edu.au>

Peter Grootenboer
Griffith University
<p.grootenboer@griffith.edu.au>

Mark Brimble
Griffith University
<m.brimble@griffith.edu.au>

We argue the importance of *praxis* in financial literacy education teaching practices that is, the moral and ethical nature of teaching and learning. Post the global financial crisis of 2008, the teaching of financial literacy has become a priority for many countries. Indigenous communities are often the target of broad FLE strategies and/or government policies. We present a case for praxis in financial literacy education by drawing on interview data following a financial literacy 'train the trainer' workshop in an Indigenous community in Canada that failed to gain traction.

Introduction

Improving financial literacy is a global concern with many countries establishing initiatives and strategies to help citizens acquire the financial knowledge that is thought to be necessary to ensure effective management of personal finances over a lifetime. With financial well-being the ultimate aim of most financial literacy initiatives (Blue & Brimble, 2014), financial literacy education (FLE) promotes financial skills and knowledge. Therefore, a plethora of FLE initiatives established by government, industry, workplaces and community are available, although there is concern about the effectiveness and appropriateness of many of these programs (Worthington, 2013).

In Australia and Canada, FLE *train-the-trainer* multiple day workshops have been offered to Community organisations working with low-income individuals. These workshops are usually developed and financially supported by financial institutions and this approach to training and learning is an area of concern that will be addressed in this paper, particularly vis-à-vis the *praxis* of FLE. Indeed, despite the trainers best intentions to financially educate vulnerable individuals, there is a risk that the individuals receiving this training are being misguided into thinking their financial problems could be 'fixed' if only they could acquire the basic personal financial skills such as developing a budget (Pinto, 2009; Willis, 2008). While these tools and skills do provide financial awareness, they do very little to change behaviour and are unlikely to assist an individual move from their current financial circumstances (Lyons, Chang & Scherpf, 2006).

In this paper we examine the trend to educate some of the most vulnerable individuals in society with generic FLE. The impetus for this study was the ineffectiveness of a FLE *train-the-trainer* workshop in an Indigenous community in Canada. It is important to note that the first named author of this paper is a member of the Indigenous community, and so in conjunction with fellow Community members, the approaches to learning, the relevance of FLE, and their felt needs regarding FLE were explored. Furthermore, understanding educational *praxis* as "the moral, ethical and caring dimension of teaching" (Grootenboer, 2013, p. 1) appears to be lacking from current FLE practices. In this study we examined the possibilities of what could be achieved in the Community and it was found that site-

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 117–124. Sunshine Coast: MERGA.

based and Community developed FLE was desired, relevant and important, and some of these ideas are outlined in later sections. We also discuss FLE as *praxis* in Indigenous financially excluded communities and report some findings from the interviews (both individual and group) that took place in the Community with Community members who had either attended the training and/or had an interest in FLE.

Financial literacy and financial literacy education

Financial literacy has been defined as "a combination of financial awareness, knowledge, skills, attitudes and behaviours necessary to make sound financial decisions and ultimately achieve individual financial wellbeing" (Atkinson & Messy, 2012, p. 2). Financial literacy continues to be a priority area for the OECD

> … as highlighted by three sets of principles, endorsed by G20 leaders: the G20 Principles on Innovative Financial Inclusion; the G20 High-Level Principles on Financial Consumer Protection and the OECD/INFE High-level Principles on National Strategies for Financial Education. (Atkinson & Messy, 2013, p. 9)

Regardless of whether an individual ever achieves financial well-being, the pedagogical focus of FLE is being able to equip students with the financial skills and knowledge to perform appropriate and efficient mathematical calculations when faced with everyday financial decisions. This is what Sawatzki (2013 p. 557) refers to as the ability to problem solve your way through real life "financial dilemmas". In Australian primary and secondary mathematic classrooms students will be taught these financial literacy skills under the *money and financial mathematics* component of the mathematics curriculum (Mathematics Curriculum, 2015). Therefore, education that focuses on increasing an individual's financial literacy through the acquisition of personal financial knowledge we refer to as FLE (Blue, Grootenboer & Brimble, 2014). We understand the importance of teaching FLE however, we do believe that the expectations of FLE are unrealistic and argue that they need to be reframed so that they align with the appropriate expectations and outcomes for each individual and their life choices (Blue & Brimble, 2014). Nevertheless, FLE has a role to play in reaching individuals living in financially excluded communities. What this role is, and the expectations and outcomes associated with the delivery of FLE to financially excluded communities will be explored throughout this paper.

The role of a praxis perspective and site-based education development

We align praxis in the FLE classroom/workshop to the dual purpose of education, that being for the benefit of both the individual and society (Grootenboer, 2013). Therefore, FLE teachers and/or practitioners may have a critical role in developing students' financial identities, similar to those reported about mathematics classroom teachers, but also to develop financial literacies across the Community for the benefit of the Community as a whole. Financial literacy is in the compulsory school curriculum in many countries, and it is often the mathematics teachers that are given the responsibility of teaching it (Blue, Grootenboer & Brimble, 2014). Grootenboer (2013) argues the importance of having skilful and knowledgeable practitioners from a pedagogical perspective, but that good "…teaching is more than knowledge and technique – it is a form of *praxis*" (p. 1). We view this as an important requirement in order to prevent the inequalities and marginalisation that may occur when financially vulnerable individuals, whom are financially educated, are unable to act on the financial knowledge they receive. Moreover, FLE is not the solution to poverty as, "…poverty is … an issue of low wages" (Ivanova and Klein, 2014, p. 2), long working hours and lack of access to social goods (Raffo, 2011). Part of teaching financial literacy with a praxis perspective involves understanding what FLE can and cannot offer students. It also involves understanding who benefits in a capitalist economic system and who suffers; that is that some individuals will obtain great wealth and others will face poverty - such a system guarantees these two extremes (Arthur, 2012).

Also, many researchers have argued that there is no 'one-size fits all' approach to education and each site and has specific circumstances and conditions (Kemmis, Wilkinson, Edwards-Groves, Hardy, Grootenboer & Bristol, 2014). Therefore, the move to more sustainable FLE pedagogies involves responding the specific demands of the site, what Kemmis, et al. (2014) refer to as "site-based education development". This is "when educators think together about how best to do this, in a particular school, for particular students and a particular community, they are engaging in site based education development" (p. 212). This was an important aspect of this research project, particular as the generalised 'best practice' notions of FLE had been ineffective and even damaging in the past. Indeed, it was evident that the Community members interviewed wished for FLE resources that were collaboratively developed and connected to their existing practices and site conditions.

The Study

This study took place in a Canadian Indigenous reservation and the Community is located on an island. Approximately 620 residents or 250 households live on the reservation year round and approximately 1000 members live off reserve (First Nation Profiles, 2015). There is no access to a mainstream financial institution (i.e., a bank) on the island (it is a financially-excluded Community). Both 'on' and 'off' members of the Community were included in the study. The study focused on three key themes:

1. The Community's experiences with FLE;
2. The Community's interest in FLE; and
3. The Community's perception of what FLE can/cannot achieve and its relevance.

We conducted interviews with individuals and with groups of Community members who had either attended the previous financial literacy workshops and/or had an interest in financial literacy education. These interviews took place approximately one year after a financial literacy 'train-the-trainer' workshop failed to gain traction. The workshop was run by an established organisation (funded by one of the big Canadian banks) was invited by some Community members to deliver a financial literacy train the trainer workshop after some consultations. Nineteen participants were interviewed and their audio recorded interviews were transcribed. During these interviews we explored the reasons for this workshop inability to develop new trainers and the relevance, interest and perception of financial literacy in Community. After the interviews were transcribed, the text was analysed and this involved first manually analysing the transcripts for themes and then using the NVivo software to further refine and organise the thematic structure.

Findings

After the data were analysed there were a number of themes and issues that emerged. Here we will report specifically on the findings related to the relevance of FLE in the Community; the train the trainer workshop inability to develop "new" trainers; and the practice of FLE and the issues associated with a financially-excluded Community. There were four key findings that emerged:

1. FLE is important and relevant for the Community;
2. Community members are not comfortable presenting;
3. The previous practices of FLE were unsustainable; and,
4. Living in a financially excluded Community present several challenges.

Together these make a case for a praxis approach for FLE in this Community, and also lend weight to similar considerations in other similar disadvantaged and marginalised communities and groups. We will now outline and discuss each of these findings in turn, and in doing so we will rely on the Community members own words to make each point.

FLE is important for the Community

We found FLE was important and relevant for the participants. Some Community members reported the desire to start a program for youth before they leave the Community to attend post-secondary school. For example, a Community member (Female, CM1) stated;

> It's [FLE] relevant … You know everyone wants to learn about money and everyone wants to, I guess, it's like everything revolves around it right? It brings out either positive behaviours or negative behaviours that everyone sees especially in a small community.

And, another Community member similarly commented;

> …. we've been wanting to make a program to start it with the later years in the public school and then to our high school kids… before they go off to college education. So that they know how to manage their money right away. (Female, CM2)

With this in mind, and was noted in the second quotation, many Community members wanted FLE to be an integral part of their high school education.

> … I do think that there should be more in the education system. Like you know growing up and in high school and stuff doing math most of the time it ends up being stuff you will never use again. You know I wish that more education in the education system you know focused on personal finances like how do you do your taxes and how to do business stuff and investment stuff and be smart with… I don't know anything about taxes or how it works. Like I have student loans but I don't necessarily know how it all works. I think there are a lot of things that we don't learn that are really important, you know growing up. I think that is something that can be for the Community if you are going to leave the Community and you are going to be going to school or to get a job somewhere. It is stuff you need to know. So you know maybe not for everyone, that stuff is relevant but maybe for parents to learn. (Male, CM1)

However, there was also a concern that the FLE that was undertaken in the high school was relevant and appropriate for the particular needs of the students in this site. This was as necessary for the individual students themselves, but also for the Community itself, thus highlighting the dual purposes of education noted earlier.

> Some of it went over their heads; like it was too much information and then we are looking at budgets for our work anyway just teaching people the basics financial things. (Female, CM2)

> … when a client comes to a realisation on actually what they are spending, like that budget sheet was a real eye opened for me. To see dollar for dollar were it was going and it was going to things that I didn't really need like magazines, the cigarettes, pop, junk food, anything like that. So when I [saw] that for myself it was a matter of making those choices on whether or not, prioritising what is really important. (Female, CM3)

These snippets of data are reflective of the larger data set, and together that show a clear desire for FLE in the Community. Furthermore, this FLE needs to be appropriate and designed for the particular needs of the individual and the Community more broadly.

Community members are not comfortable presenting

Although the Community members expressed a desire to be involved in the planning and development of FLE in the Community, they expressed a sense of discomfort about actually being financial educators themselves. This is somewhat ironic given the 'train the trainer' model for the previous FLE program in the Community. The goals of this previous program were to:

1. Increase the financial skills and knowledge of the "trainers" that attended this course;
2. Establish financial literacy trainers in the Community; and,
3. Increase other Community members personal finance skills and knowledge by these 'trained' members teaching the content to others in the Community.

It was reported by Community members that attended this training that their financial skills and knowledge did increase after attending the workshop however, it was also reported that no one was willing to become a financial literacy trainer after attending this workshop. Therefore, no financial literacy trainers were established in the Community and these newly acquired financial skills and knowledge were not reported to be passed onto other Community members in a workshop setting.

> No, the majority that were there thought it was a pretty good course and gave them some awareness into financial literacy and what to look at and what to expect around budgeting and all that but they all agreed that or the majority agreed that they wouldn't be willing to go out and teach people. (Male, CM2)

> They would be uncomfortable presenting and I think presenting is another piece in itself that is not specific to financial literacy but with anything. You know presenting anything, they are uncomfortable with that. I mean which we knew that was going to be case anyway. But [name of the organisation delivering the workshop] was hoping to get more facilitators and instructors out in the Communities and that is part of their goals to educate more people out there who can educate their clients or Community members. And we thought that was a good idea and that's why we brought the program here. So it's not taking off that way because people are just, really just don't want to present. (Male, CM2)

Clearly there is a need in this Community to develop and implement FLE that is morally and culturally appropriate for this particular site – a praxis approach to FLE. To this end, a brief one-off workshop that attempts to 'train the trainers' is unlikely to be effective or appropriate, even if the content can be seen to be relevant.

The previous FLE practices were unsustainable

Not surprisingly, the previous generalised FLE practices based on the 'train the trainer' workshops delivered by external agencies were unsustainable with unrealistic expectations and outcomes. We argue that it is an unsustainable practice because the generic training material that is taught as an *additional* practice for individuals to adopt. By this, we mean that it does not connect or is not incorporated into the Community members' existing practices, nor is it tailored in any way to the address the needs of the Community. Furthermore, the pedagogical practices seemed to disconnect with the Community.

> It kind of stopped we're probably the only ones that use it. Because other people don't like to do workshops or conferences. (Female, CM2)

> But, I think coming back to our discussions that we've had and designing a program or a model that we can use in our community. [One] that people would be comfortable using and delivery it in our own style. Whether it is one on one or group sessions or even if it comes down to people deciding that we would like an independent person not part of any staff to be delivering this type of information as like an advisor that comes in once in a while. Maybe more people [would be] comfortable with that because they don't have to devolve their financial information or their habits. (Male, CM2)

That said, there was one participant (Female, CM4) who spoke about using a "budget sheet" that she had incorporated into their practices at her place of employment, thus indicating that some practices were taken-up in a limited way where the individual could see the relevance and application.

> This is another personal budget sheet for them [the clients] [it] has [been] incorporated in the system, we all have it, if a person comes in requesting assistance with hydro or maybe they need an appliance or something that is just part of TSF (transitional support funding) this is part of the application and it is already in our system and we just enter the numbers in.

Living in a financially excluded Community present several challenges

With one store in the Community individuals were faced with either using the ATM to withdraw money from their account (no deposit facilities), handing their cheques (from employment and/or social assistance) over to the store and spending the required percentage to have the cheque cashed, and/or traveling on the ferry and then by vehicle into town to deposit and/or withdraw their cheque (this could also include sending your cheque into town with a trusted relative/friend to do this for you). The opportunity to turn to predatory lenders is possible as a simple task of depositing a cheque requires two modes of transportation and almost an hour of your time to get to the bank (assuming direct deposit is not set up).

> The implementation and use of direct deposit and online banking is of such importance and makes so much sense today. When you think of the current method, where members of the community are earning an income or social assistance, and they have to make this great journey to cash their cheque. Using direct deposit and online banking methods will eliminate the need to make the trip with your pay cheque from the island into the town where there is a financial institution and then you also have to wait the 3-7 day business hold day until that cheque clears, is going to prevent people from wanting to go to these pay day lenders [cheque cashers] that are charging them a massive amount of interest upfront. So it just makes so much sense to have direct deposit where they will have immediate access to their money. If payday is Friday they have access to their money on Friday it would just completely eliminate all of this because a lot of people are thinking if I've got to get off this Island and I am relying on the fact that the boat running and the weather is good. Then I've got to get into town, and then I have to wait 3-7 business days, people are probably thinking I don't have up to a week before this money clears. I need this money as soon as possible. Instead they side with thinking it is simpler and quicker to pay this person at the payday lender place $100 so I can get my money now; I am going to do it. (Female, CM5)

Clearly, the standard FLE usually offered by a major financial institution is not going to be appropriate under these conditions. Furthermore, a lack of employment opportunities, homelessness and lack of collateral in their land were identified as issues for the Community that meant that quite particular FLE was required.

> … there's not much work here so there aren't even opportunity to get money for people that have low education or don't want to leave the reserve or whatever (Male, CM3)

> I don’t know how people get there? I guess it could be pretty easy. I am just thinking if I lost my job I could be on the street pretty quickly. (Female, CM6)

FLE that focuses on specific initiatives or activities in the Community such as learning how to prepare the financial sections in grant applications and learning how to read the Band Office’s financial statements was deemed important. Also, how to navigate it financially without having collateral was identified as a major issue in the Community.

> [Indigenous people] are funded by and can only be funded by grants and that’s what we have to do each year and even the programs at the [Indigenous] admin office they have to submit their grant application every single year and this is what we are getting from the Government. Now people on the outside don’t understand this and they think we have all this potential to have great businesses and stuff like that but we have no collateral. Because we can’t do that and banking institutions rely on collateral and we don’t have any of that so no one would ever fund us and if we wanted to go into a [type of] business. We won’t be able to do it because financially no institution would finance us. So it’s a matter of okay where do we go now? (Female, CM7)

Throughout the four findings outlined above it is clear that FLE was wanted and needed in the Community, but they needed different content and teaching practices than they had experienced in the past.

Conclusions, Implications and Recommendations

The inappropriate one-size fits all approach to FLE that is often is delivered across a wide range of contexts and communities reaches only a small few who fit the generic or textbook model (Pinto, 2012). However, the example of this Community shows that this can be almost completely useless and inappropriate. These *quick fix* approaches seem particularly problematic in sites of poverty and disadvantage, particularly where it is not easy to change your circumstances (e.g., take on a higher paying job). Importantly, we recognise that FLE is not the solution to poverty. The *real* issue of poverty which involves low wages, working long hours and lack of employment opportunities are not going to be solved by FLE. Understanding what FLE can and cannot achieve is what we view as the moral and ethical aspect of teaching and learning FLE. Specifically, within a high school mathematics classroom connecting FLE to real life “financial dilemmas” is an approach that we support as some participants we interviewed wished this would have occurred in their classrooms. However, increased financial awareness will not ensure that the student can financially support themselves outside of their Community. Rather the student will be more aware of the “financial dilemmas” they are likely to experience. Thus FLE content that is relevant, age and culturally appropriate, inclusive and requires the students to critically explore multiple solutions is well placed in inquiry based high-school mathematics classrooms. Classrooms where the teachers realises that being financial literate does not leads to financial well-being and that ‘poverty’ will not be overcome by making effective financial decisions.

References

Arthur, C. (2012). Financial literacy education for citizens: What kind of responsibility, equality and engagement? *Citizenship, Social and Economics Education, 11*(3), 163-176.

Atkinson, A., & Messy, F-A. (2013). Promoting Financial Inclusion through Financial Education: OECD/INFE Evidence, Policies and Practice, *OECD Working Papers on Finance, Insurance and Private Pensions, No. 34,* OECD: France. doi: 10.1787/5k3xz6m88smp-en

Atkinson, A., & Messy, F-A. (2012). Measuring Financial Literacy: Results of the OECD INFE Pilot Study. *OECD Working Papers on Finance, Insurance and Private Pensions*, No. 15, OECD, France. doi: 10.1787/5k9csfs90fr4-en

Mathematics Curriculum: money and financial mathematics. Retrieved from: http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1#level9

Blue, L.E., & Brimble, M.A. (2014). Reframing the expectations of financial literacy education: bringing back the reality. *JASSA the Finsia Journal of Applied Finance or the Australasian Accounting Business and Finance Journal,* (*1),* 37-41.

Blue, L.E., Grootenboer, P.J., & Brimble, M.A. (2014). Financial literacy education curriculum: missing the mark or making the grade. *International Journal of Economic Education*. doi: 10.1016/j.iree.2014.07.005

Brimble, M.A., & Blue, L.E. (2013). Tailored Financial Literacy Education: An Indigenous Perspective. *Journal of Financial Services Marketing, 18* (3), 207-219.

Grootenboer, P. (2013). Praxis and mathematics education. *Pedagogy, Culture and Society, 21*(2), 321-342.

First Nation Profiles (2015). Retrieved from: http://pse5-esd5.ainc-inac.gc.ca/FNP/Main/Index.aspx?lang=eng

Ivanova, I., & Klein, S. (2014). *Working for a living wage*. Retrieved from https://www.policyalternatives.ca/sites/default/files/ uploads/publications/BC%20Office/2014/04/CCPA-BC_Living_Wage_update_2014_ revMay16.pdf.

Kemmis, S., Wilkinson, J., Edwards-Groves, C., Hardy, I., Grootenboer, P., & Bristol, L. (2014). *Changing practices, Changing education*. Singapore: Springer.

Lyons, A.C., Chang, Y., & Scherpf, E.M. (2006). Translating financial education into behavior change for low-income populations. *Journal of Financial Counseling and Planning, 17*(2), 27.

Pinto, L.E. (2009). Is financial literacy education the solution to credit crises? *Our Schools, Our Selves, 18*(4), 123-133.

Pinto, L.E. (2012). One size does not fit all: conceptual concerns and moral imperatives surrounding gender-inclusive financial literacy education. *Citizenship, Social and Economic Education 11*(3), 177-188.

Raffo, C. (2011). Barker's ecology of disadvantage and educational equity: Issues of redistribution and recognition. *Journal of Educational Administration and History, 43*(4), 325-343.

Sawatzki, C. (2013). Connecting Social and Mathematical Thinking: The Use of "Real Life" Contexts. *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, Brisbane, pp. 557-564). Sydney: MERGA.

Willis, L E. (2008). Against financial-literacy education. *Iowa Law Review, 94*(1), 197-285.

Worthington, A. (2013). Financial literacy and financial literacy programmes in Australia. *Journal of Financial Services Marketing, 18*, 227-240.

Coming to do Mathematics in the Margins

Raymond Brown
Griffith University
<ray.brown@griffith.edu.au>

Trevor Redmond
Somerville House
<TRedmond@somerville.qld.edu.au>

This paper explores teacher' 'identity' as two teachers talk about teaching mathematics in classrooms situated within two different contexts of learning – mainstream and alternative. Employing a form of discourse analysis framed within a participation approach to learning, this paper describes teacher identity in terms of the norms and practices that frame the translation of content, pedagogy and assessment in each teacher's classroom. Differences between each context of learning are highlighted and parallels drawn between similarities.

The notion of identity is difficult to theorise. It cannot be simply reduced to elements such as thinking, emotion, morality, gender, agency, or practice (Roth, 2007). However, researchers often focus on one or two elements of a notion of 'identity' in order to explore, for example, the power of agency through working mathematically (Grootenboer & Jorgensen, 2009). In many ways, what each of these studies is attempting to do is provide a believable account of individuals 'being' in a world that is observable, a world that is constituted of many contexts of activity based on distinctive principles and practices (Roth. 2007).

This paper explores the nature of the 'identities' engaged by teachers when working in mathematics classrooms that may be considered to be at the margins of mathematics teaching. 'Identity' is conceptualized in this paper as a community-forming process where adults and students express and communicate ideas according to a set of norms and practices (see Lave & Wenger, 1991).

Theoretical Framing

According to Wertsch (1998), the tension between people and the cultural means (norms and practices) at their disposal results in an on-going process of transformation and creativity that has the potential to not only change the relationship of people to the world by shaping and constraining their participation in it, but also to transform the individual person by incorporating his/her activity into new, functionally active systems that are culturally and historically situated. One means of capturing the norms and practices of a cultural group is through the use of narrative, that is, the telling of the "rich and messy domain of human interaction" (Bruner, 1991, p.4).

Narratives may be elicited as a spoken or written account of stories revealed in long sections of talk or in a single research interview (Liamputtong, 2013). The purpose of the research reported in this paper is to explore the nature of the identities constructed by teachers as they participate in face-to-face interviews designed to elicit their accounts of being a teacher. As narrative interviews are concerned with appreciating and contrasting differences in perspectives (Jovchelovitch & Bauer, 2000), the interviews of two teachers, one from a mainstream system of education and one from an alternative system of education were analysed for the purpose of exploring the identities constructed and their links with the principles and practices privileged in their context of teaching mathematics.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 125–132. Sunshine Coast: MERGA.

Method

Interview Design

In order to privilege the context of education, mainstream or alternative, interview questions were designed that conformed to the 'participation framework' employed by Vadeboncoeur (2006) to map young people's learning in different contexts. This framework situates interview questions within broad categories relating to location, relationships, content, pedagogy, and assessment, allowing different questions to be asked according to context, thus allowing the story of the interviewees to be told, but ensuring that information is provided to assist in explicating the similarities and differences across contexts. Interviews were conducted by researchers in places where interviewees felt comfortable to tell their story. Each interview lasted for approximately 30 minutes, was audio recorded and transcribed for analysis.

Participants

Mainstream School Context: The teacher who is the focus of this paper, Sam, taught mathematics to students (ranging in age from 12 to 18years of age) at an independent co-education P-12 school located in a middle-class suburb of a major city. Sam started his career, now in its 26th year, by framing his teaching of mathematics within pedagogical practices that reflected a transmission approach to teaching and learning. However, after a decade of wondering why students performed inadequately when it came to the application of mathematics to everyday situations, Sam began to view student learning within a framework that anchored his teaching to pedagogical strategies that afforded him a focus that privileged student understanding over teacher demonstration and student practice. It is well understood that the predominant pedagogical view in many mainstream mathematics classrooms is based on traditional approaches to teaching that privilege teacher demonstration and student practice (see for example, Handal & Herrington, 2003). As such, even though Sam teaches in a mainstream school his views about teaching mathematics (see Brown & Redmond, 2008, for more information) place his classroom on the margins of classroom mathematics teaching.

Alternative Education Context: The teacher who is the focus here, Lisa, is an experienced female teacher in the middle phase of schooling (students ranging in age from 12 to 16 years of age) and a long-term member (over ten years) of a 'network' of alternative education schools. It is for this reason that Lisa's classroom may be considered to be on the margins of teaching mathematics. Lisa is responsible for planning and delivering the Numeracy program in the school. The alternative education school in question is a co-educational Catholic school where teaching and learning is characterised by small class sizes, a flexible curriculum that draws on individual student interest for curriculum focus, and a democratic pedagogical approach that encourages learner empowerment and autonomy. Multi-disciplinary teams of professionals work with young people who are vulnerable and experience a complexity of inter-related needs. The learning experiences also build self-confidence and esteem in students, promote an optimistic view of their potentialities and future, and assist them to develop the knowledge, skills and attitudes necessary to enjoy a healthy and fulfilling life. Teaching and learning is grounded in principles that guide practice such as: respect, for self, others; participation; safe and legal; and, being honest (see Vadeboncoeur, 2009 for more information).

Analytic Process

To systematically analyse the interview transcripts, a framework was developed based on Vadeboncoeur's (2006) participation framework that centred on the following broad categories: location, relationships, content, pedagogy, and assessment. This framework was chosen because it allows researchers to see links between learning contexts, thus allowing for an explication of the similarities and differences in the dilemmas encountered by the interviewees across teaching contexts (Vadeboncoeur, 2006).

Analysis and Discussion

The dilemmas that we aim to highlight and discuss in this paper focus on the categories of content, pedagogy and assessment. This focus was chosen for this paper due to the emphasis given to these categories in national school curricula documents. Only those segments of text pertinent to the analysis have been tabularised.

Content: Knowledge Versus General Development

Teaching curriculum emerges from the interview transcript as being of secondary importance for Lisa; her aim is not to teach young people mathematics knowledge and skills, partly because she thinks it is too 'unsafe' of these 'kids' to learn (see Excerpt 1).

Excerpt 1: Safe to take risks

Question: Can you tell me something that explains how you work with young people?

Lisa: Um (...) it's a hard question to answer in that it changes a lot of the time. You know it's a dynamic place and I think that I have always come to the place with the understanding that I teach kids not curriculum. So the content of what I teach is not of great importance to me, it's um, it's certainly not my priority, neither sometimes is actually teaching them skills. Initially, my priority is to make them feel like they are learning. So there's a lot of work, I think I put most of my energy into that, how to re-engage, how to help a young person experience success in some way. And that may be through addition or a piece of writing, or it may be through something curriculum type, but until they feel safe to take that risk, the rest is nothing.

For Lisa, teaching curriculum and skills emerges in opposition to re-engaging 'kids' to learning, offering them opportunities where they can feel safe to try and make mistakes, where they can experience success and see opportunities (see Excerpt 2).

Excerpt 2: Experiencing success

Question: Are you preparing them (the young people) for anything?

Lisa: We open windows, we open the curtains for them that's what we do. And I think that's what you'd find here, it's not always about building up skills, but it's about seeing yourself as a learner again. It's about believing in yourself, it's about being willing to take risks. It's about experiencing success in something.

For Sam, teaching Mathematics emerges from the interview transcript as being of primary importance. His aim is not to teach young people mathematics knowledge and skills out of context, but to find topics of interest that will assist the students to develop an 'holistic view' of Mathematics where the knowledge and skills that are developed are seen as useful (see Excerpt 3).

Excerpt 3: Building a holistic view of Mathematics

Question: How have your learning experiences that you provide your students with changed?

Sam: We try things, I mean we try things that are interesting, that we find interesting, that hopefully the kids will find interesting and so we try to build things around that. To allow kids to build things around mathematics but also see the use in the mathematics. So it's a case that we are sort of doing that. So we try and build a holistic view of mathematics so that kids don't see mathematics as a bunch of functions, things that don't relate to one another and I think that is probably really important.

For Sam, teaching curriculum and skills emerges in tandem with engaging students in the learning of Mathematics, offering them opportunities where they can 'stretch themselves' and be guided and helped by a teacher who has a particular plan in mind to 'get to a particular place' in the curriculum (see Excerpt 4).

Excerpt 4: Moving understanding forward

Question: What roles do you adopt in the teaching and learning process?

Sam: I control it a fair bit because I want the kids to get to a particular place. There's only a finite time that we have to get to it. We need to move the kids thinking forward. But if you give them something where they have to stretch themselves and that sort of stuff and I think that is pretty useful when the teacher helps and guides them and gets them to move their understanding forward. I don't know how that is all going to go in the future though.

Pedagogy: Teaching Versus Socialisation

Teaching emerges for Lisa as having something to do with academic content, teaching the young people tables, to read and to spell and teaching them to solve tasks like planning a trip with a specific budget. However this is not the aim of teaching for Lisa and for those young people she teaches; she has a different benchmark for success for these young people; she would like the 'kids' to leave the school being socialised into ways of being that allow them to function in the world (see Excerpt 5).

Excerpt 5: Saying sorry and meaning it

Question: How does teaching and learning relate to what you do with young people?

Lisa: I probably have a much lower benchmark for success, than mainstream teachers. The kid that says sorry and means it is a huge success for me. Um, when you see the lights go on, "Oh that's why the zero goes there", you know those little things, or when they speak nicely to each other, those are the things that actually show me success and certainly give me a lot of joy.

Socialisation emerges for Lisa as the main goal; socialising these young people into 'proper' ways of behaving and talking; their own ways of being are not acceptable for the wider society; school is a place that teaches them how to leave these ways of being behind, how to switch them off, gain 'self-control', and instead become like others (see Excerpt 6).

Excerpt 6: Becoming like others

Question: How does teaching and learning relate to what you do with young people?

Lisa: I also think that um, we probably, most of us I would say, have a broader view of what, ah, what needs to be taught and what is teaching. I mean sitting on that chair for longer than five minutes is teaching. Not swearing in public is teaching, so our view of what teaching is, I think is, yeah, and I think that those things are as important as the academic teaching we do, socialising.

Teaching for Sam is about learning, it is about playing with the form of teaching to 'get the kids to do the learning'. It is about facilitating students' construction of understanding through engagement in meaningful and interesting tasks. Interest is the key to teaching for

Sam; he has a high benchmark for success for his students and their engagement with mathematics and he is willing to spend time, effort and resources in the pursuit of meaningful academic learning (see Excerpt 7).

Excerpt 7: Finding interesting things in different places

Question: How has your approach to teaching and learning changed?

Sam: To my mind the learning is the thing that is important and that comes first and the teaching is there to facilitate that sort of learning. I try to get the kids to build their understanding and find ways of doing it, but also get things that are meaningful and are interesting. So I guess in terms of my teaching, that's sort of where I am at. If I am not particularly interested in it then I probably won't play with it too much. But if it interests me or if I think there is something that I can drag out of it, I am happy to spend time on it, and if I am happy to spend time on it then maybe the kids will find something interesting in it, and you will find interesting things in different places.

Socialisation for Sam emerges in the context of linking mathematics to the 'outside world'. Socialising in Sam's classroom is about providing tasks that the students can relate to and 'get engaged in'. School is a place that teaches students to see relevance in and to draw out meaning in the mathematics that they learn (see Excerpt 8).

Excerpt 8: Seeing relevance in Mathematics

Question: So what do you perceive to be authentic activity in the classroom?

Sam: I think that authentic, we try to find contexts that link to the outside world that the kids can see relevance to it. But sometimes something that's authentic is something that the kids can relate to and can get engaged in. So it might not necessarily be a life related type thing, but if the kids can relate to it and can engage in it and can draw some sort of meaning from it then I probably think that can probably be considered as an authentic task.

Assessment: Scientific Versus the Everyday

For Lisa, academic knowledge appears to be of secondary importance; she seems to be very rational about what these kids can and cannot achieve – she seems to know exactly where they are and even more that they cannot go or be anywhere else in terms of their skills and knowledge (see Excerpt 9).

Excerpt 9: What you need to learn

Question: So how do you gauge your successes in teaching?

Lisa: There's a story I have that I think is a real example of that and I think this young man is quite a bright kid, um academically, and older, I think he was almost fifteen and I get a phone call in the office one day and I answered the phone, and this um, it didn't sound like a young person actually, he had quite a deep voice and he said, "is John there?" John was a teacher at the time and he was in class and I said, "can I take a message?" and he said, "when does he come out of class?" and I said such and such a time, and he said, "can you just tell him that 3000 is nowhere near enough, I have to have at least 5, I can't possibly do it on 3000", and I said, "who is this?" he said it's Pete, this kid, "oh right, what are we talking about Pete?" and then he went on to explain to me. He was doing this assignment where he had to plan for a trip with a budget of $3000, it really struck me. It was actually one of those things that I took away and reflected upon, and I thought he is going to pass that assignment and he is going to get a good mark for that assignment, and yet he couldn't use the phone properly, he couldn't say to me, "It's Pete here can I speak to John?", you know have we succeeded? You know I just had a different view of the grade he'll get for that assignment, really doesn't reflect what he has learnt or needed.

While Lisa does not see much relevance of academic knowledge for these kids, she does value some of their interpersonal skills, such as leadership and initiative; she is willing to legitimise and celebrate these over and above the academic (see Excerpt 10).

Excerpt 10: Privileging interpersonal skills

Question: Are you preparing them (the young people) for anything?

Lisa: There's one young man in my class, I have had him for five years now, and he hadn't been to school for three years before he got to us um, a chronic school refuser, extreme behaviours et cetera. This last term he got 100% attendance so I got him a little wall plaque for it. Now he comes from a family of nine and I have actually taught six of the kids. And they all have this really specific learning problem in terms of reading. So in five years with me, his reading has not improved a great deal, and basically now we are in survival reading. Can you fill in forms, what do you need to survive beyond here? But, he won the leadership award from the ADF, Australian Defence Force, just last week, the grade eleven leadership award, he ran the footy team.

For Sam, academic knowledge appears to be of primary importance. Instead of being concerned only with what students should achieve he seems to be very rational about what his students could achieve if provided the opportunity. He doesn't seem to be concerned with simply knowing where his students are in terms of learning mathematics but also with knowing what might provide them with 'a lot of usefulness' (see Excerpt 11).

Excerpt 11: Getting usefulness out of what you do

Question: So what do you perceive to be authentic activity in the classroom?

Sam: You know you try and find things that, like I had a kid that, in my year, he's in my year 11 and he's going into year twelve. Now we set up a task for them to do but he, on the way through, he collected some data of a ball bouncing and he spent so much time trying to build an equation for that and do something with it. Rather than making him do this assignment over here, I just structured the assignment so that he could do what he wanted to do. He went away and he has built some really sophisticated mathematics to be able to model a ball that's bouncing along the ground. But he wouldn't have necessarily done that if I said, "No you can't do that you have got to do this over here". But he spent a huge amount of time doing that thing. Now that's not necessarily what you do for all of the kids because maybe they can, maybe they can't, and maybe they don't necessarily show the interest, but this kid did and we were able to, at that particular point of time, provide him with the opportunity to do that and I think he probably got a lot of usefulness out of doing that.

While Sam focuses on the relevance of academic knowledge, he does value some of the interpersonal skills that his students have, such as peer interaction and communication; he is willing to legitimise these as being 'useful' aspects of the academic (see Excerpt 12).

Excerpt 12: Getting an idea of the sorts of understandings

Question: So how do you allow students to express their social selves in the classroom?

Sam: Oh that's alright, I mean I think that peer tutoring and that sort of stuff is useful. But they have to have that time at the beginning to think about whatever it is that they are doing. Think about what they know and what they can bring to the discussion table. Then that interaction between the students and teacher is really, useful. In terms of getting kids to be able to verbalise their understanding because the number of kids who say, "Oh I know what to do", but can't communicate it, um, tells me that they really don't know what they are doing, it's just the procedure that they are following and they really can't put it together. So when they actually verbalise it to somebody else or to the teacher, and they can do a good job of it, that's when you start to get an idea of the sorts of understandings that they are building and how strong their understanding is.

Conclusion

What emerges from these interviews are teachers who on the one hand have positive ideas about teaching in non-traditional learning contexts, as evidenced by their emphasis on interpersonal skills and re-engagement in general development alongside or above academic content knowledge. On the other hand this positive basis is lost by Lisa in her positioning of these young people as coming from dysfunctional families and rough world conditions and seeing them as not capable of making any real progress or re-engagement with education. In this way her goals of re-engaging these young people with education become translated into practices of socialisation; that is, teaching them how to talk nicely, how to sit still and follow the routine, how to learn to control themselves and hide their rough backgrounds. Instead of legitimising the knowledge and experiences that these young people bring to the learning context, and trying to re-engage them through those, Lisa aims to suppress these different ways of being and to teach them ways of overcoming these ways of talking and behaving. She does not seem to work on 'what could be', but on 'what is' and 'what should be', aiming to create what is appropriate and acceptable for the world of opportunities.

However this positive basis for learning and teaching is used by Sam to position his students as partners in the learning process and to see them as capable and as having potential to make real progress in their education. His goals of engaging students in authentic tasks privilege meaning making and transference of learning to their own worlds, that is, his teaching focuses on teaching students how to talk about mathematics, how to see the mathematics in real world tasks, how to work with others (teacher and students) to progress learning, and how to see difficulty and challenge as being important aspects of knowing and doing mathematics. In this way, he legitimises the knowledge and experiences that students bring to the classroom, teaching them that mathematics is an important aspect of 'who they are' and 'what they do. In many ways, Sam not only works on 'what should be' done to teach the mathematics curriculum, but also works on 'what could be' done in order to well position students for the world of opportunities.

Even though Lisa and Sam have different foci in their teaching of Numeracy/Mathematics, both Lisa and Sam are highly regarded in their respective schools. For Lisa the teaching of numeracy is about present and future health and well-being, for Sam the teaching of Mathematics is about present and future learning opportunity. Both teachers provided efficacious learning contexts. Common across both learning contexts is an engagement with epistemic, ontological, and axiological positions for young people. These contexts, albeit in different ways, mediated knowing and doing, identity and difference, through relational norms and pedagogical practices.

It is clear from reading the interview transcripts that both Lisa and Sam are teaching in the margins. Lisa works with young people who are vulnerable and experience a complexity of inter-related needs. Participation and retention are key elements in her philosophy of teaching. Sam 'battles' to teach students in a mainstream schooling system, participation and meaning making are key elements in his philosophy of teaching. Both teachers have elements to their identities that teachers of mathematics across all contexts may lay claim to. It could even be stated that, without knowing the context, either teacher could be assumed to be teaching in a mainstream or alternative context. As such, both Lisa's and Sam's approach to teaching have important contributions to make not only to developing the identities of teachers and the young people that they teach, but also to bridging the divide between mainstream and alternative contexts of learning.

References

Bruner, J. (1991). The narrative construction of reality. *Critical inquiry, 18*(1) 1-21.

Brown, R., & Redmond, T. (2008). Reconceptualising agency through teachers talking about a sociocultural approach to teaching mathematics in the classroom. In M. Goos, R. Brown, & K Maker (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 101-108). Brisbane: MERGA.

Grootenboer, P. & Jorgensen, R. (2009) Towards a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of Mathematics, Science & Technology Education, 5*(3) 255-266.

Handal, B. & Herrington A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal, 15*(1), 59-69.

Jovchelovitch, S., & Bauer, M. W. (2000). Narrative interviewing. In M.W. Bauer & G. Gaskell (Eds.), *Qualitative researching with text, image and sound : A practical handbook* (pp. 57-74). London: SAGE,

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge: University Press.

Liamputtong, P. (2013). *Qualitative research method* (4th edition). Oxford University Press: Melbourne.

Roth, W.M. (2007). Emotion at work: A contribution to third-generation cultural historical activity theory. *Mind, Culture and Activity, 14*, 40-63.

Vadeboncoeur, J. A. (2006). Engaging young people: Learning in informal contexts. *Review of Research in Education, 30*, 239-278.

Vadeboncoeur, J. A. (2009). Spaces of difference: The contradictions of alternative educational programs. *Educational Studies, 45*(3), 280-299.

Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.

"You play on them. They're active." Enhancing the mathematics learning of reluctant teenage students.

Nigel Calder
University of Waikato
<n.calder@waikato.ac.nz>

Anthony Campbell
Te Wananga o Aotearoa
<anthony.campbell@twoa.ac.nz>

This paper reports on a research project that examined the beliefs and attitudes of reluctant 16 to 18-year-old learners when using apps in their numeracy and literacy programmes. In particular, it considers the students' change of attitude towards numeracy learning. The data were consistent that the use of apps in the numeracy programme was instrumental in transforming student attitudes towards numeracy.

Introduction

The use of mobile devices such as tablets in educative settings has grown markedly in recent years. While research is emerging, the uptake has been so rapid as to limit the ongoing related research that might inform and critique this transition. Linked to the increase in mobile technology is the growth in apps that can be utilised for learning.

Some researchers contend that digital technologies offer the opportunity to re-envisage aspects of mathematical education, opening up alternative ways to facilitate understanding (e.g., Borba & Villareal, 2005; Calder, 2011). For instance, the visual and dynamic elements of engaging mathematical thinking through digital technologies repositions the ways that content and processes are engaged. Likewise, the exploration and transformation of data with digital technology affords alternative approaches to analysing statistics (e.g., Forbes & Pfannkuch, 2009). Meanwhile, the availability of apps and their inclusion in classroom programmes continues, often without critical examination.

The affordances of digital technologies for mathematics education are well documented (Beatty & Geiger, 2010). Learning through apps offers similar potential affordances for learning (Calder, 2015). Apps offer the opportunity to engage with mathematical ideas in visual and dynamic ways, with learners receiving instantaneous feedback to input. They can link various forms of information or data (e.g., numeric, symbolic and visual) and transform them simultaneously. The use of digital games in mathematics learning has been reported to facilitate engagement with spatial elements and 3-dimensional visualisation (Lowrie, 2005).

Apps predominantly present the mathematical ideas and processes in a game context, often with extrinsic motivators, which use points as rewards. Care must be taken to ensure that the apps match the teacher's intended purpose. An analysis of mathematics apps indicated that they are variable in quality and often labelled inaccurately in terms of the cognitive aspects that they are claiming to address (Larkin, 2013). This implies a need for ongoing teacher professional learning so that they can best select apps that support and enhance the mathematical learning.

Meanwhile, research has also reported that iPad usage in primary-school mathematics' programmes has led to enhanced engagement, greater reflective practice, and higher order thinking (Attard & Curry, 2012). They found that it led to increased enthusiasm, while also affording opportunities for the teacher to broaden the range of tasks they could integrate into the learning. Carr (2012) in a study with fifth grade students learning mathematics with iPads and apps, found that the apps enhanced student engagement and the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 133–140. Sunshine Coast: MERGA.

reinforcement of concepts. If the students were not working completely individually, then they also promoted active discussion (Van de Walle, Karp, & Bay-Williams, 2010).

iPad apps offer the potential for transforming student beliefs and attitudes to learning. They can foster positive attitudes to numeracy learning and be highly motivational (Attard & Curry, 2012). In a six-month trial that integrated iPads into classroom practice, Attard (2013) reported that all of the students were positive about the experience, and that the teacher indicated that this had led to improved engagement. Carr (2012) also reported that the students were more motivated and engaged compared to a control group not using the mobile technology in their programme.

Much of the discussion regarding how the use of iPads and apps influences the affective elements of the learning experience, centres on the notion of student engagement; of students being actively enthralled and motivated, often by the visual and interactive characteristics of the pedagogical medium (Carr, 2012; Li & Pow, 2011). An increased motivation to learn and an indication of students being more attentive in class have also been reported (Li & Pow, 2011). The inclusion of game-based apps in programmes has likewise enhanced engagement and is reported to have increased enthusiasm and participation (Attard & Curry, 2012).

In a study with pre-service teachers, Grootenboer (2008) reported that student beliefs and attitudes are often the accumulation of significant episodic events. Positive classroom episodes that included apps might be influential in changing student beliefs and attitudes towards mathematics. Enhancement of learning was seen to be conditional on the apps selected, the purpose intended, and, in particular, the pedagogical processes in which they were used (Calder, 2015). However, at present there is a lack of research into this relationship within the context of apps and mobile devices in the mathematics classroom.

This research project was undertaken at Te Wananga o Aotearoa (TWoA) a tertiary institute conceived and developed under the cloak of New Zealand Maori kaupapa – a set of values, principles and plans that underpin its philosophy. TWoA has Youth Guarantee programmes that deliver introductory Sport and Leisure and Contemporary Māori Arts programmes with embedded literacy and numeracy. These Youth Guarantee programmes are for 16 to 18-year-olds who leave school without any formal qualifications, and frequently have negative attitudes to school. The research question was: In what ways did the use of iPad apps influence the beliefs and attitudes of Youth Guarantee students towards numeracy and literacy? However, this paper is concerned with changes in attitude towards numeracy learning.

Methodology

A qualitative interpretive research methodology was used for this project. This involved case studies with three different Youth Guarantee classes. An interpretive lens was applied to the data that reflects the sociocultural discourses that influenced learners as they moved through cycles of interpretation, action and reflection in the learning process. A Vygotskian sociocultural perspective also theoretically informed the project. The project considered that learning is mediated by language and the use of tools. Hence, not only does the dialogue of the teacher and the learners in the classroom act as a mediator, but also the app itself acts as a mediating tool. The learner's preconceptions of the pedagogical media, in conjunction with the opportunities and constraints offered by the media itself, promote distinct pathways in the learning process. That is, mathematical activity is inseparable from the pedagogical device, derived as it is from a particular understanding of social organisation. This pedagogical device is more than an environment. It is imbued with a complexity of relationships evoked by the users and the influence of underlying discourses.

Participants

There were 41 student and eight teacher participants altogether in the original interview groups. A number of students had left by the time of the second interviews, due to shifting, finding work or other training, or not being present on the days of the interviews at their campus. They were all aged 16 to 18 and came from a variety of settings. By the nature of Youth Guarantee, they had no formal literacy and numeracy qualifications and a large proportion had left school without any qualifications. There was a mixture of ethnicities, but the great majority were Māori and Pasifika. The teachers (kaiako) were responsible for selecting the apps, in conjunction with a contracted external facilitator.

Methods

Methods used to generate the data included: Student group semi-structured interviews (two groups in each class, pre-iPad intervention); student attitudinal surveys, (post-iPad intervention); student group semi-structured interviews (two groups in each class, post-iPad intervention); teacher group semi-structured interviews; class observational data; and before and after assessments using the Tertiary Education Commission (TEC) online diagnostic tool. The survey contained both quantitative and qualitative data with 19 questions using a 5-point point-scale and three open-ended questions. The TEC online adult assessment is designed to identify learner's strengths and weaknesses in numeracy. It draws from a database of problems set in adult contexts. Typically, students do about 30 questions, but this varies due to the adaptive online nature of the Assessment Tool. The researchers only had access to the individual students' level scores, not the component parts. Level one indicated the lowest conceptual understanding.

The research was conducted in accordance with Kaupapa Wānanga: Koha (provided valued research); Āhurutanga (ensured the wellbeing and dignity of participants); Kaitiakitanga (acknowledgement of the contributions of all people associated with the research); and Mauri Ora (the potential to improve student outcomes).

Results and Discussion

Interestingly, all the students agreed that numeracy was part of everyday life. They articulated a connection between being functional in a range of everyday tasks and using numeracy. Typical responses to the question "Where do you use numeracy?" were:

> Jed: If you can't add or subtract, you're going to get ripped off by the shopkeepers, it's very important, you use it every day … Yeah, bills and power bills and stuff like that.
>
> Charlie: Yeah, it pretty much revolves round everything … you've got to know maths.

In the questionnaire data, 95% agreed or strongly agreed that maths was useful, while 90% agreed that people use maths every day. As well, 70% agreed or strongly agreed that they needed maths to get a good job, while 90% agreed or strongly agreed that maths is important. Most of the students thought that it was important for employment, that it would make them more employable and better employees. Typical responses were:

> All of one interview group: Yes, need it to find a job.
>
> Mike: I just like maths cos it helps me in the future, going to have to need it in the work force…

While they were consistent in the acceptance of mathematics as a valuable aspect of being an informed citizen, their self-efficacy and attitude towards mathematics was often negative. They attributed some of their negative attitude to their teachers. The students' perceptions of their teachers were accessed through a question asking them 'Is there a maths teacher you can remember? Tell me a bit about them'. In general, this group retained negative perceptions of their relationship with their teachers, particularly in mathematics. Some felt they were treated inequitably, for instance:

> Nel & Jenni: The teachers didn't support the dumber students …
>
> Mike: I always ... I never actually had a problem with literacy and numeracy, it was just all the teachers that spoiled it for me ... many students that have dropped out, it is all ... 90% is because of the teachers ... the way they teach ... yeah, some teachers mix it up and get you confused.
>
> Charlie: At school it was just hard and fast…they didn't really explain things. That's why I hated it at school.

This perception, coupled with the lingering negativity towards mathematics, gestures towards the need to reshape the mathematics learning experience for these particular students. Another interesting aspect from the questionnaire data was that 80% agreed that they would avoid maths if they could. However, 45% agreed or strongly agreed that maths is interesting.

TEC Online Assessment

The initial pre-app intervention assessment results for this group were low, with step 2 the most common level, and most students at steps 2 and 3. In the post-app intervention assessment, steps 3 and 5 were the most common levels, with most students at steps 4 and 5. Overall, there was greater than expected improvement between the initial and final online numeracy assessments although this cannot be attributed solely to the iPad app intervention. The mean of the initial online numeracy assessment steps was 3.3 and the mean of the final one was 3.8. In a hypothesis test for the difference in the two means, $z = 1.65$, which is significant at the 90% level. So at this level, we can conclude that there was a difference between the two means. There was a complex array of interconnected contributing aspects that would have been influential in this improvement, including those outside of the learning environment.

The following graph compares the initial and final online assessment.

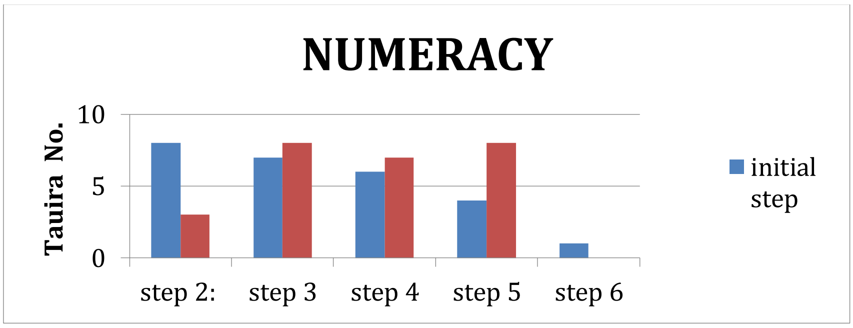


Figure 1. Student step in numeracy online assessment

Influence of using the iPad apps

The use of iPads apps within the teaching and learning process led to some changes in students' attitudes towards numeracy. Most students felt the change was positive and enjoyed learning in the visual interactive manner that the apps evoked. They enjoyed the change in pedagogical medium and found the learning fun and engaging. Typical comments were:

> Paora: It's visual. It's cool, good for each and all of us. We all had our own one to use.
>
> Hine: They're fun, easy, makes it easier. Better than plain writing.
>
> John: You play on them. They're active.
>
> Whetu: … and visual. Felt like it was more easy.
>
> Ollie: Liked the maths games. We played it as a whole class. That maths thing with the facts helps me with my learning. It makes me brainy. It's interactive.
>
> Tom: Yeah, it helped to focus, and with concentration.
>
> John: Made me more confident.

Some students found it a bit repetitive as a learning approach.

> Nel: Sometimes it gets boring.

In general though, the changes in student attitudes and engagement were positive, especially when the apps were integrated into the learning programme in an interactive way or as a class or group game or challenge. Teachers were positive about the learning experience for students, while also seeing potential learning opportunities. A prevalent teacher observation was that students were more engaged. They indicated that students enjoyed using the apps as part of the learning.

> Anthony: Super interactive, like when it came to maths, the maths games, everyone was so enthusiastic about it. And then the different games that we came up with. They are really good if you want to get team interaction games. Also, getting them into groups and working as a team. Everyone's just thoroughly enthusiastic.

They also indicated that these aspects led to greater student confidence.

> Ben: They (the students) value themselves more when they are confident. These have helped.

Another teacher commented on the use of apps for games and competitions.

> Ash: Very positive. It's good because it's more interactive so we are able to utilise it as tutors to challenge them off against each other. You see who knows what. It's more 'hands on', more practical so they are able to see the calculations and add it up on the spot as opposed to writing.

The apps games were viewed positively as a context for engaging with the mathematics. Students generally enjoyed them and the ensuing social interaction, identifying learning through games as a positive experience. For example,

> Manu: Now maths is fun, our teacher explains more. It helps learning when it's fun … games, times tables, it's a fun way to learn.

The data indicated that the use of iPad apps in the numeracy learning programme transformed student attitudes in a positive way. For some, their integration into programmes coupled with accompanying social interaction, led to students who had been very negative towards mathematics feeling confident and willing to try new approaches. While their cognition also developed over this period, this was not necessarily related to the use of iPad apps, hence the research was directed towards their beliefs and attitudes. With reluctant learners, it would be difficult to change mathematical ability significantly over a short period of time, but if attitudes towards mathematics became positive, this in time will influence conceptual understanding.

Conclusions

The data were consistent that the use of the iPad apps in Youth Guarantee numeracy programmes was received very positively by students and had been influential in transforming their attitudes towards numeracy. Consistent with other studies (e.g., Attard & Curry, 2012), they contributed to the development of positive attitudes towards numeracy. The initial interviews indicated a high proportion of negative attitudes, while the questionnaire likewise contained a relatively high proportion of responses that echoed those sentiments. The data indicated that student experiences in numeracy had been negative over a sustained period. Beliefs and attitudes are episodic in their development, and emerge through experiences that individual's respond to in varying ways. For these students, both relationships with their schoolteachers and the nature of the curriculum were influential in this disjuncture between perceiving something is important and eventually not wanting to engage with it. Comments that indicated frustration, disengagement, negativity and at times hostility were articulated. To get even a small transition in attitudes would have been significant, but there was a high proportion of attitudinal change across the pre and post data.

The reasons articulated for this change were primarily because of the repackaging of the content and processes. The iPad work was only one aspect of this, along with teacher pedagogical approaches and transformational practice. The students' learning was mediated by the use of the pedagogical device (the iPad) and the language associated with this usage. Most of the students and teachers responded that the iPad component of the programme was instrumental in the transition. The reasons for this were based around the fun and engagement aspects when engaging with the maths apps, but also through the affordances of the digital pedagogical medium. Comments such as the learning being visual, interactive and dynamic were recorded, and resonate with other reports of the learning experiences of primary-aged students who engaged mathematics through an apps pedagogical medium (Carr, 2012; Pelton & Pelton, 2012). Many found the iPad apps less threatening and easier to learn from.

The inclusion of the game-based apps in their numeracy programme made the learning more engaging for the students and in much of the data, they facilitated increased enthusiasm and participation. This is consistent with Attard's (2013) study. Some tasks which had previously been considered repetitive and boring, such as learning basic numeracy facts, were engaging for students within an apps game context. This needs to be tempered by comments that playing the same game repeatedly in time caused students to lose some motivation to play, and that several students commented that some of the games were too easy or too babyish. Nevertheless, the vast majority of the data clearly indicated that in terms of the affective dimension of learning, the use of iPad apps in the numeracy programme led to more positive dispositions towards learning, increased engagement, and enjoyment of the learning experiences in these areas. In general, this is in contrast with their attitudes towards numeracy prior to being enrolled in a Youth Guarantee programme. While increased engagement and a more positive disposition towards learning generally transform and enhance cognitive understanding, this does not always manifest simultaneously and has to be considered within a tapestry of inter-related influences.

A successful and motivational way of using the apps was when they were introduced or played as a whole class competition. There was also informal social interaction associated with this as students verbalised their feelings and mutual encouragement. Students collaborated on strategies with this approach. Hutchison, Beschorner and Schmidt-Crawford (2012) also identified that apps facilitated collaboration between students by allowing the simultaneous sharing of responses or screens. This was also observed with students in this study. The apps and iPads were used in a variety of ways that enriched the diversity of learning approaches that were possible, and facilitated both focused and incidental social interaction. Hence, the mathematical activity was inseparable from the pedagogical device and the social interaction that was facilitated through the use of the apps.

The research was relatively cohesive regarding the appeal of game-based apps for learners. Students found them engaging and motivational, and advocated their inclusion in programmes. As well, teachers reported perceptions of their positive influence on students' attitude to learning that echoed the students. Perhaps there is an element of novelty and a potential for being engaged without learning, but generally if students are motivated, more engaged, and enjoying an element of learning, they will come to understanding more readily. Central to them enhancing the learning in an engaging manner, is keeping the apps as part of a varied programme, to ensure that they are relevant and appropriate for the students, and for the development of apps to be ongoing and responsive to critical review. Case studies give insights into particular situations but are limited in terms of generalising behaviour or learning. Nevertheless, they do enhance an accumulating body of research. Another limitation of the research was that the researchers interpreted the data through their prevailing discourses and preconceptions. While awareness of this meant that consideration was given to alternative perspectives, with an interpretive approach, the researchers were unlikely to completely escape the influence of personal perspectives.

Today's learners can use digital media effectively to communicate, investigate and process ideas and personal questions. In general, they are comfortable with and interested in their use. However, just allowing these learners access to mobile technology is not sufficient to enhance learning, nor educationally ethical. It has to be resourced equitably, and have both the learners and the teachers engaged in processes that enable effective use. Effective utilisation also requires having both teachers and students involved in their ongoing evaluation and dynamic development. Teachers and students need to be influential in the development of apps and the ways they are used in the learning process. If the interrelated pedagogical processes and conceptual thinking are given primacy, then apps can enhance the learning experience and understanding of students. They certainly offer affordances that might transform the attitudes of reluctant teenage students towards learning mathematics.

Acknowledgements

We would like to acknowledge Te Wananga o Aotearoa and Wilf Malcolm Institute of Educational Research for their support of the project.

References

Attard, C. (2013). Introducing iPads into primary mathematics pedagogies: An exploration of two teachers' experiences. In In V. Steine, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 58–65). Melbourne, VIC, Australia: MERGA.

Attard, C., & Curry, (2012). Exploring the use of iPads to engage young students with Mathematics. In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), *Mathematics education: Expanding horizons Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia* (pp. 75–82). Singapore: MERGA.

Beatty, R., & Geiger, V. (2010). Technology, communication and collaboration Re-thinking communities of inquiry, learning and practice. In C. Hoyles, & J-B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain* (pp. 250–284). New York, NY: Springer.

Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication Technologies, modeling, experimentation and visualisation*. New York, NY: Springer.

Calder, N. S. (2011). *Processing mathematics through digital technologies: The primary years*. Rotterdam, The Netherlands: Sense.

Calder, N. (2015). Apps: Appropriate, applicable and appealing? In T. Lowrie & R. Jorgensen (Eds.). *Digital games and mathematics learning: Potential, promises and pitfalls.* The Netherlands: Springer.

Carr, J. (2012). Does math achievement h'APP'en when iPads and game-based learning are incorporated into fifth-grade mathematics instruction? *Journal of Information Technology Education, 11,* 269–286.

Forbes, S., & Pfannkuch, M. (2009). Developing statistical thinking: Teaching and learning. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (pp. 93–128). Wellington, New Zealand: NZCER Press.

Grootenboer, P. J. (2008). Mathematical belief change in preservice primary teachers. *Journal of Mathematics Teacher Education*, 11 (6), 479-497.

Hutchison, A., Beschorner, B., & Schmidt-Crawford, D. (2012). Exploring the use of the iPad for literacy learning. *The Reading Teacher*, *66*(1), 15–23.

Larkin, K. (2013). Maths education: Is there an app for that? In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow*. *Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 426–433). Melbourne, VIC, Australia: MERGA.

Li, S. C., & Pow, J. C. (2011). Affordance of deep infusion of one-to-one tablet-PCs into and beyond classroom. *International Journal of Instructional Media, 38*(4), 319–326.

Lowrie, T. (2005). Problem solving in technology rich contexts: Mathematics sense making in out-of-school environments. *Journal of Mathematical Behaviour, 24,* 275–286.

Pelton, F. L., & Pelton, T. (2012). Sharing strategies with teachers: iPods in math class. In *Society for Information Technology & Teacher Education International Conference* (Vol. 2012, No. 1, pp. 4363–4366).

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Allyn & Bacon.

CAS or Pen-and-paper: Factors that Influence Students' Choices

Scott Cameron
The University of Melbourne
<scottwc@student.unimelb.edu.au>

Lynda Ball
The University of Melbourne
<lball@unimelb.edu.au>

This paper reports on a study of choices about the use of a computer algebra system (CAS) or pen-and-paper (p&p) by a class of seven Year 11 Mathematical Methods (CAS) students as they completed a calculus worksheet. Factors that influenced students' choices are highlighted by comparing and contrasting the use of CAS and p&p between students. Teacher expectation of students' use of CAS and p&p reveals that, even in a small class, the students' use of CAS and p&p sometimes differed from what was expected. The analysis here indicates that there are a variety of factors that influence students' decisions, including speed of calculation and accuracy of p&p work.

Background

In Victoria, Australia, a Year 11 functions and calculus course, called Mathematical Methods (CAS) (VCAA, 2010) has integrated CAS for more than a decade. Teachers and students working in classrooms with a computer algebra system (CAS) available face choices about the use of CAS or pen-and-paper (p&p) for teaching, and learning, mathematics and for solving problems. Teachers' choices about use of CAS may influence students' opinions about CAS (Artigue & Lagrange, 1997), which is not surprising given that a teacher will institutionalise acceptable techniques in their classroom. A CAS may "gobble up" (Flynn & Asp, 2002) intermediate steps that might normally be part of a p&p solution and when students are solving problems there is a choice to be made about p&p or CAS.

In a study of senior secondary students, Geiger (2008) found that access to CAS enabled students to solve problems beyond their p&p capability. Geiger, Galbraith, Goos, and Renshaw (2002) noted that students would use CAS when symbolic features were seen to expedite processes. Ball and Stacey (2005) found that in a group of five students there were differences in preferences for CAS use; the main use was for speed. One student used CAS to compensate for weak pen-and-paper skills and CAS was also used for checking answers. This study investigated the choices that seven Year 11 students made regarding CAS or p&p use when working on a calculus worksheet. Students' use of CAS for the problems is compared with their teacher's expectation of CAS or p&p use.

Methodology

Participants in this study were a teacher, Peter (all names are pseudonyms), with four years' experience teaching Mathematical Methods (CAS), referred to as MMCAS here, and his Year 11 class in a co-educational school in Victoria, Australia. Peter was invited to be involved in the study as he was known to the first-named researcher, and had previously expressed interest in participating in the study. His class of seven students had been studying MMCAS for 8 months at the time of data collection. Peter supported the use of CAS, and all students (excluding one student, Emily) owned their own CAS.

Data reported here was collected within a wider study examining influence of attitude on use of CAS in a class activity (see Cameron & Ball, 2014). Three research instruments were used to collect data: a worksheet, semi-structured individual interviews (teacher and students), and lesson observation notes. In consultation with Peter, it was decided that the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 141–148. Sunshine Coast: MERGA.

focus for the worksheet would be calculus, the topic studied at that time. Students completed the worksheet after they had studied differentiation and anti-differentiation in class. Peter was consulted to ensure that the worksheet was an appropriate length for a 50-minute lesson. Six problems were discussed with Peter: three written by the first-named researcher and three from other sources. Peter requested minor modifications to the wording of original problems, resulting in the final six problems shown in Table 1. Two problems required the use of CAS to solve (problems 4 and 6); the other four problems could be solved using CAS or p&p.

Table 1
Worksheet Problems

Problem	Statement of problem
1	Find $f'(x)$ if $f(x) = 95 + 2.7x + 4x^2 - 0.1x^3$
2	Use first principles to find the derivative of the function $f(x) = x^3 - 4x + 1$
3	An art collector purchased a painting for $500 from an artist. After being purchased the value of this artist's paintings increase, with respect to time, according to the formula $\frac{dP}{dt} = \frac{5}{2}t^{\frac{3}{2}} + 20t + 100$, where P is the anticipated value of the painting t years after it is purchased. Find an equation that will give the price of the painting at a given time. Consequently, find the price of the painting 4 years after it was purchased.
4	Determine the anti-derivative of the function $f'(x) = \sin(2x + 1) + 7\cos(x)$
5	If $f(x) = k(x - a)(x - b)(x - c)$ what is the simplified form of the derivative $f'(x)$?
6	What are the coordinates of the stationary points for the function $f(x) = (4x + x^{-2})^2$??

Note – Problem 1 Flynn, Berenson and Stacey (2002); Problems 2, 4, & 6 First-named researcher; Problem 3 adapted from Flynn (2001); Problem 5 adapted from Flynn et al. (2002).

A semi-structured interview was conducted with Peter prior to students completing the worksheet. Interview prompts focussed upon his own use of CAS when teaching calculus, expectations of CAS use for these six problems, features of the problems that would contribute to CAS or p&p use, and difficulties that students may encounter.

Students had one 50-minute lesson to complete the worksheet. They worked individually or in groups and Peter interacted with students as they worked. While completing the worksheet, students self-reported (refer to Figure 1) both the process used (e.g. anti-differentiation) and the use of CAS or p&p. If students did not record any steps, but indicated use of CAS or p&p, the solutions were analysed to determine the steps used. The first-named researcher wrote lesson observation notes, without interacting with the participants. The focus of these notes was student/teacher and student/student interactions.

For the process of Anti-differentiation	I used: Pen and Paper	CAS	(Please circle)
For the process of Substitution	I used: Pen and Paper	CAS	(Please circle)
For the process of Solving	I used: Pen and Paper	CAS	(Please circle)
For the process of ______	I used: Pen and Paper	CAS	(Please circle)
For the process of ______	I used: Pen and Paper	CAS	(Please circle)

Figure 1. Self-reported CAS or p&p use – Jessica problem 3.

One week after completing the worksheet, individual semi-structured interviews were conducted with six students (Amy was absent). Interview prompts were based upon student responses to the worksheet and survey (not reported in this study). Interview transcripts were examined for comments to provide insight into students' choices of CAS or p&p.

Results and Discussion

Table 2

Methods Expected by Teacher and used by Students (adapted from Cameron & Ball, 2014)

		Peter	Sam		James		Emily		Simon		Jessica		Amy		Kate	
Problem	Key steps	Anticipated Method	Method	Correct	Method	Correct	Method	Correct	Method	Correct	Method	Correct	Method	Correct	Method	Correct
1	Differentiation	P	C	✓	C	✓	P	×	P	✓	C	✓	C	✓	P	✓
2	Expand $f(x+h)$	P	P	✓	P	✓	P	✓	P	✓	P	✓	P	✓	C	✓
	Simplify $\frac{f(x+h)-f(x)}{h}$	P	P	×	P	✓	P	✓	P	✓	P	✓	P	✓	P	✓
	Limit	P	P	×	P	×	P	✓	P	✓	C	✓	P	✓	P	✓
3	Anti-differentiation	P	N	×	N	×	N	×	N	×	C	✓	C	✓	C	✓
	Solve for c	P	N	×	N	×	N	×	N	×	P	✓	C	✓	C	✓
	Substitution	P	N	×	N	×	N	×	N	×	P	✓	C	✓	C	✓
4	Anti-differentiation	C	C	×	N	×	N	×	C	✓	C	✓	C	✓	C	✓
5	Expansion	R	N	×	N	×	N	×	P	✓	R		R		R	
	Differentiation	C	N	×	N	×	N	×	N	×	C	✓	C	✓	C	✓
	Simplification	R	N	×	N	×	N	×	N	×	N	×	N	×	N	×
6	Expansion	C	N	×	N	×	P	✓	N	×	R		R		C	✓
	Differentiation	C	N	×	N	×	N	×	N	×	C	✓	C	✓	C	✓
	Finding x co-ordinates	P	N	×	N	×	N	×	N	×	C	✓	C	×	C	×
	Finding y co-ordinates	P	N	×	N	×	N	×	N	×	C	✓	N	×	N	×

Table 2 provides a summary of the key steps for each problem (identified by the researcher prior to the lesson), the anticipated method (either CAS or p&p) identified by Peter and the methods used by students. Student responses for each step were coded using four categories: C (CAS was used to complete the step); P (p&p was used complete the step); N (No evidence that this step was completed); and R (CAS removed the need for the student to complete the step). Information is provided on correctness; a tick indicated that a step was completed correctly, whilst a cross indicated an incorrect response. A blank indicated that CAS removed the need to complete the step.

Problem 1

Peter expected his students to use p&p methods for problem 1, stating that "it would just be quicker to do it with p&p than type it into the calculator". He elaborated: "if they [the students] recognise the function and the task looks easy to do, they will do it with p&p". This may indicate that Peter expected his students to perform routine procedures, in this case differentiation of a function, with p&p. It is not unreasonable to expect Year 11 students to mentally calculate the derivative of a polynomial function and write the result, so it was not surprising that p&p methods were viewed as faster than CAS.

Emily, Simon and Kate used p&p (refer to Table 2) to solve problem 1, whilst Sam, James, Jessica and Amy used CAS; in interviews they gave a range of reasons for their choices. Kate, who solved the problem using p&p, made her choice based on speed, reporting that p&p was quicker than CAS. She stated, "I find myself a faster writer than pressing it into the calculator", so syntax entry was viewed as time consuming. This concurred with Peter's notion that students would use p&p methods for problem 1 for speed. Emily (who doesn't own a CAS, but borrowed Peter's during the lesson) stated that she wouldn't use CAS to differentiate, due to her lack of familiarity with syntax, as "I don't really know what the buttons do and how to use [CAS features]". It was not surprising that a student unfamiliar with CAS chose to use a p&p method for this problem and in this case, the problem was within the expected p&p range of students.

Some students chose CAS for speed, rather than p&p. Sam used CAS for speed and accuracy because "I knew that I'd probably get a more precise answer if I used my calculator and it's quicker". James, who also used CAS, valued its speed, stating that his CAS use is "… more as a time saver than anything". Interestingly, James stated that "if you can solve it [a problem] by p&p then there's not really a place for it [CAS]". It seemed that James first considered whether or not he could solve a problem using p&p and only considered CAS if p&p wasn't viable, citing speed as the determining factor. Even for this one step problem perceptions about speed of CAS and p&p varied between students.

Problem 2

Peter expected students to use p&p methods as this was how he taught differentiation from first principles in class and he anticipated his students would replicate his approach.

> Perhaps an overriding factor for students using p&p or calculator would be the initial way in class that they had been taught. If we started looking at the concept using p&p, this tends to be the way they [the students] respond to a questions and the same goes if we started off looking at a concept on the calculator.

Peter noted that "expanding can be done using the calculator" so for this problem, where calculus was the core mathematical focus, he supported use of CAS for expansion (a lengthy p&p process here).

Sam, James, Emily, Simon and Amy used p&p for problem 2 (see Table 2), so five out of seven students used the approach demonstrated (and expected) by Peter. Although Sam noted the speed of CAS for problem 1, she chose p&p for problem 2 as: "we would always do it [i.e., differentiation from first principles] with p&p". Jessica and Kate used a combination of CAS and p&p to solve problem 2, while Amy (who worked with them) chose p&p only. The use of CAS and p&p seemed to be a personal choice, even for the students who worked in groups.

Students commented that they wouldn't always replicate what the teacher demonstrated and that speed of calculation played a factor in their decisions. Even though James solved this problem using p&p, he commented that if a problem "was going to take forever to do by p&p then it's going to really help if you can do it by the calculator". Whilst students appreciated the use of CAS to quickly complete problems, it is possible that because Peter had taught them to perform differentiation from first principles using p&p that they didn't realise, or perhaps thought it was unacceptable, to use CAS in this problem.

Jessica used p&p for substitution and CAS to find a limit commenting that p&p was needed to show working expected by her teacher.

> You can't just put it in [to the calculator] and go well there's your answer, because then they'll [the teacher] go, "well where's your working out?"

Jessica determined what was acceptable in mathematics classes by analysing Peter's teaching "I think the whole point of it [Peter's teaching], is to learn how to do it by hand [i.e., p&p] so then you understand where it's coming from". This insightful comment showed that students do determine what practices are institutionalised from their teacher.

From Table 2 we can see that Peter and Kate had the same preference for use of p&p for problems 1 and 2, except for expansion where Kate used CAS. In the interview, Peter identified expansion as a potential area for CAS use, despite expecting p&p, so their choices aligned here. Again we note the variety of choices, but in this problem student choices aligned well with Peter's expectation. This could be due to the nature of the problem, where Peter favoured p&p use to "show the process of first principles".

Problem 3

Peter expected use of p&p for problem 3. He noted: "CAS could be used just as easily to find the integral, particularly for working out the fraction part, which is where the students are likely to make the mistake". This suggested that CAS use may minimise the potential for errors that might appear in p&p working for this problem.

Emily, Sam and James asked Peter for assistance with problem 3, but neither they, nor Simon, gave a solution. Amy and Kate used CAS, whilst Jessica used a combination of CAS and p&p (refer to Table 2). Although problem 3 was not addressed specifically in the interview, Jessica stated that she completed "the easy stuff [on the worksheet] by hand" and used CAS for "the more complex things". The anti-differentiation required for problem 3 may have been viewed as complex by Jessica, whereas calculating the coefficient of integration and substitution could be perceived as easy tasks. It is possible that a belief that p&p was required to show working (discussed in problem 2) for multistep problems influenced Jessica's decision to use both CAS and p&p. Kate and Amy, who worked together, used CAS to solve problem 3 so in this case collaboration may have resulted in similar CAS use.

Problem 4

Peter stated, "Students would not know how to find the anti-derivative of this function [i.e., $f'(x) = \sin(2x + 1) + 7\cos(x)$] so I'd expect them to use the calculator". Students were anticipated to encounter difficulty here as "they've never seen those functions [in problems 4 and 6]" however he noted that "students that generally persevere at questions are more likely to use their calculators than those that are less confident, or give up easily". This suggested that confident students would persevere with solving problem 4 if they had CAS available. Although problem 4 was designed to lie outside the range of students' p&p skills (see Cameron & Ball, 2014), Peter believed that where "the problem involves functions they [i.e., the students] are uncertain about, they may use the calculator " to perform the anti-differentiation. In this case a CAS extends the range of problems that students can solve.

Sam, Simon, Jessica, Amy and Kate attempted problem 4 using CAS and four of them were correct. James and Emily did not provide working for problem 4, with James stating that he did not solve the problem as "the whole sine and cosine … confuses me". Conversely, Emily felt that she could have completed the problem "if I knew more about the CAS "; so one student viewed the mathematics as problematic, while the other had technical concerns. Both students sought assistance from Peter, but did not end up providing a solution. Although Peter believed that access to CAS would enable students to solve this problem, this was not the case for these two students. This highlights that the mere presence of CAS does not guarantee that students can use it to solve problems.

Jessica used CAS to solve problem 4 as the unfamiliar function perturbed her, stating, "It [the function] threw me off. I didn't think I'd be able to do it by hand, so I thought I'll just do it on the calculator and see what it says". Simon also chose to use CAS and suggested to the researcher that this problem was outside his p&p range stating "I don't think I'd done these problems before so I'd just use the calculator". Students can use CAS to solve problems outside their p&p skills when they have an understanding about what is required to solve problems of a particular type, which in this case involved anti-differentiation. Kate used her knowledge of inbuilt features of CAS, "you can do it for any problem with that part of CAS". It is possible that perseverance was a key requirement to solve this problem, as all students stated they were unfamiliar with the function, but as Peter expected, they applied their knowledge of CAS to solve the problem.

Problem 5

Peter expected students to use p&p to solve problem 5. However, he recognised "CAS would expand it [i.e., the function] for you and then find the derivative too" suggesting that a CAS based approach was possible. Peter did not identify that CAS could "gobble up" (Flynn & Asp, 2002) the step of expansion here to enable differentiation to be performed in one step. Peter believed that students might experience difficulty "because it [i.e., the original function] is factorised ", an unfamiliar problem format for his students.

Simon, Jessica, Amy and Kate were the only students who attempted problem 5. Simon expanded the function using p&p, but did not solve the problem. Simon explained that when he encounters a difficult problem he would most likely skip the problem, rather than persevere. This was illustrated on the worksheet where Simon started solving problem 5, (expanding the expression correctly) but struck difficulties in continuing to solve the problem, writing on his worksheet "I have no idea what I am doing". This provides an example to support Peter's contention that those who use p&p are less likely to persevere with difficult problems than CAS users. In this case a student using CAS could avoid the complexities encountered by Simon by finding a derivative in one step.

Sam, James and Emily did not complete problem 5, with James asking, "Have we ever seen questions like that?" Although some students may not attempt problems with unfamiliar functions, they do not necessarily form a barrier when students have CAS and know the mathematics, as evidenced above in problem 4.

Jessica, Kate and Amy used CAS for differentiation; CAS "gobbled up" (Flynn & Asp, 2001) the step of expansion here. In the interview Kate discussed her use of CAS to explore unfamiliar problems, "I just write it down … if it looks a bit weird then I'll stick it on the calculator and see what happens". This, along with Jessica's explanation of her approach to problem 4, suggested that unfamiliar functions can be less problematic for students when they understand what is required and can use CAS for calculation and exploration.

Problem 6

Peter expected students to use CAS for problem 6. CAS use was required here as the problem was outside students' range of p&p skills, with Peter noting that this problem would be within the range of p&p skills of Year 12 students, rather than Year 11: "I suppose in Year 12 you could use the rule to find the derivative; some p&p work could be used to find when the derivative is equal to zero".

Three students (Jessica, Amy and Kate) attempted problem 6 using CAS and Emily used p&p. Emily completed the problem up to the step where the use of CAS was required. Emily was not a CAS user, so it is not surprising that she did not use CAS. Kate used CAS to expand the brackets, as she stated that she does not like brackets. She then used the expanded form and "stuck it into the calculator because it knows how to do it for me". This contrasts with her use of CAS in problem 5 where she did not expand the brackets before using CAS to differentiate; students can make decisions problem-by-problem and also step-by-step. Jessica was able to correctly solve this problem using CAS, but required the support of Peter and her peers with syntax. Her first inclination when solving was to consider whether or not p&p was a viable option, "I thought I could do it by hand, but I didn't really know … I have no idea how to do it by hand". She then used CAS, stating "I threw in the original function to find the derivative on it and once I did that, I'm pretty sure I did all the steps on CAS". The CAS use by Kate and Jessica was quite different even though they were working together. As Jessica chose to differentiate the function using the format of the function provided on the worksheet, this removed the need to expand the function (a step where Kate used CAS, prior to differentiating).

Conclusion

It might be expected that in a class of seven students there would be some consistency of CAS use as there is the opportunity for the teacher to spend considerable time with individuals discussing use of CAS and p&p. In this study, we found that even with a small class of seven Year 11 students there were considerable differences in the choices that students made about CAS or p&p for solving common problems. Students' choices sometimes aligned with the expectations of the teacher, but this was not always the case, with students using CAS more than the teacher anticipated. This highlights the complexity of a CAS-active classroom, where students are solving problems in a variety of ways, not just the way that has been demonstrated by their teacher. Artigue (2002) commented on an "explosion of possible techniques" (p. 260) in a CAS-active classroom.

Students' made problem-by-problem and also step-by-step decisions about the use of CAS or p&p. This suggested that students evaluated problems based on the perceived personal benefits of CAS or p&p throughout a problem. Speed and accuracy appeared to be two key factors influential in students' choices. Different students cited either CAS or p&p as being faster for specific tasks, so this shows that speed is very dependent on facility with p&p skills as well as technical ability for using CAS. Although students based choice of CAS or p&p on speed and accuracy, these were not the only factors influencing their choices. Some students noted a preference to use the approach demonstrated by their teacher. Where Peter identified that his students could use CAS, the students tended to use CAS. This showed that Peter had insight into range of the students' p&p skills and also that CAS enables students to solve problems outside the range of their p&p skills; some students were willing to tackle unfamiliar problems with CAS. Where students have the conceptual understanding of the key mathematical principle in a problem (e.g., differentiation) they are able to use CAS to correctly answer the problem. This may have implications for teaching, as teachers can introduce more difficult problems using CAS; scaffolding students' conceptual understanding.

The findings reported in this paper highlight a diverse use of CAS and p&p in a small class and the complex way in which students make decisions regarding the use of CAS and/or p&p. The students in this study are undertaking a subject with CAS-assumed examinations at Year 12 level, so decisions about the use of CAS or p&p for speed, accuracy or to supplement their p&p skills are important ones to consider at Year 11.

Reference List

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245-274.

Artigue, M & Lagrange, J-b. (1997). Pupils learning algebra with DERIVE: A didactive perspective. *Zentralblatt für Didaktik der Mathematik, 29*(4), 105-112.

Ball, L., & Stacey, K. (2005). Students' views on using CAS in senior mathematics. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce & A. Roche (Eds.), *Building Connections: Theory, Research and Practice* (Proceedings of the 28th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 121-128). Sydney: MERGA.

Cameron, S., & Ball, L. (2014). Attitude and CAS use in senior secondary mathematics: A case study of seven Year 11 students. *International Journal of Technology in Mathematics Education, 21*(4), 143-155.

Flynn, P. (2001). Year 11 semester two exam. Retrieved from http://extranet.edfac.unimelb.edu.au/DSME/CASCAT/ResourcesForTeachers/Teachingresources/Year_11_support_material/Y11sem2_Exam.pdf.

Flynn. P., & Asp, G. (2002). Assessing the potential suitability of "show that" questions in CAS-permitted examinations. In B. Barton, K. C. Irwin, M. Pfannkuch, and M. O. J. Thomas (Eds.), *Mathematics Education in the South Pacific* (Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 252-259). Auckland: MERGA.

Flynn, P, Berenson, L. & Stacey, K. (2002). Pushing the pen or pushing the button: a catalyst for debate over future goals for mathematical proficiency in the CAS age. *Australian Senior Mathematics Journal, 16*(2), 7-19.

Geiger, V. (2008) Learning mathematics with technology from a social perspective: A study of secondary students' individual and collaborative practices in a technologically rich mathematics classroom (Unpublished doctoral dissertation), The University of Queensland, Queensland, Australia.

Geiger, V., Galbraith, P., Goos, M. and Renshaw, P. (2002). Matching the hatch - Students' choices and preferences in relation to handheld technologies and learning mathematics. In B. Barton, K. C. Irwin, M. Pfannkuch, and M. O. J. Thomas (Eds.), *Mathematics Education in the South Pacific* (Proceedings of the 25th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 293-300). Auckland: MERGA.

Victorian Curriculum and Assessment Authority [VCAA]. (2010). Mathematics: Victorian Certificate of Education Study Design. Retrieved from http://www.vcaa.vic.edu.au/documents/vce/mathematics/mathsstd.pdf.

The Language Used to Articulate Content as an Aspect of Pedagogical Content Knowledge

Helen Chick
University of Tasmania
<Helen.Chick@utas.edu.au>

Mathematical knowledge in classrooms is mediated through the use of both technical and informal language. This paper is a report of a study of the language use of teachers as they examine students' work and discuss teaching for the topic of fraction operations. This provides a window on their pedagogical content knowledge and also on the way in which language is used to make sense of mathematical knowledge, either personally or for students. It was found that some mathematical knowledge appeared to be *taken as understood*, perhaps because the expected words were used.

Introduction and Background Literature

Recently the author was reading some material that reported on teachers' solutions to a particular class of proportion problems. What was striking about these solutions, which came from a country different from her own, was the consistency of the terminology and representations used by the teachers as they presented their mathematical reasoning. In addition, this consistency of language seemed to give the teachers a greater capacity to articulate the mathematics: they were fluent in the way they used the terminology in support of their solutions. Their content knowledge of mathematics appeared to be enhanced by this precision of language, and the language was used effectively in support of their further discussions about how they might help students with similar problems. This suggests that the terminology and representations used by teachers might give insights into their pedagogical content knowledge (PCK).

There are a number of frameworks associated with the knowledge for teaching mathematics, and they categorise that knowledge in slightly different ways. The knowledge quartet of Rowland, Huckstep, and Thwaites (2005) is a little different from some of the other frameworks, in that it views certain aspects of knowledge dynamically. The more static component—foundation—includes "the meanings and descriptions of relevant mathematical concepts" (p. 265), which includes relevant terminology. One of the more dynamic knowledge-in-action aspects—transformation—also makes reference to language, and considers how "the teacher's own meanings and descriptions are transformed and presented in ways designed to enable students to learn it" (p. 265), with the acquisition of essential vocabulary explicitly mentioned as part of the teacher's work. The Hill, Ball, and Schilling framework (2008, p. 377) for mathematical knowledge for teaching highlights common content knowledge, which includes terminology, although it might be argued that some terminology (e.g., the use of the word *whole* with respect to fractions) is really only used by teachers, rather than being in general mathematical use. Finally, the PCK framework of Chick, Baker, Pham, and Cheng (2006) identifies at least two components that allude to language use: *knowledge of representations* (which might be construed to include language, since language is used to signify and represent ideas), and *knowledge of explanations*, which requires use of appropriate language.

Boero, Douek, and Ferrari (2002) wrote about the role of natural and symbolic languages in mathematics, and assert that "only if students reach a sufficient level of familiarity with the use of natural language in the proposed mathematical activities can

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 149–156. Sunshine Coast: MERGA.

they perform in a satisfactory way" (p. 242). They also highlighted the teachers' role in increasing students' linguistic competencies, including in discussing solutions. Boero et al. discussed language as a mediator between mathematical objects, properties, and concepts and the development of *theoretical systems*, by which they seem to mean a connected conceptualisation of bigger mathematical ideas (such as *rational numbers and operations* in the case of this paper). In addition, they highlighted the role of language as a tool for the validation of statements.

The area of fractions is one of the first mathematical topics that moves beyond the concrete arena of natural numbers, where operations are readily visualised and described, often with words that are part of everyday language. With fractions come new technical words, such as *numerator*. The conceptualisations of part-whole relationships and operations such as addition and division must be mediated through language use. In their seminal chapter on rational numbers Behr, Harel, Post, and Lesh (1992) explore the complexity of the domain, and highlight the idea of a *unit*". When a fraction is defined in relation to a whole, it requires the conceptualisation of new units, namely the individual parts of the whole determined by the denominator. Thus, in the fraction 4/5, we must think of 1/5 as the unit (determined by creating 5 equal pieces from the whole), and consider 4 of these units. Simultaneously interpreting the information supplied by the denominator and the numerator is required in order to see a fraction as a single quantity. A complete understanding of the domain of rational numbers requires, among other things, understanding of how fractional quantities operate on other quantities and how we compute efficiently with such quantities (including finding valid algorithms).

Ma (1999) found that primary teachers from the United States (US) and China varied in their capacity to make sense of, for example, fraction division. In giving explanations of their reasoning, the Chinese teachers seemed to have consistent terminology with which to refer to the components, operations, and procedures (e.g., referring to *quotient* and *dividing by a number is equivalent to multiplying by the reciprocal*), whereas the US teachers were less consistent and, indeed, unsure about the language (e.g., *change them into sync*, *flip over and multiply*). In addition, the Chinese teachers could more often provide mathematical reasons to justify the processes used.

With this in mind the present research examines the language use and mathematical reasoning made evident in teachers' discussions of students' work with fractions. Specifically, it looks at the consistency of language and the way in which the mathematical ideas inherent in the terms and operations were discussed and justified.

Method

Data were gathered during focus group discussions involving some invited experienced teachers and the researchers (including the author, and colleagues from the *Powerful Knowledge* project of which this study was a part). There were primary and secondary focus groups sessions with around four to five teachers each in both Tasmania (Tas) and New Zealand (NZ), and a session with four primary teachers in Victoria (Vic). The teachers were purposely selected, and were known to the researchers as having an interest in mathematics teaching. The focus groups were intended to explore the knowledge that is brought to bear in teaching mathematics, and to provide an opportunity to access the tacit and explicit knowledge on which teachers draw in the act of teaching.

The stimuli for the focus group discussions were items covering a range of school level topics and pedagogical content knowledge issues. The researchers presented items in turn, and discussion ensued about the nature of and responses to the situation. The intent was to explore the kind of knowledge that is needed for these situations, rather than to gauge any or all of the teachers' capacities to respond. Because the nature of knowledge for teaching mathematics was the focus of the data gathering, the researchers themselves were not independent of the discussion and contributed to the conversations and, thus, to the data. Discussions were audio-recorded and later transcribed.

Two of the items used with the focus groups have been selected for analysis for this study, both coming from the domain of fraction operations. The task shown in Figure 1 was presented to the New Zealand and Tasmanian primary teachers only, and concerned a misconception that is reinforced by the child's choice of representation. To determine an appropriate response to the student's ideas, the teacher might choose to draw on other alternative representations (as suggested by some of the options in the question in Figure 1), or may choose to work with the child's self-selected representation. There are advantages and disadvantages associated with each of these approaches (see Chick, 2011).

A student says that 1/4 + 1/4 is 2/8. She uses counters to show this as follows:

Given what the student has just shown you, which of the following representations of 1/4 + 1/4 is most likely to help her to see that 1/4 + 1/4 = 1/2?

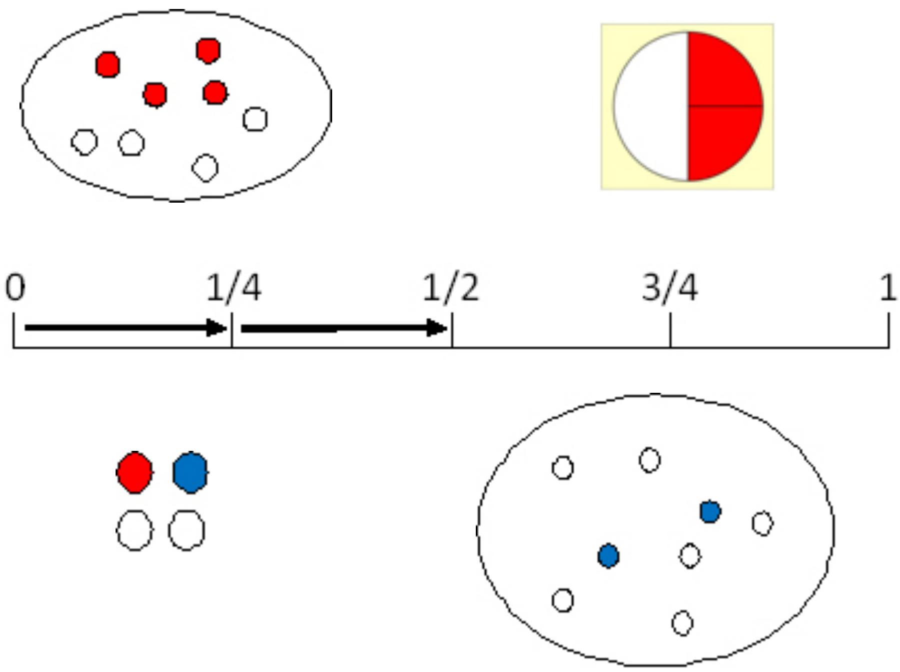


Figure 1. The *modelling fraction addition* item.

The second item, shown in Figure 2, was presented to the secondary and Victorian primary groups, and involved a student's computation of the quotient of two fractions, obtained by using an alternative to the standard *invert-and-multiply* algorithm.

When asked to describe how they determined $\frac{2}{3} \div \frac{3}{4}$

a student wrote the following on the classroom whiteboard:

$$\frac{2}{3} \div \frac{3}{4}$$
$$= \frac{8}{12} \div \frac{9}{12}$$
$$= 8 \div 9$$
$$= \frac{8}{9}$$

Figure 2. The *division of fractions* item.

The transcripts of the focus group interactions were examined to investigate the mathematical language used when discussing the issues associated with these teaching situations. In particular, the key foci for the analysis were:

- commonalities and differences in language across individuals;
- strengths and inadequacies in language use;
- the extent to which language aligned well with the content under discussion; and
- the extent to which language was used to address mathematical issues implicitly and explicitly.

Results

Modelling Fraction Addition

One of the key issues underpinning the fraction addition situation is the identification of the whole. This terminology came up regularly in conversation, almost always as *whole* but once as *unit of analysis*.

> … if we've already talked about our understanding of fractions is, "How many equal parts make that whole, so how many do you need to make that whole? And what part of our fraction tells us how many equal parts make the whole?" [NZ, Primary]

> I like that we have to move away from counters because in some ways the counters are talking … what's the unit of analysis. [Tas, Primary]

In the situation illustrated in Figure 1, the student does not appear to have misconceptions associated with a specific fraction of a given whole, although there was some initial doubt about this in one focus group, after one teacher had suggested that the student needed more experience with the idea of a "quarter of a group". Another teacher counter-argued that the student's basic conception of a quarter was sound, saying "she has actually represented … one counter out of four … she sees that one out of four is … it's working with four counters as a whole, and here's the one". The specific issue associated with the misconception illustrated in Figure 1 is that, in order to be added, the two quarters and their sum must be represented and conceptualised in reference to the same whole. This was not always clearly articulated among the teachers who considered the situation, and the first extract below actually preceded the push to focus on the basic conceptualisations, with the concluding comment about there being two groups not followed up at first.

> She's got two wholes here … but I think I'd have to go right back to the beginning of fractions with her … yeah, forget about the adding, go back to a quarter of that, of one, and then a quarter of another group and so that you can transfer the quarter of the group to here, and realise it's a quarter of another group of what she's done. [Tas, Primary]

> She doesn't catch on that four in one quarter and eight in one-eighth is [in reference to] the same one whole. [Tas, Primary]

The second quote above, which arose later in the conversation among the Tasmanian teachers, gets to the heart of the matter but without specifying that the problem of the identification of the whole even applies with the original two quarters which were not shown with respect to a common whole.

Although it was intended that the focus group discussion should address Figure 1 (while also allowing this to be a springboard for other discussion), the New Zealand focus group spent some time discussing teachers' experiences with their own students' work. In so doing, they discussed aspects of the role of the whole. For example, one teacher told of the way students discussed the equivalence of mixed and improper fractions, and another described a student's approach to showing that one-third cannot be the same as three-eighths. In these accounts, the discussion of the whole and fractions was fluent and correct. The problematic nature of the whole in Figure 1 was hinted at only once, however, in the following.

> If a child said to me, "Well, here's my whole" … you know, but then you'd be saying, "Well, what fraction is that? If that's your whole, what's your fraction?" You know, "What's the fraction?" so, "Really is it a quarter plus a quarter?" … So it's making sure that they really understand, that you're on the same wavelength in what you each consider the whole to be. [NZ, Primary]

In this case, if the words *are* referring to the need for a consistent whole—and it is not entirely clear that they are—then here the language has not been explicit about the mathematical details of the problem. However, two of the other teachers concurred with the statement, suggesting that some aspect of this contribution was understood and received agreement. This hints at some *taken as understood* common understanding held by the teachers, but the lack of clarity in the language begs the question of whether or not the *same* understandings were actually held.

Division of Fractions

In the division of fractions scenario shown in Figure 2, the student's computation resembles aspects of the addition algorithm, in that a common denominator and resulting equivalent fractions are found for the two fractions prior to continuing with the division process. The final answer is correct; the issue is whether or not the student's method is valid, and, if so, on what grounds.

On seeing the student's solution, one of the Victorian primary teachers seemed to be distracted by the seemingly incorrect use of common denominators.

> Very common, see it all the time, a mixture of algorithms that they've learnt, looks like the algorithm of changing the denominators, and then [indiscernible] algorithms, numerator to the numerator, denominator to the denominator, so it's just a mix up of things that they've got in their head going on, and they're just applying them … randomly. [Vic, Primary]

Here the familiar aspects of one algorithm in a context different from its usual application appeared to lead to an assumption that the student's work is incorrect. The phrase "the algorithm" may suggest the teacher believed in an *authorised* way of doing things (this is speculative, but it is known that some people view the *standard* algorithm as the *correct* way of solving a problem). There was no explicit use of language associated with *equivalent fractions* although this idea is, perhaps, implicit in the phrase "changing the denominators". The use of the words "numerator" and "denominator" in the latter part of the quote seemed to be in reference to actions/changes that are not clearly specified; the use of the word "randomly" at the end suggests that the teacher thought that the student's actions are muddled rather than purposeful. There was no disagreement with this interpretation from the other teachers in the group, and the next phase of discussion turned to whether or not fraction division was part of the primary school curriculum.

The use of a common denominator was also disconcerting for the Tasmanian secondary teachers, with one saying "It looks like to be they're confusing it with the addition algorithm," but some at least recognised that the answer was correct. As the following extracts show, the teachers wanted to have the student explain the thinking behind the solution in order to discern the student's rationale. It is, however, ambiguous as to whether or not any of the teachers thought the approach was valid and applicable more generally. Although the words *numerator*, *denominator*, *common denominator*, and *algorithm*, were used correctly in general, they seemed to be used superficially and the deeper meanings were not considered in any attempt to explain and justify the student's valid approach.

> Well, firstly, I would ask them why they did what they did. You know, I think that's really important that kids understand how to do something, why it works and when you use it. And in this case I am intrigued that they've got the right solution by not the standard algorithm so I would ask them why they did it like that … and then engage them in a conversation about, "Let's weigh up some of the advantages and disadvantages of the different strategies you're looking at now." And see if they would actually change their mind about what they've done there. [Tas, Secondary]

Later in the discussion this teacher was able to articulate more about what was going on mathematically, but there was still much that was implicit in the explanation.

> If this kid knew why they were doing that and they explained it in a way that makes sense to me, I want to add that to my library because in many ways it makes more sense than what we teach them because it is linked and connected to the addition and the subtraction one quite well because if you actually do the eight divided by nine and the twelve divided by twelve … so you're nearly there. [Tas, Secondary]

The first of the following teachers suggested that the student thinks you can "lose the twelves", but did not examine if there are mathematical reasons that make this a valid step. The vanishing twelves, from the second line of the student's solution to the third, was a point of concern for the other teachers, too, expressed in different ways.

> You'd still have to ask why because maybe that's not … there's something else is going on, that you can go about, you know, maybe they just noticed that you lose the twelves or something when they've been doing it elsewhere. But you really have to ask them first what's happening. … If they continued on the 'correct' algorithm, putting over the common denominator wouldn't have mattered. … I mean that's the thing. I mean it wasn't needed, but wasn't incorrect, but there's some reason they seem to think somehow you, once you put them over the common denominators you can forget the denominator. [Tas, Secondary]

> When I got through to here, I thought, "Oh! Clever kid!" like, you know how when we're teaching, we're teaching the algorithms and they've got to remember, "Okay, when I divide fractions, what do I do again?" and the recall of that. I find that in adding fractions they get under control putting over common denominators fairly well and I think they do that a lot in primary school too before they come into grade 7. I think this is really clever in that again, we're using the same sort of thought pattern of putting it over a common denominator again. Where I would be asking questions, even though I'd still ask … why did you do that, like how did you get from at the second line to the third line? What happens there? [Tas, Secondary]

> Yeah, assuming like that's over one, or we're having it over the same denominator means that they magically disappear or- that there is a lack of reasoning between those two steps. [Tas, Secondary]

The disappearing twelves can, in fact, be explained intuitively by realising that twelfths are the units for each fraction. Since each fraction is expressed in terms of the same unit (as shown by the same denominators), then the quotient is obtained from the quotient of the number of units in each of the dividend and divisor (i.e., 8 and 9, respectively).

The New Zealand secondary group also discussed asking the student to explain why he/she changed the fractions to equivalent fractions with a common denominator, with at least one misled by this out-of-place application of part of the addition algorithm. A few minutes later one of the teachers claimed that the solution was "Absolutely right. It's a perfectly valid way to do it." One of the teachers had had a student take a similar approach in class, and there was some extended discussion about the applicability of the method.

> T1: I can't remember whether the student actually had the understanding or whether they just … but they were definitely using their prior knowledge of adding fractions. … I did a little bit more work on it myself, and um, I do use it sometimes, but it's a bit limited in its use, because the numbers have to work, but it is a valid method that's come out. … So using equivalent fractions, and then, … I think the student was saying, I think, … [they] were going eight divided by nine is, is, yeah, to get the eight divide, and twelve divided by twelve is one … However, it works, and I think when I did some more work on it myself, it does work, but you can get into a bit of a tangle.
>
> T2: So it works for every whole [sic] number.
>
> T1: Um, I don't know. I think …
>
> T2: I can't see why it wouldn't. … It's just an algorithm. …

Note that the word *algorithm* seems to have been endowed with an authority that implies that it makes things work, rather than as something that requires validation. Later the researcher interviewer tried to probe the deeper mathematical reasoning ("If we think of the multiplication algorithm for fractions, we multiply numerators, we multiply denominators … So, if we're dividing two fractions, why not … divide the numerators, divide the denominators?") which was followed by some general discussion about setting up class explorations of the phenomenon, but again the discussion seemed to tacitly assume or agree with the method without confirming its validity.

Discussion and Conclusions

The teachers, for the most part, appeared to have a shared vocabulary with respect to basic fraction numeration and operations. There was some informality associated with the vocabulary on occasions (e.g., the phrase "numerator to the numerator, denominator to the denominator" used by one of the primary teachers, and "putting over a common denominator" by the secondary teachers), although there were no egregious errors of terminology. At times, however, some of the usage seemed to be *taken as understood*, in that teachers used some expressions that had the potential to be ambiguous in meaning, but there was an apparent assumption that they all understood what was meant by the terms (e.g., exactly which whole was meant by *whole* in the addition problem, and no one felt the need to question whether or not there was a reason behind the *loss* or *forgetting* of the denominator in the division problem).

There was very little detailed articulation of the fundamental principles underpinning the students' thinking. It may be that this assumption of shared meanings inhibits explicit examination of the underlying definitional mathematical meanings and implications. This is not to say that single words that capture complex ideas are not powerful, but our familiarity with them might make it difficult to get back to and express the component foundational principles, and it is these that may be necessary in order to make sense of students' work and help them develop better understanding. For example, it was striking that there was little talk about the way that the numerator and denominator quantify the value of the fraction, by defining the size of the unit components (as determined by the denominator) and the number of such units being considered (as given by the denominator). The teachers seemed to understand fractions and the meaning of the numerator and denominator, but did not ever articulate that meaning explicitly.

In the discussion of the division problem, the meaning of *division* was completely taken as understood; the focus of the discussion was on computational operations and what could and could not be done with the fraction components and the operations, rather than on what it might mean for one fraction to be divided by another. It also seemed to be taken as understood—perhaps because the student *had* obtained the correct answer—that $(a \div b) \div (c \div d)$ is equivalent to $(a \div c) \div (b \div d)$.

The mixed results seem to suggest that perhaps it is time to talk about the way we talk, and the language we use in teaching mathematics. When working with pre-service teachers, it may be that we assume some things are taken as understood. Maybe, as a consequence, we are not as precise about defining and being consistent with the language we model, nor give enough emphasis to how that language allows us to discuss mathematical meanings, nor appreciate that these meanings allow us to justify mathematical procedures.

Acknowledgements

This research was supported by Australian Research Council grant DP130103144. Other researchers on this project included Rosemary Callingham, Kim Beswick, Ian Hay, Tom Nicholson, and Tim Burgess. Our thanks to the teachers for contributing to this study.

References

Behr, M. J., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio and proportion. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296-333). New York: Macmillan.

Boero, P., Douek, N., & Ferrari, P. (2002). Developing mastery of natural language: Approaches to theoretical aspects of mathematics. In L. English (Ed.), *Handbook of International Research in Mathematics Education* (pp.241-267). London: Lawrence Erlbaum.

Chick, H. L. (2011). God-like educators in a fallen world. *Proceedings of the 2011 annual conference of the Australian Association for Research in Education* (Hobart, TAS, 28 Nov – 1 Dec, 2011). Retrieved from: http://www.aare.edu.au/.

Chick, H. L., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 297-304). Prague: PME.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39,* 372-400.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*, 255-281.

Specialised Content Knowledge: Evidence of Pre-service teachers' Appraisal of Student Errors in Proportional Reasoning

Mohan Chinnappan
University of South Australia
<mohan.chinnappan@unisa.edu.au>

Bruce White
University of South Australia
<bruce.white@unisa.edu.au>

That the quality of teachers' knowledge has direct impact on students' engagement and learning outcomes in mathematics is now well established. But questions about the nature of this knowledge and how to characterise that knowledge are important for mathematics educators. In the present study, we examine a strand of *Specialised Content Knowledge,* SCK (Ball, Thames and Phelps, 2008) of a group of pre-service teachers in the domain of proportional reasoning. In particular, we were concerned with teachers' knowledge of evaluation of the plausibility of students' claims and errors. Our preliminary results indicate that the participants, as a group, had developed a sense of student error but experienced difficulty in explaining the source of these errors.

Introduction

High school students' engagement with mathematics and their learning outcomes have come under increasing scrutiny from teachers and curriculum policy makers. This issue has received increasing attention against the backdrop of a declining enrolment trend in senior mathematics subjects. While students seem to be showing interest in studying general mathematics subjects, there is an appreciable decline in enrolment in mathematically demanding subjects. In order to arrest and reverse this pattern, it is critical that teachers and teacher educators understand the multitude of factors that could afford or hinder a higher level of student participation than is evidenced hitherto.

The quality of instruction that students receive in their mathematics classroom must surely feature as a significant factor that could impact on students' learning and development of mathematics proficiency. While the quality of mathematics instruction could be analysed from a number of angles, the kind of knowledge that teachers bring to and activate prior to and during teaching can be expected to have a significant influence on students' engagements with mathematics concepts and problem solving skills. In this regard, we argue that, the development of a nuanced understanding of processes and content of mathematics that is taught in our secondary classrooms is a necessary first step in characterising quality of mathematics instruction. As teachers are at the forefront of subject delivery and assessment of student performance, it is imperative that researchers focus on teacher knowledge and how that knowledge impacts on their decisions.

Conceptions of Student Learning of Mathematics- Framework of Schema

In discussions about teaching it is imperative that we unpack notions of student learning and understanding of mathematics. Our conception of student learning is built around the construct of *mathematical schemas*. Mathematical schemas are organised knowledge clusters or chunks of knowledge that are built on and around core mathematics concepts, principles and procedures. Schemas provide an important theoretical tool to facilitate discussions about deep and surface understanding in mathematics. Schemas that are sophisticated can be expected to have more concepts and links between concepts, thus

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 157–164. Sunshine Coast: MERGA.

reflecting deep understandings. Students who have large and extensive mathematical schemas are expected to also show fluency in the use of procedures and the use of multiple strategies for problem solving. Drawing on the work of Mayer (1975), Chinnappan, Lawson, & Nason (1998) analysed understanding of mathematics concepts in terms of schemas that have *internal* and *external* connectedness. Thus, in our study of quality of teaching and its relationship to teacher knowledge, we work on the assumption that teachers need to build extensive, deep and well-connected mathematics schemas themselves in the first instance in order to support their students to construct similar schemas. The question is what are the constituents of such schemas for effective mathematics teaching? In order to answer this question, we need to consider the broad categories of knowledge that teachers need to access prior to and during their teaching.

Teacher Knowledge and Teaching Mathematics

In his seminal work on analysing teacher knowledge, Shulman (1986), hypothesised the role of two key components of knowledge that teachers need for effective practice: Content Knowledge (CK) and Pedagogical Content Knowledge (PCK). The identification of CK and PCK strands provided the initial prompt for educators to explore how these two core knowledge bases could support mathematics teaching. Following several lines of inquiry (Ball, Hill, & Bass, 2005; Chinnappan & Lawson, 2005; Walshaw, 2012), there is an emerging consensus that effective mathematics classroom practices are driven by a robust body of teachers' mathematics content and pedagogical content knowledge.

Research interest in knowledge that teachers bring to support learning has gained momentum by recent empirical evidence that teachers' mathematics content knowledge contributes significantly to their students' achievement (Bobis, Higgins, Cavanagh, & Roche, 2012; Senk, Tatto, Reckase, Rowley, Peck, & Bankov, 2012). In broad terms, mathematics content knowledge refers to knowledge of the concepts, principles, procedures and conventions of mathematics, while pedagogical content knowledge involves teachers' understanding of students' mathematical thinking (including conceptions and misconceptions) and representing mathematics content knowledge in a learner-friendly manner.

The pioneering work of Shulman led Ball and her associates (Hill, Rowan, & Ball, 2005; Ball & Hill, 2008) to focus on mathematics teachers and fine tune the knowledge strands that are necessary for teaching mathematics effectively. The outcome of their work was the development of a number of new strands of knowledge clusters for mathematics practice that was collectively referred to as *Mathematics Knowledge for Teaching,* MKT (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008).). We regard MKT as providing a macro schema for understanding and describing teacher knowledge that is critical to their work. Within MKT, there are two main categories of knowledge: Content (Subject-matter) Knowledge and Pedagogical Content Knowledge. The Content Knowledge category was decomposed into Common Content Knowledge (knowledge of mathematics common to most educated adults), Specialised Content Knowledge (specific and detailed knowledge of mathematics required to teach it), and Knowledge at the Mathematics Horizon. In our attempts to better understand teacher knowledge that is necessary for supporting school mathematics, we have been inspired by the above dimensions of teacher knowledge for teaching mathematics that was proposed by Ball and colleagues.

Ball *et al.*'s (2008) conceptualisation of MKT led researchers to develop tasks in order to measure the various components. However, most of this effort has been invested in conceptualising and measuring MKT in the context of primary mathematics. Ball (personal communication) has suggested that there is a need to analyse the character of MKT for junior and senior secondary mathematics. In the present study, we attempt to fill this void by focussing on investigating one strand, namely, Specialised Content Knowledge (SCK) of prospective junior secondary mathematics teachers. SCK is an important strand for two reasons. Firstly, this strand has been shown to correlate with high levels of student learning outcomes, particularly at the primary levels (Ball & Hill, 2008). Secondly, it has been shown that SCK tends to be underdeveloped in most teachers including future teachers of mathematics (Hill, Rowan, & Ball, 2005; Hill *et al*., 2008; Chinnappan & White, 2013).

SCK in Number and Algebra

In discussions about SCK, the mathematics community is concerned with mathematical content that is unique to teaching. This knowledge base includes structuring and representing mathematics concepts, identification of the mathematics that underpins an instructional task and anticipation of different ways students might think about concepts including their misconceptions (Steele, 2013). SCK of a teacher also includes their ability to appraise and analyse unconventional solution methods of their students. In this regard, Ball *et al*. (2008:400) suggested ‘looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general’ as an important component of teachers’ SCK. In the present research, we take up this particular aspect of SCK in the context of a problem that involved proportional reasoning. Our research was guided by the following question: What is the nature of SCK of prospective teachers in the domain of proportional reasoning that involved evaluation of plausibility of student errors?

Methodology

Design

We have adopted a case study design for this study involving groups of pre-service teachers (PSTs) engaging in discussions about a given proportional problem. This design was considered to be appropriate as we aimed to gain an in-depth understanding of a phenomenon - evolving teacher knowledge within groups, as suggested by Yin (2009) and Zevenbergen (2004). The groups of PSTs constituted the units of analysis for the study.

Participants

A cohort of 8 PSTs participated in the study. The participants were enrolled in a Master of Teaching which is a professional Masters leading to a teaching qualification and were then employed in Government schools across South Australia. The participants came from a variety of backgrounds, many had industry experience, some were recent graduates and a number had PhDs. The PSTs had completed two core mathematics methods courses and twelve weeks of professional experiences before the commencement of the study. In this report we provide data generated within one group (4) of the PSTs.

Task

We were conscious that the task that we provided for our PSTs to interact with will engender multiple opportunities to activate their SCK. In a study about teacher preparation, Beswick and Goos (2012) developed a set of mathematical problems that were used to assess content knowledge. From this set, we selected a proportion problem, namely, the Cordial Mixture Problem (CMP) for the present study (Figure 1).

The following question was given to Year 8 students:

Some children are making batches of cordial by mixing together sweet concentrate and water.

Sally uses 4 cups of sweet concentrate and 13 cups of water.

Myles uses 6 cups of sweet concentrate and 15 cups of water.

One student has a misconception and thinks these cordial mixes will have the same sweetness.

Which of the following cordial mixes might this student ALSO think will have the same sweetness?

☐ Aisha uses 8 cups of sweet cordial mix and 26 cups of water.
☐ Carly uses 10 cups of sweet cordial mix and 19 cups of water.
☐ Deng uses 8 cups of sweet cordial mix and 20 cups of water.
☐ Erin uses 10 cups of sweet cordial mix and 28 cups of water.

Figure1. Cordial Mixture Problem

The CMP is regarded as a rich context for the externalization of teachers SCK for the following reasons. Firstly, in examining the solution to the problem, teachers could activate a range of intuitive knowledge about the solution to the given problem as well as examine the mathematics underlying the solution, both of which were regarded as core elements of SCK by Sullivan (2011).

The conclusion by the Year 8 student (Figure 1) that the cordial mixes have the same sweetness suggest that the student have added 2 to both the number of cups of sweet and cups of water respectively. This indicated the use of additive thinking by the student. In contrast, the activation of multiplicative reasoning, in context, would involve comparing the ratio between cups of sweet to cups of water between Sally and Myles respectively (4:13 to 6:15). Such a comparison of relationships would have led the student to the correct conclusion that the ratios are not equal, and therefore, the two cordial mixes are not of same concentration.

In analysing CMP and its solution offered in terms of concepts such as ratio, proportion, additive and multiplicative thinking, we suggest, constitute PSTs' SCK. At the core of this knowledge is reasoning about the multiplicative relationship that exists between base ratios within the given proportional context. That the student had used additive thinking suggests PSTs' awareness of how the student could have reached the erroneous conclusion, a component of their SCK.

Procedure

Participating PSTs were organised into groups of four, they were given a number of questions to complete individually and then asked to discuss their solutions to these problems including the CMP. In sharing their responses, each member of the group was also invited to comment on the problem, identify potential solutions from their students' perspective and issues related to teaching and learning about the given problem. In prompting the participants along the above lines of analysis, our expectation was our PSTs will focus on the key concepts that underpin the different representations and solution paths all of which constitute SCK underpinning the CMP. Each group was allowed a maximum of 30 minutes to complete this activity.

Results and Analysis

We provide transcripts of PSTs' discussions in two excerpts below.

Excerpt 1

PST1: Me to because it was up to us to see the pattern as to see the other pattern, because I've been telling the kids that maths is all about patterns and things,
PST2: Would you see misconception patterns?
PST1: so interesting for us to have to work out where the misconception pattern was.
PST3: So I ended up with 10 cups and 19 cups
PST1: Yes.
PST3: Yeah.
PST4: I interpreted this in a different way I think because...
PST2: I did too.
PST4: ... because the children interpreted two, two groups of concentrated water in the top I think, and down the bottom there I presumed that we had to choose between two of those, because on the top they use the difference of 2 in both sides.
PST2: Two, two on both sides.
PST4: So down the bottom I used the difference of 2 and 2 so I said Aisha the top one and Erin in the bottom one.
PST2: I saw that pattern to, that's the pattern that I saw.
PST3: OK, I think I see what you mean
PST2: I think it's the one where 10 and 19 come from.
PST1: That was my answer.
PST3: Because when you've got a difference of 4 cups of sweet water, you've got 4 and 13, and then it goes 6 15, so then I went 8 17 and 10 19.
PST2: Sorry, I don't understand, you got 4 and 13.
PST4: I just said the...
PST3: 6 and 15 which is the next one.
PST1: You add 2 again.
PST3: Yep, and then the next one up would be 8 17, so you'd have 8 cups which is 17 cups of water.
PST2: 8 to 17.
PST2: And then you'd go to 10 and 19, so I could see that both, both ways could be
PST1: What was your way?
PST4: I just said OK, there was a difference of 2 in both those 2 and 4 to 6, 13 to 15, so then I looked down the bottom and I said, *Right, we have to choose two of those,* so 8 and 10, 26 and 28.
PST2: So you guys looked at it, *what one out of this lot would be the same as those two?*
PST3: Yeah.
PST4: That's not how I interpreted that.
PST2: Which two would be the same
PST1: We picked two others.
PST1: Yeah.
PST1: Oh!
PST1: See I thought that was a separate thing.
PST2: So I think they're both right.
PST3: Yeah, I think they both are right it depends how you read it
PST1: Good old English.
PST4: Mm, it's a bit ambiguous isn't it, not crystal?
PST1: So what would we talk to about that student?
PST2: We'd have to find out why.
PST3: Yeah.
PST4: Why that student thinks
PST1: Well they're not dividing all are they, they're basically not.
PST3: They've just picked a pattern.
PST1: They're not getting a ratio at all.

PST3: That’s right, they’ve just assumed, they’ve made an assumption that it’s going to be the same, maybe you need to be set up to see that it’s not the same.
PST1: They’re doing differences so you need to go back and show them that they should be doing division with this sort of thing, for ratios

The discussion above highlights some interesting insights into the SCK of these PST’s. Initially the discussion centered on the identification and importance of pattern and their ability to identify not only the correct pattern but also an incorrect pattern that the students may have used (turns 3 and 12). However there were two interpretations of the question and so the discussion then focused on interpretation of the question and how easy it was to read the question in a different way than was intended (turns 4, 7 and 11). It also highlighted that it does take some time to see the problem in a different way to how you initially interpret it. Interestingly both were able to answer the question based on their interpretation (turns 16 and 25). However this was a distractor from the intended discussion and made us question the value in not having the researcher as part of the discussion. The discussion then returned to what the student had done and they identified that the student had not used a ratio at all and that they needed to set up a situation where it would not work and that the student would need to use a ratio, although no detail was give how they would do this (turn 46). The discussion appears to show that the students were able to identify the problem and had some idea of what they needed to do but did not have the breadth of SCK required to draw upon to give specific examples of how they would help the students.

The participating PST’s were also asked to comment on the effectiveness of the process used – use of CMP as a prompt for externalizing SCK). Below is a short extract of their discussion.

Excerpt 2

PST1: Yep. So I think it’s easy to just focus, like just focus on just getting the right answer and like you said, when you’re time poor you focus on just trying to get them the basics.
PST2: That’s right.
PST1: Instead of stepping back for a second and throwing one of these out there and saying, OK, we’ve done all these ratios and stuff, let’s look at this one.
PST2: Let’s move on because the higher-order ones, they have that
PST1: This is how you check they understand it, right?
PST2: Yeah, yeah.
PST1: That they haven’t just learnt your tricks and processes.
PST2: That’s right.
PST1: That they’ve
PST3: Yeah, they have to sort of figure out, you know, what they’re doing.
PST1: Yes.
PST3: Get an understanding.
PST1: And equally it’s, go through this process because if you’re just marking a test, that’s the wrong answer and you put a cross, something wrong, then obviously you’re not going learn anything. If you don’t know where they went wrong, if you can’t follow it, you can’t help them out, so

Comments from Excerpt 2 indicate that PSTs are aware of the need to examine aspects of students’ thinking that goes beyond procedural knowledge (turns 47, 56 and 58). This could be evidence of the PST’s activation of PCK but one that is reliant on SCK about proportional reasoning. The exchanges also indicate that the PSTs found the process to be useful for them and that they were able to make the connections between the type of problem that they were using and the outcomes that they expect to get. We suggest that the process had made participants think about the SCK that is involved in analyzing CMP although it was at times difficult to distinguish between exchanges involving SCK and PCK.

Discussion

This study that is reported here was motivated by our desire to better understand the state of SCK by a cohort of prospective teachers of numeracy who were enrolled in our teacher education program. We worked on the assumption that by providing opportunities for pre-service teachers to externalise their SCK in informal situations, as teacher educators, we will be in a stronger position to understand the quality of this knowledge. Such data were expected to generate guidelines for supporting their future learning needs in developing their SCK further.

The preliminary data indicate that prospective mathematics teachers' SCK is somewhat tenuous in the particular area of proportional reasoning, an area of mathematics that has been shown to continue to present challenges for both teachers and students (Lamon, 2011; Beswick & Goos, 2012). However, given that the participants are in the early stages of their professional development, there were important signals to suggest that our PSTs have formed precursors of powerful SCK. For example, there is strong evidence that our teachers were keen to explore the ratio schema in which the CMP was anchored.

From a schemas perspective, the CMP acted as an effective prompt for internal and external schemas (Mayer, 1975) about concepts of co-variation, ratio, additive and multiplicative relationships. In this context, understanding of the concept of ratio is part of students' internal schema, whereas deducing the equality of ratios in proportional thinking is a component of external schema. Both schemas are reflective of knowledge that is unique to the work of teachers as suggested in the framework of MKT (Ball *et al*., 2008).

Our preliminary study along this line of inquiry examined SCK in the context of matrices (Chinnappan & White, 2013) among prospective mathematics teachers. The results of that study provided evidence that the quality of representations can be used as a key indicator in studies of SCK. In the present study, we suggest that the analysis of additive vs multiplicative representation of CMP or similar problems by PSTs could be a useful way to extend the current study.

The data that we present here is drawn from four PSTs who were asked to study and comment on the given CMP. As pointed out earlier, the group discussion was conducted in an informal manner with limited intervention from the investigators. While this strategy for data collection was effective, from a methodological perspective, the above arrangement may not have provided an optimal environment to obtain a more complete picture as to the state of the participants' SCK and information for charting its evolution. In a future large scale study, we intend to fine-tune this weakness by involving the researcher in engaging the participants by the use of semi-structured questions to probe the participants both during and post group discussions.

A challenge in the present study was that the conceptualisation of SCK had to be grounded in one specific area of secondary mathematics in order to generate fine-grained data. Within the domain of numbers and algebra, there are numerous areas that are ripe for the exploration of teachers SCK. However, pinning down one area within these strands was problematic for us in order to be able to make general claims.

Capturing the nuances of SCK is also limited by the fact the knowledge is developmental in nature and that any description of this knowledge is only valid at the time of the investigation. Thus future studies should also track the growth of SCK and map a trajectory of the growth by providing different proportional reasoning problems and examine the connection to PCK.

References

Ball, D. L., & Hill, H. C. (2008). Measuring teacher quality in practice. In D. H. Gitomer (Ed.), *Measurement issues and assessment for teaching quality* (pp. 80-98). Thousand Oaks, CA: SAGE Publications.

Ball, D., Hill, H. C., & Bass, H. (2005). Knowing Mathematics for Teaching. *American Educator, 36*(3), 14-46.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Beswick, K. & Goos. M. (2012). Measuring pre-service primary teachers' knowledge for teaching mathematics. *Mathematics Teacher Education and Development*, *14*(2), 70-90.

Bobis, J., Higgins, J., Cavanagh, M., & Roche, A. (2012). Professional knowledge of practicing teachers of mathematics. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Mathematics Education Research Group of Australasia: Research in mathematics education in Australasia 2008-2011* (pp. 313-341). The Netherlands: Sense Publishers.

Chinnappan, M., & White, B. (2013). Developments in teacher knowledge for teaching algebra. In M. Inprasitha (Eds.), Innovations and exemplary practices in mathematics education. *Proceedings of the 6th East Asia Regional Conference on Mathematics Education*, Vol. 3 (pp. 202-210). Phuket: Centre for Research in Mathematics Education, Khon Kaen University.

Chinnappan, M. & Lawson, M. (2005). A framework for analysis of teachers' geometric content knowledge and geometric knowledge for teaching. *Journal of Mathematics Teacher Education*, *8*(3), 197-221.

Chinnappan, M., Lawson, M., & Nason. R (1998). Representing the connectedness of mathematical knowledge. In C.Kanes, M.Goos and E.Warren (Eds.), *Proceedings of the 21st Conference of the Mathematics Education Research Group of Australasia*, (pp. 161-168). Griffith University Press: Mathematics Education Research Group of Australasia.

Hill, H.C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371- 406.

Hill, H. C., Blunk, M., Charalambous, C., Lewis, J., Phelps, G., Sleep, L.,, & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511.

Lamon, S. J. (2011). Rational numbers and proportional reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Charlotte, NC: Information Age.

Mayer, R. E. (1975). Information processing variables in learning to solve problems. *Review of Educational Research*, *45*, 525-541.

Senk, S., Tatto, M T., Reckase, M., Rowley, G., Peck, R., & Bankov, K. (2012). Knowledge of future primary teachers for teaching mathematics: An international comparative study. *ZDM – The International Journal on Mathematics Education, 44*(3), 307-324.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *4*(15), 4-14.

Sullivan, P. (2011). *Teaching Mathematics: Using research-informed strategies*. Retreived on January 28 2015, from http://research.acer.edu.au/aer/13.

Steele, M. D. (2013). Exploring the mathematical knowledge for teaching geometry and measurement through the design and use of rich assessment tasks. *Journal of Mathematics Teacher Education*, *16*, 245–268, DOI 10.1007/s10857-012-9230-3

Walshaw, M. (2012). Teacher knowledge as fundamental to effective teaching practice. *Journal of Mathematics Teacher Education*, *15*(3), 181-185.

Yin, R. K. (2009). *Case study research: Design and methods*. SAGE Publications: Thousand Oaks, CA.

Zevenbergen, R. (2004). Study groups as a tool for enhancing students' content knowledge. *Mathematics Teacher Education and Development*, *6*, 4-22.

Learning from Lessons: Studying the Construction of Teacher Knowledge Catalysed by Purposefully-designed Experimental Mathematics Lessons

Doug Clarke
Australian Catholic University
<doug.clarke@acu.edu.au>

David Clarke
University of Melbourne
<d.clarke@unimelb.edu.au>

Anne Roche
Australian Catholic University
<anne.roche@acu.edu.au>

Man Ching Esther Chan
University of Melbourne
<mc.chan@unimelb.edu.au>

A central premise of this project is that teachers learn from the act of teaching a lesson and that this learning is evident in the planning and teaching of a subsequent lesson. In this project, the knowledge construction of mathematics teachers was examined utilising multi-camera research techniques during lesson planning, classroom interactions and reflection. Our goal is a refined understanding of classroom events that create opportunities for teacher learning. This paper reports what one Year 5 teacher appeared to learn from the process.

Literature and Conceptual Framework

Our overarching research question is: In what form and by what process do teachers learn from the experience of teaching mathematics lessons? This paper focuses on two sub-questions:

(i) When reflecting on a recently taught lesson, which lesson elements or events do teachers consider most salient and how do these influence subsequent lesson planning?
(ii) What forms of teacher knowledge and beliefs are foregrounded in the process of reflection on a lesson, and how do these contribute to subsequent lesson planning?

In recent years, a great deal of research has been conducted that provides evidence for what many intuitively believe to be true—that ultimately the teacher is the key to improved student learning (Fennema & Franke, 1992; Hattie, 2003). Artzt and Armour-Thomas (1999) identified “dimensions” of the lesson as “those broad aspects of instructional practice that define critical areas of teachers’ work during the enactment of the lesson” (p. 214). These dimensions are Tasks, Learning Environment, and Discourse. While using the framework of Artzt and Armour-Thomas in creating the experimental lesson plans used in this study, we have also drawn upon the literature of effective teaching of mathematics (Anthony & Walshaw, 2009; Sullivan, 2011), as many of the insights from this research elaborate the categories of Artzt and Armour-Thomas.

Despite the growing recognition of the centrality of the teacher’s role to student learning, teacher knowledge, and teacher learning remain under-theorised. This project takes as its starting point one of the most widely cited models of teacher learning (Clarke & Hollingsworth, 2002, see Figure 1), as this provides an orienting framework for the first research question. Central to this model is the mediating role played by Salient Outcomes (those outcomes of classroom practice to which the teacher attaches significance), which provide both the basis for change in beliefs and knowledge and, once changed, the motivation to engage in classroom experimentation in recognition of changes in those outcomes considered salient by the teachers.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 165–172. Sunshine Coast: MERGA.

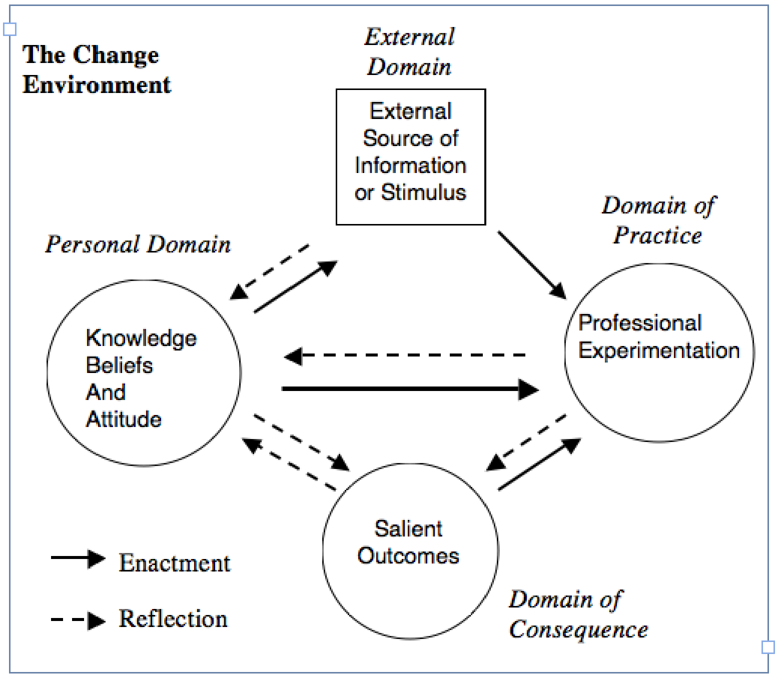


Figure 1. The Interconnected model of teacher growth (Clarke & Hollingsworth, 2002).

Shulman (1987) distinguished between Mathematical Content Knowledge (MCK) and Pedagogical Content Knowledge (PCK) and this distinction informed the design of the TEDS-M instrument used in this study (Tatto et al., 2012). Our thinking was also informed by the work of Van Es and Sherin (2002), who have developed a substantial body of research on "teacher noticing". Related work on decision-making by Schoenfeld (2011) can be usefully integrated with the idea of "adaptive expertise" (Hatano & Inagaki, 1986) to extend the Clarke-Hollingsworth model, by providing a mechanism for both reflection and enaction within a model of teacher learning.

Research Design

Three middle school teachers with at least five years' classroom experience were recruited in Melbourne to participate in the study, drawing upon available networks of teachers known to the researchers.

Data Generation

A key element in this research design is the provision of purposefully-designed experimental mathematics lessons, which provide the initial context for this study of teacher selective attention, reflection, and learning. During a preparatory (pre-active) interview, the teacher was asked to complete the same mathematics tasks as those employed in the lesson about to be taught. The teacher then annotated a provided lesson plan with respect to any aspects of the lesson that the teacher believed would require adaptation or which might represent a particular challenge for either the students or the teacher. A pre-lesson interview just before the lesson focused on the teacher's thinking regarding the lesson to be taught. An open-ended interview protocol offered teachers the opportunity to discuss (unprompted) such things as: key mathematical or pedagogical points, likely student difficulties, anticipated important moments in the lesson, intended student learning outcomes, and so on.

The teacher taught the lesson to their usual class. The lesson was filmed using a two-camera configuration: (a) The teacher camera recorded all teacher actions and statements throughout the lesson; and (b) the whole class camera recorded the entire class continuously throughout the lesson.

The original lesson plan categories were used to structure the teacher's reflection on the lesson. Initially, the teacher was asked to: (i) comment on each lesson component; and (ii) to identify salient events in the lesson (activities or actions that the teacher believed were important for some reason). The interviewer encouraged the teacher to explain why the chosen events were important. Each event was then viewed on a synchronised, split-screen video record of the lesson and the teacher was invited to make any comments suggested by viewing the video supplementary to those already made.

Teachers were then asked to develop a written plan for "a follow-up lesson" (Lesson 2) using a structured template provided by the researchers. It was intended that Lesson 2 offer the opportunity to build on the first lesson, in relation to content, student understanding, and student engagement. A second pre-lesson interview followed the protocol for the corresponding Lesson 1 interview in every respect. In addition, the teacher was asked to describe any way in which the teaching of Lesson 1 had influenced their thinking about Lesson 2.

The teacher then delivered the second lesson to their usual class. Once again, the lesson was filmed using a two-camera configuration. Again, the original lesson plan categories were used to structure the teacher's reflection in a post-lesson interview of which the latter half was video-stimulated. After this process had been completed, the teacher was asked to identify anything that she had learned over the course of the two lessons.

One week after the filming was completed, teachers were given a written assessment of content knowledge and pedagogical content knowledge and a beliefs survey adapted from the test instrument developed for the 17-country TEDS-M study (Tatto et al., 2012). Given that the capacity of any individual to learn from a specific experience is dependent on their existing knowledge, it was important to establish general measures of the teachers' knowledge. This information could then provide part of any explanation for the teacher's subsequent capacity to learn from the experience of teaching a lesson.

The study design attempts to maximise authenticity by investigating teacher learning "in situ" – that is, teachers in interaction with students with whom they are familiar and for whose learning they are responsible. The teachers' subsequent learning from any lesson will be dependent on their existing knowledge of their students and of the mathematics curriculum relevant to that grade level. It was hypothesised that this existing knowledge, together with teacher beliefs and values, would determine those classroom events, objects and people to which the teacher chose to attend. This, in turn, would influence the teacher's in-the-moment decision-making, shaping the way in which the teacher translated the lesson plan into classroom activity. Further, the teacher's knowledge, beliefs and values would critically inform their evaluation of the effectiveness of any particular lesson activity and the significance attached to those lesson outcomes they considered salient.

Data Analysis

The analysis reported in this paper drew primarily on interview data with a particular teacher, supplemented by results from the TEDS-M instruments. All interviews were fully transcribed, and were coded by at least two of the authors, who worked together closely in the early stages. In coding the teacher responses, our overall guiding question was: What do the teachers notice or pay attention to in preparing for teaching, and in reflecting on the lesson? This is closely tied to both research questions. Text which provided information on this question was coded in four broad categories: Mathematical content (what reference does the teacher make to mathematical content?); Students (what aspects of students' knowledge, behaviour or needs do teachers refer to?); Instruction (what instructional actions or considerations do teachers refer to?); and Teachers (what aspects of themselves do teachers make reference to?). Several excerpts attracted more than one of these codes.

Where two coders assigned different codes to the same interview excerpt, we adopted an inclusive approach—any text which was given a code by at least one coder was included in that category. All four authors were then involved in drawing out particular themes from within the four broad categories. Making sense of the data involved both direct interpretations and categorical aggregation (Stake, 1995). Examples of such themes include student engagement, the adoption of new lesson structures, connections to the everyday, and the role of measurement benchmarks. These themes could then be related to the data on knowledge and beliefs from the TEDS-M instruments.

Results

From one perspective, the *Learning from Lessons* project can be seen as an investigation into the mechanisms by which teachers develop the "wisdom of practice" conceptualised by Shulman (1987). The theoretical basis for the project derives, as has been discussed, from the Clarke-Hollingsworth (2002) model of teacher growth, in which a key determinant of teacher learning is the particular classroom outcomes (e.g., student performances or lesson efficiencies) that the teacher considers to be "salient". Both the study design and the associated analytical framework take the following connective chain as fundamental: Teacher Change (and therefore Teacher Growth or Learning) is critically dependent on those classroom events to which the teacher chooses to attend while teaching a lesson. This selective teacher attention is a direct reflection of the classroom outcomes the teacher considers to be salient. Decisions of salience reflect the teacher's system of values and beliefs. Teacher selective attention is also significantly determined by teacher knowledge. Put simply, a teacher's attention is directed towards those things that the teacher knows and believes to be important. Any understanding of teacher learning in the classroom must start from the documentation of those things to which teachers attend. In terms of the Clarke-Hollingsworth model, teacher attention reflects teacher judgements of salience and constitutes a key mechanism providing the matter for teacher reflection.

It became evident in our analysis of teacher interviews and the classroom videos that while teacher *attention* might be identified with some confidence, consequent *learning* was much more difficult to document empirically. In the following discussion, the findings with regard to the operationalisation of teacher *in situ* learning will be illustrated with examples drawn from a single teacher ("Tracey") of a particular Year 5 class. In discussing this teacher's learning, we found it useful to draw a distinction between the development of teacher knowledge and the on-going refinement of teacher adaptive practice. In empirical terms, this distinction corresponds to the difference between a declarative "claim to know" (the individual's epistemic stance) and an observable (or recounted) change in the individual's practice. We found evidence of both types of learning in our data.

Our principal source of evidence for learning was the body of interview data. As has been outlined, five interviews were conducted with each teacher. The illustrative results that follow are reported as (i) those things to which the teacher chose to attend in her interviews before and after each lesson; and (ii) those epistemic claims or reported changes in practice that can be taken to constitute teacher learning.

The Teacher: Tracey

Tracey has been teaching for 13 years, following the completion of a BEd in 1998. This study took place in her second year of teaching Year 5. Based on questionnaire and test data, Tracey answered approximately 80% of the TEDS-M mathematics content knowledge (MCK) items correctly, with 60% of the items addressing pedagogical content knowledge (PCK) answered correctly. She described herself as fairly confident in teaching mathematics (6/10) and, specifically, more confident that she could address the needs of low-attaining students (7/10) than high-attaining students (6/10). She described her instructional approach as focusing on putting ideas into a practical context very often. In her responses to TEDS-M beliefs items, she did not endorse “Learning mathematics through following teacher directions”, but strongly affirmed “Mathematics as a process of inquiry” and “Learning mathematics through active involvement”. From her questionnaire responses, Tracey can be described as having a conceptual orientation, rather than a calculational orientation (Philipp, 2007). These personal attributes of knowledge and belief help us to understand both the patterns in Tracey’s attention and the form taken by her consequent learning.

Teacher Selective Attention

The targets of teacher attention were classified as concerning: Instruction, Mathematics, the Student, or the Teacher. On the basis of our analysis, we were able to detect distinct characteristics of the teacher’s attention associated with each of these four categories.

Instruction. The lesson provided by the researchers dealt with student estimation of mass and the subsequent lesson developed by Tracey dealt with student estimation of angles. In interview, Tracey made specific and repeated reference to three features of the lesson structure: the “hook” or story shell used to engage students and situate their activity at the beginning of the lesson; the three-part structure (estimation and measurement, discussion, and further estimation) (see Lovitt & Clarke, 1988); and the summing up phase of the lesson. She chose to utilise the same features in her second lesson.

Other considerations about instruction for the planning of this lesson included the timing or pacing of the lesson (e.g., “when they’re all sitting and each thing is being measured, it might be a bit time-consuming there and the kids might get a bit bored”) and how she might group the students for the activity (e.g., “I did think about that because I was going to place them, perhaps, with someone with high ability skills but I just thought also the conversations that they're going to have are probably just as important and I'd like them to be with, perhaps, people they're comfortable with. So I'm just going to let them choose their pairs.”). One aspect that she felt did not go as well as she had hoped was the summing up phase (e.g., “I'm very aware of it in all lessons, not just this one that the reflection at the end is really the key. And, perhaps, I didn't leave enough time for that in this lesson and it quite often happens that the time for reflection is not there.”). This concern to provide sufficient time for an adequate reflection at the end of the lesson led her to reduce the number of opportunities to estimate in each round (from five to three).

Mathematics. In discussing her planning for both lessons, Tracey made frequent and quite detailed mention of mathematics content, however mathematics was much less frequently mentioned in her post-lesson reflections. A persistent emphasis was the role of referents (her term "benchmarks") by which the students could make judgements in relation to estimating quantities. The other persistent emphasis was "Measurement Units" – which is understandable, given the focus of both lessons.

The theme "connection to the everyday" appeared to be implicitly connected to a concept of embodied learning (although not articulated by Tracey in those terms). This was clear in Tracey's discussion of whether or not to include a discussion of the relationship between a gram, a cubic centimetre, and one millilitre of water. This seemed to constitute a significant focus of reflection for her and also a form of learning. For example: "And then I thought that the idea of water and how heavy water is and relating it to the millimetre and the cubic centimetre might be something that interests them which is something we could run with" (Pre-Lesson Interview 1) and "So I had to make a decision there to, perhaps, we'll bring up the water thing and water being equivalent, mls and grams, bring that up later in another lesson" (Post-Lesson Interview 1). The prioritisation of "connection to the everyday" is also consistent with Tracey's responses to the TEDS-M beliefs questionnaire.

Compared to Lesson 1, the Lesson 2 Pre-Lesson Interview was more concerned with the curriculum, probably because the responsibility for choosing the topic had been handed to Tracey. The interview included many references to the curriculum (AusVELS was cited) and Tracey tried to work out how the lesson would connect with the curriculum. Tied up with this was her uncertainty over the students' prior knowledge.

The students. Except for the preparatory interview, Tracey gave consistent attention to student engagement/disengagement. In the first pre-lesson interview, she discussed the importance of the pacing of the lesson so that the students did not get bored. After teaching the first lesson, she noticed the disengagement of the students towards the end of the lesson and reported that her instructional decisions for the lesson were determined by the students' performance and engagement during the lesson: "So I guess it's the kids' response and how they're performing during the lesson and their engagement I think helps me decide mostly when I need to move on and that." Similar comments regarding student engagement and interests were made in the second pre-lesson interview.

Tracey's concern for student engagement was consistent with the attention she gave to student knowledge in planning her first lesson: "I just thought I might make sure that. . . they understand what mass is. . . they may not have done mass for a while and they confuse it with volume or something" and the second lesson: "I'm not sure of previous knowledge about angles so we're going in blind a bit so I just did a little recap."

Tracey seemed to create more opportunities for students' reflection at the end of the second lesson compared to the first one. "I think hearing them reflect on the lesson last time, I think that was important" and "I think it was the way they verbalised it and also the others were paying more attention this time around as well. Whereas the last time they weren't and that. …"

The teacher. There were very few statements where Tracey referred to her own capabilities, confidence or feelings. She did comment in relation to the topic of angles, that "perhaps, it's my own lack of knowledge about angles as well. I couldn't quite maybe explain it as clearly as I …" She noted that for the second lesson (which she had prepared) that she "got more ownership of this so I knew exactly where I wanted to go." Apart from these, self-referential statements by Tracey were rare.

Discussion

On the basis of our data, teacher learning could be identified in the form of developed knowledge or adaptive practice. Examples of each were available.

Evidence of knowledge development included Tracey's comments on mathematical content and curriculum, new instructional strategies, the prior knowledge of her students, and the significance of particular elements of lesson structure. We would argue that the distinction between declarative knowledge and adaptive practice is an important one. Tracey not only articulated new forms of declarative knowledge, this knowledge was frequently described or actually enacted in the form of adaptive practice.

Within the category of adaptive practice, Tracey attached particular value to the three-part structure, whereby students are given two opportunities to estimate, and to the "hook" which was not new to her but something she had not used often. The third aspect was the reflection at the end of the lesson, which she had added to the original lesson template. This lesson feature was clearly important to her as she worked hard to improve this stage in the second lesson. Each of these can be interpreted as indicative of adaptive practice.

Her final interview emphasised the importance of drawing student attention not just to the measurement units as such but to the role of the "benchmarks" in helping them to make better estimates. It seems reasonable to suggest that this constitutes a form of learning for Tracey, whose interview statements suggested that she was likely to be giving the same emphasis to this strategy in her future teaching of any topic in measurement.

Tracey's interviews illustrate how her professional learning was tied to particular practical aspects of the lesson, but ones with instructional implications, such as measurement benchmarks, and connections to the real world. Further, when responsible for the choice of topic, Tracey paid significant attention to location in the curriculum and to student prior learning. In particular, after the lesson, she was more inclined to reflect on the mathematics her students did or did not know prior to the lesson (i.e., their preparedness) to a greater extent than the mathematics they actually learned during the lesson.

Conclusions and Implications

To a significant extent, our analysis has addressed the question: What are the dominant emphases in Tracey's interviews, how do these change, and is there evidence of learning? This question represents the pragmatic challenge addressed by the research design employed in this study.

The analysis of data pertaining to Tracey has demonstrated both the efficacy of the approach and also validated the intended connectedness of the data sources. For example, the teachers' personal attributes of knowledge and belief, as documented through the TEDS-M instruments and the teacher interviews, did provide insight into the patterns in the teacher's attention and the form taken by any consequent learning. These consistencies align well with the hypothesised connection between teacher knowledge, beliefs and values, teacher selective attention, and teacher learning. It is through the documentation of these connections that we hope to identify the mechanisms underlying Shulman's *wisdom of practice* (Shulman, 1987) and the processes of reflection and enaction that mediate change in the Clarke-Hollingsworth model (Clarke & Hollingsworth, 2002). Once a process is understood, it may become possible to increase its effectiveness.

In respect of practical implications of this research: teachers are busy people, and the opportunities for reflection, if not structured by others, are sometimes lost. We can envisage a teacher professional learning program where a group of teachers choose a lesson from a bank of recommended lessons, adapt the lesson as necessary for their students and then teach it. A questionnaire, using similar prompts to those used in our post-lesson interviews, could catalyse teacher reflection. Teachers would then construct an appropriate follow-up lesson to the provided lesson, and teach it, completing another reflective questionnaire. The teachers would meet as a group to share their experiences.

It is our opinion that the research design of this project proved capable of generating the data needed to document at least two broad forms of teacher learning from the experience of teaching lessons: developed knowledge and adaptive practice. It does appear that teachers learn from the activity of teaching lessons. Our challenge is therefore to better understand that process in order to optimise its occurrence. The effectiveness of the research design in catalysing teacher reflection has significant potential for future adaptation to professional learning contexts.

Acknowledgment

We gratefully acknowledge the Spencer Foundation (USA) and the Australian Research Council for funding this study, the Science of Learning Research Centre for the provision of technical support and data storage, and the Australian Council for Educational Research and the International Association for the Evaluation of Educational Achievement for permission to use the TEDS-M items.

References

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Educational Series 19. Brussels: International Academy of Education.

Artzt, A. F., & Armour-Thomas, E. (1999). A cognitive model for examining teachers' instructional practice in mathematics: A guide for facilitating teacher reflection. *Educational Studies in Mathematics,* 40, 211-235.

Clarke, D. J., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*, 947-967.

Fennema, E., & Franke, M. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). Reston, VA: National Council of Teachers of Mathematics.

Hatano, G., & Inagaki, K. (1986). Two courses of expertise. In H. Stevenson, H. Azuma, & K. Hakuta (Eds.), *Child development and education in Japan* (pp. 262-272). New York, NY: W. H. Freeman and Company.

Hattie, J. A. (2003, October). *Teachers make a difference: What is the research evidence?* Paper presented at the Australian Council for Educational Research Conference on Building Teacher Quality, Melbourne.

Lovitt, C., & Clarke, D. (1988). *The Mathematics Curriculum and Teaching Program. activity bank* (Vol. 2). ACT: Curriculum Development Centre.

Philip, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: NCTM.

Schoenfeld, A. H. (2011). *How we think: A theory of goal-oriented decision making and its educational applications*. New York, NY: Routledge.

Shulman, L. S. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Stake, R. (1995). *The art of case study research.* Beverly Hills, CA: Sage Publications.

Sullivan, P. (2011). Teaching mathematics: Using research-informed strategies. *Australian Education Review, 59.* Melbourne, VIC: ACER.

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodriguez, M., Ingvarson, L., Reckase, M., & Rowley, G. (2012). *Policy, practice, and the readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher education and Development Study in Mathematics (TEDS-M).* Amsterdam, Netherlands: International Association for the Evaluation of Educational Achievement.

Van Es, E., & Sherin, M. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571-596.

Inclusive Practices in the Teaching of Mathematics: Supporting the Work of effective primary teachers

Barbara Clarke
Monash University
<Barbara.clarke@monash.edu.au>

Rhonda Faragher
Australian Catholic University
<Rhonda.faragher@acu.edu.au>

The practices of effective primary school teachers including students with Down syndrome in their mathematics classes are largely unexplored and many teachers feel unprepared to teach students with intellectual disabilities. A study with cohorts in Victoria and the ACT is underway and here we report a subset of findings concerning the support teachers claim to require. There was an identified need for mathematics specific resources and strategies but a strong endorsement of inclusion as an appropriate practice in primary mathematics.

Introduction

As the focus of teaching has shifted toward a more child centred approach, there is much discussion about how to differentiate both teaching and curriculum to suit the needs of different learners. This is even more pronounced when considering the teaching of children with intellectual impairment within the context of inclusive classrooms. Mathematics teaching has traditionally been approached with an assumption of the development of sequential skills and is seen as a greater challenge to differentiate than other areas of the curriculum. A current research project is investigating practices of effective primary school teachers who were including children with Down syndrome in the teaching of mathematics in the primary school. In this paper, we report early findings from the project that identified the support teachers required to do their work.

Down syndrome is one of the most commonly occurring conditions leading to intellectual disability (Selikowitz, 1997) affecting approximately 1 in every 660 live births in Australia (Centre for Development Disability Health, 2005). In Australia, the majority of these children attend mainstream primary schools and are taught in classrooms alongside their age peers (Gothard, 2010). We have been studying learners with Down syndrome in previous work (Faragher & Clarke, 2014) and became interested in the classroom experiences of these children and the teachers who worked with them. The project described in this paper studied the classroom practices of primary mathematics teachers who were experienced with teaching in inclusive classrooms, as they taught classes including children with Down syndrome. At the start of the project, none of the teachers had taught a student with Down syndrome before. We followed the teachers' journey through a school year and studied a number of aspects of their work. Here we report on the support that these teachers claimed to require to successfully teach mathematics to the learner with Down syndrome alongside the rest of the class.

Literature

Including students with Down syndrome in regular mathematics classrooms has a relatively recent history. In Australia, the Disability Discrimination Act of 1992 and the companion Disability Standards for Education 2005 provided legislative protection to ensure learners with disabilities had the same rights as all other learners in Australia to education in their local school.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 173–180. Sunshine Coast: MERGA.

> The education provider must take reasonable steps to ensure that the course or program is designed in such a way that the student is, or any student with a disability, is able to participate in the learning experiences (including the assessment and certification requirements) of the course or program, and any relevant supplementary course or program, on the same basis as a student without a disability, and without experiencing discrimination. [Standard 6.2 (1), p. 23, Disability Standards for Education 2005]

With such a recent history of teaching practice, the challenge arises for teachers in the design and delivery of programs such that learners are able to participate and be assessed on classwork alongside their peers. It may be rare for these teachers to have experienced inclusive education in their own schooling. Australian research indicates that the majority of pre-service teachers feel underprepared on graduation for teaching students with special educational needs (Department of Education, Science and Training, 2006). Therefore, opportunities to develop expertise may come largely through experience, and appropriate professional learning will be important at that point. The nature of such professional learning is a focus of the current project.

Inclusive Education

Making adjustments and supporting learners with significant intellectual disabilities in mathematics can take many forms. We were particularly interested in inclusive education practices. We adopted the definition of inclusive education to be the practice of "welcoming, valuing and supporting the diverse learning needs of all students in shared general education environments" (Thousand & Villa, 2000, p. 73).

Inclusive education can be seen as a philosophy, process and practice (Cologon, 2014b, p. xviii). As a **philosophy**, it honours human diversity – all people, without exception, have value and a deserved place in an education setting. We are not doing some a favour; we are welcoming the contribution of all.

As a **process**, inclusive education differs from other processes for educating learners with disability. *Segregation* refers to education apart, such as in special schools or separate classrooms within a school (e.g., a special education unit). *Mainstreaming* (not to be confused with mainstream schools such as local schools, which are those that are not targeted to a specific group) refers to the process of enrolling students in a general classroom setting, but without adjustments or support for the requirements of the learner. Another common approach is *integration* where the student is present and adjustments may be made, but the setting itself does not change. The child who cannot fit in, cannot take part. An example of this approach would be where the child is physically in the same room as the rest of the class but does different work with the assistance of an aide. D'Alessio (2011, p. 102) referred to this as 'micro-exclusion'. Another commonly used model of integration is the practice of co-location (Slee & Allan, 2001) where students attend some lessons, such as art classes but are withdrawn for other lessons, often mathematics.

As a process, inclusive education is different to segregation, mainstreaming and integration. It involves "both social and academic inclusion, free from discrimination in any form" (Cologon, 2014a, p. 12). How this is done relies on the **practices** of inclusive education. Here the concern is with approaches, strategies and activities that are founded on the philosophy and processes of inclusive education. In our work, we have been particularly concerned with the practices of teachers from the academic inclusion perspective.

Inclusive Mathematics Education

The research literature provides little indication of what happens in inclusive mathematics classrooms. A recently published systematic review of observational research into inclusive education practices (McKenna, Shin, & Ciullo, 2015) identified just five studies published between 2000 and 2013 relating to mathematics classrooms. Observational studies are those that “seek to document how schools utilize instructional procedures based on policy change and research” (McKenna et al., 2015, p. 2).

McKenna et al. (2015) reported a number of findings of mathematics instructional practices documented in the reviewed studies. Each of the five studies observed lessons from the Number strand with very little time devoted to other areas of mathematics, if at all. A second finding attended to support for understanding. Teachers were observed to skip over work that might be thought difficult, and when students required assistance, they were told to try harder or given the answer. One of the studies, by contrast, observed students explaining their mathematical thinking in journals. Overall, observed opportunities for students to verbalise and discuss their mathematics were limited.

The third finding referred to types of instruction. Explicit instruction was observed and is a commonly recommended technique for teaching students with mathematics learning difficulties (Westwood, 2000). The approach was described as “Within a structured class, teachers systematically delivered mathematics lessons using specific procedures—introducing objectives, reviewing previously learned concepts, modeling new skills, and providing guided and independent practice. In this way, teachers applied procedure-based mathematics instruction to support students with LD [learning difficulties]” (McKenna et al., 2015, p. 8). While explicit instruction is considered an important approach by some mathematics education researchers, it is not regarded as solely effective, with researchers recommending a balanced approach including opportunities for strategic thinking and reasoning along with explicit teaching in numerical techniques (e.g., Baroody, 2006).

Another finding of the McKenna et al. review was that only two of the five studies reported observed use of visual support for learning number. As students with Down syndrome generally find support from visual strategies (Couzens & Cuskelly, 2014), it is concerning that visual strategies were not common place in the inclusive classrooms observed in these studies.

With so few studies providing observational evidence of inclusive mathematics education practices, further research is clearly needed. We are unaware of any studies explicitly investigating mainstream classrooms including learners with Down syndrome.

Mathematics Education for Learners with Down Syndrome

In reviewing current understandings of the mathematical development of learners with Down syndrome there are two cautions. First, people with Down syndrome have diverse phenotypes – they are not all alike. In common with other characteristics, educational attainment in general and mathematics attainment in particular, vary greatly from individual to individual. Second, diagnosis-specific knowledge can be a barrier to inclusive practice. This may seem counter-intuitive, but as Cologon notes, this approach can lead teachers to “become focused on the label and not the child, thus they implement inappropriate strategies that do not suit the child” (Cologon, 2014b, p. xix). All the same, there are some common traits exhibited by many learners with Down syndrome that can be helpful to consider in making adjustments to the mathematics curriculum.

Almost all research into the development of mathematics by learners with Down syndrome has studied aspects of Number. Very few studies from other areas of the discipline exist (Faragher & Clarke, 2014). Considerable difficulties with the development of number concepts have been documented by many researchers (Bird & Buckley, 2001). Unfortunately and incorrectly, many of these authors extrapolate difficulties with number concepts to difficulties with mathematics in general, leading to a very pessimistic view of what might be possible for students with Down syndrome to accomplish. Some studies have emerged (Faragher, 2014; Monari Martinez & Benedetti, 2011; Monari Martinez & Pellegrini, 2010) that suggest that other areas of mathematics, including algebra, may be within the grasp of learners with Down syndrome if they have access to a calculator and have been taught how to use it.

From the reading of the literature, the following aspects were considered important for teachers beginning their work including learners with Down syndrome: a shared understanding of inclusive practice as defined earlier in this paper; an understanding of Down syndrome; effective use of resources in mathematics education, particularly with respect to visualisation strategies; and appropriate use of calculators. In the present study, this was a starting point for our work with teachers.

This study had an overarching research question: What is the nature of inclusive mathematics education for learners with Down syndrome in primary classrooms? Here we report findings on the following sub-question: What are the teacher identified support needs to effectively include a child with Down syndrome in primary mathematics?

Background and Design of the Project

How does the teacher of a Year 4 student who is not able to reliably count a collection of 10 objects productively include the child in the teaching and learning of fractions? Teaching is complex but the challenges in these contexts are even greater. Such students are often assisted by a teacher aide and other advice and support are provided. What does a teacher need to know and be able to do in order to enhance the mathematics learning of children with Down syndrome in inclusive settings? How do teachers balance the needs of a range of children within the regular classroom with external curriculum expectations?

A research project - *Supporting the Mathematics Learning of Children with Down Syndrome in Inclusive Settings* - was conducted in 2014 by the authors, funded by Gandel Philanthropy and undertaken through the Australian Council for Educational Research Foundation, to explore these and related questions.

The project involved two groups of teachers, one in Melbourne and one in Canberra. In Melbourne, schools were identified by education officers of the Down Syndrome Association, were chosen based on their reputation for inclusive practice, and were currently including students with Down syndrome in their programs. In Canberra, links with parents and contacts within the local Down Syndrome Association were used to identify schools where inclusion was being effectively implemented. Parents were initially contacted and once ethics approval was obtained, the schools were approached.

An initial workshop was held at the beginning of the school year in each location, which included both professional learning and project planning. Teaching teams were introduced to a task based assessment interview for students, revised from an instrument used in a previous project (Faragher & Clarke, 2014). This was intended to be used at the beginning and the end of the year with each child. Relevant research findings on learners with Down syndrome and effective mathematics teaching were shared. We were particularly interested in capturing effective practice. For our study, this involved classroom observations, collection of work samples from students, teacher reflection journals, and interviews with teachers. There was a cycle of professional learning followed by school observation which was undertaken twice over the year of the project. Interviews with the teaching teams after the observations informed the content of the subsequent meetings in the middle of the year. The final meeting of teaching teams was an opportunity to reflect on the year and gather summative data on inclusive practice.

As the study unfolded, it was apparent that teacher participants had a range of expertise with inclusive practice and a variety of approaches were evident. In some cases, not all represented inclusion as defined above. This had implications for support needs which we consider further in the remainder of this paper.

Some Initial Findings

In this paper we focus on the support needs identified by members of the teaching teams at the beginning and again at the end of the school year. Both classroom teachers and teacher aides completed a questionnaire at the first professional development meeting in February and in the final meeting in November. For the initial questionnaire, we prepared two versions – one for teachers and one for teacher aides. Sixteen teachers and 12 teacher aides filled out the initial questionnaire. However, based on the models of inclusion and the varying roles of the aides that were evident through our observations and conversations, we gave the same questionnaire to all team members at the end of year. They were asked however, to note any questions they did not think were applicable to their context and role. Final questionnaires from 22 participants have been analysed to date. A small number of teams or team members were unable to attend the final meeting and these are being followed up at the time of writing.

In the initial questionnaire, teachers were asked the following question: What do you expect to be the most challenging aspect of teaching mathematics to the child with Down syndrome in your classroom? Please provide 3 in order of expected challenge.

The previous item to this had identical wording with the word “mathematics” deleted to elicit general responses, including those related to syndrome specific concerns. It was also designed to ensure that the mathematical focus of this item was reinforced.

A detailed analysis is not included here, but of the 16 teachers, syndrome specific perceptions (such as “stubbornness”) and management focused responses were given as the most challenging by 3 teachers. The remaining 13 teachers identified the major challenge related to mathematics and particularly to planning and teaching for differences.

The teachers were then asked - *What help do you think you need to support the mathematics learning of the child in your class with Down syndrome?* Of the 16 responses, 12 made specific reference to their need for greater knowledge of the mathematics learning of children with Down syndrome, with the next most frequent reference (7) focused on the need for support with resources including planning. Four teachers expressed the need for assessment information and strategies.

In the final questionnaire, after the teaching teams had been working with the child for almost a year, the item on the most challenging aspect was repeated. Overall the responses were more extensive and gave specific reference to the child and their learning. This was to be expected as they now had greater knowledge and experience. For the teachers, need for specific resources (4 responses) and challenges related to the varying mathematical ability of the student (7 responses) were identified as the most challenging. There was an increased emphasis on challenges related to student attitude and behaviour. These clearly had an impact on their teaching. These were not just behavioural and syndrome specific but also related directly to engagement and motivation in mathematics. These were the major challenge for the four other teachers. The following open responses indicate some of the issues involved.

> Variations in his engagement to learning - the days where he is wanting to participate vs the days where he is being resistant and not wanting to do anything.

> The child's ability to sometimes be able to show their understanding of a concept and not be able to do it on other occasions means that you can never be sure where to start with individual instructions.

The teachers had experienced close familiarity with one of the challenges faced by students with Down syndrome – motivation to engage with learning (Gilmore & Cuskelly, 2014). Teachers were clearly concerned with children engaging with mathematics learning and were not content to allow the learner to opt out. Early work by Wishart (1993) identified the predilection for avoidance of learning by even very young children with Down syndrome. The teachers in our study were determined to not accept this as a situation that was immutable and instead sought support for strategies to overcome this detrimental learning approach.

By the time of the administration of the final questionnaire, the teaching teams had gained considerable expertise and our goal was to tap that knowledge before teachers moved to new classes, most often without the learner with Down syndrome. In the final questionnaire, all participants were asked the following open response question: *What advice would you give others who are including a child with Down syndrome in mathematics classrooms*? The responses are summarised in Table 1.

The advice most referred to (by 11 out of 19 responses) involved the explicit encouragement to emphasise inclusion. Sample responses were:

> Include them in the grade and modify if need but never to exclude them as that can affect their learning and confidence.

> Children with DS should be included in all sessions. Provide opportunities for the child to complete small tasks independently so that they can feel success and achievement.

> Include the student in all sessions as the rest of the group. Get them to be as involved in the activity as much as they can.

We were struck by the frequency of the advice concerning support for inclusion. Research (Cologon, 2014b; Department of Education Science and Training, 2006) suggested that initially teachers seek syndrome specific strategy advice. However, Forlin and Chambers (2011) indicated that "there is also a growing body of research that has identified positive attitudes as being equally important as, if not more important than, knowledge and skills as prerequisites for good inclusive teachers" (2011, p. 18).

The comments from the members of teaching teams indicate that their experiences and associated support have given them a confidence that including children with Down syndrome in primary mathematics classrooms is an achievable goal.

Table 1
Categories and frequency of responses to question - What advice would you give others who are including a child with Down syndrome in mathematics classrooms?

Response category	Frequency
Explicit encouragement to emphasise inclusion in mathematics	11
Providing concrete/visual and related materials generally additional to regular mathematics classroom needs	7
Importance of relationships and collaboration within the team including the parents	7
Be prepared to repeat as needed or find smaller steps to support mathematics learning	4
Sharing with others including school visits and professional development sessions	2
Ensure engagement including making mathematics fun and interesting	2
Be prepared to give extra support	1
Don't panic	1

Note: More than one category was evident in some responses

The following quote is representative of the important components of support identified by the participants:

> Attend PDs related to mathematics for reluctant learners; work collaboratively with the child's teacher aide. Perhaps visit other schools with children with DS. Plan effective maths lessons that cater for all children's needs. Be well resourced.

Conclusions and Implications

Our topics for inclusion in the initial professional learning that we extracted from the literature were judged worthwhile by the teachers and included: a shared understanding of inclusive practice; an understanding of Down syndrome; and effective use of resources in mathematics education, including calculators. They also identified the need for a greater emphasis on improving their own knowledge related to the mathematics learning of children with Down syndrome. Advice suggested for teachers preparing to teach in such settings was overwhelmingly positive in relation to the value of inclusive mathematics teaching.

It is important to acknowledge the complexities of teaching in this environment and the need for a range of support. As responses of the teachers and aides indicate it is difficult to predict what the behaviour of the children will be, what they know and how they will respond to mathematics lessons on any particular day. Indeed, identifying ways to circumvent behaviours that are detrimental to learning remain a challenge for research. Having said that, we were encouraged by the creative ways that teachers engaged in both the teaching and the sharing of their developing expertise, and as we continue analysing our data we hope to provide greater insights into mathematics teachers and teaching in these inclusive classrooms.

References

Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, *13*, 22-31.

Bird, G., & Buckley, S. (2001). *Number skills for individuals with Down syndrome - An overview*. Hampshire: The Down Syndrome Educational Trust.

Centre for Development Disability Health. (2005). *Down syndrome: Information sheet for children and adolescents*. Retrieved from http://www.cddh.monash.org/assets/downsynd.pdf

Cologon, K. (2014a). Better together: Inclusive education in the early years. In K. Cologon (Ed.), *Inclusive education in the early years. Right from the start* (pp. 3-26). South Melbourne, Vic: Oxford University Press.

Cologon, K. (2014b). Preface. In K. Cologon (Ed.), *Inclusive education in the early years: Right from the start* (pp. xviii - xix). Melbourne, Vic: Oxford University Press.

Couzens, D., & Cuskelly, M. (2014). Cognitive strengths and weaknesses for informing educational practice. In R. Faragher & B. Clarke (Eds.), *Educating learners with Down syndrome. Research, theory and practice with children and adolescents* (pp. 40-59). London, UK: Routledge.

D'Alessio, S. (2011). *Inclusive education in Italy: A critical analysis of the policy of Integrazione Scolastica*. Rotterdam: Sense Publishers.

Department of Education Science and Training (2006). *Survey of former teacher education students: A follow-up to the survey of final year teacher education students*. ACT: Author.

Faragher, R. (2014). Learning mathematics in the secondary school: Possibilities for students with Down syndrome. In R. Faragher & B. Clarke (Eds.), *Educating learners with Down syndrome: Research, theory and practice with children and adolescents* (pp. 174-191). London, UK: Routledge.

Faragher, R., & Clarke, B. (2014). Mathematics profile of the learner with Down syndrome. In R. Faragher & B. Clarke (Eds.), *Educating learners with Down syndrome:Research, theory, and practice with children and adolescents* (pp. 119-145). London: Routledge.

Forlin, C., & Chambers, D. (2011). Teacher preparation for inclusive education: increasing knowledge but raising concerns. *Asia-Pacific Journal of Teacher Education*, *39*(1), 17-32. doi: 10.1080/1359866X.2010.540850

Gilmore, L., & Cuskelly, M. (2014). Mastery motivation in children with Down syndrome. Promoting and sustaining interest in learning. In R. Faragher & B. Clarke (Eds.), Educating learners with Down syndrome: Research, theory, and practice with children and adolescents. (pp. 60-82). London, UK: Routledge.

Gothard, J. (2010). *Greater expectations. Living with Down syndrome in the 21st century*. Perth, WA: Fremantle Press.

McKenna, J. W., Shin, M., & Ciullo, S. (2015). *Evaluating reading and mathematics instruction for students with learning disabilities: A synthesis of observation research*. Learning Disability Quarterly, 22(January), 1-13. doi: 10.1177/0731948714564576

Monari Martinez, E., & Benedetti, N. (2011). Learning mathematics in mainstream secondary schools: experiences of students with Down's syndrome. *European Journal of Special Needs Education*, 26(4), 531-540. doi: 10.1080/08856257.2011.597179

Monari Martinez, E., & Pellegrini, K. (2010). Algebra and problem solving in Down syndrome: A study with 15 teenagers. *European Journal of Special Needs Education*, 25(1), 13-29. doi: 10.1080/08856250903450814

Selikowitz, M. (1997). *Down syndrome: The facts* (2nd ed.). London: Oxford University Press.

Slee, R., & Allan, J. (2001). Excluding the included: A reconsideration of inclusive education. *International Studies in Sociology of Education*, 11(2), 173-192. doi: 10.1177/0731948714564576

Thousand, J., & Villa, R. A. (2000). *Inclusion. Special Services in the Schools*, 15(1-2), 73-108. doi: 10.1300/J008v15n01_05

Westwood, P. (2000). *Numeracy and learning difficulties: Approaches to teaching and assessment*. Melbourne: ACER Press.

Wishart, J. G. (1993). Learning the hard way: Avoidance strategies in young children with Down's syndrome. *Down Syndrome Research and Practice*, *1*(2), 47-55.

Supporting Students to Reason About the Relative Size of Proper and Improper Fractions

Jose Luis Cortina
Universidad Pedagógica Nacional, Mexico
<jose.luis.cortina@mac.com>

Jana Visnovska
The University of Queensland
<j.visnovska@uq.edu.au>

Fractions are a well-researched area; yet, student learning of fractions remains problematic. We outline a novel path to initial fraction learning and document its promise. Building on Freudenthal's analysis of the fraction concept, we regard *comparing*, rather than *fracturing*, as the primary activity from which students are expected to make sense of fractions. Analysing a classroom design experiment conducted with a class of 14 fourth grade pupils, we identify two successive mathematical practices that emerged in the course of the experiment and indicate how their emergence was supported.

In this paper, we analyse findings from a classroom design experiment aimed at supporting fourth grade students' understanding of fractions as numbers that quantify *relative size* (Thompson & Saldanha, 2003). We focus on the second part of the experiment, in which we were successful in supporting students' reasoning about fractions as numbers that quantify magnitude values that can be smaller than, as big as, or bigger than one. This kind of reasoning is seldom expected from novice fraction learners, as it has been widely documented that conceiving a fraction as a number that accounts for a quantity that is bigger than one (i.e., a whole) can present a major conceptual challenge (Steffe & Olive, 2010).

In the experiment, we tested an instructional approach in which students were never oriented to relate fractions to the equal partition, division, or segmentation of a whole—as it is typically done. Instead, building on Freudenthal's (1983) insights about fraction as comparer, we engaged students in tasks in which the entities that fractions quantify were always separate from the reference unit.

Theoretical Background

Much of the research on fractions adopts a cognitive perspective on learning (Lamon, 2007; Post, Carmer, Behr, Lesh, & Harel, 1993; Steffe & Olive, 2010; Tzur, 1999), where the primary focus is on understanding (and modelling) the learning processes. For instructional design purposes, we found it useful to approach learning from a situated perspective and view it as changes in the forms of students' participation in classroom mathematical practices (Cobb, 2003). We interpret learning as being shaped by means of support, which therefore constitute the explicit focus of our research.

This particular perspective made us aware of another commonality among otherwise diverse studies on fraction learning: the instructional tasks used almost exclusively fall within what Freudenthal (1983) characterises as *fraction as fracturer* situations, where a whole, often a food item, is being cut or split into equal-sized parts. We elaborate elsewhere (Cortina, Visnovska, & Zuniga, 2015) how these types of instructional support result in fraction images that are counterproductive to developing mature understanding of fractions.

Our instructional approach is based on a different type of situations that call for fractions use. In these situations, fractions are used to compare aspects (e.g., lengths) of "objects which are separated from each other or are experienced, imagined, thought as

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 181–188. Sunshine Coast: MERGA.

such" (Freudenthal, 1983, p. 145). Understanding whether these *fraction as comparer* situations can effectively support student learning is the focus of our research.

Methodological Approach

The classroom design experiment was conducted in a fourth grade classroom in a public school serving low-income students in southern Mexico. The classroom consisted of 14 students, ages 9 and 10. The experiment included 13 instructional sessions, each lasting about 90 minutes. A set of individual pre- and post- interviews was conducted with all the students to document the individual learning. The sessions and interviews were video recorded. In addition, all student work was collected, and a set of field notes was kept.

The design experiment consisted of three phases: planning, classroom experimentation, and retrospective analysis (Gravemeijer & Cobb, 2006). During the planning phase, a hypothetical learning trajectory (HLT) was formulated. In it, we conjectured that it would be possible to support students, early on, to make sense of unit fractions as numbers that account for the relative size of things that are separate from a reference unit; for instance, the length of a rod relative to the length of a unit of measure (see Figure 1).

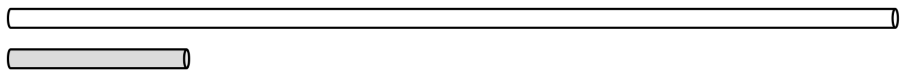


Figure 1. A reference unit and a rod that is $^1/_5$ of its length.

In addition, students would reason about the relative size of unit fractions, primarily, in terms of how many iterations of their size would be necessary to produce the size of one. Hence, a $^1/_5$ rod would have a length such that it would be necessary to iterate it five times to obtain a length as long as the reference unit (see Figure 2).

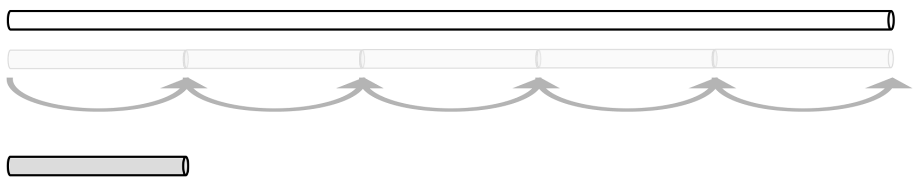


Figure 2. A fifth as a rod of such a size that five iterations of its length are necessary to obtain the length of the reference unit.

The second phase consisted of the actual experimentation in the classroom, and of conducting an ongoing analysis of the student learning. The ongoing analysis served to assess and adjust the HLT in light of ongoing classroom events.

In the final phase of the design experiment, a retrospective analysis of the actual learning trajectory undertaken by the students was conducted, with the benefit of hindsight. We analysed the data using an adaptation of constant comparative method described by Cobb and Whitenack (1996) that involves testing and revising tentative conjectures while working through the data chronologically. As new classroom episodes were analysed, they were compared with conjectured themes and categories, resulting in a set of the theoretical assertions that remained grounded in the data. Given the scope of this paper, we include representative episodes and interactions, where possible, as we build our argument. The viability of the ongoing analysis was revised to account for how the mathematical activity actually evolved in the classroom. This retrospective analysis resulted in reformulation of the HLT, so that the emergence of two identified mathematical practices would be explicitly supported in subsequent iterations of the design.

We now summarise the first mathematical practice (Cortina, Visnovska, & Zuniga, 2014), which involved reasoning about the relative size of unit fractions in ways consistent with what Tzur (2007) called *the inverse order relationship among unit fractions*. Hence, when comparing two unit fractions (e.g., $^1/_7$ vs. $^1/_{10}$) all of the students came to consider the one with the smaller denominator ($^1/_7$) as the one quantifying the bigger size. We then turn to the main focus of this paper—the second mathematical practice—where students could reason about fraction comparisons.

First Mathematical Practice: The More Times it Fits, the Smaller it has to Be

As we have elaborated elsewhere, the first mathematical practice emerged between days 1 and 4 of the design experiment. At the beginning of the instructional intervention, most of the pupils reasoned about the relative size of unit fractions following what Baroody (1991) called *the magnitude comparison rule*. They regarded unit fractions represented by numbers that would come later in the counting sequence as always accounting for larger sizes. Hence, a tenth would represent, for the students, a bigger quantity than a seventh.

Central to the instructional activities with which we helped students make sense of how big numbers can sometimes account for small sizes, was a narrative about how ancient Mayan people measured. The students were presented with a measuring stick (24 cm long) and told that that some archaeologists believed that ancient Mayans used this stick as a tool for measuring lengths. Students were then each given a replica of the stick and were asked to use it to measure the lengths of different things. This activity served to raise a question of how to account for the lengths that the stick did not cover exactly. On day 3, students were presented with the solution that the ancient Mayans could have come up with to systematically and precisely account for such lengths. It involved producing *smalls*: rods of a specific size relative to that of the length of the stick.

Each student then engaged in producing the smalls by cutting plastic straws. For the *small of two*, students were told that its length needed to be such that when used to measure the stick, the measure would have to be exactly two (i.e., a rod $^1/_2$ as long as the stick). Pupils made their small of two, with teacher guidance, by iterating a straw along the stick and adjusting its length. It took about 15 minutes for all the students to produce their small of two. Students were then told that the *small of three* would have a length such that it would fit exactly three times along the stick. Before making it, the teacher briefly discussed with the students if they expected the small of three to be longer or shorter than the small of two. A similar process was followed to produce the smalls of four, five, and six. Then, students were given leeway to produce more smalls, until the session ended. Some made as many as ten.

The activity of producing the smalls (unit fractions), and reasoning about their relative size helped the students develop imagery that was consistent with the inverse order relation (Cortina, et al., 2014). By day 5, pupils made sound comparisons between the sizes of smalls, even if they had not physically seen them. For instance, they regarded a small of 14 as being necessarily bigger than a small of 20. They also seemed to have a clear image of what the size of a small might be, so that when asked about the size of a *small of a hundred*, they responded that it would be very small, and gestured with their hands and fingers to show a tiny length.

Second Mathematical Practice: Reasoning about Fraction Comparisons

The second mathematical practice involved reasoning about fractions as representing lengths that could be either smaller than, as big as, or bigger than the reference unit. These were initially the length of paper-strips that were actually measured by the students, using the smalls. For instance, they could be the length of a paper-strip that was four times as long as the length of the small of three (i.e., $^4/_3$ as long as the stick). Later on, they were presented only as written measures, expressed with conventional fraction notation: $\frac{a}{b}$

The following excerpt from day 11 is representative of students' reasoning at this point of the design experiment. The teacher wrote the fractions $^{99}/_{100}$ and $^5/_5$ on the chalkboard using conventional notation. Several students raise their hands to answer. The teacher pointed at Lourdes.

Lourdes:	Five smalls of five is bigger because ninety-nine smalls of one hundred is smaller.
Teacher:	And why is that?
Lourdes:	Because the bigger the number is it has to be smaller (gesturing with her hands a tiny size) so it fits.
Teacher:	But ninety-nine is a lot, no?
Lourdes:	Yes, but it needs to be small to fit in the stick.
Carlos:	(jumping in) and there is not enough to fill it.
Teacher:	Marisol?
Marisol:	I think that five smalls of five is bigger because ninety-nine smalls of one hundred is smaller because it is not enough to fill the stick.
Teacher:	It is not enough to fill the stick. Carlos?
Carlos:	Ninety nine smalls of one hundred is not going to be enough to fill the stick because it is missing one small for it to be one hundred smalls of one hundred, and five of five do fill the stick.

This excerpt depicts several important aspects of students' reasoning in the second mathematical practice. First, it shows how, following what pupils had done in the first mathematical practice, the denominator of a fraction was construed as the length of a rod, relative to the length of the reference unit. Lourdes' comment about the smalls of one hundred being little, illustrates this point. As for the numerator, it was interpreted as a number that accounted for iterations of the length of the smalls, which accumulated into a length. Carlos' comment about 99 smalls of one hundred not being enough to fill the stick is illustrative of this second point.

In Lourdes' responses above, it is possible that she was only taking into consideration the relative size of the smalls involved, and not how many times each small was iterated. This kind of reasoning had emerged several times in the classroom. However, each time it was treated as inadequate or incomplete by the class. In this instance, Carlos decided to jump in and add the important missing facet of the argument. Over time, instances of reasoning about *relative size of smalls only* faded out.

The excerpt also shows how students first came to assess the relative size of a fraction in terms of it representing a length that was enough, or not, to *fill* (cover) the length of the reference unit. It is hence worth highlighting that students were not comparing the fractions relying on numeric facts and patterns (e.g., in the first fraction, the numerator was smaller than the denominator). Instead, they were comparing them quantitatively.

Importantly, the second mathematical practice was not limited to the realm of proper fractions. For instance, in the same session, a few minutes before the conversation above took place, students were asked to compare $^{12}/_{13}$ with $^{6}/_{5}$. All but two of the students chose the latter fraction as the one expressing the bigger length, and their justifications of this choice were mathematically sound. This is how one of the students justified his choice:

Eduardo: Because you need thirteen smalls of thirteen to fill the stick, and with twelve it's not enough. And in the other you need five, but they are six and it even goes further.

This contribution illustrates how, once the second mathematical practice was established, students easily construed both proper and improper fractions as numbers that soundly accounted for the size of a length. By using the comparer approach to fraction instruction from the outset of the design experiment, we had oriented pupils to construe the entities that unit fractions quantify as being separate from the reference unit and, thus, susceptible of being iterated *unrestrictedly*. For the students then, there was no natural boundary (e.g., the length of the unit whole) limiting the extent to which a small could be iterated. The iteration of a small of five ($^{1}/_{5}$) more than five times did not become, at any point of the design experiment, a troublesome issue for any of the students.

Supporting the Emergence of the Second Mathematical Practice

The second mathematical practice we just described emerged from the previous one. The retrospective analysis revealed that two shifts in student reasoning were critical in the emergence of this practice and required supporting: students first needed to come to view the smalls as capable-of-being-iterated measurement units in their own right. The second shift involved students coming to make sense of a new representation introduced by the teacher (see Figure 3) as actually representing the iterations of the smalls.

In the HLT we formulated during the planning phase of the design experiment, we conjectured that the activity of producing the smalls would rather easily lead students to make sense of the equivalence of multiples of unit fractions with *one*. In other words, we conjectured that students would somewhat effortlessly recognise that two iterations of the small of two, $^{2}/_{2}$, would render the same length as three iterations of the small of three, $^{3}/_{3}$, four iterations of the small of four, $^{4}/_{4}$, and so on. During the design experiment, we came to realise that, for the students, making sense of this basic equivalence was not trivial. The following excerpt illustrates how students were thinking about the smalls on day 5.

Teacher: Carlos, how long is the small of three?

Carlos: It has to measure three times that stick… the straw has to measure three times that stick. Until it gives you three.

It is worth noticing that Carlos used the expression *to measure* to describe the act of iterating a straw along the stick. This use of the expression sounds strange in English and in Spanish. Nevertheless, students commonly used it in this way, at this point of the design experiment. Carlos seemed to construe iterating, essentially, as a means to gauge and fix the length of a small. This should come as no surprise, since this is how iterating was used in the activity of producing the smalls. What was initially surprising to us was that even after the accurate smalls were produced, students did not automatically come to see them as units of measure in their own right. Instead, the smalls initially represented to them *only* the result of the construction process.

Aiming to help the students reason about the equivalencies between iterating the length of the smalls a certain number of times, and the length of the stick, we provided the students with a *Measurement Kit* (see Figure 3), which included a stick and four smalls (wooden rods representing 1, $^1/_2$, $^1/_3$, $^1/_4$, and $^1/_6$), and a printed sheet. The sheet had five bars the length of the stick, four of them segmented to match the sizes of the smalls. We conjectured that the sheet would become a useful a resource for reasoning about equivalence and inequalities with *one* (e.g., $1 = {}^4/_4$, $1 > {}^5/_6$) and with other fractions (e.g., ${}^5/_6 < {}^4/_4$).

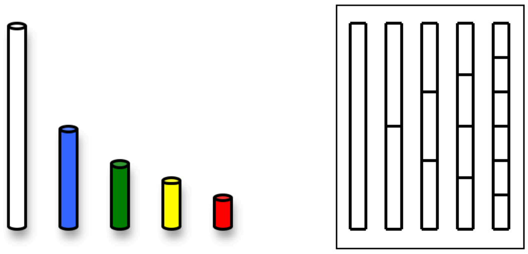


Figure 3. The *Measurement Kit* included a white stick (24 cm), four rods (blue, 12 cm; green, 8 cm; yellow, 6 cm; and red, 4 cm), and the printed sheet. Colours and sizes of rods correspond to plastic straw smalls.

When we first engaged students in activities aimed at supporting them to reason about the relative lengths produced by iterating the smalls, we noticed some unanticipated complications. The first was that in the new type of activities, when the students started to use the rods as a means to measure, they seemed to approach them as if they were independent. They did not reason with the fact that the smalls were produced from the same stick. As a consequence, students would not consider that the specific number of iterations of each small would have to necessarily render the length of the stick. For instance, they would not anticipate that a paper strip that measured two smalls of two would necessarily also have to measure three smalls of three.

The second complication, related to the first one, was that students did not readily regard the printed sheet as a useful resource for determining equivalencies between measures made with smalls. By and large, when it was first introduced, the pupils did not see the sheet as record of the iteration of the smalls, relative to the length of the stick.

It was through engaging students in activities that involved measuring paper strips of different sizes, using different smalls, and by constantly referring them to the sheet, that we eventually succeeded in helping the students recognise the equivalent relation between the iteration of each small and the length of the stick. As we illustrated above, by day 11, most of the students could make correct comparison between the sizes of two fractions, using the equivalence with the stick as a benchmark, even between fractions whose denominators they had not physically produced.

In the finial interviews, it was apparent that all of the students could do correct comparisons between fractions, using the equivalence with the stick as a benchmark. Four of them could do so only when encouraged by the interviewer to reason about the fractions as numbers that accounted for the iterations of smalls, and to reflect whether the outcome of the iteration would produce a length equal to that of the stick. The remaining ten students could do the comparisons rather easily, and could explain their answers in ways similar to Marisol, Carlos, and Eduardo in the excerpts presented above.

In the retrospective analysis we realised that the complications we faced were the result of shortcomings of our original instructional design. On the one hand, we should have provided students with activities that would have allowed them to more directly recognise and reason about the equivalence between iterating the smalls and the length of the stick. On the other hand, we should have introduced the printed sheet in a way that would have allowed students to construe the segmentations on the bars as marks left by the iteration of the smalls more easily. These realisations formed the basis for our revisions of HLT.

Discussion and Conclusions

Student learning documented above is not currently typical in mathematics classrooms. The two mathematical practices that emerged in the classroom with novice fraction learners, within the three weeks over which the design experiment took place, correspond to overcoming the two *developmental hurdles* in fraction learning that Norton and Hackenberg (2010) identified in their review of research in the field. We take the relatively smooth emergence of these practices as an indication of the potential of the tested instructional approach.

The presented analysis of the actual learning trajectory helped us to understand how the emergence of the two classroom mathematical practices was supported in the classroom design experiment, and which forms of student reasoning were crucial to the emergence of these practices. The design research cycle would not be complete without the formulation of the new, revised, HLT that would present a starting point in the next iteration of testing and refinement of instruction. With the hindsight we gained through the analysis, the revisions would include the following:

1. The students did not automatically come to see the reciprocal relation between the size of a small and the size of the stick, as a result of the process by which the small was produced. However, students can be supported to come to see smalls as units of measure in their own right, for instance by engaging in activities, in which they use smalls to construct strips of paper of the pre-determined length, such as $^3/_5$, $^5/_5$, or $^7/_5$.
2. The Measurement Kit sheet did not initially have any history for students and we struggled in supporting them in creating meaning for it and using it effectively. With the hindsight, we would now have students construct this sheet in a series of activities, rather than providing the ready copy to them. We collected some informal indications that this approach is superior.

Our understanding of the shortcomings of our initial design conjectures that led to these revisions constitutes the key theoretical contribution within the type of research we conduct (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). It is reasonable to expect that in the upcoming design iterations, our (and others') improved understanding of how specific means of support shaped forms of student reasoning (including their confusions) will lead to a more effective design. This is the pathway along which we can envision that understanding of fractions as numbers that quantify relative size would become possible for all students.

References

Baroody, A. J. (1991). Meaningful mathematics instruction: The case of fractions. *Remedial and Special Education, 12*(3), 54-68.

Cobb, P. (2003). Investigating students' reasoning about linear measurement as a paradigm case of design research. In M. Stephan, J. Bowers, P. Cobb & K. Gravemeijer (Eds.), *Supporting students' development of measuring conceptions: Analyzing students' learning in social context* (pp. 1-23).

Cobb, P., Confrey, J., diSessa, A. A., Lehrer, R., & Schauble, L. (2003). Design experiments in education research. *Educational Researcher, 32*(1), 9-13.

Cobb, P., & Whitenack, J. (1996). A method for conducting longitudinal analyses of classroom videorecordings and transcripts. *Educational Studies in Mathematics, 30*, 213-228.

Cortina, J. L., Visnovska, J., & Zuniga, C. (2014). Unit fractions in the context of proportionality: Supporting students' reasoning about the inverse order relationship. *Mathematics Education Research Journal, 26*(1), 79-99.

Cortina, J. L., Visnovska, J., & Zuniga, C. (2015). Equipartition as a didactical obstacle in fraction instruction. *Acta Didactica Universitatis Comenianae Mathematics, 14*(1), 1-18.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht, The Netherlands: Kluwer.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney & N. Nieveen (Eds.), *Educational design research: The design, development and evaluation of programs, processes and products* (pp. 45-85). New York: Routledge.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Charlotte, NC: Information Age Pub.

Norton, A., & Hackenberg, A. J. (2010). Continuing research on students' fraction schemes. In L. Steffe & J. Olive (Eds.), *Children's fractional knowledge* (pp. 341-352). New York: Springer.

Post, T., Carmer, K. A., Behr, M., Lesh, A., & Harel, G. (1993). Curriculum implications of research on the learning, teaching and assessing of rational number concepts. In T. P. Carpenter, E. Fennema & T. Romberg (Eds.), *Rational numbers: An integration of research*. Hillsdale, NJ: Erlbaum.

Steffe, L. P., & Olive, J. (2010). *Children's fractional knowledge*. New York: Springer.

Thompson, P. W., & Saldanha, L. A. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, G. Martin & D. Schifter (Eds.), *Research companion to the principles and standards for school mathematics* (pp. 95-113). Reston, VA: National Council of Teachers of Mathematics.

Tzur, R. (1999). An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning. *Journal for Research in Mathematics Education, 30*, 390-416.

Tzur, R. (2007). Fine grain assessment of students' mathematical understanding: Participatory and anticipatory stages in learning a new mathematical conception. *Educational Studies in Mathematics, 66*, 273-291.

Proportional Reasoning as Essential Numeracy

Shelley Dole
University of the Sunshine Coast
<sdole@usc.edu.au>

Annette Hilton
Aarhus University, Denmark
<anhi@dpu.dk>

Geoff Hilton
The University of Queensland
<g.hilton@uq.edu.au>

This paper reports an aspect of a large research and development project that aimed to promote middle years school teachers' understanding and awareness of the pervasiveness of proportional reasoning as integral to numeracy. Teacher survey data of proportional reasoning across the curriculum were mapped on to a rich model of numeracy. Results provided evidence of extensive and creative teaching of proportional reasoning in all learning areas. The capacity of such tasks and activities for promoting student numeracy is theorised.

Background

Numeracy is an enabling skill for life and work and means being able to apply mathematics in everyday situations. Many everyday life tasks require proportional reasoning; that is, the capacity to understand and interpret situations of comparison in relative terms (e.g., scaling recipes, currency conversions, calculating discounts). In fact, proportional reasoning has been described as one of the most commonly applied mathematics concepts in the real world (Lanius & Williams, 2003). Yet students' persistent and continued difficulties with proportion and proportion-related tasks are well documented (e.g., Lamon, 2007). An explicit focus on proportional reasoning in all school subject areas, including mathematics, may have great potential for achieving successful development of this essential life skill and therefore numeracy improvement.

Proportional reasoning is being able to make comparisons between the entities in ratio and proportion situations in multiplicative terms (Behr, Harel, Post & Lesh, 1992). The development of proportional reasoning is a gradual process, underpinned by increasingly more sophisticated multiplicative thinking and the ability to compare two quantities in relative (multiplicative) rather than absolute (additive) terms (Lamon, 2005). For example, a proportional reasoner can see that the relationship between the numbers 2 and 10 additively as a difference of 8, but also multiplicatively as 10 being the result when 2 is multiplied by 5. The essence of proportional reasoning is understanding the multiplicative structures inherent in proportion situations (Behr et al., 1992). Students' difficulties in developing proportional reasoning have been attributed to the teaching of mathematics topics in isolation (English & Halford, 1995) and an elementary school curriculum that does not promote multiplicative structures (Behr et al. 1992). There have been calls for change to the way rational number topics are taught in primary school, with greater attention to the active development of students' multiplicative thinking (Behr, et al. 1992; Lamon, 2005; Yetkiner & Capraro, 2009). How this may occur, however, is still unclear.

Theoretical Framework

In Australia, numeracy has been defined as being able to "use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 189–196. Sunshine Coast: MERGA.

community and civic life" (AAMT, 1997, p. 15). More recently, a much richer description of numeracy has been proposed by Goos (2007) that draws together the myriad definitions of numeracy and simultaneously highlights the absolute necessity of numeracy being a core goal in education, encapsulated into a compact readily-identified triangular figure. The numeracy model has been elaborated elsewhere (see Goos, Geiger & Dole, 2010). It highlights the fact that numeracy is situated within a context, and includes mathematical knowledge, tools, dispositions, and a critical orientation. The model has been found to be extremely useful for analysing the numeracy demands of a school mathematics curriculum (Goos, Geiger & Dole, 2010); to support teachers' curriculum planning (Goos, Dole & Geiger, 2011), to trace changes in teachers' understanding of numeracy (Goos, Geiger & Dole, 2011), in the analysis of the design of numeracy tasks to draw implications for pedagogy (Goos, Geiger & Dole, 2013), and for exploring the role of digital technologies in numeracy teaching and learning (Geiger, Goos & Dole, 2014). In this study, we use the numeracy model to analyse proportional reasoning tasks and activities to theorise their capacity for supporting students' numeracy capabilities.

As stated previously, the essence of proportional reasoning is multiplicative thinking, an awareness of how two quantities are related in a multiplicative rather than an additive sense. The American Association for the Advancement of Science (AAAS) (2001) Atlas of Scientific Literacy identified two key components of proportional reasoning: Ratios and Proportion (parts and wholes, descriptions and comparisons, and computation) and Describing Change (related changes, kinds of change, and invariance). Lamon (2007) outlined central core ideas for proportional reasoning as rational number interpretation, measurement, quantities and co-variation, relative thinking, unitising, sharing and comparing, and reasoning up and down. These two sources highlight the encompassing nature of proportional reasoning and the fact that it is more extensive than simple rules or calculation procedures. In the absence of knowledge of ways to promote proportional reasoning, teachers may revert to skill-based approaches that will hamper students' proportional reasoning development and capacity to use proportional reasoning in complex and unfamiliar situations. Tasks requiring proportional reasoning are a continual stumbling block for so many students in many areas of the curriculum, which suggests the need for a broad-spectrum, multi-pronged strategy for action.

This paper addresses the following research question:

> What is the nature of cross-curricular proportional reasoning tasks in relation to their capacity to promote students' numeracy?

Design and Approach

This project involved approximately 90 teachers from five school clusters comprising secondary schools and their feeder primary schools in geographical proximity. Over the two years of the project, clusters met together eight times, once per school term (four per year). We drew upon the Loucks-Horsley, Stiles, Mundry, Love, & Hewson (2010) framework for designing professional development to guide our approach for project meetings. In between cluster meetings, teachers were to devise learning plans tailored to their own school context, as a result of input from the professional learning seminars and to report back to the cluster at the next meeting. In between professional learning seminars, the researchers visited project teachers in their classrooms, offered support and advice, and assisted with planning and implementing ideas. As such, a design-based research approach (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003) was taken in this study as it aimed to investigate and build theory about the enrichment of teachers' numeracy-related subject matter knowledge and practice as well as the improvement of students' numeracy levels. A large corpus of data was collected over this project, and included results from a researcher developed pen-and-paper pre- and post-test diagnostic assessment instrument specifically tailored for this project, to classroom observations, teacher interviews and focus groups, individual student interviews, and teacher feedback surveys.

The data reported in this paper is from a teacher survey, which was administered during the second year of the project (second meeting in Year 2). Teachers were provided with a large sheet of paper containing a table of cells with each curriculum subject area displayed as column headings. Teachers were asked to reflect upon activities and tasks they had implemented in their classrooms that had either been directly focused on promoting their students' proportional reasoning, or opportunities they had seized (teachable moments) for emphasising proportional reasoning to their students. Survey data were analysed three ways. First, the responses were collated into a master list of tasks and activities to give a direct count of the number of proportional reasoning moments described by teachers for each learning area. Second, similar responses in each learning area were collapsed to highlight the different types of proportional reasoning moments that teachers had identified according to each learning area. Third, the proportional reasoning moments were categorised as aligning with particular elements of the numeracy model to give a sense of how proportional reasoning activities might serve to promote numeracy.

Results

A total of forty survey responses were collected, comprising responses from six teachers of Grade 4, nine teachers of Grade 5, nine teachers of Grade 6, twelve teachers of Grade 7, and four teachers of Grades 8-10 (secondary school). Survey return was dependent upon attendees at the workshop at the time. In total, these teachers identified 395 instances of proportional reasoning opportunities, teachable moments, tasks, and activities across the learning areas, including five instances in "Other" areas. In many cases, repetition was seen in the examples provided, so a second level analysis removed repetition, resulting in 284 distinct proportional reasoning moments identified in the learning areas. These results are presented in Table 1.

Table 1 shows that teachers identified proportional reasoning moments in all learning areas, with most counts in Mathematics followed closely by Science. Without accounting for repetition, in the learning areas of Health and Physical Education (HPE), Studies of Society and Environment (SoSE), and The Arts, proportional reasoning moments were identified approximately half as many times as for Mathematics and Science, with the learning areas of English, and Design and Technology approximately one-third as many times as for Mathematics and Science. After repetition had been taken into account, these amounts were similar, except for English and Languages other than English (LOTE) where there was little repetition of examples given by teachers. English examples thus were approximately half the number of examples given for Mathematics and Science.

Examples of proportional reasoning in Mathematics included money, fractions, angles, determining the better buy, using maps, and scale. In Science, proportional reasoning moments included comparing rates for generating electricity, comparing shadows, making predictions based on data, planets, energy, and ramps. Examples of proportional reasoning moments in HPE included balancing diets, ball games and speed, comparing heart rates at rest and after exercise; SoSE examples included devising timelines, latitude and longitude, house plans, paper usage, and percent per capita to population, needs and wants and natural resources; The Arts examples included drawing and body proportions, mixing paint, devising dance steps, perspective drawing, and cartoon drawing; Design and Technology examples included computer usage per country per gender per age group, analysing product packaging, gear ratios, book making, and water quality analysis; English examples included making posters with words in proportion to importance, creating task timelines, analysing ballads, and spatial information in a range of texts. In LOTE, identified proportional reasoning moments included land mass of Japan compared to Australia, time zones, financial exchange rates, and place value associated with other number systems. Examples of proportional reasoning in the 'Other' category included: looking at teacher time on analysing national test data and the amount of time given to planning and developing curriculum, students creating their own study planner, seating plan for the classroom, and staff discussion time on student diagnostic test results.

Table 1
Number (and percentage) of initial identified proportional reasoning (PR) opportunities, teachable moments, tasks, and activities for each subject area, with second analysis removing instances of repetition

Learning Area	Number of initial PR moments identified	PR moments after repetition removed
English	27 (7%)	26 (9%)
Languages other than English (LOTE)	11 (3%)	11 (4%)
Health and Physical Education	43 (12%)	32 (11%)
Studies of Society and Environment	49 (12%)	34 (12%)
Mathematics	98 (25%)	57 (20%)
Science	89 (23%)	67(24%)
The Arts	41 (10%)	32 (11%)
Design and Technology	32 (8%)	20 (7%)
Other	5 (1%)	5 (2%)
Total	395	284

For the third level of analysis, three members of the research team analysed each task separately and then met together to compare classification. Differences in classification were discussed and agreement attained through establishment of guidelines for classification (described below). There was high agreement between researchers with only five instances of differences in classification. Each proportional reasoning moment was considered in relation to the definitions of elements within the numeracy model: *Mathematical Knowledge* (problem solving, estimation, concepts, and skills), *Tools* (representational, physical and digital), *Contexts* (a real-world situation), *Dispositions* (confidence, flexibility, initiative, and risk), and *Critical Orientation* (questioning, hypothesising, interpreting results to make informed decisions).

Although each of the proportional reasoning moments could be categorised as relating to several of the numeracy elements, classification was determined on the basis of emphasis. As such, proportional reasoning moments that were classified as *Mathematical Knowledge* included: finding unknown angles, problem solving using ratio examples, designing fair tests, moon phases, mixing paint, examples that predominantly link to mathematics content and process knowledge. Proportional reasoning moments classified as *Tools* included tasks that predominantly required the use of tools for completion: drawing a pulse rate graph, layers of the earth models, enlarging and reducing images on the computer, and drawing a circle graph.

Table 2
Proportional Reasoning moments categorized according to elements of the Numeracy Model

	Critical Orientation	Context	Mathematical Knowledge	Tools	Dispositions	Total
English	3	6	17	0	0	26
LOTE	0	0	11	0	0	11
HPE	7	9	15	1	0	32
SoSE	12	8	14	0	0	34
Maths	4	17	32	4	0	57
Science	7	23	35	2	0	67
Arts	0	7	18	7	0	32
Tech	4	4	12	0	0	20
Other	1	3	0	0	1	5
Total	38 (13%)	77(27%)	154 (54%)	14(5%)	1 (0.4%)	284

Proportional reasoning moments classified as *Context* were those that specifically located the task within a real context, and included: comparing the proportion of time spent on various themes in a movie, shortcuts to the school oval, orienteering using maps, cooking to create food (as opposed to determining ingredients for fictitious recipes), fuel use on *Mr Brown's* motorbike (as opposed to calculating fuel use for any bike), and exploring why penguins huddle. This category was difficult to determine in some instances as the context provided opportunity for developing a critical orientation, for using tools, and developing mathematical knowledge. However, the authenticity of the context was the determining factor for classification. For example, the calculations for Mr Brown's motorbike related directly to Mr Brown as the students' classroom teacher. This is a real context for the application of mathematics. Some of the proportional reasoning moments listed by teachers clearly linked to the development of a critical orientation, and included: bullying – the victim feels small while the bully looms large; advertising – the size of photos and words for emphasis or persuasion; gambling debt and proportion of club profit; carbon production versus power use and a home audit. In the analysis, there was only one proportional reasoning moment that could be categorised as linking to the *Dispositions* element of the numeracy model, and this was in relation to students creating their own study planner as this was deemed a task where students had autonomy over the outcome, which was very personal to them. It could be conjectured that many of the contexts of the proportional reasoning moments also provided opportunities for development of students' positive dispositions, and this has been found to be the case in other research (Goos, Dole & Geiger, 2011; Geiger, Goos & Dole, 2014), but we surmised that this was not the main focus of teachers' thoughts as they completed this exercise. Table 2 provides a summary of classification of all proportional reasoning moments according to the elements of the numeracy model.

The data presented in Table 2 indicate the high level of potential mathematical knowledge students in this study would be exposed to through engaging in the proportional reasoning tasks identified by their teachers. Table 2 also shows the range of contexts, beyond mathematics in which teachers were incorporating proportional reasoning moments. Of all learning areas, Science appeared to be one that teachers found most contexts for proportional reasoning moments, as well as in mathematics. Surprisingly, teachers identified many proportional reasoning moments in the learning area of English, three of which would potentially promote a critical orientation. Of all subject areas, the learning area of SoSE had the most proportional reasoning moments associated with developing a critical orientation, suggesting that students were engaging in meaningful learning experiences as active and responsive citizens.

Discussion

Survey data suggest strong evidence of a cross-curricular approach by teachers in designing and implementing tasks that promote students' proportional reasoning. Forty teachers nominated 395 instances of proportional reasoning tasks, activities, and learning opportunities across all areas in the curriculum. Whilst Mathematics was the subject area most nominated, this was followed by Science, but this accounted for only approximately 50% of tasks. Tasks and activities associated with subject areas of The Arts, Health and Physical Education, and Studies of Society and Environment were nominated approximately 10% each with just fewer than 10% of tasks located in the subject area of English. Data collection occurred in the final year of the teacher workshops, suggesting that with a greater understanding of the nature of proportional reasoning, project teachers were more responsive to triggers for potential proportional reasoning tasks they could use in their classroom that extended beyond mathematics.

Using the numeracy model to frame analysis of the nominated tasks and activities, we saw richness beyond simply the development of mathematical knowledge, although just over half of the tasks were identified as promoting this numeracy dimension. Just over one-quarter of the tasks primarily were rated as being situated in an authentic context. This means that students were developing and applying proportional reasoning in real situations, in accordance with how numeracy should be developed (Steen, 2001). The most note-worthy outcome of the analysis was that approximately one-eighth of tasks were categorised foremost as relating to the numeracy dimension of a critical orientation. This means that students were being provided with a critical numeracy education that included opportunities to critique, make critical interpretations of mathematical information, use mathematics in a reflective way, and use mathematics to operate powerfully in the world (Stoessiger, 2002). The example of describing how one feels when one is being bullied is a stunning proportional reasoning moment that has a strong social message that would have a profound impact on students.

Clearly, through the high number of counts of tasks that targeted mathematics knowledge in the data, it would appear reasonable to suggest that the students in our project teachers' classrooms were in a much stronger position for developing multiplicative thinking and engaging in processes that comprise proportional reasoning. From the activities listed, we surmised that students would be engaging in rational number interpretation, measurement, exploring quantities and co-variation, relative thinking unitising, sharing and comparing, and reasoning up and down; mathematical processes core to proportional reasoning (Lamon, 2007). Classroom observations that were omitted here due to space limitations provide further evidence of this.

In relation to our research question that guided our analysis here, it appears that cross-curricular proportional reasoning tasks can be grounded in authentic contexts through the nature of the learning area in which they are located, that they have the capacity to promote mathematics knowledge, tools, dispositions, and a critical orientation. As such, the development of proportional reasoning can occur in all learning areas, and as a result, has the capacity to promote students' numeracy. The long history of students' difficulties with proportional reasoning tasks has led to repeated calls for change to the teaching of proportional reasoning in the curriculum (Lamon, 2007; Sowder, Armstrong, Lamon, Simon, Sowder & Thompson, 1998). Proportional reasoning is generally regarded as something that is located in topics of ratio and proportion, although it has long been identified as something that cuts across subject areas and is most frequently applied in real life (Ahl, Moore & Dixon, 1992; Boyer, Levine & Huttenlocher, 2008; Lanius & Williams, 2003). Taking a cross-curricular approach to proportional reasoning in this project provided teachers with an alternative approach to developing students' proportional reasoning capabilities. The fundamental cause of students' difficulties with proportional reasoning has been levelled at a lock-step mathematics curriculum that teaches topics in isolation (e.g., English & Halford, 1995; Sowder et al., 1998). In this project, teachers circumvented the traditional pathway to rational number teaching, creating new and diverse learning activities that not only have the potential to promote students' proportional reasoning, but also to enhance their numeracy capabilities.

Conclusion and Implications

The research in this paper relates to current educational issues in three ways. First, numeracy is an educational priority on a global scale. The academic debate around defining numeracy has now turned to cross-curricular teaching of numeracy. Our research here shows the creative ways that teachers designed authentic numeracy tasks across all curriculum areas. Second, a focus on proportional reasoning framed within a rich model of numeracy drew teachers' attention to fundamental mathematics content for proportional reasoning that they incorporated into their task/lesson design. Teachers designed a myriad of cross-curricular tasks, showing the pervasive nature of proportional reasoning throughout the curriculum. Teachers' gradual and continued awareness of proportional reasoning highlights its elusive nature. Third, the theoretical frame of this study, combining key research from the fields of proportional reasoning and numeracy, provides a frame for analysis to determine the richness of numeracy tasks whilst simultaneously illuminating essential mathematics content knowledge for proportional reasoning.

In sum, this research has argued that numeracy, as a major goal of education, is essential curriculum, and that proportional reasoning is an essential component of numeracy. The theoretical model highlights core mathematical content knowledge for proportional reasoning whilst simultaneously serving to assess the richness of numeracy practices. Through a targeted approach to numeracy from the basis of promoting proportional reasoning, data presented here suggest that rich numeracy practices can be enacted in the classroom in all learning areas.

Acknowledgement

The research reported in this paper was funded by the Australian Research Council, in partnership with the Department of Education and Child Development, South Australia,

and individual schools from Education Queensland. The views expressed here are not necessarily the views of these bodies.

References

Australian Association of Mathematics Teachers Inc. (AAMT). (1997). *Numeracy = Everyone's business. Report of the Numeracy Education Strategy Development Conference*. Adelaide: AAMT.

Ahl, V. A., Moore, C. F., & Dixon, J. A. (1992). Development of intuitive and numerical proportional reasoning. *Cognitive Development*, *7*, 81-108.

American Association for the Advancement of Science (AAAS). (2001). *Atlas of Science Literacy: Project 2061*. AAAS.

Behr, M. J., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio, and proportion. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learnin*g (pp. 296-333). New York: Macmillan.

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology*, *44*(5), 1478-1490.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, *32*(1), 9-13.

English, L. D., & Halford, G. S. (1995). *Mathematics education: Models and processes*. Mahwah, NJ: Lawrence Erlbaum Associates.

Geiger, V., Goos, M., & Dole, S. (2014). The role of digital technologies in numeracy teaching and learning. *International Journal of Science and Mathematics Education*. DOI 10.1007/s 10763-014-9530-4.

Goos, M. (2007).*Developing numeracy in the learning areas (middle years)*. Keynote address delivered at the South Australian Literacy and Numeracy Expo, Adelaide.

Goos, M., Dole, S. & Geiger, V. (2011). Improving numeracy education in rural schools: A professional development approach. *Mathematics Education Research Journal*, *23*(2), 129-148.

Goos, M., Geiger, V., & Dole, S. (2013). Designing rich numeracy tasks. In C. Margolinas (Ed.), *Task Design in Mathematics Education* (Proceedings of the 22nd annual conference of the International Commission on Mathematical Instruction, Oxford, pp. 589-598). ICME.

Goos, M., Geiger, V., & Dole, S. (2010). Auditing the Numeracy Demands of the Middle Years Curriculum. In L. Sparrow, B. Kissane & C. Hurst (Eds.), *Shaping the Future of Mathematics Education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia (pp. 210-217). Fremantle, Australia: MERGA.

Goos, M., Geiger, V., & Dole, S. (2011). Teachers' personal conceptions of numeracy. In B. Ubuz (Ed.), *Proceedings of the 35th conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 457-464). Ankara, Turkey: PME.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learnin*g (pp. 629-668). Charlotte, NC: Information Age Publishing.

Lamon, S. J. (2005). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum.

Lanius, C. S., & Williams, S. E. (2003). Proportionality: A unifying theme for the middle grades. *Mathematics Teaching in the Middle School*, *8*(8), 392-396.

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics* (3rd ed.). Thousand Oaks, CA: Corwin.

Sowder, J., Armstrong, B., Lamon, S. J., Simon, M., Sowder, L., & Thompson, A. (1998). Educating teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education*, *1*, 127-155.

Steen, L. (2001). The case for quantitative literacy. In L. Steen (Ed.), *Mathematics and democracy: The case for quantitative literacy* (pp. 1-22). Princeton, NJ: National Council on Education and the Disciplines.

Stoessiger, R. (2002). An introduction to critical numeracy. *Australian Mathematics* Teacher, *58*(4), 17-20.

Yetkiner, Z. E., & Capraro, M. M. (2009). *Research summary: Teaching fractions in middle grades mathematics*. Retrieved from http://www.amle.org/TabId/270/ ArtMID/888/ ArticleID/326/Research-Summary-Teaching-Fractions-in-Middle-Grades-Mathematics.aspx

A Case Study of the Pedagogical Tensions in Teacher's Questioning Practices When Implementing Reform-Based Mathematics Curriculum in China

Lianchun Dong
Monash University
<donglianchun@gmail.com>

Wee Tiong Seah
Monash University
<weetiong.Seah@monash.edu >

David Clarke
The University of Melbourne
<d.clarke@unimelb.edu.au >

This study examines a teacher's questioning strategies in mathematics classrooms in China when implementing reform-based mathematics curriculum. It explores teacher's strategies to deal with the tensions involved in the creation of opportunities for students to express and communicate mathematics ideas while ensuring the productivity of mathematics communication and the accomplishment of the lesson goals in a limited period of time. By doing so, this study has implications for teacher education and professional development in terms of how to strengthen the links between intended mathematics curriculum reforms and teacher's actual practices in mathematics classrooms.

Introduction

In current mathematics curriculum reform movement, not only the mathematics knowledge need to be upgraded to the most fundamental and useful in today's world, but also the pedagogical principles and should be improved so as to support the implementation of reform-based curriculum (Sullivan et al., 2013).

As an important pedagogical strategy in delivering mathematics curriculum, to provide students with sufficient opportunities in classroom interaction and communication has been well accepted by most nations. When implementing mathematics curriculum, teachers are encouraged to effectively use questioning strategies in classrooms to elicit students' mathematical ideas and to scaffold students' construction of mathematics knowledge. Although the use of questions in mathematics classrooms is not new for most nations, the effectiveness of this strategy has been challenging (Boaler & Brodie, 2004). This is not only because that question asking per se is a sophisticated art ((Boaler & Brodie, 2004), but also because that there are pedagogical tensions involved in teachers' strategies regarding the creation of opportunities for students to express and communicate mathematics ideas while ensuring the productivity of mathematics communication and the accomplishment of the lesson goals in a limited period of time (Sherin, 2002).

This study intends to investigate the ways in which a secondary school teacher employed questioning practices when implementing the reform-based curriculum. It explores the teacher's strategies to create opportunities for students to express and communicate mathematics ideas while ensuring the productivity of mathematics communication and the accomplishment of the lesson goals. By doing so, this study has implications for teacher education and professional development in terms of how to strengthen the links between intended mathematics curriculum reforms and teachers' actual practices in mathematics classrooms. Given that the challenges in employing effective questioning strategies are also experienced by mathematics teachers worldwide (Kosko, Rougee & Herbst, 2014), this case analysis of a Chinese teacher could also provide some implications for teachers in other countries to improve their instructional practices.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 197–204. Sunshine Coast: MERGA.

Methodology

A case study design was adopted in the present study so as to undertake a detailed analysis of mathematics lessons delivered by the participating teacher. Since this study aims to reveal the detailed and in-depth features of teacher questioning practices in mathematics classrooms, there is a need to utilise a case study design which could provide tools for researchers to explore complex phenomena within their contexts (Baxter & Jack, 2008).

Meanwhile, the IRF (Initiation-Response–Follow up) framework was utilised to analyse the teacher's discourse process of initiating questions and building up on student responses. Classroom lessons could be interpreted as a process of alternations between verbal and nonverbal behaviour that are jointly created by teachers and students and these alternations are characterised by interactional sequences of three interconnected parts: teacher initiation, student response and teacher follow up or IRF (Cazden, 2001). While the IRF structure has been criticised as limiting the potential of teacher-student dialogue in promoting students' conceptual learning in mathematics classrooms (i.e., Kyriacou & Issitt, 2007), more and more researchers have pointed out the IRF pattern includes more possible variations that could fulfil a diverse range of pedagogical purposes (Drageset, 2014, Franke, et al., 2009)

Considering the above analysis, the IRF structure in the classrooms was identified and then teacher questioning practices were examined within the IRF structure. It intended to develop a comprehensive framework with regard to teacher questioning and thereby to analyse what kinds of verbal questions were initiated by the teachers to elicit mathematical information and in what ways the teachers took students' verbal contributions into consideration so as to facilitate students' construction, acquisition and articulation of mathematical knowledge.

Setting and Participants

Data were drawn from video-recorded observations of one Chinese mathematics teachers' lessons in junior secondary level. The language of instruction is Mandarin Chinese. The participant is from the city of Nantong, Jiangsu Province in southeastern China and he is recognised as competent according to local criteria. As the teacher intended to implement the reform-based curriculum by providing more opportunities with students to express and communicate mathematics in classrooms, group learning was introduced into the classroom and the self-learning guide was also used. It includes three main sections in the self-learning guide, namely the review of mathematics knowledge relevant to the new topic, the construction or exploration of the new mathematics knowledge by problem solving, and the reflection and summary. In each section, the students are provided with some questions or tasks.

One day before a particular lesson, the teacher passed out the self-learning guide to every student, asking them to learn the new topic on their own and then to accomplish the tasks in the self-learning guide independently. On the next day, the students handed in the self-learning guide to be corrected by the teacher, who would leave written feedback in detail and then pass out the corrected self-learning guide to students before the lesson. It is worthwhile to point out that the teacher's feedback is not just simplistic evaluation of students' answers, but some detailed comments which could help students to reconsider their answers, encourage students to make connections with some previous mathematical knowledge, or challenge students to think more deeply.

For each lesson, the teacher had established a very regular structure, which consisted of four distinct parts. Firstly, at the beginning of the lesson, the teacher asked students to exchange ideas on the tasks in the self-learning guide in groups, as well as on the answers to the tasks. Secondly, after discussion and exchange in groups, one group was selected by the teacher to present in public the unanimously agreed ideas they had achieved on how to solve the tasks and each member of this selected group was responsible for one part of the whole group's presentation. Thirdly, after each member had accomplished his/her part of the presentation, other students were encouraged to give comments and ask questions. The teacher would generally get involved in this part and direct the public discussion. Fourthly, when the whole group had completed the presentation, the teacher always gave a lecture to sum up the presentation and discussion, as well as the main mathematical points in this lesson. When the data were collected, the teacher had been teaching his class in this way for around two years. A whole unit of consecutive lessons was collected and the analysis of the first three lessons is presented in this study. The details of the lessons are listed in Table 1.

Table 1
Lesson Topics Delivered by the Participating Teacher

Teacher	Year level	Lesson content	Time
		Lesson1 An introduction to quadratic functions	45mins
CHN	8	*Lesson2* Investigating the graph of $y=ax^2$	45mins
		Lesson3 Investigating the graph $y=a(x-h)^2+k$	45mins

Data Analysis

The term "question" refers to what the teacher says to elicit students' verbal responses related to mathematical content. Questions that were not mathematical were excluded unless they were associated with other mathematical "talk". Questions immediately repeated using the same wording was counted only once.

Three types of occasions when the teacher interacted with students by using questions were identified initially. When the student/s replied to the teacher's questions and the teacher did not respond the interactions were categorised as Question-Answer (Q&A) pairs. IRF (Teacher initiation-Student response-Teacher follow up) sequences (Cazden, 2001) were those where the teacher responded to students' answers that were triggered by the teacher's previous question. There are two types of IRF sequences: (1) IRF (single) in which the teacher asks a question and then gives a closed follow-up move (such as evaluation) to students so as to complete the current discussion, and (2) IRF (multiple) in which the teacher asks a question and then gives an open follow-up move (such as clarification or elaboration) to students so as to continue the current discussion. The episodes of Q&A pairs, the sequences of IRF (single) and IRF (multiple) were transcribed prior to the analysis.

When analysing the teacher's questions, a distinction was highlighted between initiation questions and follow-up questions. Initiation questions are those questions asked by the teacher for initiating purposes, such as to start conversation or discussion. In contrast, follow-up questions are those questions asked for the purposes of following up, such as in response to students' answers or contributions to the teacher's previous questions. In this study, the Q&A pair contains teacher initiation questions and student responses and the IRF sequence includes the teacher initiation question, the student response, and the teacher follow-up question.

A coding system was developed to categorise the initiation questions and follow-up questions. Instead of inventing the name of each category in advance, those questions documented in our data were analysed first and then attempts were made to provide names to describe these different kinds of questions. The coding systems are presented in Table 2 and Table 3 where examples are shown in italics. Abbreviations for these categories are also provided.

Table 2
Sub-categories for Initiation Questions

Category	Description and Example
Understanding check (UND)	Questions used to check whether students can follow the teacher. "Is everyone OK with how I get from the 2nd line to the 3rd line?"
Evaluation (EVA)	Questioning requiring students' comments. "Now let's look at these two descriptions, which one do you agree with?"
Review (REV)	Questions used to elicit the previously learnt or mentioned mathematics knowledge. "Now what do I know about squares and their area?"
Information extraction (INF)	Questions requiring students to identify and select information from text descriptions, graphs, tables, or diagrams. "What is (b), what's the mathematical word for what (b) is asking you to find?"
Link/ application (LIN)	Questions requiring students to provide examples or application of mathematical knowledge. "Could you list some examples?"
Result/product(R ESL)	Questions requiring results of mathematical operations or the final answer of the problem solving. "What is the square root of 80?"
Strategy/ procedure (STR)	Questions used to elicit the procedures or strategies of problem solving. "How can we solve this problem?"
Explanation (EXP)	Questions requiring students to provide explanations. "How would it be interpreted from the perspective of a function?"
Comparison (COM)	Questions requiring the comparison. "Is this different from the previous questions?"
Reflection (REC)	Questions requiring the reflection after mathematical activities. "What mathematics have we already used in solving triangles?"
Variation (VAR)	Questions requiring students to consider the variations of mathematical tasks. "So what if I got a hundred and twenty seven in that answer?"

The development of the coding system in this study was informed by coding systems proposed by some previous researchers (Boaler & Brodie, 2004; Hiebert & Wearne, 1993). Some categories' names were borrowed from these studies, but because the distinction between initiation questions and follow-up questions was considered in this study, some new categories of questions were also identified and labelled. A test-retest method was used to check the reliability of the coding systems and the elapsed time between the first and second coding was two months.

Findings

The coding systems presented above were used to analyse the selected lessons taught by the participating teacher. In total, 121 initiation questions and 116 questions were asked by the Chinese teacher in the three lessons which cover 135 minutes altogether. On average, the Chinese teacher raised approximately 1.8 (237/135) questions in every minute and for every initiation question, the Chinese teacher used approximately one (116/121) follow-up question.

Table 3
Sub-categories for Follow-up Questions

Category	Description & Example
Clarification (CLA)	Questions requiring a student to show more details about his/her answers or solutions. "How did you get this 16?"
Justification (JUS)	Questions requiring students to justify their answers "Why did you choose this method to solve this problem?"
Elaboration (ELA)	Questions requiring for additional information especially when the students fail to fully achieve the teacher's goals. "In other words, the green line becomes the what?"
Extension (EXT)	Questions used to extend the topics under discussion to other situations or to connect the knowledge under discussion with the previous knowledge. "Would this work with other numbers?"
Supplement (SUP)	Questions used to request for supplement. "Did anyone do this problem in a different way?"
Cueing (CUE)	Questions used to direct students to focus on key elements or aspects of the situation in order to enable problem-solving. "What is the problem asking you to find?"
Refocusing (REC)	Questions used to guides students to refocus on the key points, especially when students are off track. "But what was the question, if this was a textbook question, what would it look like?"
Repeat/ rephrase (REP)	The teacher repeats or rephrases the question asked in the last turn.
Agreement request (AGG)	Questions used to check whether the rest of the class agrees with the student who gives the answer. "So would you agree that the height of this one is going to be a hundred and forty nine?"

The detailed information in terms of the breakdown of initiation questions and follow-up questions is shown separately in Figure 1, Figure 2 and Figure 3. For the abbreviations in these figures, please refer to Table 1 and Table 2.

Figure 1 shows the proportion of each type of initiation question that was asked in the three lessons and this outlines the teacher's initial purposes when asking initiation questions. As is shown in Figure 1, although 11 types of initiation questions were identified in the three lessons, several types of initiation questions are predominant in each lesson. For lesson 1, the teacher's initiation questions were mainly asked for understanding check (UND), review (REV), and explanation request (EXP).

For What Initiating Purposes did the Teacher Ask a Question?

There are more variations in Lesson 2 where the teacher asked initiation questions for understanding check (UND), review (REV), explanation request (EXP), evaluative comments request (EVA), and reflection request (REF). Two types of initiation questions, namely review (REV) and explanation request (EXP), take up more than 60 percent of all the initiation questions asked in Lesson 2. Among all three lessons, questions for review (REV) and explanation request (EXP) were the two common types of initiation questions with significant proportions. Apart from these two common types, questions were also asked for understanding check (UND), evaluative comments request (EVA), and reflection request (REF) rather frequently, even though these types did not turn up with a significant proportion in every lesson.

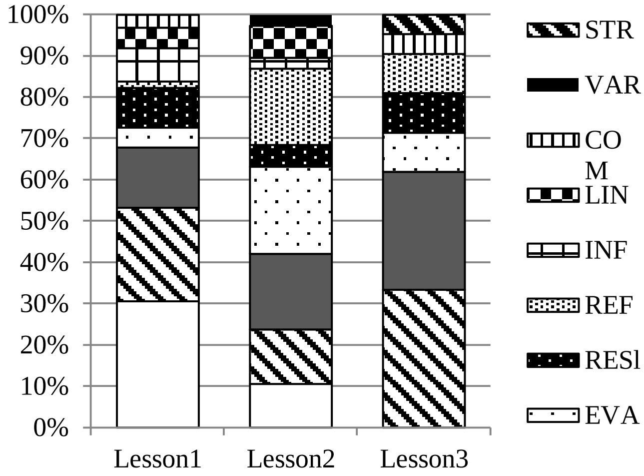


Figure 1. The breakdown of the teacher's initiation questions
Note: Refer to Table 2 for the abbreviations.

Theoretically, all these types have the potential of allowing students to express mathematics except the questions for understanding check (UND) which usually request a yes or no answer. In particular, explanation requests (EXP), evaluative comments requests (EVA), and reflection requests (REF) are more likely to elicit students' mathematics ideas on the basis of which the teacher could thereby provide facilitation and request elaboration. In this way, mathematics communication could occur between the teacher and students. However, it would depend on this teacher's strategies whether the students' responses could be used to build up mathematics communication. To investigate how the teacher dealt with the students' responses is examined in next part as well as the extent to which the teacher built on students' responses after the asking initiation questions.

Figure 2 presents the proportion of initiation questions asked on three types of occasions and it shows to what extent the teacher's initiation questions lead to the sequences of teacher-student mathematics communication. Five types of initiation questions (EVA, EXP, COM, REF, and LIN) were asked with higher chances of leading to IRF (multiple) in which the teacher tended to build on students' responses and therefore create more opportunities for students to communicate mathematics. In Figure 1, it was shown that questions for review (REV) and explanation request (EXP) are the two common types of initiation questions with significant proportions.

To What Extent Did the Teacher Build on Students' Responses?

Figure 2 reveals almost 90 percent of the questions for review were asked by the teacher without giving follow-up moves that could lead to the sequences of mathematics communication. In other words, when the teacher asked initiation questions for review, instead of having opportunities to communicate mathematics in discourse sequences, the students normally just need to respond with answers to the questions. In contrast, the initiation questions for explanation requests were mostly asked by the teacher with the following support through which the students could communicate mathematics. As is shown in Figure 2, around 85 percent of questions for explanation requests were asked within the IRF (multiple) structure in which the teacher tended to give open follow-up moves after initiation questions to students so as to continue the current discussion. The detailed approaches in terms of the follow-up moves used by the teacher to facilitate students to communicate mathematics are presented in Figure 3.

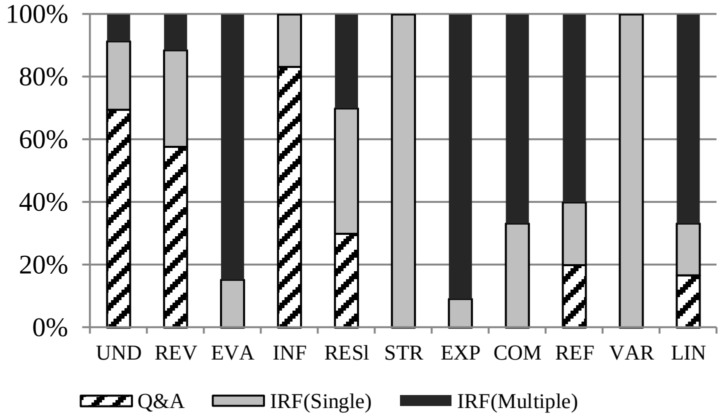


Figure 2. The teacher's initiation questions on three occasions
Note: Refer to Table 2 for the abbreviations

In What Ways Did the Teacher Build on Students' Responses?

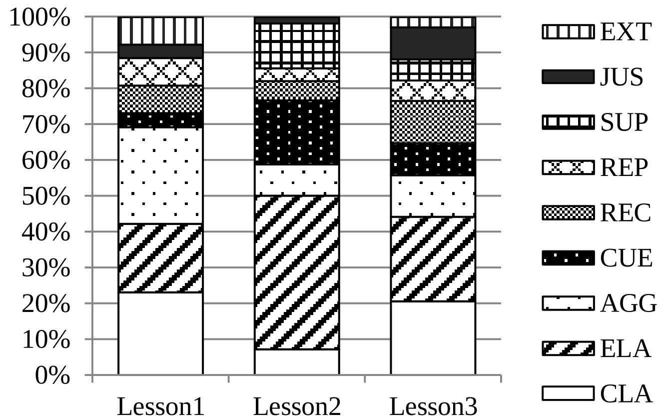


Figure 3. The breakdown of the teacher's follow-up questions
Note: Refer to Table 3 for the abbreviations

Figure 3 shows the proportion of follow-up question types employed by the teacher in the three lessons. Nine types of follow-up questions were identified in the three lessons and once again the majority of the follow-up questions consist of several types of question. In Lesson 1, the follow-up questions were mainly asked for clarification (CLA), elaboration (ELA) and agreement requests (AGG). For Lesson 2, the teacher posed follow-up questions mainly for elaboration (ELA), giving cues (CUE), and supplement requests (SUP). And questions for clarification (CLA), elaboration (ELA), agreement requests (AGG) and refocusing (REC) constitute the main body of follow-up questions in Lesson 3. Among the three lessons, the teacher tended to choose elaboration questions as a tool to facilitate students' expression and communication of mathematics ideas.

Conclusion

Compared with the traditional mathematics curriculum, the reform-based mathematics curriculum requires teachers to provide students with more opportunities to communicate mathematics. But it brings pedagogical tensions because teachers also need to control the flow of the communication to ensure the productivity of mathematics communication.

The participating teacher adopted a new way of delivering mathematics lessons by introducing the self-learning guide. The students had attempted to solve the tasks in the self-learning guide prior to the lessons, thus in the classroom the teacher and students had sufficient time to talk deeply about the corresponding strategies to solve these tasks. Meanwhile, students' discussion on these tasks also became the springboard on which the participating teacher could ask further questions to elicit students' deeper thinking and promote students' construction of mathematics knowledge. All of these made contributions to the creation of a discourse-rich classroom.

This study separately examined initiation questions and follow-up questions asked by the teacher. By exploring the proportion of these questions and the context in which these questions were used, it is possible to analyse the actual opportunities provided for the students to communicate mathematics. This helped understand more clearly the nature of the discourse-rich classroom. This study showed that a considerable proportion of elicitation and facilitation were used by the teacher to promote the communication and construction of mathematics. But it also reflected some suppression of learning when the teacher attempted to fulfil the lesson goals.

This has implications for mathematics teacher education in terms of supporting the implementation of intended mathematics curriculum. For one thing, the reform in terms of learning materials, such as the introduction of self-learning guide, could assist the shift from traditional classroom into inquiry-based classroom. The design of the tasks in the self-learning guide could help to ensure the depth and productivity of mathematics discussion and communication in classroom. Also, more assistance is needed to help the teacher use the self-learning guide more efficiently.

References

Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The qualitative report*, *13*(4), 544-559.

Boaler, J., & Brodie, K. (2004). The importance, nature and impact of teacher questions. *Proceedings of the 26th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* Vol. 2, pp. 773-781.

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education*, *27*(1), 174-186.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning*. Portsmouth, NH: Heinemann.

Drageset,O. (2014). How Students Explain and Teachers Respond. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia pp. 191–198)Sydney: MERGA

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Hiebert, J. & Wearne, D. (1993).Instructional Tasks, Classroom Discourse, and Students' Learning in Second-Grade Arithmetic, *American Educational Research Journal, 30*(2), 393-425

Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom? *Mathematics Education Research Journal*, *26*(3), 1-18

Kyriacou, C., & Issitt, J. (2007). Teacher-pupil dialogue in mathematics lessons. *BSRLM Proceedings*, 61-65.

Sullivan, P., Jorgensen, R., Boaler, J., & Lerman, S. (2013). Transposing reform pedagogy into new contexts: complex instruction in remote Australia.*Mathematics Education Research Journal*, *25*(1), 173-184.

Sherin, M.G. (2002). A balancing act: developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education*, *5*(3), 205–33

Improving the Effectiveness of Mathematics Teaching through Active Reflection

Kerryn Driscoll
Clifton Springs Primary School
<driscoll.kerryn.k@edumail.vic.gov.au >

A small study of active reflection was undertaken with 21 primary students in a Prep and Year 1 classroom. To provide feedback from the students on their views about their personal learning and ways they could be better supported to learn mathematics a simple survey was supplemented by one-to-one interviews. Students' perceptions of their learning of mathematics, their identification of the mathematics they were learning; and what they felt would better cater for their learning needs. Ongoing reflections by students and the teacher throughout the year provided further data for consideration. The findings show that young students are self-aware, they understand what they are learning and they have ideas about ways their teacher could support their learning. These findings fed into the teacher-reflection process and professional learning, which in turn led to some changes in classroom practice.

Introduction

To improve my practice as a teacher, I became involved in a Teacher-Led Research project. This research was underpinned by the *evidence-based professional learning cycle* (Timperley, 2008) where teachers measured the impact of their professional learning on their practice. My aim was to inquire more deeply into my practice in an attempt to understand how to improve student learning. This paper reports the results of an aspect of the work that we were doing as a school community involving active reflection, based on feedback from our students.

My research question was: What do I need to do differently in my teaching practice to address the perceived needs of my students in mathematics? In the words of Hattie (2012), "my role, as a teacher, is to evaluate the effect I have on my students … it is to understand this impact, and it is to act on this knowing and understanding" (p. 19). This requires gathering "defensible and dependable evidence from many sources" (p. 19). The evidence gathered included results of a survey, interviews and classroom observations.

Background

Reflective teacher practice is a topic that has long been explored by researchers (Shulman, 1986; Shulman, 1999). It is suggested that teachers learn "via disciplined critical reflections of their own practice" (Shulman & Shulman, 2004, p. 258). Shulman pointed out the importance of learning from our experiences and suggests that "critical analysis of one's own practice and critical examination of how well students have responded are central elements of any teaching model" and "at the heart of that learning is the process of critical reflection" (Shulman & Shulman, 2004, p. 263-264). Accomplished teachers are capable of learning from their own and others experiences through active reflection according to Shulman.

Absolum also used the term 'active reflection' and suggested that this applies to the teacher, to the student, and to both taking time together "to think, review and enhance the learning process" (2006, p, 23). Through a learning-focused relationship with students "the student focuses on what has to happen *now* that will best help me learn?" and "the teacher

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 205–212. Sunshine Coast: MERGA.

focuses on what do I need to do *now* to best help this student learn?" (Absolum, 2006, p. 28) Teachers are, as a profession, very reflective considering whether their students are learning as well as they could and what adjustments they might make to their teaching to support the learning even further (Absolum, 2006). Teachers constantly make decisions based on reflection about class management, activities and resources and their students. A major OECD study points out that, "individuals who are reflective also follow up such thought processes with practice or action" (OECD, 2005, as cited in Absolum, p. 143).

Through reviewing and reflecting, problems can be identified and something can be done to resolve the situation. If teachers are to establish a firm foundation for improved student outcomes, they need to "integrate their knowledge about the curriculum, and about how to teach it effectively and how to assess whether students have learned it" (Timperley, 2008, p. 11). Teachers can then respond constructively to what the evidence is telling them and make the necessary changes to their practice, and as they "take more responsibility, and as they discover that their new professional knowledge and practice are having a positive impact on their students, they begin to feel more effective as teachers" (Timperley, p. 9, 2008).

My purpose in collecting the data reported here was to consider ways in which I could further support students learning of mathematics, thereby becoming more effective. I was able to do this through my own reflection on the teaching and learning process, the adjustments I made, and how I taught my students "to use reflective strategies to strengthen their own capacity to learn" (Absolum, 2006, p. 143).

Methodology

Three data sources were collected and examined for this research: an initial student survey determined student needs, feelings and beliefs about their learning in mathematics, a five minute video of a semi-structured student interview with a subgroup of 7 students, and regular daily student reflections following mathematics lessons. The daily reflections were continued throughout the year. These daily comments by the students enabled me to reflect on the teaching and learning in my classroom and to consider what I needed to do to better support the learning and what adjustments I needed to make to suit individual needs.

Survey Design

Data collection began with a survey to gauge how students felt about learning mathematics. It was administered early in the school year to one class of Prep/Year 1 students, 11 students in their first year at school (Prep) and 10 students in their second year at school (Year 1). There were 10 male and 11 female students. The purpose of the survey was to establish how students felt about their learning in mathematics; whether they could identify the mathematics they were learning and whether they felt I, as their teacher, could better cater for their learning needs. The survey was designed to be easy for younger students to interpret and quick to complete. I read each question aloud, students completed a Likert scale by colouring one of four faces ranging from smiley to sad (see Table 1). A scale of four options was chosen to prevent students who were unsure selecting the middle option as a default.

Video Interviews

The week following the survey, seven students were selected as a subgroup, representing a range of mathematical thinking and experiences across the year levels, for video interview, to elaborate their written responses. Survey questions were revisited and additional questions were included during semi-structured interviews. Examples of additional questions were, '*Do you feel like you can extend yourself*' and '*What do you do when you have tried something and it doesn't really work?*'

Table 1
Collated survey results (n= 21)

Class Survey Colour the face that shows how you feel.				
1. How do you feel about learning mathematics?	15	3	0	3
2. Do you always know what you are learning?	9	8	2	2
3. Do you know when you are successful?	10	6	4	1
4. Does your teacher help you to learn?	14	4	1	2
5. How do you feel when something is difficult?	8	4	2	7
6. How do you feel in our classroom?	17	2	0	2

Table 2
Interview group results (n = 7)

Distribution of responses of subgroup of students. Colour the face that shows how you feel.				
1. How do you feel about learning mathematics?	4	2	0	1
2. Do you always know what you are learning?	3	3	1	0
3. Do you know when you are successful?	5	2	0	0
4. Does your teacher help you to learn?	4	2	1	0
5. How do you feel when something is difficult?	2	2	1	2
6. How do you feel in our classroom?	4	2	0	1

Student Reflections

Classroom-based student observation data were videorecorded two to three times weekly throughout the year, during or at the conclusion of the mathematics lesson, as students reflected on their learning. In total 120 videos were collected. These videos were viewed, some were transcribed and all were analysed to determine patterns and similarities. This provided valuable feedback and evidence that facilitated my reflection and gave me direction on how I could better cater for student learning needs. Students were deliberately introduced to and provided with a variety of prompts to support this. Some examples of prompts included:

What am I most pleased with in my learning today?
What did I find tricky and why? What did I do about it?
Where do I need to go next with my learning?
How could the teacher have helped you more?

Findings

Survey responses and elaborated interview responses gave a sense of students' perceptions about learning mathematics and insight into individual students' feelings and attitudes at the beginning of the school year, while classroom observations of student reflections provided feedback that assisted my reflection and continued throughout the year.

As evidenced in Table 1 most students surveyed enjoyed learning mathematics. The three students who were unhappy were often provided with extra support in the mathematics classroom. Seventeen (80%) students felt that they knew what they were learning and when they had achieved success. Eighteen students (85%) felt that the teacher helped them to learn. Nine (43%) students felt neutral or sad when they encountered difficulty. It is interesting to note that 4 (19%) students felt neutral when they encountered difficulties and 8 (38%) felt positive. Results from the subgroup of students are presented in Table 2 for comparison. The summary shows similar distribution patterns to the whole class results.

Analysis of the interviews provided insights as the students elaborated their survey responses. It became clear that the seven students in the subgroup had a sense of where they saw their learning in mathematics going and how I as their teacher could help them. For example, Ben said, “You could help me learn higher numbers and then I could try and plus them.” Nina suggested, “Maybe you could just tell me what the whole thing was again because sometimes I forget.” All seven students agreed that they knew what they were learning by referring to the whiteboard or being a good listener. Literature supports (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, & Rowley, 2002) the idea that if a teacher has a clear focus and if the children understand the purpose of the lesson it is likely to lead to a better understanding and better learning outcomes.

All of the subgroup of students knew that it was important to learn mathematics and felt happy that they could extend themselves through prompts offered. For example, Zena said, “If we are choosing numbers, sometimes we choose an easier number, or a bigger number to challenge ourselves.” Four of the seven students preferred to work with a partner and all students expressed the need to persist when things get tough. Some of their comments included, “I just try again” or “I just have a go of doing it” or “I try my own way.” One student’s responses are highlighted below.

The Story of Eric

Eric (aged 6 years) was selected for interview as one of a subgroup. He began the year with reluctance and had difficulty settling into the school routine. The fact that he was very nervous and hesitant when sharing his thoughts with the class, made his story even more interesting. Eric had a limited understanding of number and could not count past 13. However, during his interview Eric expressed a liking for mathematics, particularly learning how to count and working with a partner. He felt that his learning was progressing well because he listened a lot and knew what I was saying. Eric knew that if he had “done a lot of work” he was successful. He knew that you need to learn mathematics because “if you don’t learn maths you won’t be smart” and he thought that I could help him more by telling him what to do when he “got the stuff mixed up.” When asked, “What do you do when you have tried something and it doesn’t really work?” He replied, “I look at the page what (sic) I’ve done and it hasn’t really like been all done. I fix it up and try and remember what we did.” In other words Eric would try to evaluate his thinking by interpreting what he had done, review it and then think back to what was expected of him, and try and complete the task. This reflection process demonstrates some interesting thinking for a young student and was one of the many pieces of evidence that I reflected on.

The third source of evidence were the daily reflections by the students based on how they saw their learning in mathematics was progressing and how they felt their learning could be improved. On each occasion students responded to one of the reflective question prompts. This daily feedback led me to act to better support my students. One question I used weekly is, "*How could the teacher have helped you more?*" This is the underpinning question of the project.

As I reflected on the student's comments some interesting patterns and similarities arose and seven themes emerged from the data. These elements contributed to and continued to inform decisions I made as to how to be a more effective teacher. Each theme will be discussed in the section that follows.

Provide a Variety of Prompts, Models, Materials and Supports. Feedback suggested that several students were lacking in confidence and needed support, not only when writing numbers or words when recording their findings, but also with skills. Nathan wanted help working out how much his numbers made altogether and Nina wanted help counting, "not by ones but by a quicker number." These types of messages informed my future planning and practice. A variety of support was implemented through coaching, scaffolding, guiding and providing a range of mathematical models and tools.

Use a Variety of Assessment Tools. The difficulty that some students found with recording their mathematical thinking in writing indicated that alternate means of capturing evidence of mathematical growth and achievement were necessary, especially for students in the early years of school. As Eric expressed in his way, "If you are writing the words and if you can't remember, it's about holding it and if you are writing the number and you can't really write it, it's hard because you are doing two things at the same time." Therefore in addition to students written work samples, assessment evidence was captured on video where students demonstrated their skills. Formal assessments such as the Mathematics Online Interviews were also used. Each of these provided powerful evidence and gave a very clear picture of mathematical knowledge and growth.

Maintain High Expectations of Students. Some students expressed the need for more of a challenge and needed to be provided with further opportunities that allowed them to "gather, discover and create mathematical knowledge and skills" (Siemon, Beswick, Brady, Clark, Faragher &Warren, 2011, p. 14). Ben desperately wanted to work with bigger numbers and Eric wanted to learn "to count to numbers like the grade sixes count up to, like one thousand." These responses reinforced the fact that high expectations for all students, even those who experience difficulty, are important.

Choose Quality Tasks that Challenge and Allow for Differentiation. It was obvious that "for worthwhile learning in mathematics, students need mathematically appropriate, engaging and challenging tasks" (Roche, Clarke, Sullivan, & Cheeseman, 2013, p. 32) that encourage students to think deeply and problem solve. Eric suggested he would have liked to work with more three-digit numbers and not just two-digit numbers in subtraction. This comment reinforces the importance of choosing mathematical learning experiences, which can be differentiated, and "specifically plan to support students who need it and challenge those who are ready" (Sullivan, 2011, p. 27).

Encourage Persistence. According to Roche and Clarke (2014) the term 'persistence' can be described as "actions that include concentrating, applying themselves, believing that they can succeed, and making an effort to learn" (p. 3). Responses from these students indicated they were willing to persist. They demonstrated this in several ways. When Ben was faced with a struggle he commented that instead of asking for help "I tried to make the idea different and tried my own way." Encouraging persistence, promoting effort and allowing students to persist help them to see it as valued. A further example of this was demonstrated when Eric admitted to continuing to work, "While you said we had to come to the floor," because he desperately wanted to solve the problem.

Provide Clear Instructions, Highlighting the Focus and Expectations. Feedback from Zena suggested that I "could explain the work better" and Nina thought I could "say it two times so we know and we don't do the wrong thing." Keeping in mind that I purposefully "hold back from telling them everything" (McDonough, 2003, p. 34) and "setting up tasks with a certain amount of uncertainty in how they have to go about the task is a way to make them engage mindfully and bring their sense making to the activity" (Askew, 2012, p. 108). As a result of this feedback emphasis has been placed on more specific and engaging whole class introductions with interactions between the teacher and the whole class.

Create a Learning-Focused Relationship with Students. Fundamental to this project was the notion of a learning-focused relationship where "both the student and the teacher know that by working together the quality of student learning will be much better and the standard of achievement much higher" (Absolum, 2006, p. 28). I established a relationship with these students "based on openness, honesty and mutual respect" (Absolum, 2006, p. 142). Reflecting and acting on student responses in an environment where students felt supported in their learning emphasised that learning was valued in this classroom. Jack pointed out, "At the start of the year I was pretty scared to do it, but the more you do it the better you get," which indicates his increased confidence over the period of the project. Terrri summed it up by saying, "You have to co-operate and work together."

Discussion

This study demonstrates how active reflection by the teacher can enhance the learning process of students and teachers. While there are limitations of sample size and scope in these data, I would argue that it is possible to learn from the findings. Another constraint of the study may be that student interpretation differed to an adult view. However, the reflective processes and the student responses provided a starting point for me to reflect and act. As the teacher it was my responsibility to make changes to my practice in the light of the students' feedback and decide what I could do better. I was also challenged to discover any unproductive beliefs that I might hold in relation to effective conditions for learning.

Survey results at the beginning of the year indicated that 85% of students enjoyed learning mathematics, however 15% felt less positive and 10% of students felt unhappy in the classroom. This was one aspect that I attempted to change through careful planning, assessment and preparation of resources to support those students who needed it. Interestingly 15% of students felt I needed to do more to help them learn. This then led me to investigate student opinions in more depth.

Through semi-structured interviews students were encouraged to elaborate further on this, as well as other aspects. Several students suggested they would like their learning extended and could give examples of what it was they would like to be able to do, for example, learn how to count to 100 or learn how to count by 3s. Surprisingly when presented with a challenge they claimed they were not discouraged and were able to express several ways of over-coming this. Responses such as these provided direction and increased my awareness of how and what I needed to do to adapt to provide opportunities for these students to feel challenged, but at the same time supported. By offering enabling and extending prompts and posing tasks that created a challenge I was able to support those experiencing difficulty and extend those that needed it. One of my students, Eric shared the fact that he wanted to learn to count to 100 and through support such as this he was able to count past 100 by the end of the year, with confidence.

Reflecting on my practice prompted me to make small changes to practice, and observe resulting improvements in student outcomes. “When this happens, teachers come to expect more of their students—that they will learn more quickly or deeply than they had previously believed possible” (Timperley, p.18). Eric’s story is evidence of this. Following the initial interview there was a dramatic improvement in Eric’s confidence and his willingness to reflect and clearly articulate his feelings and achievements. He confidently self-assessed his work and was extremely motivated to achieve his goals. He could express where he needed to go next with his learning and was always eager to contribute to the class plenary, and frequently selected his own reflection question. His increased confidence and enthusiasm when he reflected on his learning was inspiring and rewarding to see. By the end of the year he had been assessed as operating six months above expected level. He could confidently count by 2s, and 5s and 1s and 10s past 100. He could read, record and order two and three digit numbers. As a result of attending to student feedback, careful consideration went into planning appropriate tasks and scaffolds such as labelled photos of classroom resources and displays of key words and numbers were used to support students like Eric to facilitate their effective learning.

Evidence was provided on how the teacher could cater better for student needs through opportunities for students to reflect on this and further learning became evident. Initially this took time to establish in the classroom. While I acknowledge that there is a concern that this is my classroom and these are my students, nevertheless the feedback that I received is worthwhile and substantially influenced my planning and reviewing of mathematics lessons. Students as young as six can reflect on their own thinking when asked to do so with familiar material (Schraw & Moshman, 1995).

The messages I received from my students as they reflected actively on their learning are important in relation to promoting effective teaching and learning. It is important that reflection by both the teacher and the student happen over the course of each lesson: whether at the beginning, throughout the lesson or at the conclusion. Some obvious frustrations that were experienced by some of the students in relation to recording, feeling suitably challenged, and obtaining clear instructions were highlighted, and as a result I made adjustments to my planning and teaching, responding to their feedback.

Through active reflection I was able to consider the successfulness of the lesson in terms of student engagement, appropriate challenge, strategies used by the students, and intended learning. I examined my practice in light of listening to my students.

> While there is a need for teachers to impart information … and while teachers do and should know more than students, there is a major need for teachers also to listen to the students’ learning. This listening can come from listening to their questions, their ideas, their struggles, their strategies of learning, their successes, their interactions with peers, their outputs, and their views about the teaching. (Hattie, 2012, p. 163)

Reflective questions can often reveal the complexity of what is happening in the classroom and give students the opportunity to be heard. Responses the children provided enabled me to see “learning through the eyes of students” (Hattie, p. 14) and make necessary adjustments to facilitate effective teaching in mathematics.

Conclusion

This present study sought to investigate ways in which active reflection could be used to create the opportunity for improved teacher practice in mathematics. Daily reflection enabled me to promote and sustain the conditions to further support student learning. Throughout this study I adapted my teaching practice taking into consideration ways in which I could support students who struggled with understanding what was expected of them, who had difficulty recording their findings, and who needed help or extension. These improvements resulted from feedback and reflections from students. I have incorporated these elements into my practice and will continue to incorporate them into my planning. Being actively reflective empowered students to see the value of reflecting and the impact it has on improving the teaching and learning in their classroom. Thinking about the concluding remarks for this paper made me aware that not only has my practice changed as a result of active reflection, and the students have seen the value of active reflection, but in addition I realise that the habit of mind involved in active reflection has caused a change in the way I think about teaching and learning. Constantly in the back of my mind every decision I make is now based on the question: How could I be more effective?

Acknowledgement

I wish to express my appreciation to Dr Jill Cheeseman for her comments on earlier drafts of this paper.

References

Absolum, M. (2006). *Clarity in the classroom, using formative assessment, building learning-focused relationships.*, Auckland, New Zealand:Hodder Education.

Askew, M. (2012). *Transforming primary mathematics*. London, United Kingdom: Routledge, Taylor & Francis Group.

Clarke, D. M., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., & Rowley, G. (2002). *Early Numeracy Research Project: Final report*, February 2002. Fitzroy, Victoria: Mathematics Teaching and Learning Centre, Australian Catholic University.

Hattie, J. (2012). *Visible learning for teachers maximizing impact on learning*. London and New York: Routledge.

McDonough, A. (2003). *Effective teachers of numeracy in early years and beyond* Paper presented at the 40th Annual Conference of the Mathematical Association of Victoria, Monash University. Retrieved from http://www.mav.edu.au/files/conferences/2003/synopsis2003.pdf

Roche, A., & Clarke, D. (2014). Teachers holding back from telling, A key to student persistence on challenging tasks. *Australian Primary Mathematics Classroom*, *19*(4), 3-8.

Roche, A., Clarke, D., Sullivan, P., & Cheeseman. J. (2013). Strategies for encouraging students to persist on challenging tasks: Some insights from work in classrooms. *Australian Primary Mathematics Classroom*, *18*(4), 27-32.

Schraw, G. M., & Moshman, D. (1995). Metacognitive theories, *Educational Psychology Review*, *7(*4), 351-371.

Shulman, L. S., & Shulman, J. H. (2004). How and what teachers learn: A shifting perspective. *Journal of Curriculum Studies, 36*(2), 257-271.

Shulman, L. (1999). Knowledge and teaching: Foundation of the new reform. In J. Leach & B. Moon (Eds.), *Learners and pedagogy* (pp. 61-77). Milton Keynes: Open University.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher,* 15(2), 4-14.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher R., & Warren, E. (2011). *Teaching mathematics foundations to middle years*, Oxford University Press.

Sullivan, P. (2011). *Teaching mathematics: Using research-informed strategies. Australian Education Review: no. 5*. ACER press Retrieved from http://research.acer.edu.au/aer/13

Timperley, H. (2008). *Teaching professional learning and development: Educational practices series-18*: http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_18.pdf

Promoting Teacher Growth through Lesson Study: A Culturally Embedded Approach

Marlon Ebaeguin
The University of Melbourne
<m.ebaeguin@student.unimelb.edu.au>

Lesson Study has captured the attention of many international educators with its promise of improved student learning and sustained teacher growth. Lesson Study, however, has cultural underpinnings that a simple transference model overlooks. A culturally embedded approach attends to the existing cultural orientations and values of host schools. This paper reports on the author's implementation of Lesson Study in a Philippine public school and the growth teachers experienced as a result of their participation.

With its long tradition, Lesson Study (LS) is the most prevalent practice-based form of teacher professional development in the Japanese primary schools. It is a school-based collaborative activity that involves a continuous cycle of planning, demonstrating, and improving a lesson (Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stephens, 2011). It is a good catalyst for teacher growth as it allows the teachers to interact with the curriculum, their own and colleagues' content and pedagogical content knowledge.

Over the past decade, there has been a vast interest on Lesson Study (LS) with many international educators implementing it in their local context. Though the author strongly believes that LS is a powerful tool for effecting teacher growth through understanding of student thinking, an uncritical transfer to a different national context may prove to be problematic. Teaching and learning are profoundly cultural activities (Stigler & Hiebert, 1999) that there certainly are aspects of LS that may not be readily embraced by the teachers in the importing context (Ebaeguin & Stephens, 2014).

Two Approaches of Implementation

There seem to be two approaches to the implementation of LS outside Japan–the fidelity approach and the culturally embedded approach. A fidelity approach to implementation means bringing LS to another context by demonstrating how it is done in Japanese schools and faithfully executing the same procedures with the local teachers. This approach makes several assumptions. First is that a simple transference or 'copy-paste' model works across cultural contexts. Second is that LS is a package of procedures, that may be taught to and learned by the teachers in a seminar/workshop, after which, the teachers are expected to have acquired the skills, to be able to participate in LS, and to be able to integrate it in their regular practice. Third assumption is that all teachers are capable of and open to changing their beliefs and practices to meet the requirements of LS. Finally, a fidelity approach assumes that the school structure and administrative support for LS present in Japanese schools are easily replicable in any context. Certainly, an implementation that is as faithful as possible to LS is desirable as this would assure realisation of the benefits of LS, but this, of course, is very ideal.

A fidelity approach fails to recognise that culture is expected to contribute to the forms of acceptable pedagogy, teacher-student interactions, classroom practices and teacher professional development programs (Ebaeguin & Stephens, 2014). Though it is possible, of course, to learn the procedures and to acquire the skills needed to participate in LS, the author believes that this would lead to something that is short-lived and without continuity

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 213–220. Sunshine Coast: MERGA.

like most sporadic professional development training teachers would have. This increases the chances of developing misconceptions on LS (Yoshida, 2012; Fujii, 2014). In addition, considering the high demand of work LS puts on the teachers, it can be expected that not all teachers would be very receptive to new ideas and practices and open to modifying their own beliefs and practices.

A culturally embedded approach to implementation assumes that there are aspects of LS that may not be transferrable to another context. It acknowledges that LS, having originated from Japan, has cultural underpinnings that explain its success in the Japanese school system and that these, however, may not necessarily be present in another context. It recognises that when LS moves into another culture, it is likely to change and be adjusted to fit the local context of the importing culture. In a study conducted by Dudley (2012), LS was regarded as a useful method for professional learning in England because of the culture of collaborative enquiry in the schools. However, the nature of research questions in LS, which is always based on the school's aims and values, conflicted with England's tradition of action research in which the research question varies from project to project. While retaining the elements of learning as a professional community, LS's purpose for them became a means to creating new practice knowledge (Dudley, 2012). A culturally embedded approach entails identifying which aspects of LS could be supported by existing practices and beliefs of the importing culture, and which ones may not be easily embraced or may need to be modified. The goal is not to turn teachers in the importing culture to be a Japanese teacher but rather come up with an adaptation of LS that would be easily supported by the teachers and the school system. This means a continuous and sustainable professional growth for the teachers. One obvious weakness of this approach, though, is that it will not guarantee realisation of the same benefits, in terms of level or quality, as with LS in Japan. As each school would have their own practices and beliefs within the same cultural context, the adaptations and speed of realising the growth would also vary. Despite this weakness, however, the author believes that a culturally embedded approach to implementation, aside from being more critical, promotes a more systemic professional development for teachers. Having said this, an important question to ask now is how, then, can LS change if it is to work well in another country?

Methodology

In a prior study, Ebaeguin and Stephens (2014) provided insights on how culture may contribute to the success of LS implementation in Japan. That study, they described and used Hofstede's dimensions of national culture—Power Distance Index (PDI), Individualism/Collectivism (IDV), Masculinity/Femininity (MAS), Uncertainty Avoidance Index (UAI) and Long-term Orientation (LTO). PDI pertains to hierarchy in the system which influences interaction between stakeholders and distribution of key roles, while IDV deals with propensity towards collaboration. MAS distinguishes between achievement and competitiveness or harmony and consensus. UAI relates to openness to change and innovation, while LTO is associated with having future-oriented or short-term perspectives (for a more detailed discussion on Hofstede's dimensions of national culture, please refer to Hofstede, Hofstede, & Minkov, 2010). Based on these dimensions, their instrument Values Survey Module for Teachers 2012 (VSMT12) was administered to their sample of Japanese teachers to identify orientations that support LS in Japan (see Figure 1). Their second instrument, the Mathematics Teachers Perceptions of a Good Mathematics Lesson (MTPGML), asked the sampled Japanese teachers to rate nine attributes of a good mathematics lessons implied by JLS, as Not Important (NI), Undecided (U), Important (I), Very Important (VI), or Essential (E). This showed the sampled Japanese teachers' endorsements of key aspects of planning a good mathematics lesson and was used to identify value orientations that would be conducive for LS implementation (see Table 1).

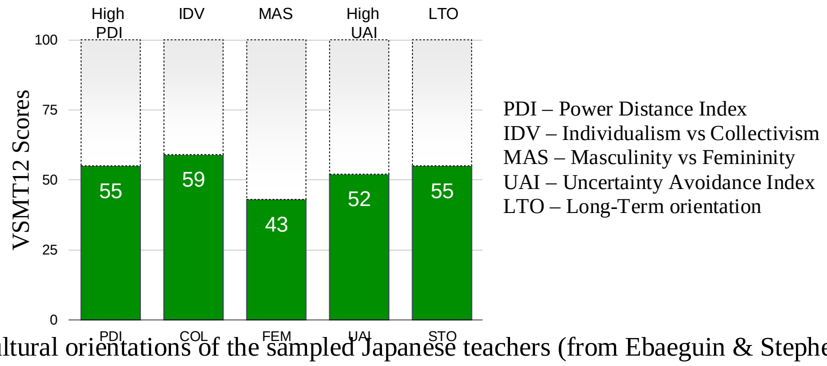


Figure 1. Cultural orientations of the sampled Japanese teachers (from Ebaeguin & Stephens, 2014)

Table 1
Sampled Japanese mathematics teachers' endorsement of some key aspects of planning a good mathematics lesson (from MTPGML, Ebaeguin & Stephens, 2014)

	Japan (%) n=16				
Items	NI	U	I	VI	E
1. Using/researching on curriculum materials (national curriculum, textbooks, course syllabus, scope and sequence, etc.) in planning out your lessons.	0	0	13	25	63
2. Working with other teachers to plan a lesson.	0	19	38	25	19
3. Having other teachers/colleagues in the classroom to observe my teaching.	0	0	25	38	38
4. Identifying in advance the range of expected student responses to the task including likely wrong responses in a problem-solving lesson.	0	0	0	25	75
5. Writing a detailed lesson plan incorporating the range of expected student responses.	0	6	31	31	31
6. Talking about and sharing successful mathematics lessons with colleagues.	0	0	44	44	13
7. *Relying on my own opinion as to whether a lesson has been successful or not.	0	44	50	6	0
8. Evaluating a lesson through analysing collected samples of students' solutions and attempted solutions.	0	0	19	31	50
9. Getting involved in school research.	0	6	6	19	69

Notes: Shading indicates combined percentages of *Very Important* (VI) and *Essential* (E) ≥ 50%.
* Lower values are important for this item (from data in Ebaeguin & Stephens, 2014)

The moderate orientations for each of Hofstede's dimensions (see Figure 1) can be expected to provide support for LS implementation. Table 1 highlights the consistency in the strong valuing of aspects of lesson planning across the sampled Japanese mathematics teachers. These cultural and value orientations provide an environment that is conducive for LS implementation (Ebaeguin & Stephens, 2014) in Japan. These orientations, however, cannot be assumed outside Japan.

.

This study replicates the methodology and instruments used in the aforementioned study. It focuses on a LS implementation in a public high school in the Philippines using a culturally embedded approach and the growth the participating mathematics teachers experienced. In this study, School A, a Philippine public high school was recruited. The school was chosen to maximise the participants in the school to meet the minimum requirement of fifty responses for one of the instruments to be administered. The author worked with eight participating mathematics teachers in the school, meeting them twice a month over a period of seven months; and implemented three cycles of LS. The small sample limited the analysis of the data to simple descriptive statistics.

There were two phases to the study. First phase involved administration of two questionnaires, VSMT12 and MTPGML. VSMT12 was administered to all the teachers in School A. The results were used to identify the teachers' existing cultural orientations which then informed the strategies employed to promote attitudes conducive for LS. For example, if the group appeared to be hierarchical, novice teachers may find themselves either unable to participate or assigned much of the work. The author needed to employ strategies that distribute the participation evenly amongst experienced and novice teachers.

The MTPGML questionnaire was administered only to the mathematics teachers. The results from this questionnaire showed the extent of the mathematics teachers' endorsements of key aspects of LS and were used to inform the focus of the training that was given to the teachers. For example, if most of the teachers rated working with other teachers to plan a lesson U or NI, more sessions that involve collaborating with colleagues in planning a lesson need to be provided. This questionnaire was again administered to the same teachers after the research intervention. Results from the pre- and post-intervention administration of this questionnaire were compared to identify what teacher growth occurred after their experience of the adapted LS.

The second phase of the study involved execution of the intervention program. An intensive workshop on LS was given to the participating mathematics teachers prior to the regular monthly meetings. The focus in these regular monthly meetings is based on the results of the pre-intervention administration of MTPGML. For example, if majority of the teachers rated anticipating student responses to be NI or U, activities that would require them to think like their students will be given. At the end of these monthly meetings, the teachers were also asked to write short reflections about the session. At the end of the intervention, when the teachers would have already gone through 2-3 cycles of LS, post-intervention administration of MTPGML and exit interviews were done. The author interviewed the participating teachers individually and asked them to talk about four core themes/tasks. First part asked the teachers to talk about how important professional development is for them and how their experience of LS helped them grow. In the second part, the teachers were shown their pre-LS and post-LS ratings in MTPGML. The teachers were asked to talk about the items where their endorsement changed and to give examples on how this affected their regular practice. The third part of the interview required the teacher to analyse a mathematical task and a set of student solutions. The last part of the interview asked the teachers to raise any issue or comments they had/have in their experience of LS. The next section will report on results in School A.

Results and Discussion

Hofstede cautioned against comparisons of replicated studies with his published scores, for doing so requires matched samples. This is not part of the aim of this paper. The author utilised Hofstede's dimensions to anticipate possible affordances and/or barriers that culture may bring to the plate when implementing LS outside Japan.

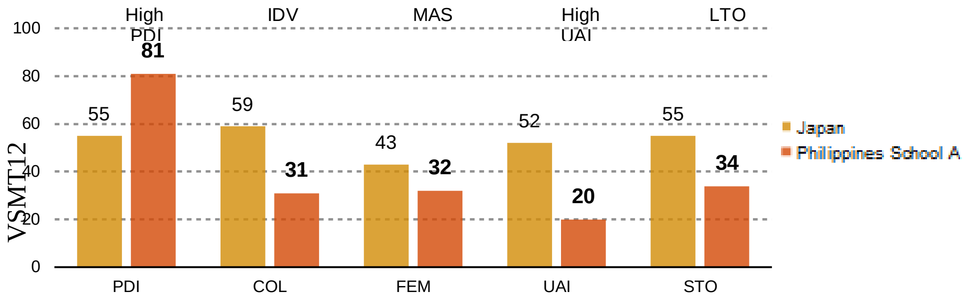


Figure 2. VSMT12 scores for Japanese sample (adapted from Ebaeguin & Stephens, 2014) and School A

From Figure 2, it can be seen that School A is very high in PDI, moderately collaborative, moderately feminine, low uncertainty-avoidance and moderately short-term oriented. Obvious barriers would be the very high PDI, very low UAI, and the moderate short-term orientation. Despite being quite collaborative, the very hierarchical nature of the teachers may affect the level of participation of the teachers. Novice teachers may tend feel intimidated and remain passive in discussions, whereas, the seasoned teachers may feel the need to assert themselves and dominate the exchanges. Also, how the teachers see the author, either as an outside resource person who is there to train them or a colleague/fellow educator who is there to work with them, may affect the teachers' involvement. To address this, the author designed the trainings and meetings such that everyone's opinions will be heard, tasks are distributed fairly to everyone, and that the novice teachers were given the chance to take on more important roles. The author was also consultative when making decisions. Low UAI was also an impediment because the teachers may not have seen the value in making detailed lesson plans, for example, or rehearsing lessons prior to actual teaching of the lessons. Moderate short-term orientation wais also a challenge as the teachers may see LS as a one-shot activity like seminars or workshops they go to because they did not see the long-term benefits of engaging in this activity. To avoid a low level of commitment from them, it was important to have immediate superiors involved such as the mathematics department head: to keep them in the program; to make them realise the learnings they get from each session so they could feel they are improving; and to constantly remind them of other possible benefits they can get such as prepared lessons or activities which they can use for the next school years. Note that the study did not aim for the Philippine teachers to have the same orientations with their counterparts in Japan, but to create an environment that would be supportive of a LS implementation.

Figures 3—6 below show some results of MTPGML for the sampled Japanese teachers (Ebaeguin & Stephens, 2014) and the pre- and post-intervention of teachers in School A. As mentioned in the methodology section, analysis of data from this instrument has been limited to simple descriptive statistics due to the small sample. Furthermore, the results from the Japanese sample did not serve as the goal for the teachers in School A post-intervention, but to highlight the differences between what the sampled Japanese teachers and School A teachers, pre-intervention, value when planning a lesson and to anticipate aspects of LS School A teachers may struggle with or may not readily embrace. These became the focus of some sessions with the teachers.

.

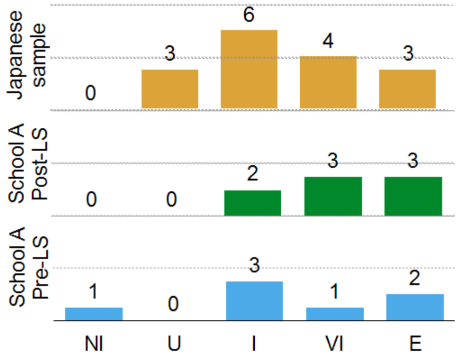


Figure 3. Japanese sample and School A teachers' endorsement for 'Working with other teachers to plan a lesson.' (adapted from Ebaeguin & Stephens, 2014)

There was one teacher in School A who thought collaborative lesson planning was not important. Public school teachers in the Philippines are provided with outlined lessons plans by the Department of Education and have the leeway to execute the lesson in their respective classes. This was not the case with LS which is a collaborative activity that provides teachers, whether novice or experienced, an opportunity to share their expertise when designing a lesson. Post-LS, several teachers in School A shifted towards a positive endorsement. In his exit interview A7, a novice teacher from School A, said:

> "It gave me an idea how important it is to collaborate with other teachers in planning a lesson…Majority of us plan our own lessons…but through LS, there was a realisation that I am not alone in the academe and you can collaborate with others teachers. Since they are the more experienced ones, they can advise me about the best strategies and methods to use in teaching a topic… [Because of this] my relationship with my fellow year 7 teachers improved…the superiority complex of some teachers lessened."

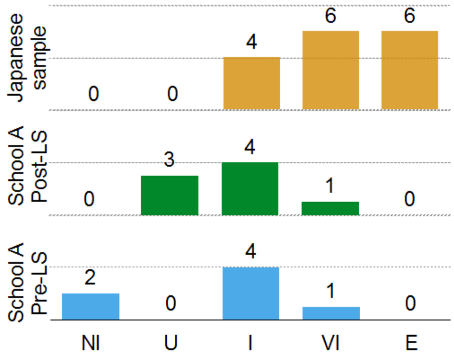


Figure 4. Japan sample and School A teachers' endorsement for 'Having other teachers/colleagues in the classroom observe my teaching.' (adapted from Ebaeguin & Stephens, 2014)

Figure 4 shows that there were two teachers in School A, both classified as experienced teachers, who thought having colleagues observe their teaching was NI. Notice also that none of them rated it as E which was very different from their Japanese counterparts who all rated this item as at least I. This aversion of School A teachers may be attributed to the fact that classroom visits are used by department heads, supervisors and principal to evaluate the teacher's performance. This is very different from LS where the lesson, not the teacher, is the focus of the observations. Looking at the post-LS results, it may seem that there was not much shift in endorsement except for the two experienced teachers, A1 and A5, who initially rated it as NI shifting to U. This may be something that School A teachers needed more convincing on because of the teacher evaluation scheme employed by the school and required by the Department of Education. A2. A novice teacher, though, said in her interview *"I was able to learn different teaching strategies. I was able to observe how other teachers teach so I was able to improve my own practice."*

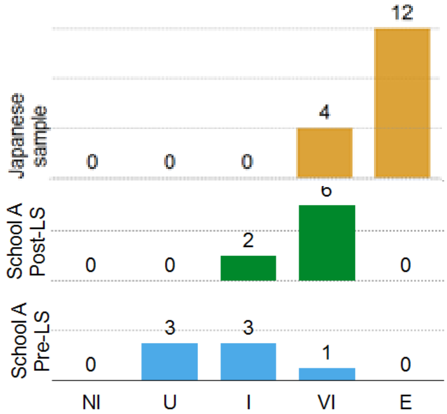


Figure 5. Japanese sample and School A teachers' endorsement for 'Identifying in advance the range of expected student responses in a problem-solving lesson.' (adapted from Ebaeguin & Stephens, 2014)

In Figure 5 there is a striking difference of value orientation between the Japanese sample and School A teachers. Every Japanese teacher rated "Identifying in advance the range of expected student responses in a problem-solving lesson" to be at least VI, whereas, three School A teachers rated U and none rated E. This could be attributed to the low UAI. In planning a lesson, teachers need to anticipate possible correct solutions, misconceptions and needed support for their students. Post-LS, there was a clearer endorsement of this item with the majority rating it VI. A1, an experienced teacher, said:

> "It helped me to construct a good lesson plan wherein we need to consider the students' anticipated responses. It's only now that I realised the need to consider these because you can use them to develop the flow of your lesson by connecting the students' ideas from one another."

A5, another experienced teacher, further added:

> "Readiness in dealing with my students every day. Usually, we only expect the correct answer to be given. When a wrong answer is given, we assume that all the wrong responses are the same. But through LS, we consider all possible student responses so we're able to prepare responses in case a wrong response comes up."

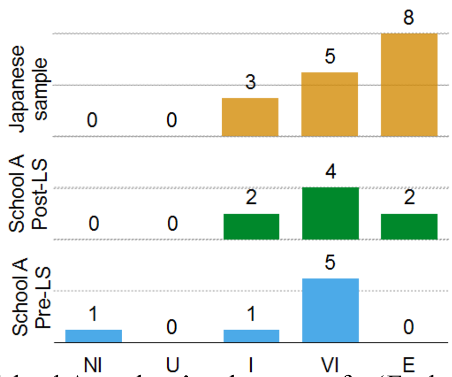


Figure 6. Japanese sample and School A teachers' endorsement for 'Evaluation of a lesson through analysing collected samples of students' solutions and attempted solutions.' (adapted from Ebaeguin & Stephens, 2014)

In Figure 6, though a majority of the teachers in School A rated "Evaluation of a lesson through analysing collected samples of students' solutions and attempted solutions" at least

.

I, none rated it E, compared to the Japanese sample where the majority rated E. In the Philippines, more often than not, lessons are evaluated based on test scores. If most students get passing scores, then the lesson is considered successful. In LS, collecting student artefacts is important because this allows the teachers to understand how students think. Other student responses that were not anticipated prior to teaching of the lesson are then integrated into a revised version of the lesson plan. Post-LS, the lone teacher who rated this NI shifted to at least I and two teachers who rated it as E. A4, a seasoned teacher shared *"In LS, you get to look at the different answers from the students some of which may appear wrong but, on a closer look, may be correct. Even the teachers are learning."*

Conclusion

Culturally grounded aspects of LS contribute to how they are embraced by the teachers in the importing culture. A culturally embedded approach to implementation builds on the teachers' cultural and value orientations in order to facilitate a locally appropriate implementation of LS. Knowing the disparities in the orientations with the importing culture allowed us to be more strategic in our implementation and focus our intervention where necessary. The shift in endorsement of some key aspects of LS, and reflections from teachers in School A, provided solid evidence of teacher professional growth. This shows that LS, if adapted and implemented critically is a successful and sustainable program that will provide teachers with opportunities for professional growth.

References

de Mooij, M. (2010). *Global marketing and advertising: Understanding cultural paradoxes*. Los Angeles: SAGE.

Dudley, P. (2012) Lesson study development in England: from school networks to national policy, *International Journal for Lesson and Learning Studies*, 1(1), 85–100.

Ebaeguin, M. & Stephens, M. (2014). Cultural challenges in adapting lesson study to a Philippines setting. *Mathematics Teacher Education and Development,* 16(1), 43–64.

Fernandez, C., & Yoshida, M. (2004). *Improving mathematics teaching and learning: The Japanese lesson study approach (Studies in Mathematical Thinking and Learning Series)*. Mahwah, NJ: Lawrence Erlbaum Associates.

Fujii T. (2014). Implementing Japanese lesson study in foreign countries: Misconceptions revealed. *Mathematics Teacher Education and Development,* 16(1), 65–83.

Hofstede, G., Hofstede G. J., & Minkov, M. (2010). *Cultures and organizations: Software of the mind: Intercultural cooperation and its importance for survival*. New York: McGraw-Hill.

Hofstede, G., Hofstede, G. J., Minkov, M., & Vinken, H. (2008). *Values survey module 2008*. Retrieved from http://www.geerthofstede.nl/vsm-08

Lewis, C. (2002). What are the essential elements of lesson study? *The California Science Project Connection*, 2(6), 1, 4.

Lewis, C., & Tsuchida, I. (1998, Winter). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator*, 12–17; 50–52.

Stephens, M. (2011). Ensuring instruction changes: Evidence based teaching: How can lesson study inform coaching, instructional rounds and learning walks? *Journal of Science and Mathematics Education in Southeast Asia*, 34(1), 111–113.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York, NY: Free Press.

Yoshida, M. (2012). Mathematics lesson study in the United States: Current status and ideas for conducting high quality and effective lesson study. *International Journal for Lesson and Learning Studies*, 1(2), 140-152.

The Self-Efficacy of students with Borderline, Mild and Moderate Intellectual Disabilities and their Achievements in Mathematics

Agbon Enoma
Curtin University
<a.enoma@postgrad.curtin.edu.au>

John Malone
Curtin University
<J.Malone@curtin.edu.au>

The relationship between the self-efficacy of 23 High School students with intellectual disability (ID) and their achievements in Mathematics was evaluated using a modified version of the self-efficacy instrument developed by Joet, Bressoux and Usher (2011). Four different number sense assessment tools were administered pre- and post- six months of instruction to measure their Mathematics achievement. Relevant data analyses were carried out with *Minitab* statistical software. While the mean self-efficacy was found to be about 65%, the correlation between self-efficacy and the mathematics achievements of students was weak.

The definition of intellectual disability (ID) and its levels of severity have undergone many revisions over the years in response to emerging research outcomes which have changed how ID is perceived. There is a shift in thinking from a deficiency model which suggests the problem resides in the individual with ID to the environment or support/needs model that focuses on what adjustments needed to be made to support people with ID. The term 'intellectual disability' (ID) (previously known as mental retardation) has been used interchangeably in the literature for 'intellectual developmental disorder' (American Psychiatric Association - APA, 2013, p.33) and 'intellectual impairment' (Wen, 1997, p. 2). One current definition describes ID as a form of disability that is "characterised by significant limitations in both intellectual functioning and in adaptive behaviour, which covers many everyday social and practical skills. This disability originates before the age of 18" (American Association on Intellectual and Developmental Disabilities - AAIDD, 2010, p. 1). Similarly, the APA (2013, p. 33) defines ID as a disorder that is characterised by: (a) deficits in intellectual functioning; (b) deficits in adaptive functioning; and (c) intellectual and adaptive deficits occurring during the developmental period.

For several decades IQ scores have been employed widely in describing the levels of severity of ID including borderline (IQ 84 to 71), mild (IQ 70 to 55), moderate (IQ 54 to 35), severe (IQ 34 to 20) and profound (IQ below 20) (Wen 1997, p. 4). This IQ-based classification is being phased out and to be replaced by needs-based severity codes. The APA (2013, pp. 33-36) has introduced needs-based severity codes that consist of mild, moderate, severe and profound ID. This categorisation is based on adaptive functioning rather than IQ scores and with functional limitations evaluated across conceptual, social and practical skills domains as detailed in the *Diagnostic and Statistical Manual of Mental Disorders (Fifth Edition)*. The AAIDD (2010) has also introduced its own support-based severity codes of ID consisting of intermittent support, limited support, extensive support and pervasive support which are based on the intensity of support needed by the individual with ID. A summary description of these codes as provided by Reynolds, Zupanick, and Dombeck (2015, pp. 33-34) includes: (1) Intermittent support (equivalent to mild ID under APA standards) – "they may only require additional supports during times of transition, uncertainty, or stress"; (2) Limited support (equivalent to moderate ID under APA standards) – "with additional training, they can increase their conceptual skills, social skills, and practical skills. However, they can still require additional support to navigate everyday situations"; (3) Extensive support (equivalent to severe ID under APA standards)

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 221–228. Sunshine Coast: MERGA.

– "… they will usually require daily support"; and (4) Pervasive support (equivalent to profound ID under APA standards) – "daily interventions are necessary to help the individual function. Supervision is necessary to ensure their health and safety. This lifelong support applies to nearly every aspect of the individual's routine". The IQ-based classification was used in this study as that was the practice in place at the commencement of this study 3 years ago at the school where this study was conducted.

Self-efficacy has been defined as "beliefs in one's capabilities to mobilise the motivation, cognitive resources, and courses of action needed to meet given situational demands" (Gist & Mitchell, 1992, p. 184). This involves the "convictions that one can successfully carry out given academic tasks at designated levels" (Bong, 2004, p. 288). Embedded in this definition of self-efficacy is the affirmation of the importance of motivation and cognitive ability. Motivation and cognitive factors are essential ingredients of self-efficacy. Azar, Lavasania, Malahmadi and Amani (2010) have acknowledged that motivation and cognitive ability influence achievements among other factors. All around us today, there are everyday examples of mathematics impacting on our lives including shopping, using the phone, transport, money, cooking and many others (Gouba, 2008). Students with ID require some functional knowledge of Mathematics to achieve some degree of independence in their lives. For example, the ability to read time is essential to employees arriving at work on time and keeping their job ("if the short arm of a clock points to 3 and the long arm to 12, what is the time?"). Also, it is important to be able to identify one's phone number (functional mathematics) and name (functional literacy) on a bill to avoid paying the bill of a previous tenant in a rented accommodation ("identify your phone number (from a given set of numbers)". Self-efficacy has been found to be a good predictor of Mathematics achievements among mainstream students (Pajares, 1996).

There is a copious amount of information in the literature on the self-efficacy beliefs of individuals in mainstream educational settings. The first author has searched the literature for studies on the effects of self-efficacy of students with ID on their Mathematics achievements and it appears that no study of this nature exists.

Rationale

This study sought to establish: (1) the relationship between mathematics self-efficacy and intellectual disability; and (2) the relationship between the self-efficacy of students with borderline, mild and moderate ID and their achievements in Mathematics.

Method

Participants

Twenty-three High School students from Years 8 to 12 consisting of three, thirteen, and seven borderline, mild and moderate ID respectively participated in the study. The Mathematics self-efficacy instrument used in this study was an adaptation of the instrument described by Joet, Bressoux and Usher (2011). It was modified to make it relevant and appropriate to students with borderline, mild and moderate ID by including functional numeracy questions – from questions 4 to 25 (Table 1). Only questions 1 to 3 were retained from Joet, Bressoux and Usher's (2011) original Self-Efficacy items. The modified instrument (Table 1) had 25 items and each item was rated along five response categories including completely true (weighted 5), very true (weighted 4), moderately true (weighted 3), slightly true (weighted 2) and not at all true (weighted 1) (Table 1).

Statistical analyses were undertaken with *Minitab 17* (Minitab Statistical Software, 2010; Aylin, 2010). The self-efficacy instrument was administered orally and clarifications provided where necessary to ensure the participants understood the questions.

Table 1
Mathematics Self-Efficacy Items for students with ID

No.	Item
1	I am capable of solving math problems
2	I can solve geometry problems (e.g. identify shapes, calculate area and perimeter)
3	I am capable of getting good grades in math
4	I can solve addition problems involving single digit numbers
5	I can solve double-digit addition problems
6	I can subtract single-digit numbers
7	I can subtract double digit numbers
8	I can multiply single-digit numbers
9	I can multiply double-digit numbers
10	I can divide single-digit numbers
11	I can divide double-digit numbers
12	I can identify a number’s place value
13	I know how to write numbers with their symbols up to 20
14	I know how to calculate the area of a rectangle
15	I am capable of measuring the sides and diagonals of a rectangle
16	I know how to add metres and centimetres
17	I know how many centimetres make a metre
18	I know how many cents make a dollar
19	I know how many minutes make 1 hour
20	I can count from 1 to 10
21	I can count from 1 to 20
22	I can count from 1 to 50
23	I can count from 1 to 100
24	I can count from 1 to 1000
25	I know my 12 times table

The self-efficacy instrument was administered at the commencement of the school year (Self-Efficacy 1). The students went through 6 months of instruction after which a second round of self-efficacy assessment (Self-Efficacy 2) was carried out. To measure the mathematics achievements of the students, the authors administered Test 1 - IMPELS (Enoma & Malone – in press), Test 2 - the Delaware Universal Screening Tool for Number Sense Grade 2 (Delaware Department of Education, 2010), Test 3 - Streamlined Number Sense Screening Tool (Jordan, Glutting & Ramineni, 2008) and Test 4 - Number knowledge Test (Okamoto & Case, 1996; Okamoto, 2004) on each occasion that the self-efficacy assessment was conducted.

Results and Discussion

Table 2 showed that 20 students (about 86%) of participants achieved >50% in the Self-Efficacy 1 assessment. When the self-efficacy assessment was repeated after 6 months of teaching (Self-Efficacy 2 – Table 2), similar results were obtained.

Table 2
Comparing pre-instruction Efficacy (Self-Efficacy 1) with Tests 1, 2, 3 & 4

Student	Year Level	Severity of ID	Self-Efficacy 1 (%)	Test 1 (%)	Test 2 (%)	Test 3 (%)	Test 4 (%)
1	10	Borderline ID	56.00	84.91	75.00	96.00	64.77
2	10	Borderline ID	84.80	99.58	75.00	98.00	65.91
3	8	Borderline ID	62.40	70.65	75.00	99.00	56.82
4	11	Mild ID	73.60	53.25	58.33	84.00	50.00
5	10	Mild ID	81.60	71.07	50.00	98.00	71.59
6	9	Mild ID	74.40	75.68	41.67	90.00	37.50
7	8	Mild ID	77.60	98.74	58.33	99.00	60.23
8	9	Mild ID	65.60	98.32	66.67	99.00	56.82
9	10	Mild ID	86.40	98.95	83.33	93.00	39.77
10	11	Mild ID	76.00	95.81	58.33	100.00	70.45
11	11	Mild ID	73.60	85.95	75.00	100.00	54.55
12	8	Mild ID	76.80	85.53	83.33	86.00	73.86
13	9	Mild ID	72.00	93.71	66.67	100.00	67.05
14	9	Mild ID	66.40	54.72	41.67	74.00	25.00
15	9	Mild ID	55.20	48.63	75.00	93.00	29.55
16	10	Mild ID	47.20	90.14	66.00	84.00	56.82
17	9	Moderate ID	60.00	41.30	16.67	56.00	29.55
18	12	Moderate ID	48.00	63.94	50.00	98.00	43.18
19	10	Moderate ID	64.80	62.68	66.67	99.00	52.27
20	8	Moderate ID	38.40	27.46	25.00	43.00	22.73
21	9	Moderate ID	52.80	51.36	66.67	90.00	50.00
22	11	Moderate ID	76.00	56.39	75.00	100.00	56.82
23	10	Moderate ID	58.40	60.97	75.00	84.00	39.77

Test 1 = IMPELS (Enoma & Malone, 2015 – in press), Test 2 = the Delaware Universal Screening Tool for Number Sense Grade 2 (Delaware Department of Education, 2010), Test 3 = Streamlined Number Sense Screening Tool (Jordan, Glutting & Ramineni, 2008), Test 4 = Number knowledge Test (Okamoto & Case, 1996).

However, it was observed that some students with relatively high self-efficacy achieved low marks in mathematics as indicated by a student with a self-efficacy score of 60% achieving 41% in the mathematics Test 1 (Table 2). This suggests possible cognitive limitation or some degree of over-confidence or both. Similarly, some students with low self-efficacy achieved high marks in mathematics. An example of this case was demonstrated by a student who had a self-efficacy score of 23.2% and achieved 70.27% in

Test 1 (Table 3). An additional example of the low self-efficacy-high marks scenario was displayed by another student who achieved a relatively low self-efficacy score of 47.2% but achieved 90.14% in the mathematics Test 1 (Table 2). The situation described in the latter two examples has manifold implications: (1) students in this category possess some level of mathematics anxiety, (2) students in this group have the potential to do relatively well in mathematics and (3) As a result of mathematics anxiety, this cohort of students may not always perform to their potential in mathematics.

Table 3: Comparing post-instruction Efficacy (Self-Efficacy 2) with Tests 1, 2, 3 & 4

Name	Year Level	Severity of ID	Self-Efficacy 2 (%)	Test 1 (%)	Test 2 (%)	Test 3 (%)	Test 4 (%)
1	10	Borderline ID	78.40	95.39	83.33	98.78	73.86
2	10	Borderline ID	64.80	99.58	100.00	100.00	77.27
3	8	Borderline ID	76.80	77.99	75.00	98.78	80.68
4	11	Mild ID	83.20	97.06	66.67	97.56	65.91
5	10	Mild ID	86.40	99.79	83.33	100.00	76.14
6	9	Mild ID	68.80	97.69	66.67	84.15	45.45
7	8	Mild ID	65.60	93.08	75.00	100.00	67.05
8	9	Mild ID	68.80	98.74	50.00	98.78	65.91
9	10	Mild ID	61.60	88.68	83.33	93.90	53.41
10	11	Mild ID	72.00	96.86	91.67	100.00	76.14
11	11	Mild ID	73.60	91.19	91.67	100.00	59.09
12	8	Mild ID	72.80	92.87	66.67	98.78	73.86
13	9	Mild ID	72.00	99.16	58.33	100.00	68.18
14	9	Mild ID	65.60	87.00	8.33	78.05	43.18
15	9	Mild ID	23.20	70.27	25.00	84.15	34.09
16	10	Mild ID	56.80	91.19	83.33	98.78	50.00
17	9	Moderate ID	60.00	41.30	8.33	68.90	29.55
18	12	Moderate ID	53.60	83.23	58.33	97.56	47.73
19	10	Moderate ID	64.80	88.68	91.67	100.00	47.73
20	8	Moderate ID	40.80	37.32	41.67	45.73	22.27
21	9	Moderate ID	69.60	61.32	25.00	93.90	54.55
22	11	Moderate ID	76.00	90.36	75.00	98.78	56.82
23	10	Moderate ID	44.00	90.78	75.00	93.90	52.27

Test 1 = IMPELS (Enoma & Malone, 2015 – in press), Test 2 = the Delaware Universal Screening Tool for Number Sense Grade 2 (Delaware Department of Education, 2010), Test 3 = Streamlined Number Sense Screening Tool (Jordan, Glutting & Ramineni, 008), Test 4 = Number knowledge Test (Okamoto & Case, 1996).

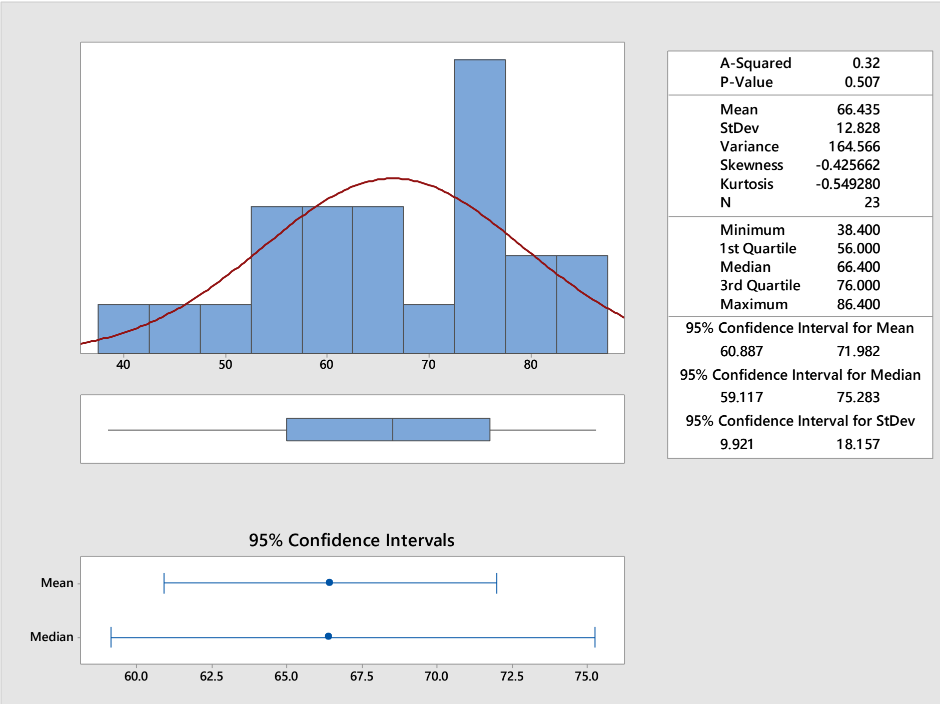


Figure 1: Distribution of Self-Efficacy 1 scores of students (pre- instruction).

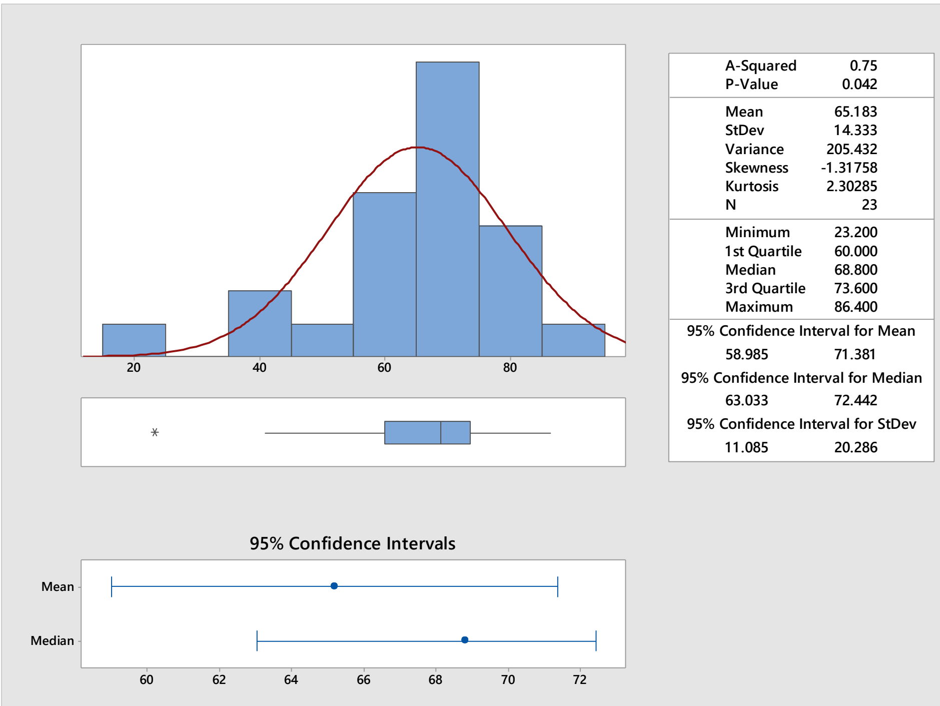


Figure 2: Distribution of Self-Efficacy 2 scores of students (post-instruction).

Students' self-efficacy scores ranged from 38.4% to 86.4% for Self-Efficacy 1 (Figure 1) and 23.20% to 86.40% for Self-Efficacy 2 (Figure 2). The mean self-efficacy scores were about the same, ie 83.04 (66.43%) for Self-Efficacy 1 (Figure 1) and 81.48 (65.18%) for Self-Efficacy 2 (Figure 2). Such impressive average self-efficacy scores of 66.43% (Self-Efficacy 1) and 65.18% (Self-Efficacy 2) demonstrate a belief in the majority of the students in their capabilities to do well in Mathematics. While self-efficacy has been acknowledged as an important factor in academic accomplishments because of its positive relationship with effort and persistence (Bandura, 1993), it must borne in mind that individuals can only perform within the limit of their cognitive abilities.

Linear Regression Graphs

Considering the sample size was less than 30 (n = 23), Anderson-Darling normality tests were undertaken on both pre- and post-instruction data using MINITAB 17 statistical software (Minitab Statistical Software, 2010). The outcome was a mixed group of normally and non-normally distributed data. As a result, Pearson and Spearman Rho correlation coefficients were calculated. Pearson's pre-instruction correlation coefficients of 0.57 (P = 0.005), 0.33 (P = 0.122), 0.49 (P = 0.015), 0.48 (P = 0.02) and post-instruction correlation coefficients of 0.50 (P = 0.01), 0.37 (P = 0.07), 0.51 (P = 0.01) and 0.71 (P = 0.00) were obtained for Tests 1, 2, 3 and 4 respectively. Similarly, Spearman Rho pre-instruction correlation coefficient of 0.58 (P = 0.003), 0.24 (P = 0.26), 0.38 (P = 0.06), 0.47 (P = 0.02) and post-instruction correlation coefficients of 0.50 (P = 0.015), 0.27 (P = 0.20), 0.48 (P = 0.02) and 0.708 (P = 0.00) were obtained for Tests 1, 2, 3 and 4 respectively. The relationship between students' scores in Self-Efficacy 1 and their achievements in Mathematics showed a weak Pearson correlation coefficient (R) of 0.57, 0.33, 0.50 and 0.48 for Tests 1, 2, 3, and 4 respectively. Similar results were obtained for Self-Efficacy 2 with correlation coefficients of 0.50 (Test 1), 0.38 (Test 2) and 0.51 (Test 3). The only exception was Test 4 with a correlation coefficient of 0.71.

Figure 3 shows the relationship between Self-Efficacies 1 and 2 and the full scale IQ scores of students prior to instruction. Achievements in the self-efficacy assessments correlated weakly with their full scale IQ scores. Pearson correlation coefficients (R) of 0.36 and 0.30 were obtained for Self-Efficacy 1 (conducted at the beginning of the school year) and self-efficacy 2 (conducted 6 months after). This result shows that self-efficacy is an individual attribute as some students with high full scale IQ demonstrated lower self-efficacy than those students with IQ scores below them. The reverse was also true for some students.

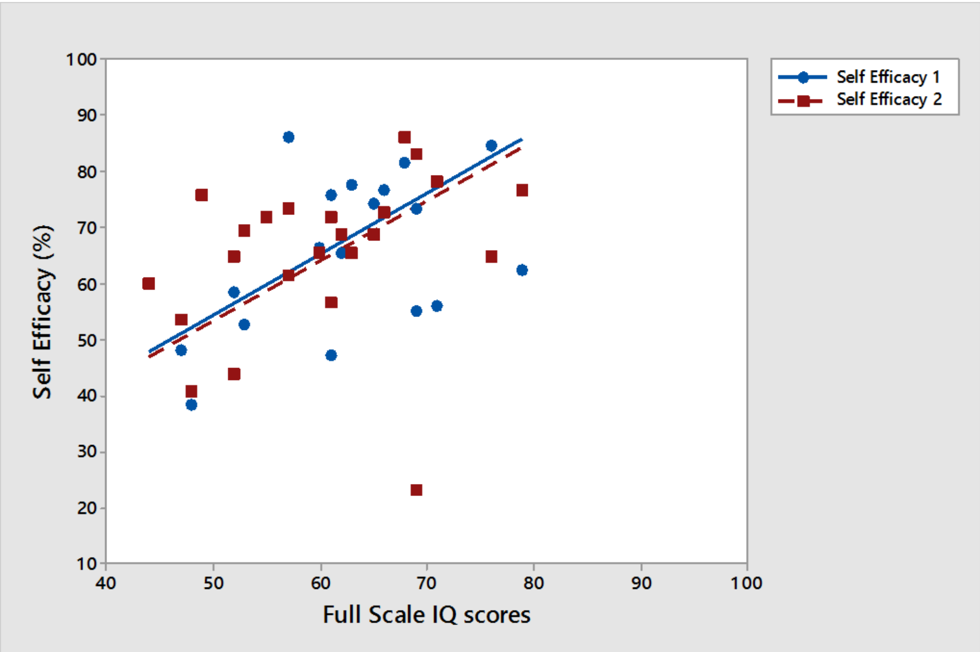


Figure 3: Relationship between Self-Efficacy and Full Scale IQ scores of students prior to instruction

Conclusion

The study found no strong correlation between the mathematics self-efficacy of students with ID and their achievements in Mathematics or with the categories of ID. The various scenarios that emerged from the study include students with low mathematics self-efficacy that achieved high scores in the tests, students with high mathematics self-efficacy that achieved low scores in the tests, students with high mathematics self-efficacy that achieved high scores in the tests and students with low mathematics self-efficacy that achieved low scores in the tests. These results further reinforced the importance of individualised mathematics education for students with ID.

References

American Association on Intellectual and Developmental Disabilities (2010). *Intellectual Disability: Definition, Classification, and Systems of Supports.* Washington (USA): American Association on Intellectual and Developmental Disabilities.

American Psychiatric Association. (2013). *Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition.* Arlington, USA: American Psychiatric Association.

Aylin, A. (2010). Minitab. *Wiley Interdisciplinary Reviews: Computational Statistics, 2*(6), 723-727.

Azar, H.K., Lavasania, M. G., Malahmadia, E. & Amania, J. (2010). The role of self- efficacy, task value, and achievement goals in predicting learning approaches and mathematics achievement. *Procedia Social and Behavioral Sciences*, *5*, 942–947.

Bandura, A. (1993). Perceived self-efficacy in cognitive development and functioning. *Educational Psychologist*, *29*, 117-148.

Bong, M. (2004). Academic motivation in self-efficacy, task value, achievement goal orientations, and attributional beliefs. *Journal of Educational Research, 97*, 287-297.

Delaware Department of Education (2010). *Delaware universal Screening Tool for Number Sense Grade 2*. Delaware (USA): Delaware Department of Education.

Enoma, A. O. S & Malone, J. (2015 - *in press*). The Development and Evaluation of an Individualised Learning Tool for Mathematics students with Intellectual Disability: IMPELS – *in press*

Gist, M., & Mitchell, T. R. (1992). Self-efficacy: A theoretical analysis of its determinants and malleability. *Academy of Management Review, 17*, 183-211.

Gouba, L. (2008). *The Importance of Mathematics in Everyday Life*. South Africa: African Institute for Mathematical Sciences.

Joet, G, Bressoux, P. & Usher, E. L. (2011). *Journal of Educational Psychology*, *103*(3), 649-663.

Jordan, N. C, Glutting, J. & Ramineni, C. (2008). A Number Sense Assessment Tool for Identifying Children at risk for Mathematical Difficulties. In A. Dowker (Ed.), *Mathematical Difficulties: Psychology and Intervention* (pp. 45-58). San Diego, CA: Academic Press.

Minitab 17 Statistical Software (2010). [Computer software]. State College, PA: Minitab, Inc.

Okamoto, Y. (2004). The “Number Knowledge” Assessment. *National Longitudinal Survey for Children and Youth – Cognitive Measures*, 1-9.

Okamoto, Y. & Case, R. (1996). Exploring the microstructure of children’s central conceptual structures in the domain of number. In R. Case & Y. Okamoto (Eds.), *The role of central conceptual structures in the development of children’s thought: Monographs of the Society for Research in Child Development* (Vol. 1-2, pp. 27-58). Malden, MA: Blackwell Publishers.

Pajares, F. (1996). Self-Efficacy Beliefs and Mathematical Problem-Solving of Gifted Students. *Contemporary Educational Psychology*, *21,* 325-344.

Reynolds, T; Zupanick, C. E. & Dombeck, M. (2015). *Intellectual Disabilities*. Retrieved 18 January 2015 from: http://www.mhmrcv.org/poc/center_index.php?id=208&cn=208

Wen, X. (1997). *The definition and prevalence of intellectual disability in Australia*. Australian Institute of Health and Welfare, Canberra, AIHW Catalogue Number, Dis 2, 1-59.

Identifying Core Elements of Argument-Based Inquiry in Primary Mathematics Learning

Jill Fielding-Wells
University of Tasmania
<Jill.Wells@utas.edu.au>

Having students address mathematical inquiry problems that are ill-structured and ambiguous offers potential for them to develop a focus on mathematical evidence and reasoning. However, students may not necessarily focus on these aspects when responding to such problems. Argument-Based Inquiry is one way to guide students in this direction. This paper draws on an analysis of multiple primary classes to describe core elements in Argument-Based Inquiry in mathematics.

The inclusion of inquiry-based pedagogies into classroom mathematics teaching has the potential to engage students in mathematics in authentic ways (Fielding-Wells & Makar, 2008). Students engage with inquiry as they are supported to work with ambiguous and ill-structured problems (Makar, 2012); ill-structured problems being considered those which have no *correct* solution, may have multiple solutions, or have unclear solution processes (Eraut, 1994). An advantage of working with such questions is that:

> their inherent ambiguity allows for multiple interpretations of a question, a range of pathways, and numerous solutions with varying degrees of efficiency, applicability and elegance. This requires students to focus on decision-making, analysis and justification. Rather than a 'correct' answer or strategy, there is a claim which requires evidence, explanation and defense – in short, an argument (Fielding-Wells & Makar, 2012, p. 149).

Blair (2012) describes a view of argumentation that essentially sees it as a form of inquiry in which argumentation is utilised to explore a problem and to arrive at a solution through examination of the evidence and grounds that can be employed towards solving the problem. By implementing such a model of argument, students may be explicitly focussed on obtaining evidence, using evidence to make a claim, and articulating how the evidence leads to the claim through reasoning. Thus, argumentation offers potential to purposefully direct students engaged in inquiry to focus on the discipline content, and the ways in which the discipline content can be used, to respond to a problem or dilemma.

Argumentation in not new in mathematics, there is a great deal that mathematicians do that incorporates reasoning and argument. For example, mathematical proof must stand up to rigorous, critical, dialectical argument by other mathematicians and be open to argument as attempts are made to examine, generalise, extend, and simplify the proof. Another area of argumentation research in mathematics has been as it applies to procedure (see, for example, Goos, 2004; Yackel & Cobb, 1996). Here it is "the strategies used for figuring out, rather than the answers, that are the site of the mathematical argument" (Lampert, 1990, p. 40).

There is a third type of argumentation, one that would appear to have been the focus of less research and that is the use of argumentation to address authentic, ill-defined mathematical problems in which *neither* the procedural pathways *nor* the solutions are limited in terms of 'correctness'; that is, *inquiry* (Anderson, 2002). This is the focus of the research described in this paper and which differs from the existing body of literature somewhat in that both the solution process and the answers are considered the site of the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 229–236. Sunshine Coast: MERGA.

argument. Hence, the term *Argument-Based Inquiry* (ABI) has been adopted to describe this view of argumentation.

Argumentation

Toulmin, Rieke and Janik's (1984) seminal work on argument structure enables an argument to be identified by components of claim, grounds, backing, warrants, and so forth. However, such a structure focuses on the components of an argument rather than providing a focus on evaluating the logic or strength of their claim. A simpler model than that proposed by Toulmin et al. would appear to be indicated for children, such as the Claim-Evidence-Reasoning model devised by McNeill and associates (McNeill & Martin, 2011; Zembal-Saul, McNeill, & Hershberger, 2013). This enables a more general focus on the core components of classroom argument. The claim and evidence components align with Toulmin et al.'s claim and grounds: claim being the conclusion that addresses the original question and evidence being the scientific data that supports the claim. In explaining their model, Zembal-Saul et al. (2013) maintain that the data needs to be both appropriate and sufficient to support the claim. The third component, reasoning, encompasses the warrants and backing; that is, the logic that enables the grounds to be used to establish the claim (McNeill & Krajcik, 2012).

The Nature of Argumentation

Various theories of argumentation can be found in the literature with Toulmin et al.'s (1984) classical work on argumentation structure forming a basis for most. For instance, van Eemeren and Grootendorst (2004) extended Toulmin et al.'s structural (product) approach to pragma-dialectical argumentation that incorporated the process of argument also. Lumer (2010) and Siegel and Biro (1997) proposed a model of Epistemic Argumentation, which distinguished itself through a focus on the strength and validity of an argument, based on epistemic criteria (Nettel & Roque, 2012) rather than structure and use of emotive devices. It is this theory of argumentation that was adopted throughout the research described here: the rationale being that science (and mathematics) value accuracy, logic and verifiability over persuasive devices seen in other forms of argument. Furthermore, the ability to challenge the argument is offered on an epistemic level, giving potential rise to challenge about what is acceptable evidence and reasoning within a discipline (Simon & Richardson, 2009).

Traditional ways of teaching do not provide a classroom culture that is necessarily conducive to the introduction of ABI practices and thus there are many practical considerations to developing such approaches. In order to facilitate the research undertaken, argumentation was introduced into primary classrooms that were already fluent in the use of inquiry-based learning (IBL) in mathematics. What signature elements of Inquiry-Based Argument can serve to guide children's mathematical argumentation?

Methodology

The larger research study from which this report stems was conducted using Design-Based Research; selected because this methodology entails engineering forms of learning and then systematically studying the learning within its context, which was ideal for the research purpose (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

Participants and Data Collection

In total, five classes of students were engaged in Inquiry-Based Argumentation units with some carrying out one unit and others as many as three across the course of a year. A total of nine units were recorded in full from classes (at the Prep, Year 1, Year 3, Year 5 and Year 7 levels) at a metropolitan government primary school in Queensland. This school is a relatively large primary school with approximately four drafts of each year level. At the commencement of the research, the school site had been part of an IBL research project for seven years and involved a number of teachers at the school. The teachers were all experienced with teaching using IBL; however, due to changing grouping over years some students were quite familiar with learning through inquiry whereas others had little or no experience. Teachers were provided with ongoing guidance and support; however, beyond a need for a Question-Evidence-Conclusion focus, the teachers were largely responsible for implementing their approach to the inquiry question as they chose.

Data and Data Analysis

A selection of videotaped units (approximately 5-10 lessons each) was analysed using a process adapted from Powell, Francisco and Maher (2003). In line with their approach, the lesson videos were viewed and logged, lesson-by-lesson, along with time stamps, excerpts of students' work, and still shots of teaching materials to capture the essence of the lessons. Critical events, such as those that demonstrate a particular struggle or advancement in the inquiry were noted and transcribed in more detail. Logs were coded using adapted grounded theory (Corbin & Strauss, 2008) and this enabled cross comparison between the units for particular events and patterns in the development of the inquiry. In particular, commonalities and differences were highlighted in order to develop an overarching narrative of the ABI process. Four units that were felt to demonstrate deep engagement with ABI pedagogy were transcribed in full. For consistency of the story, all classroom illustrations provided are drawn from one unit: *Biased Bingo* (Year 3): a teaching unit designed around the game of addition bingo, which addressed the question *'What is the best card for addition bingo?'* In the game, all possible combinations of the sum of two numbers (1 to 10) were each written on a slip of paper and placed in a box. Children had a card consisting of a 5 x 5 array of self-selected numbers (their predictions of what will be called). In order to address the problem, they needed to decide on the best numbers they could place on their card.

The purpose of the grounded coding was to enable the development of substantial codes to describe, name, or classify aspects of the study (Flick, 2009). The codes assigned were grouped into common themes and codes that were essentially duplications were amalgamated. These codes were clustered where appropriate into code categories and substantive categories and used to map themes and relationships.

Results and Discussion

The analysis undertaken illustrated four key components or threads at the most basic level of ABI; that is, that were consistent across all classes and ages. While more advanced components were also able to be identified in older classes engaging in more than one ABI unit, the essential and consistent elements noted are the focus of this paper. At the very simplest level, mathematical argumentation was characterised by students addressing of a purposeful inquiry question, the advancing of evidence which was used to form a claim, the justifying of the evidenced claim through epistemically acceptable reasoning, and acknowledgement of context. While the elements are presented here sequentially, in practice the teacher drew attention to different components and the relationships between different components, as required. Each of these elements will be addressed in turn.

Addresses a Purposeful Inquiry Question

In order to present an argument, the students first required a question they could address. Questions were variously provided by the teacher, by the students or, most often, in a vague and unrefined way by the teacher and then refined by the students with teacher guidance to a topic that was mathematically researchable. The excerpt below illustrates a teacher working to help the students unpack the question being posed.

Mrs T:	Can you create a bingo card with the BEST chance of winning? What does 'best' mean?
Jess:	The best chance of winning doesn't mean like every number that gets pulled out that one person will always get that number. It means that like most of the time when you pull out a number that that person will have that number. If they have a like a good bingo card they have worked out like how many of each number they need to have to have a really good chance of winning.
[unidentified student]	The best chance of winning is the most likely chance that it is going to get called out.

The inquiry question in this instance was posed by the teacher but in such an ill-structured way that the students needed to engage with it determine the meaning. The question need not be posed by the teacher. In another unit, a student's question was adopted after it was posed spontaneously in class. Students are capable of formulating their own inquiry questions even from a young age, although research indicates there is a need to teach students how to pose their own questions with a focus on what makes a *good* question (Allmond & Makar, 2010). While this may be time consuming it does more closely match authentic practices and teaches students an important skill – how to mathematise a problem so that it can be addressed.

The word purposeful has been added to the element *addresses a purposeful inquiry question*. In this instance, a purposeful question is deemed one that seeks to address a genuine problem. By purposeful, it is meant that the question has a genuine reason for being asked. Often when students are provided a question, the teacher already has a known answer. Because of this, even if the question is open-ended, students may not engage purposefully as they have no real need to persuade their audience (the teacher) of the answer or a method (Sandoval & Millwood, 2007).

Advances Evidence to Enable the Forming of a Claim

In scientific/mathematic argument, evidence or data is sought and then attempts are made to make sense of it and to make a claim based on all the evidence, both supportive and contradictory (Sampson & Clarke, 2006). This is distinct from the role that evidence may have in advocative argument, where a claim is made and then evidence is presented in order to support or add weight to the claim. In ABI, the teacher needs to focus students on the obtaining of evidence to make a claim.

Mrs T:	... I wanted to just come back to our question, because our question was '*What Bingo card would give you the best chance of winning*?' …? Who can remember what you were doing yesterday and what you were hoping to achieve, or what were you trying to find out?

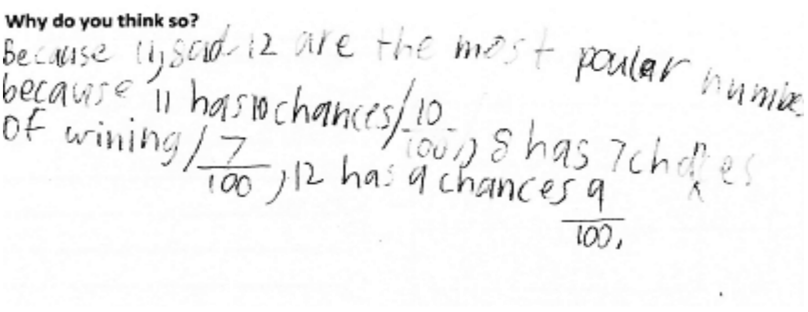
Gen:	If other numbers other than 12 would be pulled out mostly.
Mrs T:	Yes. Some people said, 'Wow, another 12 another 12' and so everyone decided 'OK 12 comes out the most' but we weren't really sure of that, so you guys had to find?
Students:	Evidence.
Mrs T:	So you went off to find some evidence for that [writes evidence on whiteboard]. To prove that. So, while you were finding evidence, what did you find? What did you discover along that track?
Byron:	That 18 was second most popular …
Gen:	That um if you did 12, there was eleven of them. And when we did 11 there was ten. And you keep taking 1 from each one and then it makes how many ...
Mrs T:	I am hearing people saying, oh well actually, 10 is the most common. And I heard someone say, 'No, 11 is'' And Bethany saying, '12' …
	So now that you look at your book, can you tell me, from the evidence that you have got there, which number, definitely, and I mean definitely. Can you prove to me, which number is the most common? Or numbers. You can Jasmine, from your evidence there could you show me, and could you prove it to the rest of the class?

In this instance, the teacher is focussing on the students need for evidence to support their claims: one commonality throughout all the units observed was the repeated and consistent focus of each teacher to bring students' attention back to the need for evidence in order to lead them to a claim, but also the need to represent the evidence in ways that assisted students to see patterns in their evidence that would lead to a claim.

It was evident throughout the units analysed that students needed to envisage the evidence they could use to address the problem, plan to obtain that evidence, organise or represent the evidence, and then interpret and analyse it in order to make and support a claim.

Justifies the Claim through Epistemically Acceptable Reasoning

Students need to use reasoning that is based on evidence to justify the making of a claim. There is potential for the connection between evidence and claim to be omitted, largely because the connection is either thought to be implicitly understood, or is left unaddressed unless challenged. However, this does not meet the purposes of IBA in mathematics, as the reasoning is the site of the actual mathematical understandings, connections, proofs, or concepts. In one class, the students engaged in three units over the year, and, by the end of that period, were explicitly stating their reasoning in terms of the mathematical underpinnings. However, this was not a stage typically reached by classes engaging in only one unit. Thus, a more *typical* response is shown from the Year 3 class:



Because [I ?] said 12 are the most popular number because 11 has 10 chances 10/100, 8 has 7 chances of winning 7/100, 12 has 9 chances 9/100

While the suggestion here is that the signature components for argumentation should include claim-evidence-reasoning (McNeill & Martin, 2011; Zembal-Saul et al., 2013), it is only essential that the teacher be able to recognise these components, particularly in younger students, and that these components may be elicited, for example verbally, pictorially, diagrammatically, or concretely. However, to have the students accustomed to providing evidence for and justifying their responses even at an earlier age would likely position the students for more formal learning and reasoning at a later time.

Reflects the Context

The final element is the necessity of the claim, evidence, and reasoning to reflect the question context. In a unit contextualised outside of mathematics, there should be a reflection of what the student's response means in the context. While the claim would reflect the context and the reasoning would require a mathematical basis, the evidence may be constrained or guided by the context and this could potentially influence the evidence at several stages: envisaging (How many trials of the Bingo should we make?) and interpreting (What does the evidence mean in light of the context? How can anomalies be interpreted in light of the context?).

Justine:	I keep losing on a 10.
Mrs T:	This is an interesting comment. Laura says 'I keep losing on a 10'. How many times have you lost on 10?
Justine:	Two.
Mrs T:	So if this was happening as you predicted and as you expected, do you think Laura could have been winning?
Students:	Yes.
Mrs T:	Because she's been waiting for a 10 and it hasn't happened although I would have predicted, or I would have expected that we would have had more 10's. So would her choosing two 10's have been a reasonable sort of assumption to make? Do you think that would have been a good idea when she was making her card?
Students:	Yes.

In this instance, these students have determined that ten is one of the highest frequency outcomes. However, in playing the game, ten has not been drawn as often as expected. The students recognise that is brought about through chance and accept that Laura has still designed a card that has a good chance of winning. Context plays an important role in the interpretation of mathematical evidence. In this instance, we see that students are able to take the numbers as drawn (experimental data) and explain why it doesn't behave as they predicted. According to Borasi (1992):

> Mathematical applications require not only good technical knowledge but also the ability to take into account the context in which one is operating, the purpose of the activity, the possibility of alternative solutions, and also personal values and opinions that can affect one's decisions. Unfortunately, none of these elements is usually recognised as relevant to mathematical activity by people who have gone through traditional schooling. (p. 160)

Conclusion

The purpose of this research was to begin to identify some key components of Argument-Based Inquiry as it might take place in primary mathematics classes. Four components that appear essential are suggested: the addressing of a purposeful inquiry question; the advancing of mathematical evidence to enable a claim to be made (in the illustrated unit the students' bingo cards formed the basis of their claim); the justification of a claim through epistemically acceptable reasoning; and, the acknowledgement of context. It is suggested that these components are likely present, or a requirement, of all ABI in mathematics. However, at the level of the youngest children, there may not be an explicit acknowledgement of *claim*, *evidence*, *reasoning*, and *context* by the children. However, it is essential that the teacher can identify these components and guide students towards there development.

Argumentation structures and practices offer the means to focus students on the need for quality evidence and thus encourage students to focus deeply on mathematical content. Much of the work with argumentation that has already occurred in mathematics is associated with justification of procedural choices to arrive at a correct answer, or on the defence of the answer itself. By contrast, mathematical ABI offers the opportunity for students to engage in ill-structured, ambiguous problems that have neither a defined solution path nor a single correct answer. Thus, while this is only a small beginning, there appears to be potential for argumentation to be effective in deepening student focus on developing mathematical evidence and reasoning in inquiry-based learning environments.

Acknowledgements

The author would like to thank the teachers involved for their willingness to share their expertise, time and classrooms. The author acknowledges financial support from an Australian Postgraduate Award and from the Australian Research Council (DP120100690) in conjunction with Education Queensland and The University of Queensland.

References

Allmond, S., & Makar, K. (2010). Developing primary students' ability to pose questions in statistical investigations. In C. Reading (Ed.), *Proceedings of the 8th International Conference on Teaching Statistics*. Voorburg, The Netherlands: International Statistical Institute.

Anderson, R. D. (2002). Reforming science teaching: What research says about inquiry. *Journal of Science Teacher Education, 13*(1), 1-12.

Blair, J. A. (2012). Argumentation as rational persuasion. *Argumentation, 26*(1), 71-81. doi: 10.1007/s10503-011-9235-6

Borasi, R. (1992). *Learning mathematics through inquiry*: Portsmouth, NH: Heinemann.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Corbin, J. M., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, CA: Sage Publications.

Eraut, M. (1994). *Developing professional knowledge and competence.* New York: Routledge.

Fielding-Wells, J., & Makar, K. (2008). Using Mathematical Inquiry to Engage Student Learning within the Overall Curriculum In J. Adler & D. Ball (Eds.), *Eleventh International Congress of Mathematics Education (ICME11) Mathematical Knowledge for Teaching* (pp. 1-17). Monterrey, Mexico, 6-13 July 2008.

Fielding-Wells, J., & Makar, K. (2012). Developing primary students' argumentation skills in inquiry-based mathematics classrooms. In K. T. Jan van Aalst, Michael J. Jacobson, & Peter Reimann (Eds), *The Future of Learning: Proceedings of the 10th International Conference of the Learning Sciences [ICLS 2012] - Volume 2 Short Papers, Symposia, and Abstracts.* (pp. 149-153). Sydney, NSW, Australia: International Society of the Learning Sciences.

Flick, U. (2009). *An introduction to qualitative research* (4 ed.). London: SAGE.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Lumer, C. (2010). Pragma-dialectics and the function of argumentation. *Argumentation, 24*(1), 41-69. doi: 10.1007/s10503-008-9118-7

Makar, K. (2012). The pedagogy of mathematical inquiry. In R. Gillies (Ed.), *Pedagogy: New developments in the learning sciences* (pp. 371-397). Hauppauge, NY: Nova Science.

McNeill, K. L., & Krajcik, J. (2012). Middle school students' use of appropriate and inappropriate evidence in writing scientific explanations. In M. Lovett & P. Shah (Eds.), *Thinking with data* (pp. 233-266). New York: Lawrence Erlbaum Associates.

McNeill, K. L., & Martin, D. M. (2011). Claims, evidence, and reasoning. *Science and Children, 48*(8), 52-56.

Nettel, A. L., & Roque, G. (2012). Introduction. *Argumentation, 26*(1), 1-17. doi: 10.1007/s10503-011-9238-3

Sampson, V., & Clarke, D. (2006*). Assessment of argument in science education: A critical review of the literature*. Paper presented at the International Conference of the Learning Sciences, Indiana.

Sandoval, W. A., & Millwood, K. A. (2007). What can argumentation tell us about epistemology? In S. Erduran & M. P. Jimenez-Aleixandre (Eds.), *Argumentation in science educatio*n (pp. 71 - 88): Springer.

Siegel, H., & Biro, J. (1997). Epistemic Normativity, Argumentation, and Fallacies. *Argumentation, 11*(3), 277-292.

Toulmin, S., Rieke, R., & Janik, A. (1984). *An introduction to reasoning* (2 ed.). New York: Macmillan.

van Eemeren, F. H., & Grootendorst, R. (2004). *A systematic theory of argumentation: The pragma-dialectical approach*. New York: Cambridge University Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Zeidler, D. L. (1997). The central role of fallacious thinking in science education. *Science Education, 81*(4), 483-496.

Zembal-Saul, C., McNeill, K. L., & Hershberger, K. (2013). *What's your evidence? Engaging K-5 students in constructing explanations in science*. Boston, MA: Pearson.

STEM Education: What Does Mathematics Have To Offer?

Noleine Fitzallen
University of Tasmania
<noleine.fitzallen@utas.edu.au>

The emphasis on science, technology, engineering, and mathematics (STEM) education in recent times could be perceived as business as usual or as an opportunity for innovation and change in mathematics classrooms. Either option presents challenges for mathematics educators who are expected to contribute to the foundations of a STEM literate community. A greater understanding of the implications of a STEM education for mathematics education is needed. This paper seeks to add to conversations about the implications of STEM education for the learning and teaching of mathematics.

Ongoing calls for strengthening the nation's skills in Science, Technology, Engineering, and Mathematics (STEM) (Australian Industry Group, 2013; Marginson, Tytler, Freeman, & Roberts, 2013) are fuelled by the imperative to foster national and global economic growth. For this to occur, it is acknowledged that it is necessary to generate more graduates who have the capacity to pursue science-based careers in the future (Office of the Chief Scientist [OCS], 2013). In Australia, however, it is considered that there is:

> …too little time on average spent teaching science in primary school; declining interest in the study of STEM disciplines in senior secondary school; limited growth, even decline in particular areas of the natural and physical sciences, in branches of engineering and information technology at tertiary level; and STEM skill shortages in the workforce. (Office of the Chief Scientist, 2013, p. 10)

As a result, there have been efforts to promote STEM education and what it has to offer. For example, the publication, *Australia's Future: STEM Launches Stars into Orbit* (OCS, 2014), showcases a diverse range of young, high achieving scientists who have forged careers in STEM fields. The profiles presented give brief descriptions of each individual's motivation, interest, experience at school, and pathway to the chosen career. Common to many of the profiles are participation in extension activities while at school, such as the Science, Mathematics, Physics, or Chemistry Olympiads. These are learning opportunities made available to students who are identified as exceptional or talented in those fields of study. This sort of publication serves to highlight the diversity of innovation and career pathways possible in Australia but does little to support teachers to enact a curriculum that will fulfil the expectations of "developing a scientifically literate and numerate society... [and] nurturing student interest in science and influencing their study and career choices" (OCS, 2013, p. 14) in regular mathematics classrooms.

To take advantage of career opportunities in the future, individuals need to develop 21st century skills, which include critical thinking, team work, problem solving, creativity, analytic reasoning, and communication (Bowman, 2010). These are evident when individuals "can manage their own wellbeing, relate well to others, make informed decisions about their lives, become citizens who behave with ethical integrity, relate to and communicate across cultures, work for the common good and act with responsibility at local, regional and global levels" (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2013, p. 3). These learning, literacy, and life skills are epitomised in the General Capabilities detailed in the Australian Curriculum, which are Numeracy, Literacy, Information and Communication Technology (ICT) Capability, Critical and Creative Thinking, Personal and Social Capability, Intercultural Understanding, and

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 237–244. Sunshine Coast: MERGA.

Ethical Understanding. These capabilities align with the notions of literacy that apply to the STEM disciplines: scientific literacy, technological literacy, and engineering literacy, and mathematical literacy (Sneider & Purzer, 2014). However, schools vary in the extent to which they incorporate the development of those skills. Hence, it is worth exploring ways in which STEM education can be implemented effectively.

What is STEM?

In its simplest form, STEM is an acronym for the four independent disciplines of science, technology, engineering, and mathematics, which often involve traditional disciplinary coursework. This view is reflected in the way in which the Australian Curriculum is structured with separate subject areas for each of the disciplines, with the exception of engineering, which is addressed implicitly in the *Australian Curriculum: Technology* and the *Australian Curriculum: Science* (ACARA, 2015). In the US, engineering is included in the *Next Generation Science Standards* (National Academy of Sciences, 2013). It is acknowledged explicitly along with Life science, Physical science, and Earth and space science.

Implementing the school curriculum in subjects that are discreet areas of study limits students' opportunities to develop the 21st century skills touted as being necessary to take advantage of career opportunities in the future (Rennie, Wallace, & Venville, 2012). Relabelling the subject areas and referring to them collectively through the use of the STEM acronym only maintains the "status quo educational practices that have monopolized the [education] landscape for a century" (p. 21) and does not result in changes to educational practices (Breiner, Johnson, Harkness, & Koehler, 2012; Sanders, 2009). At the other extreme, Moore and Smith (2014) suggest there is the potential for STEM to be a "discipline" in its own right. This view is idealistic and is likely to draw much criticism but raises the question "Should STEM be a discipline in the Australian curriculum?" Sanders (2009) suggests this is not viable. One of the reasons given is because the demand on teachers to have sufficient content knowledge as well as pedagogical content knowledge across all of the four STEM disciplines and their integration is too great. Not achieving these demands may result in inadequate content knowledge in some areas, which has the potential to impact negatively on teachers' ability to implement integrative pedagogical approaches in meaningful ways (Treacy & O'Donoghue, 2014).

Conceptually and by its very nature, STEM is interdisciplinary because it is comprised of other disciplines (Treacy & O'Donoghue, 2014). Smith and Karr-Kidwell (2000) conceptualise the interdisciplinary nature of STEM as "a holistic approach that links the [individual] disciplines so that learning becomes connected, focused, meaningful, and relevant to learners" (p. 24). A complementary but different holistic view is offered by Shaughnessy, who suggests "STEM education refers to solving problems that draw on concepts and procedures from mathematics and science while incorporating the teamwork and design methodology of engineering and using appropriate technology" (2013, p. 324). This view harnesses the characteristics of each of the disciplines in an interrelated manner.

Another interrelated integrative approach, *Authentic Integration*, is suggested by Treacy and O'Donoghue (2014). Their model is underpinned by four main characteristics: knowledge development, synthesis and application; focused inquiry resulting in higher order learning; application to real-world scenarios; and rich tasks (Figure 1). This model is applicable to the individual STEM disciplines as well as the integrative notion of STEM. It focuses on the way in which each of the characteristics supports the other characteristics and does not rely on the inquiry processes and ways of working that apply to particular disciplines.

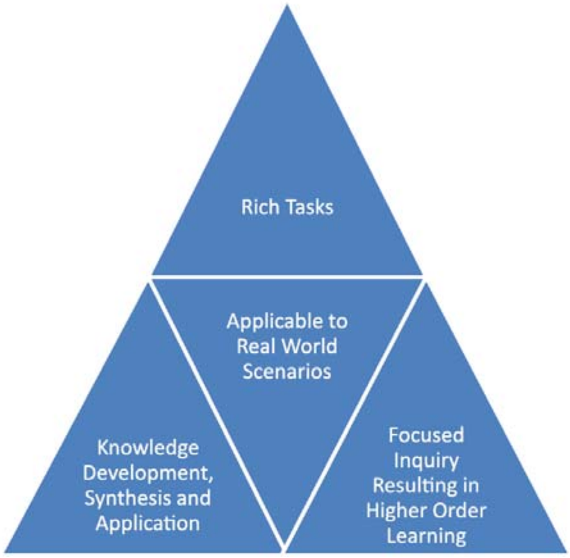


Figure 1. Authentic Integration model (Reproduced from Treacy & O'Donoghue, 2014, p. 710).

Other notions of STEM do not rely on making connections across all four disciplines collectively. They suggest that making connections between/among any two or more STEM disciplines or between/among a STEM discipline and one or more other school subjects is taking an integrative approach (Sanders, 2009). This view gives teachers the freedom to enact STEM through the disciplines with which they are most familiar but does not assure all the disciplines will be addressed sufficiently (Shaughnessy, 2013). Regardless of the extent the disciplines are integrated, the main aim is to support student learning in the traditional content areas (Cardella, Purzer, & Strobel, 2014) and support students to connect content and concepts from the STEM disciplines to create new knowledge (Ostler, 2012).

What Does an Integrative STEM Education Have to Offer?

The integration of STEM subjects is advocated by many as it is seen as a way of engaging students in real-world problems, promoting recall, and enhancing knowledge transfer (e.g., Berry, Chalmers, & Chandra, 2012; Moore & Smith, 2014; Ostler, 2012). It provides ways of placing the learning of mathematics within meaningful contexts and promotes the use of hands-on activities that link to real world problems (Treacy & O'Donoghue, 2014). A potential product of an integrative STEM education is described by Sneider and Purzer (2014) as:

> …a person who has sufficient knowledge and skills in all four fields to participate and thrive in the modern society with confidence and the capacity to use, manage, and evaluate the technologies prevalent to everyday life, as well as the capacity to understand scientific principles and technological processes necessary to solve problems, develop arguments, and make decisions. (p. 9)

It is very desirable for students to develop these capabilities, which are analogous with the expectations of the Australian Curriculum (ACARA, 2015). It is reported that integrative approaches improve students' interest and learning in STEM (e.g., Bottge, Grant, Stephens, & Rueda, 2010; Moore & Smith, 2014; Moore, Stohlmann, Wang, Maruyama Tank, & Roehrig, 2014; Mulligan & English, 2014) yet little evidence is available about the effectiveness of the varying integrative approaches. An exception is a study that reported on a meta-analysis of 98 studies that were identified as investigating the effects of integrative approaches among STEM subjects (Becker & Park, 2014). The meta-analysis included a synthesis of the results reported for seven types of integration: E-M-S-T, E-S-T, E-T, M-S-T, E-M, E-S, M-S, S-T. The results for the different types of integration varied considerably. Integrating the four disciplines, E-M-S-T, showed a large effect size (1.76), whilst the effect size for integrating engineering and mathematics (E-M) was small (0.03). Also low was the effect size for mathematics (0.23) when integrating mathematics, science, and technology (M-S-T). A study that integrated technology and science in high school had a very large effect size (2.80). Becker and Park suggest that "the types of integration may be the key factor that impact the effects of the integrative approaches among STEM subjects" (p. 31).

Unsurprisingly, Becker and Park (2014) found the effect size for the primary years was greater than in the secondary and college years. The flexible school and classroom structures in the primary years facilitate the implementation of integrative approaches and according to Sanders (2009), teachers should take advantage of the unique opportunity offered to stimulate students' interest in STEM as early in their education as possible. Becker and Park do not go on to elaborate on why the effect sizes for mathematics were low in the integrative STEM studies. This leaves the reasons for the low results open to speculation. Among other reasons, low results may be due to the lack of teacher content knowledge (Treacy & O'Donoghue, 2014) or due to a lack of focus on the mathematics (Schmidt & Houang, 2007).

Becker and Park's (2014) results are offered with a note of caution. They admit that the number of studies included in their study is considered low for a meta-analysis. The extensive search they undertook revealed that many studies did not report on the mathematics achievement of students. For example, a study of K-5 students conducted by Hefty (2015) describes a school program that implemented tasks that focused on the engineering design process. The author highlights the mathematics outcomes targeted in each activity, such as measurement of height and angles in the *Laser Light Maze* activity, but does not detail the specific curriculum outcomes addressed nor does he report on the students' achievement according to those mathematics outcomes. Hefty reports learning gains in mathematics achievement that exceed the district and state averages but does not go as far as providing specific details about that achievement or how the results were determined. He also reports that "teachers notice carryover from engineering to mathematics lessons" (p. 427). This implies the students gain a lot from the activities but to be convincing that an engineering design integrative approach in STEM education is going to impact positively on the learning of mathematics outcomes, more evidence is required.

What are the Implications for the Learning and Teaching of Mathematics?

The implementation of an integrated STEM education raises many challenges for the teaching and learning of mathematics but "transforming the current educational paradigm toward a STEM education perspective" (Breiner et al., 2012, p. 3) has the potential to "foster the connectedness that reflects the way the world works outside of school and assist students to develop the knowledge and ability to deal with change and challenge in sensible ways" (Rennie, Wallace et al., 2012, p. 1). There does not, however, appear to be one teaching approach established for the implementation of STEM education (Berry et al., 2012; Herschbach, 2011; Rennie, Venville, & Wallace, 2012) that will optimise the opportunities for students to develop the STEM skills proposed by Sneider and Purzer

(2014). The individual disciplines do, however, have dominant practices that complement the implementation of STEM education. Those practices include problem-based learning, project-based learning, scientific inquiry, and engineering design (Rennie, Venville et al., 2012). Research has focused on these discipline practices (e.g., English, Dawes, & Hudson, 2013; Moore et al., 2014; Rennie, Venville et al., 2012) but noticeably absent from the literature is the conceptualisation of mathematics practices and concepts that have the potential to contribute to the understanding of other disciplines (Rennie, Venville et al., 2012).

Although the connectedness and applicability of mathematics to real-world contexts and across disciplines is fostered when integrative approaches are adopted (Berry et al., 2012) the implementation of these approaches has the potential to disrupt the coherence of mathematics learning programs (Schmidt & Houang, 2007). Schmidt and Houang contend that coherence in the delivery of the mathematics curriculum is critical when seeking to improve the student achievement. Their view of coherence was determined from an analysis of the Third International Mathematics and Science Study conducted in 1997. Coherence is present when the content covered increases in complexity from simple mathematics and routine computational procedures with fractions, say, to deeper structures, such as understanding the rational number system and its properties. This development occurs both over time within a particular grade level as concepts and ideas are introduced for the first time, and then are built upon through the years as students progress across grades (Schmidt & Houang). This implies that both coverage of the content and depth of understanding are the focus of learning opportunities.

What Does Mathematics Have to Offer STEM?

According to Shaughnessy (2013), "the M will become silent if not given significant attention" (p. 324) when implementing integrative STEM education programs. The difficulty of improving outcomes in mathematics when implementing integrative approaches (Becker & Park, 2014) warrants particular consideration, especially in relation to the coherence and coverage of the mathematics curriculum (Schmidt & Houang, 2007).

It is not uncommon to find in the literature reports that suggest STEM education learning opportunities provide the context for enhancing the development of mathematical skills (e.g., Alfieri, Higashi, Shoop, & Schunn, 2015; Hefty, 2015; Magiera, 2013; Smith et al., 2013). These examples, however, do not acknowledge the reciprocal nature of the relationship between mathematics and STEM. They provide examples of STEM education opportunities that support the development of mathematical ideas and concepts but do not exemplify the way in which mathematics can influence and contribute to the understanding of the ideas and concepts of other STEM disciplines.

In some cases, the mathematics is incidental to the purpose of activities. For example, an activity that requires students to explore the characteristics of aluminium baseball bats to develop an understanding of the denting strength of the bats was designed to support students' development of problem solving skills (Magiera, 2013). The mathematics of measures of centre was applied to determine the average size of aluminium crystals on the surface of baseball bats. In this example, the mathematics was not pivotal to the success of the activity but contributed to the outcomes identified. The activity did not extend to using or developing the proportional reasoning skills needed to understand the concept of density that is directly related to the strength of the bats.

Alfieri et al. (2015) also describe a proportional reasoning activity suitable for middle school students where the mathematics is incidental to the STEM context. They use the context of an animated robotics game, *Expedition Atlantis*, to provide situations where proportional reasoning calculations are made. For example, students calculate how many times the wheel of an underwater robotic device need to turn in order to move a particular distance. Although robotics is a common STEM context where mathematics is used to develop students' understanding of how to manipulate and move machines (e.g., Allen, 2013; Silk, Higashi, Shoop, & Schunn, 2010), in this case, the context of robotics is used to motivate students to make practice the mathematical procedures targeted.

Another activity, *Exploring Slope with Stairs and Steps* (Smith et al., 2013), utilised mathematics in an instrumental way. This activity involved using rates of change to develop an understanding of the concepts of slope and steepness within an engineering context. In this case, the mathematics associated with rates of change was pivotal to the understanding of the construction of stairs, which also applies to other STEM contexts, such as road construction and the safe descent of vehicles down a hill. Another activity where the mathematics is instrumental to student understanding of the concepts involved in a STEM context is related to the sale of muffins (Baron, 2015). The author states, "Selling muffins introduced the students to quadratic functions" (p. 335) and described the way in which using functions in this context provided the opportunity to review the vocabulary of functions encountered previously, such as coefficient, variable, and exponent. Baron's aim was to use relevant contexts to apply and model quadratic functions. What difference would it make if the teacher's purpose were to use quadratics and functions to make decisions about selling muffins and what would be the role of the mathematics in that scenario?

Silk et al. (2010) suggest that making subtle changes in the design and setup of lessons makes a substantive difference in what students learn. This is demonstrated in their activity that required students to synchronise the movements of robots when dancing. Originally, the activity was designed to get students "to make a dance routine that would incorporate a range of different moves (at different distances, angles, and speeds) and a range of different size robots (that varied on their wheel size and track width)" (p. 25). The project team expected students to generalise their understanding of proportional reasoning to solve the problem. They found that the majority of students used guess-and-check strategies to continually tweak the parameters in their programs until the robots looked synchronised with each other. It was not until Silk et al. made the role of the mathematics in the activity explicit to the students through the redesign of the activity that the students were able to use the mathematics purposefully and connect with the underlying general relationships associated with making the robots move synchronously.

Conclusion

Mathematics is often mentioned as underpinning the other disciplines of STEM because it serves as a language for science, engineering, and technology (Schmidt & Houang, 2007). Is that sufficient acknowledgement of the potential role of mathematics in STEM learning contexts? Stating that mathematics underpins the other disciplines sets mathematics up in a supporting role in integrative STEM education contexts. Ideally, mathematics should be given more standing and be considered an enabler or imperative for the advancement of understanding of concepts in other disciplines. Silk and his colleagues suggest, "One way to do this is to repeatedly foreground" the desired [mathematical] content while temporarily pushing other concepts into the background" (p. 23). A shift in focus from the incidental nature of mathematics in learning activities to a focus on the instrumental nature of the mathematics may be one way of making the mathematics more explicit within STEM learning contexts and activities. Research is required to determine which integrative approaches put mathematics most effectively to the forefront of learning experiences.

In Australia, the push for improved STEM education outcomes comes at a time when there is no integrative STEM curriculum to support its implementation into learning programs. Fortunately, the Australian Curriculum also includes the General Capabilities, which embody the learning, literacy, and life skills considered to be 21st century skills (ACARA, 2015; Bowman, 2010). Incorporating explicit teaching of the capabilities within integrative STEM contexts has the potential to enhance further the outcomes from learning activities. Although structured in disciplines, integration of the Australian Curriculum is possible by addressing the General Capabilities in learning activities across the curriculum together with additional STEM content outcomes. Such an approach is achievable within the school and classroom structures that are dominant in Australia. How this impacts on student learning of the key concepts in mathematics and the other disciplines warrants further research.

Acknowledgements

Thanks to Jane Watson for providing valuable feedback throughout the writing process.

References

Alfieri, L., Higashi, R., Shoop, R., & Schunn, C. D. (2015). Case studies of a robot-based game to shape interests and hone proportional reasoning skills. *International Journal of STEM Education, 2,* Article 4. DOI 10.1186/s40594-015-0017-9

Australian Curriculum, Assessment and Reporting Authority. (2013). *General capabilities in the Australian Curriculum.* Sydney, NSW: ACARA.

Australian Curriculum, Assessment and Reporting Authority. (2015). *The Australian Curriculum, Version 7.3, January 5, 2015*. Sydney, NSW: ACARA.

Australian Industry Group. (2013). *Lifting our science, technology, engineering and maths (STEM) skills.* Sydney: Author.

Baron, L. (2015). An authentic task that models quadratics. *Mathematics Teaching in the Middle School, 20*(6), 335-340.

Becker, K., & Park, K. (2011). Effects of integrative approaches among science, technology, engineering, and mathematics (STEM) subjects on students' learning: A preliminary meta-analysis. *Journal of STEM Education, 12*(5 & 6), 23-37.

Berry, M., Chalmers, C., & Chandra, V. (2012). STEM futures and practice: Can we teach STEM in a more meaningful and integrated way? *Instructional innovations and interdisciplinary research in STEM education*. *2nd International STEM in Education Conference, Beijing, November 24-27*. Retrieved from http://stem2012.bnu.edu.cn/data/long%20paper/stem2012_82.pdf

Bottge, B. A., Grant, T. S., Stephens, A. C., & Rueda, E. (2010). Advancing the maths skills of middle school students in technology education. *National Association of Secondary School Principals Bulletin, 94*(2), 81-106.

Bowman, K. (2010). *Background paper for the AQF Council on generic skills.* Retrieved from http://www.aqf.edu.au/wp-content/uploads/2013/06/Generic-skills-background-paper-FINAL.pdf

Breiner, J. M., Johnson, C. C., Harkness, S. S., & Koelher, C. M. (2012). What is STEM? A discussion about conceptions of STEM in education and partnerships. *School Science and Mathematics, 112*(1), 3-11.

Cardella, M. E., Purzer, S., & Strobel, J. (2014). Looking ahead. In S. Purzer, J. Strobel, & M. E. Cardella (Eds.), *Engineering in pre-college settings: Synthesizing research, policy, and practices* (pp. 419-425). West Lafayette: Purdue University Press.

English, L. D., Hudson P., & Dawes, L. (2013). Engineering-based problem solving in the middle school: Design and construction with simple machines. *Journal of Pre-College Engineering Education Research, 3*(2), Article 5.

Hefty, L. J. (2015). Applying mathematics during engineering design challenges can help children develop critical thinking, problem solving, and communication skills. *Teaching Children Mathematics, 21*(7), 422-429.

Herschbach, D. R. (2011). The STEM initiative: Constraints and challenges. *Journal of STEM Teacher Education, 48*(1), 92-122.

Magiera, M. T. (2013). Model eliciting activities: A home run. *Mathematics Teaching in the Middle School, 18*(6), 348-355.

Marginson, S., Tytler, R., Freeman, B., & Roberts, K. (2013). *STEM: Country comparisons.* Melbourne: ACOLA.

Moore, T. J., & Smith, K. A. (2014). Advancing the state of the art of STEM integration. *Journal of STEM Education: Innovations and Research, 15*(1), 5-10.

Moore, T. J., Stohlmann, M. S., Wang, H-H., Maruyama Tank, K., & Roehrig, G. H. (2014). Implementation and integration of engineering in K-12 STEM education. In S. Purzer, J. Strobel, & M. E. Cardella (Eds.), *Engineering in pre-college settings: Synthesizing research, policy, and practices* (pp. 419-425). West Lafayette: Purdue University Press.

Mulligan, J., & English, L. (2014). Developing young students' meta-representational competence through integrated mathematics and science investigations. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 493-500). Sydney: MERGA.

National Academy of Sciences. (2013). *Next generation science standards: For states, by states.* Washington: National Academies Press.

Office of the Chief Scientist (2013). *Science, technology, engineering and mathematics in the national interest: A strategic approach.* Canberra: Australian Government.

Office of the Chief Scientist (2014). *Australia's future: STEM launches stars into orbit*. Canberra: Australian Government.

Ostler, E. (2012). 21st century STEM education: A tactical model for long-range success. *International Journal of Applied Science and Technology, 2*(1), 28-33.

Sanders, M. (2009). STEM, STEM education. STEMmania. *The Technology Teacher*, *68*(4), 20-26.

Schmidt, W. H., & Houang, R. T. (2007). Lack of focus in the mathematics curriculum: A symptom or a cause? In T. Loveless (Ed.), *Lessons learned: What international assessments tell us about math achievement* (pp. 65-84). Washington: Brookings Institution Press.

Shaughnessy, M. (2013). Mathematics in a STEM context. *Mathematics Teaching in the Middle School*, *18*(6), 324.

Silk, E. M., Higashi, R., Shoop, R., Schunn, C. D. (2010). Designing technology activities that teach mathematics. *The Technology Teacher*, *69*(4), 21-27.

Smith, J., & Karr-Kidwell, P. J. (2000). The interdisciplinary curriculum: A literary review and a manual for administrators and teachers.

Smith, T. M., Seshaiyer, P., Peixoto, N., Suh, J. M., Bagshaw, G., & Collins, L. K. (2013). Exploring slope with stairs and steps. *Mathematics Teaching in the Middle School, 18*(6), 370-377.

Sneider, C., & Purzer, S. (2014). The rising profile of STEM literacy through national standards and assessments. In S. Purzer, J. Strobel, & M. E. Cardella (Eds.), *Engineering in pre-college settings: Synthesizing research, policy, and practices* (pp. 419-425). West Lafayette: Purdue University Press.

Rennie, L., Wallace, J., & Venville, G. (2012). Exploring curriculum integration: Why integrate? In L. Rennie, G. Venville, & J. Wallace (Eds.), *Integrating science, technology, engineering, and mathematics: Issues, reflections, and ways forward* (pp. 1-11). New York: Routledge.

Rennie, L., Venville, G., & Wallace, J. (2012). Reflecting on curriculum integration: Seeking balance and connection through a worldy perspective. In L. Rennie, G. Venville, & J. Wallace (Eds.), *Integrating science, technology, engineering, and mathematics: Issues, reflections, and ways forward* (pp. 123-142). New York: Routledge.

Treacy, P., & O'Donoghue, J. (2014). Authentic integration: A model for integrating mathematics and science in the classroom. *International Journal of Mathematical Education in Science and Technology, 45*(5), 703-718.

The Challenge for Non-first-language-English Academic Publishing in English Language Research Outlets

Vince Geiger
Australian Catholic University
<vincent.geiger@acu.edu.au>

Rudolf Straesser
Giessen University/Australian Catholic University
<Rudolf.Straesser@math.uni-giessen.de>

This paper is a reflective critique of practice within the field of mathematics education in relation to the challenges faced by non-first-language-English speaking academics when they attempt to publish in English language research outlets. Data for this study are drawn from communications between a German and an Australian academic as the Australian assisted the German in negotiating aspects of translation bound by syntactic, semiotic, cultural, and colloquial language considerations. The paper concludes by raising questions about the issue of the use of English as a universal language for the dissemination of new knowledge and offers possible solutions to the problem.

Introduction

There is general acknowledgement that the use of "home" languages in multilingual mathematics classrooms, where children are not yet fully fluent in the language utilised to conduct instruction, limits students' access to, and acquisition of, mathematical concepts (Setati & Planas, 2012). Further, it has been argued that language policies and dominant language ideologies affect students' learning of mathematics through the dynamics of power in bilingual classrooms, as well as the bilingual students' views of access to mathematics (Civil & Planas, 2012). Despite the recognition that the use of "home" languages limits non-multilingual students access to mathematical ideas and their participation in learning communities defined by mathematics classrooms, there appears to be no similar recognition that the access to new knowledge in mathematics education and participation in forums in which the acceptance of these new ideas are debated is more difficult for non-English-first-language mathematics educators because of the dominance of English language journals in this field. A similar issue was raised in the Education Research forum hosted by the web based research social network Researchgate where a participant, Professor Attila Szabo of Eötvös Loránd University, asked the question:

> Does language-mastery barrier trim scientific knowledge and the chance of publication? What is your experience in your field?
> (https://www.researchgate.net/post/Does_languagemastery_barrier_trim_scientific_knowledge_and_the_chance_of_publication_What_is_your_experience_in_your_field, 19 Oct 2013).

Aligned with this question is an issue identified by McKay (2002).

> The increasing number of bilingual speakers of English means that many speakers of English will be using English alongside one or more other languages that they speak, and hence their uses of English may be more specific and limited than monolingual speakers of English ... [thus there is] need to avoid comparing bilingual speakers of English to native speakers, and rather to recognise the many strengths of bilingual users of English who have a rich linguistic repertoire to serve their communication needs. (p. 139)

This paper is a reflective critique of practice within the field of mathematics education in relation to the question posed by Professor Szabo and the issue raised by McKay (2002). The paper draws on the specific experiences of a German academic (Rudolf), while developing a publication for an English language research outlet, and an Australian researcher (Vince) who provided a language check of the paper. Data for this study are

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 245–252. Sunshine Coast: MERGA.

drawn from communications between the German and Australian academics as the Australian assisted the German in negotiating aspects of translation bound by syntactic, semiotic, cultural, and colloquial language considerations. This situation gave rise to the following research question:

What challenges must non-first-language-English mathematics educators negotiate in order to be published in internationally recognised English language research outlets?

In addressing this question we will: (1) provide a review of relevant literature; (2) examine the countries of origin of internationally recognised research publication outlets; (3) analyse exchanges between the two academics related to initial text proposed by the German academic and the suggested edits offered by the Australian academic in order to categorise the types and forms of language divergence, inconsistency and opaqueness between German and Australian English; and (4) offer suggestions that will support effective research publication collaborations between English and non-English-first-language researchers in the future.

Literature Review

While a considerable corpus of research literature exists on teaching and learning mathematics in classrooms with students and teachers who do not have English as first language, there is a paucity of research, or even commentary, on the challenges faced by non-first-language-English-speaking academics when attempting to engage with the broader community of mathematics educators. For example, a search of MERGA publications including, *Mathematics Education Research Journal*, *Mathematics Teacher Education and Development*, and the *Proceedings of the annual conference of MERGA*, using ESL as a keyword yields only seven results, all of which are concerned with the teaching and learning of students in mathematics classrooms (e.g., Miller & Warren, 2014). Similarly, a search of relevant literature from within the US context reveals research related to mathematics and English Language Learners (ELL) but none related to the challenges faced by non-first-language-English academics. Consensus on what is known about mathematics ELLs is that mastery of content, the principles of literacy, and language acquisition are tied together—content, literacy, and language acquisition go hand-in-hand (e.g., Roberson & Summerlin, 2005). It is unclear how such findings would translate to the challenges faced by non-first-language-English academics.

By expanding the search to include non-research publications, a number of handbooks developed specifically for the purpose of providing advice to non-first-language-English speakers (e.g., Glasman-Deal 2009; Burnham & Hutson, 2007) as well as other material available on the internet (e.g. the webpage from Nature Education, 2014) were found.

Taking *Science research writing for non-native speakers of English: A guide for non-native speakers of English* (Glasman-Deal 2009) as an example, advice is specific to particular publication types, such as journal articles, and offers only a single structure for writing empirical articles, that is: Introduction; Methodology; Results; Discussion; and Conclusion. There is no advice on how to structure and write other forms of research publications, such as theoretical or discussion. Within this standardised structure for a research paper, the handbook offers suggestions for appropriate vocabulary and use of grammar as these related to different sections of an article. For example, advice is provided about grammar and writing skills within the Discussion/Conclusion section (see pp. 154-159), where there is a discussion on the use of modal verbs like “should, must, can, ought to, may, could”.

A different approach is offered in *Scientific English as a foreign language* (Burnham & Hutson, 2007) which offers specific advice on how to avoid predictable mistakes in English. Based on the experience of consulting with non-English speaking colleagues, the authors offer 59 comments on mistakes to avoid, for example, common mistakes associated with the use of commas, colons and semicolons, and dashes" (see pp. 34-38). Other examples include discussion of pairs of words with overlapping semantics, such as: locate and localise; borrow and loan; teach and learn; make and do; and, experience and experiment (see pp. 7-14).

Other advice is available from online collaborative learning spaces such as Scitable by Nature Education (2014, http://www.nature.com/scitable) or Unilearning (n.d., http://unilearning.uow.edu.au/academic/2e.html) where self-education modules such as *English communication for scientists* (Nature Education, 2014) are available. This particular module identifies obvious potential difficulties related to spelling and grammar and also flags challenges associated with: (1) expressing concepts associated with a single word in a native language in English where the concept may not exist; (2) expressing, in a precise manner, the subtleties associated with concepts that are similar but not the same as in a native language and English; (3) clarifying the meaning of words that have similar forms but different meanings (so-called false friends) in native language and English; and (4) using two different words in English for two meanings rendered by the same word in the native language.

Each of these problems, and associated advice, connects with broader questions related to the use of English as an international language for the communication of ideas and new knowledge among scientific communities. The issue is a complex one and a matter of debate among academics from non-English speaking countries. Ammon (2001), for example, poses the question of whether English should be accepted as the international language of science or if it should be a general means of communication within scientific communication. He argues that accepting English as the universal language is problematic as there are doubts about whether English can mirror the subtleties of research originally completed in other languages. It has also been noted (e.g., Baldauf, 2001) there are even differences between the way English speakers from different countries use their language, contributing to a lack of clarity in some scientific reports.

The issue of a universal language of science is taken further by McKay (2002) in arguing:

> ...the teaching and learning of an international language must be based on an entirely different set of assumptions than the teaching and learning of any other second or foreign language.

As the assumptions McKay refers to are rarely raised when non-first-language-English speakers attempt to publish in English language publications it is likely, native English speakers underestimate the size of the challenge faced by colleagues from non-English speaking countries.

In summary, the review of literature indicated that while there is general advice available to non-first-language-English-speaking academics on publishing in English language journals, there appears to be no specific advice to those academics whose native tongue is not English about publishing in English language journals devoted to mathematics education. It was also noted that while publication advice is available in handbooks and online forums with a focus on assisting non-first-language-English academics in publishing their work, there appears to be limited, at best, research literature available on this topic.

English Language Journals in Mathematics Education

In attempting to gain a sense of the proportion of internationally recognised English language mathematics education journals in relation to non-English language journals, we conducted a search using the SCImago journal rankings. SCImago is an internationally recognised source of journal metrics. Only journals that achieve a benchmark metric are listed. While other agencies exist that also provide metrics data on journals, space in this paper does not permit a comparison between different journal rankings. SCImago has been selected, in this instance, because of its currently unsurpassed capture of journals in the social sciences – including mathematics education.

The search was initiated by using Mathematics (miscellaneous) as key words, which yielded 386 results. From the resulting list of journals, we followed-up with a manual search of the journals known to publish articles related to Mathematics Education / Didactics of Mathematics. This resulted in the 20 journals listed in Table 1.

Table 1
Ranked journal listed in SCImago under mathematics education

Title	Country
Educational Studies in Mathematics	Netherlands
For the Learning of Mathematics	Canada
International Journal of Computational and Mathematical Sciences	France
International Journal of Mathematical Education in Science and Technology	United Kingdom
International Journal of Mathematics and Mathematical Sciences	United States
International Journal of Science and Mathematics Education	Netherlands
Journal for Research in Mathematics Education	United States
Journal für Mathematik-Didaktik	Germany
Journal of Mathematics Teacher Education	Netherlands
Mathematical Intelligencer	United States
Mathematics Education Research Journal	Netherlands
Mathematische Semesterberichte	Germany
Notices of the American Mathematical Society	United States
PRIMUS	United Kingdom
Pythagoras	South Africa
Research in Mathematics Education	United States
Revista Matematica Iberoamericana	Spain
Teaching Mathematics and its Applications	United Kingdom
Technology, Knowledge and Learning	United States
ZDM - International Journal on Mathematics Education	Germany

Examination of this list shows that the country of origin of these journals stands at 11 English language countries (55%), 4 journals residing in the Netherlands (25%), 3 in Germany (15%), and one each in Spain, and France (5% each). Further scrutiny reveals that the 4 journals listed as emanating from the Netherlands are English language journals (e.g., MERJ). One Journal (ZDM), from Germany, only publishes English texts. Consequently, 16 out of the 20 listed journals are English language (80%) – demonstrating a dominance of English language journals in the field of mathematics education.

Approach and Analysis

In addressing the intent of this paper, we describe and analyse, in the form of a heteroglossic discourse (Bakhtin, 1981), correspondence between a non-first-language-English academic, Rudolf (the second author of this paper), who had been invited to be the respondent to a keynote address at a prestigious international mathematics education conference, and Vince, a first-language-English mathematics educator (the first author of this paper) who provided Rudolf with advice on a draft of his response. Rudolf's response was required to be written in English and published in the conference proceedings as a complement to the paper written by the keynote speaker. Rudolf contacted Vince for advice on his use of English language within the paper.

In outlining and describing this collaboration, we present the original text provided by Rudolf to Vince, Vince's suggested edits to this text, commentary from both Rudolf and Vince related to the nature of the suggested edits, and the fashion in which edits were received. Rudolf initiated the conversation as he was seeking:

> A language check from Vince to be sure that my text was correct English, understandable in terms of the mathematics education arguments, and respecting the terminology in use in this scientific community.

While Vince was very happy to assist he was concerned about exerting too much influence on the text:

> I was, of course, more than willing to assist a colleague. After a long association with Rudolf, I was aware of the difficulties non-first-language-English speakers face in having their work published in prestigious English language outlets. But the request also brought with it challenges for me, as there was a dilemma associated with changing Rudolf's text so that it was not just understandable but also acceptable within the conventions of native English, while at the same time also preserving the author's voice.

Initial Texts and Edits

In this section, categories that represent the types of advice Vince provided to Rudolf are illustrated via excerpts from the text sent to Vince for comment. Each excerpt is accompanied by comments, from both Rudolf and Vince, which are intended to exemplify the type of editorial suggestions made by Vince and Rudolf's responses.

Words that are Not Suited to the Context or are Unfamiliar in English

Original text from Rudolf with comments from Vince

> This definition ***transports*** the three categories of competencies (as defined in a longer, interdisciplinary project sponsored by OECD)...
>
> Comment [Vince]: Do you mean "outlines or "describes" or "communicates"?
>
> Vince: The use of the word transports did not appear to make sense in the context within it was used. Thus I made suggestions to Rudolf about alternatives.
>
> Rudolf: As a German, I was not aware of "transport" being a word not used in this context. Consequently, I had no problem in changing to Vince's first suggestion "outlines", which perfectly met my intentions.

Later, Rudolf used a word that was unfamiliar to Vince.

Looking into Comparative International Surveys (“CIS”), this difference will prove helpful to better understand what CISs do. Following the competence approach of ***sensu*** Chomsky, CISs only gather information on performance.

Comment [Vince]: Are you sure of this word?

Vince: I was simply unfamiliar with the word sensu. I was aware that it is a Latin word used in some scientific disciplines but I did not know if this was a term in common usage in European education research literature or a term drawn from Rudolf’s home tongue. Thus, I asked if it was the right word rather than assuming it was incorrect and offering suggestions.

Rudolf: Checking the word sensu in English dictionaries, I had to realise that it is not a word commonly used in English (even if I was sure I had read it in an English text). So I changed this expression into “The competencies (in the sense of Chomsky and his followers) ...”

A direct translation from a native tongue can seem out of place when viewed by a first-language-English speaker. The challenge associated with this type of choice of words is consistent with the mistakes in English language usage identified by Burnham and Hutson (2007), who provide advice of the selection of words with overlapping semiotics.

Formal Versus Informal Expressions and Literal Translations of Words

At times, Rudolf’s translations took the form of informal expressions when the expectation is that formal language is used in academic papers. There were also examples of literal translations that seemed awkward to a native English speaker.

Original text (Rudolf): In xxx’s plenary, ***I do like*** two messages which I want to highlight and ***bolster up***:

Edited text (Vince): In xxx’s plenary, I would like **to support** two messages, in particular, which I want to highlight and **reinforce**.

Vince: Rudolf used *I do like* – an informal expression in English. I made a suggestion I thought would captured the sense of the original wording, while shifting the expression towards a more formal form. I was, however, concerned about altering the original meaning. Also, *bolster up* appears to be a literal translation of a word Rudolf thought of in German when writing this sentence. I thought the word reinforce would be less jarring to a native English speaker’s ear.

Rudolf: Here, I simply trusted my Australian colleague, who must have a better feeling / knowledge on which words to use.

While it is no surprise that the use of colloquial language in a native tongue would creep into a translation, we did not find and specific advice in the literature. There was, however, advice to be found on the internet (e.g., Unilearning, n.d.).

Use of Punctuation

Rudolf’s use of punctuation was different to that commonly seen in English. In the text below, the use of colons attracted Vince’s attention.

Original text (Rudolf): The first one is a repetition of this year’s conference theme: teaching and learning mathematics have to be discussed in a lifelong perspective, or: mathematics education is an issue “across the life span”.

Edited text (Vince): The first one is a repetition of this year’s conference theme, which the teaching and learning of mathematics must be discussed from a lifelong perspective or that mathematics education is an issue “across the life span”.

Vince: I noticed that Rudolf was using a colon to introduce a pause into his text. I was aware that colons can be used to indicate a long pause is needed in reading a text, but colons are most commonly used in English to mark the beginning of a list, or to mark the beginning of a quote (as in this paragraph). I made a suggestion on how to rewrite the text so that colons were not necessary.

Rudolf: I know that I have a personal over-use of colons. So I simply followed my colleague's advice – not realising that this is a more general issue.

That punctuation mistakes are a challenge for non-first-language-English authors has been identified by Glasman-Deal (2009) and Burnham & Hutson (2007). Again it is no surprise that the conventions of punctuation usage differ across cultures of mathematics education. It is a difficult problem to alleviate as authors, essentially, must unlearn the grammatical conventions of their native tongues in order to write coherently in English.

Words with Different Meanings in their Native Educational contexts

Original text (Rudolf):

The same issue is relevant for a researcher in ***Didactics*** of Mathematics.

Vince: The meaning of the word *didactics*, in English, is often associated with direct teaching methods. In Europe it has a broader meaning including considerations of content knowledge and pedagogy. While I noted the use of the word, which I might have edited if the paper was for an Australian publication, I left the text alone as I knew it would be understood in the context in which it was to be presented.

Rudolf: Vince was correct in noting that *didactics* carries a meaning different in my research community from the use in an Australian and English research tradition. In France (where it is *Didactique*), Germany and Scandinavia, Didactics of Mathematics is the name of the scientific discipline analysing the teaching and learning Mathematics (to make a long story short). It does not have the negative connotation as it has in the Anglo-Saxon tradition.

The same or similar words with different meanings in different languages, as exemplified in the above exchange, has been identified as a frequently encountered difficulty by Burnham & Hutson (2007, e.g. see no. 7). Such words are known as *false friends* in German and other languages (for specific examples and a comment see Nature Education, 2014). In linguistic terms, even if words seem to be the same (in terms of vocabulary), they may be quite different in terms of their related semantic field.

Conclusion

The excerpts from the initial text, the editing suggestions, and the author's reactions have provided some examples of the challenges which non-first-language-English authors from Mathematics Education / Didactics of Mathematics may face when publishing in English language research outlets. While space has prohibited a comprehensive list of such challenges, other issues exist, for example, the use of the original (in our case German) grammar while writing in English. These challenges potentially limit the full participation of non-first-language-English academics in the international mathematics education community.

A number of initiatives are needed to address this situation. The very least of which is that journal editors of English language journals need to be sensitive to this challenge. But what additional help could be provided in order to meet this challenge? Is it possible to identify a committed group of English language colleagues willing to help non-first-language-English authors with writing scientific papers? We believe this to be particularly important for early career researchers. Could the solution outlined and described in this paper become part of the institutionalised support structure of English language journals?

While this might be a plausible solution, it is only possible once English language colleagues are aware of the linguistic and cultural differences faced by non-first-language-English academics, and consequently themselves.

Our discussion has drawn attention to the problems non-first-language-English authors face when attempting to publish in English language research outlets, however, this issue is symptomatic of the situation in which English has become the universal language of science. As identified by McKay (2002), these challenges are far greater than simply learning English as a second language and then conducting a translation from their native tongue into English. Through the preceding discussion, we have attempted to raise the sensitivity of colleagues, in the mathematics education community, to this issue while, at the same time, offering one possible solution through the way in which this paper has been generated. Further systematic research is required, however, to find the best possible solutions.

Acknowledgement

We would like to thank Sara Rezat, Justus-Liebig University Giessen, Germany, for her helpful suggestions related to linguistics and translation.

References

Ammon, U., (Ed.) (2001). *The dominance of English as language of science: Effects on other languages and language communities*. Berlin - New York: Mouton de Gruyter.

Bakhtin, M. (1981). *The dialogic imagination*. Austin, TX: University of Texas Press.

Baldauf, R. B. J. (2001). The use by Australian university science staff of language skills. In U. Ammon (Ed.), *The dominance of English as language of science: Effects on other languages and language communities* (pp. 139-165). Berlin - New York: Mouton de Gruyter.

Burnham, N. A., & Hutson, F. L. (2007). *Scientific English as a foreign language*. Retrieved 15 March 2015 from http://users.wpi.edu/~nab/sci_eng/

Civil, M., & Planas, N. (2012). Whose language is it? Reflections on mathematics education and language diversity from two contexts. In S. Mukhopadhyay & W. H. Roth (Eds.), *Alternative forms of knowing (in) mathematics: Celebrations of diversity of mathematical practices* (pp. 71-89). Rotterdam, The Netherlands: Sense Publishers.

Glasman-Deal, H. (2009). *Science research writing for non-native speakers of English: A guide for non-native speakers of English*. London: Imperial College Press.

McKay, S. L. (2002). *Teaching English as an international language: Rethinking goals and perspectives*. New York: Oxford University Press.

Miller, J. & Warren, E. (2014). Exploring ESL students' understanding of mathematics in the early years: Factors that make a difference. *Mathematics Education Research Journal 26*(4), 791-810.

Nature Education (2014). English communication for scientists: Writing Scientific Papers. *Scitable: A Collaborative Learning Space for science*. Retrieved 15 March 2015 from http://www.nature.com/scitable/ebooks/english-communication-for-scientists-14053993/writing-scientific-papers-14239285

Roberson, S., & J. Summerlin (2005). *Mathematics and English language learners in high school: A review of the literature*. Retrieved 15 March 2015 from: http://www.tsusmell.org/downloads/Products/Resource%20Library/MELL_LitReview_HS.pdf

Setati, M., & Planas, N. (2012). Mathematics education across two different language contexts: A political perspective. In O. Skovsmose & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (167-186). Rotterdam, The Netherlands: Sense Publishers.

Unilearning (n.d.). *Academic writing: Checklist of language to avoid in academic writing*. Retrieved 20 March 2015 from http://unilearning.uow.edu.au/academic/2e.html

The Impact of Let’s Count on Children’s Mathematics Learning

Ann Gervasoni
Monash University
<ann.gervasoni@monash.edu>

Bob Perry
Charles Sturt University
<bperry@csu.edu.au>

Linda Parish
Australian Catholic University
linda.parish@acu.edu.au>

Let’s Count is an early mathematics program that has been designed by The Smith Family and the authors to assist educators in early childhood contexts in socially disadvantaged areas of Australia to work in partnership with parents and other family members to promote positive mathematical experiences for young children (3-5 years). A longitudinal evaluation of *Let’s Count* was undertaken in 2012-2014 involving 337 children in two treatment groups and 125 children in a comparison group. This paper shares preliminary results from the evaluation. Overall the findings demonstrate that *Let’s Count* was effective.

Introduction

Children’s dispositions towards learning mathematics and their formal mathematics knowledge vary considerably when they begin school, partly because of a diversity of experiences and opportunities to explore mathematical contexts and ideas prior to school. There is a wide variation in how well young children will be positioned to benefit from mathematics teaching when they begin school. Many children living in socially disadvantaged communities will be vulnerable. This raises concern about how families, educators, and communities can best promote mathematics learning in early childhood so that all children benefit; and about how to support those who are less favourably positioned than others when beginning school.

The Smith Family (2013), an Australian children’s charity, commissioned *Let’s Count,* an initiative aimed at promoting positive mathematical experiences for young children (3-5 years) in ways that position them to learn mathematics successfully when they start school. This paper reports some initial findings from the *Let’s Count* longitudinal evaluation which has been conducted by the authors. It examines whether participation in *Let’s Count* is associated with increases in children’s performance on mathematics tasks, and explores the implications of the findings for the children’s transition to school. The key research questions investigated were:

1. For which mathematics tasks was participation in *Let’s Count* associated with increased performance?
2. What was the nature of the mathematics underpinning the tasks for which there was a difference?

Disadvantaged Communities and Mathematics Learning

When communities are designated by governments as *disadvantaged*, there can be expectations that, on average, children will not perform as well academically as children from more *advantaged* communities (Caro, 2009). This expectation extends to pre-school children (Carmichael, McDonald, & McFarland-Piazza, 2013; Rimm-Kaufman, Pianta, Cox, & Bradley, 2003). Carmichael et al. (2013) concluded that “the socio-economic status of the community in which the family resides was the strongest home microsystem

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 253–260. Sunshine Coast: MERGA.

predictor of numeracy performance, explaining 10.5% of the variance in the home-community microsystem model". (p. 16)

In contrast, there is also evidence that many young children begin school as capable mathematicians who already surpass many of the first year expectations of mandated mathematics curricula or textbooks (Bobis, 2002; Clarke, Clarke, & Cheeseman, 2006; Ginsburg & Seo, 2000; Gould, 2012; Hunting et al., 2012). For example, Gould (2012) concludes from his study of the results of the mandated Best Start assessment in New South Wales (NSW Department of Education and Communities, 2013) that the expectation in the Australian Curriculum – Mathematics (ACARA, 2013) that students can make connections between the number names, numerals and quantities up to 10 by the end of the first year at school "would be a low expectation for at least half of the students in NSW public schools" (p. 109). Even in *disadvantaged* communities (Ginsburg & Seo, 2000) and rural and regional communities (Hunting et al., 2012), many children show that they are powerful mathematicians before they start school. The examination of children's knowledge presented in this paper will consider whether this is also true for children who participated in *Let's Count*.

Let's Count

Let's Count is an early childhood mathematics initiative commissioned by The Smith Family (an Australian children's charity) to promote positive mathematical experiences for young children (3-5 years). The focus of *Let's Count* is building partnerships between early childhood educators and families who live in disadvantaged communities so that opportunities are cultivated for children to engage with the mathematics encountered as part of their everyday lives, talk about it, document it, and explore it in ways that are fun and relevant to them. Such an approach is designed to enable children to learn powerful mathematical ideas in ways that develop positive dispositions to learning and mathematical knowledge and skills. *Let's Count* was piloted in 2011 in five socio-economically disadvantaged communities spread across Australia. In 2013-2014, The Smith Family delivered a revised *Let's Count* program in additional *disadvantaged* sites in 2013 and 2014 (Gervasoni & Perry, 2013).

Let's Count involves two professional learning modules for early childhood educators: (1) Noticing and exploring everyday opportunities for mathematics; and (2) Celebrating mathematics. Between modules, the educators meet with families to discuss ways that they can encourage children to notice, explore and discuss the mathematics that they encounter in everyday situations, including through games, stories and songs.

One method for evaluating the effectiveness of *Let's Count* was to measure participating children's mathematical growth across their preschool year and contrast this with a comparison group of children whose families had not participated in *Let's Count*. This comparison group was from the same economically disadvantaged communities and provided baseline data in 2012 prior to the introduction of *Let's Count* in 2013-2014.

Method

The *Mathematics Assessment Interview (MAI)* (Gervasoni et al., 2011) is used extensively throughout Australia to measure the mathematical knowledge of children when they begin school and throughout schooling and was used in the *Let's Count* longitudinal evaluation. The *MAI* is a task-based assessment interview, formerly known as the *Early Numeracy Interview* (Clarke et al., 2002), the development of which has been widely reported (e.g., Bobis et al., 2005). The tasks in the MAI are designed to correspond to a research-based learning trajectory in nine mathematics domains: Counting, Place Value, Addition and Subtraction Strategies, Multiplication and Division Strategies, Time, Length and Mass Measurement, Properties of Shape, and Space Visualisation (Clarke et al., 2002).

The interview includes a Foundation Section for school beginners, or any students who have difficulty counting a collection of 20 objects. This Foundation Section was the starting point for assessing the pre-school children in the *Lets Count* longitudinal evaluation. Children were assessed in the domains of Counting, Place Value, Addition and Subtraction Strategies, Multiplication and Division Strategies, Time and Length Measurement, Properties of Shape, and Space Visualisation. Interview stress on the children is reduced through scripted instructions that the interviewer only continues with the next task in any domain (e.g., Place Value) for as long as the child is successful. The interview was conducted by specifically trained interviewers, and independently coded to obtain the data examined in this paper.

Participants

The participants in the *Let's Count* longitudinal evaluation included three groups of children and their parents/caregivers and pre-school educators. The children are the key focus of this paper. Three groups of children including a Comparison Group of 125 children who were assessed in December 2012 and eligible to start school in 2013, and the 2013 and 2014 *Let's Count* groups. The comparison group children attended 10 low SES Early Childhood centres in regional Victoria (5) and New South Wales (5).

The 2013 *Let's Count* Group comprised 142 children eligible to start school in 2014, whose educators and families were going to participate in *Let's Count* during 2013. These children were assessed using the *MAI* in March and November 2013. Of the 142 children assessed in March, 117 were assessed in November. These children came from the same 10 Early Childhood centres as the 2012 Comparison Group.

The 2014 *Let's Count* Group comprised 195 children eligible to start school in 2015, whose educators and families were going to participate in *Let's Count* during 2014. They were assessed in March and December 2014 using the *MAI*. Of the 195 children assessed in March, 172 were assessed in December. These children came from 17 low SES Early Childhood centres in regional Victoria (6), regional NSW (8), and metropolitan Perth, Western Australia (3).

Assessment of Young Children's Mathematics Knowledge Using the MAI

The children were assessed by a team of interviewers who were all familiar with the assessment instruments and with working with young children. All children's responses to the *MAI* tasks were recorded on a detailed record sheet completed by the interviewers. The record sheets were then analysed by independent coders, with all responses entered into an SPSS database. The responses for each task were coded as correct or incorrect, and where appropriate, children's strategies for solving the tasks were also coded. These data were further analysed to calculate the percentage of children in each cohort who were successful with each task and the percentage of students using particular strategies to solve the tasks. The performance of the *Let's Count* children were compared within groups and with the Comparison Group to determine whether any differences between the performances of groups was statistically significant. This paper focuses on the results of these comparisons for the whole number tasks.

Results

The analyses presented in this paper focuses on whether participation in *Let's Count* was associated with improved performance in the Whole Number and Foundation Detour aspects of the Mathematics Assessment Interview. Table 1 shows the results for tasks involving small sets for the children in the 2012 Comparison Group and for the 2013 and 2014 *Let's Count* Groups. Of importance for the analysis was identifying any tasks for which there was a significant difference in performance associated with participation in *Let's Count*.

Table 1
Percentage Success on Tasks with Small Sets

Tasks	Significance: Comparison (Dec, 2012) to (Dec, 2013) (χ^2, p)	Significance: Comparison (Dec, 2012) to (Dec, 2014) (χ^2, p)	Comp Dec 2012 (n=125)	LC Dec 2013 (n=117)	LC Dec 2014 (n=172)
Tasks with Small Sets					
Count a collection of 4 teddies	NS	NS	95	96	97
Identify one of two groups as "more"	NS	NS	90	92	87
Make a set of 5 teddies when asked	7.043, $p<0.01$	10.735, $p<0.01$	77	90	91
Conserve 5 when rearranged by child	NS	6.748, $p<0.01$	79	88	90
Make collection of 7 (when shown number 7)	11.016, $p<0.01$	23.852, $p<0.01$	63	84 (n=92)	89 (n=135)
Knows one less than 7 when 1 teddy removed	12.018, p<0.01 12.018, p<0.01	24.804, p<0.01	61	82 (n=85)	88 (n=131)
Knows one less than 7 without recounting		24.804, p<0.01	25	40 (n=85)	33 (n=131)
One to one Correspondence					
Know 5 straws needed to put 1 straw in each of 5 cups	NS	NS	88	87	95

The results in Table 1 suggest that most children, whether or not they participated in *Let's Count*, were able to accurately count small collections, identify which of two groups was *more* and demonstrate one to one correspondence. These are all important ideas associated with Level 1 in the Australian Curriculum. *Let's Count* made a positive difference to children's ability to accurately make a set of 5 and 7 items and to work out how many teddies remained when one teddy was removed from the set of 7 teddies. Thus it appears that Let's Count was associated with children's increased abilities to produce small collections (as opposed to count collections that someone else produced) and to problem solve with these collections.

The ability to recognise and produce repeating patterns has been noted as an important aspect of young children's algebraic reasoning (Papic, Mulligan, & Mitchelmore, 2011). The next set of results report on this aspect of mathematics. The results in Table 2 show that almost all children can name the colours in a pattern prior to beginning school. However, participation in *Let's Count* was positively associated with increases in children's ability to both match and continue patterns.

Table 2
Percentage Success in Pattern Tasks

Tasks	Significance: Comparison (Dec, 2012) to (Dec, 2013) (χ^2, p)	Significance: Comparison (Dec, 2012) to (Dec, 2014) (χ^2, p)	Comp Dec 2012 (n=125)	LC Dec 2013 (n=117)	LC Dec 2014 (n=172)
Pattern Tasks					
Name colours in pattern	NS	NS	98	99	96
Match pattern	5.623, p<0.05	8.824, p<0.01	72	85	86
Continue pattern	5.102, p<0.05	14.765, p<0.01	34	48	56
Explain pattern	NS	8.464, p<0.01	34	42	51

The tasks in Table 3 involve rote counting, counting collections of 20 items, and ordering numerals. The results show that participation in *Let's Count* was not associated with improvements in children's ability to count to 10 or order numerals from 1-9. Participation was associated with improvements in children accurately counting at least 20 items and in ordering numerals from 0-9. These are certainly the more cognitively challenging tasks in Table 3.

Table 3
Percentage Success with Counting and Ordering Numerals

Tasks	Significance: Comparison (Dec, 2012) to (Dec, 2013) (χ^2, p)	Significance: Comparison (Dec, 2012) to (Dec, 2014) (χ^2, p)	LC Comp Dec 2012 (n=125)	LC Dec 2013 (n=117)	LC Dec 2014 (n=172)
Counting Tasks					
Rote count to 10	NS	NS	87	93	95
Rote count to 20	NS	6.117, p<0.05	45	55	59
Count a collection of at least 20	8.079, p<0.05	13.165, p<0.01	37	55	58
Count a collection of at least 20 & when one item is removed knows total without recounting	8.079, p<0.05	13.165, p<0.01	8	16	11
Ordering Numbers Tasks					
Order numeral cards 1-9	NS	NS	48	60	54
Order numeral cards 0-9	10.354, p<0.01	5.924, p<0.05	31	52	45

The final cluster of tasks involves calculations (see Table 4). Children use small plastic teddies to model the calculation context. The first two tasks involve adding two groups of teddies. The third task requires children to place two teddies in each of 4 cars and then work out the total number or teddies. This task can be solved using multiplicative or additive reasoning, but the strategy used has not been distinguished here.

Table 4
Percentage Success on Calculation Tasks Involving Materials (Teddies)

Tasks	Significance: Comparison (Dec, 2012) to (Dec, 2013) (χ^2, p)	Significance: Comparison (Dec, 2012) to (Dec, 2014) (χ^2, p)	Comp Dec 2012 (n=125)	LC Dec 2013 (n=117)	LC Dec 2014 (n=172)
Calculation Tasks					
Adds 5+3 with materials	NS	17.081, $p<0.01$	49	63	72
Adds 9+4 with materials	9.664, $p<0.01$	7.627, $p<0.05$	25	42	40
Calculates total for 2 teddies in 4 cars	NS	12.005, $p<0.01$	58	64	76

The results presented in Table 4 demonstrate that participation in *Let's Count* was associated with more successful performance on these calculation tasks, although this was more often significant for the 2014 group.

Discussion and Conclusion

Examination of the data demonstrates that participation in *Let's Count* was associated with statistically significant differences in young children's performance on a diverse range of mathematics tasks. What distinguished these tasks was the higher level of mathematics reasoning in which the *Let's Count* children engaged. For example, there were significant differences in the proportion of children who could produce small collections and problem solve with these collections when the *Let's Count* cohorts were compared to those children who did not access *Let's Count*. Producing a specified quantity requires more sophisticated number understanding than simply counting a collection that has been provided. This is demonstrated by the findings in Table 1 showing that almost all children in the 2012 Comparison Group and *Let's Count* groups were successful in counting a collection of four teddies, but only 77% of the Comparison Group could make a set of five teddies and 63% could make a set of seven teddies. In contrast, for the 2013 and 2014 *Let's Count* children, the percentage of correct responses was significantly higher, with over 90% of children correctly making a set of five teddies and over 80% correctly making a set of seven teddies. The *Let's Count* groups were also more successful with working out the total in a larger group of 20 items and in finding solutions for addition and multiplication tasks.

The ability to see and understand patterns has a strong correlation to early algebraic thinking (Papic et al., 2011), which in turn "promotes structural development, relational understanding and generalisation … laying the foundation for mathematical thinking (Papic et al., 2015, p. 221). This highlights that significance of our finding that children in the *Let's Count* groups were more likely than the comparison group to successfully match, continue, and explain a pattern.

There were some significant differences across the three groups of children in the counting domain, particularly in the more demanding tasks of counting to 20, recognising one less, and ordering numerals. The *Let's Count* groups were more successful in ordering numerals (0-9) from smallest to largest, while performance did not differ across the *Let's Count* cohorts and the Comparison Group when children were ordering the numerals from 1-9. This suggests that the children participating in *Let's Count* had a better understanding of zero.

All three calculation tasks provided statistically significant differences between the Comparison Group and the *Let's Count* cohorts, particularly the 2014 group. Perhaps this shows that greater realisation of the mathematics in young children's worlds provides them with opportunities to experience such calculations.

Overall, the findings highlight the extent of many children's mathematics knowledge prior to beginning school. Sometimes, this knowledge exceeds what the children will be asked to *learn* in the first year of school (Gervasoni & Perry, 2015; Gould, 2012). While these data demonstrate that children's knowledge is diverse, it is also apparent that the *Let's Count* children's everyday home and pre-school experiences provided them with a *flying start* as they made the transition to learning mathematics at school. Of interest in extending this research is investigating how successfully these children learn school mathematics and under what conditions the positive impact of *Let's Count* persists.

Acknowledgements

The research reported in this paper was funded by The Smith Family and the Origin Energy Foundation as part of the *Let's Count Longitudinal Evaluation*. The authors acknowledge gratefully the contribution of educators, parents, children, and research assistants.

References

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2013). *The Australian curriculum: mathematics v2.4.* Retrieved from: http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10

Bobis, J. (2002). Is school ready for my child? *Australian Primary Mathematics Classroom. 7*(4), 4-8.

Bobis, J., Clarke, B.A., Clarke, D.M., Gould, P., Thomas, G., Wright, R., & Young-Loveridge, J. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27-57.

Carmichael, C., MacDonald, A., & McFarland-Piazza, L. (2013). Predictors of numeracy performance in national testing programs: Insights from the longitudinal study of Australian children. *British Educational Research Journal.* doi:10.1002/berj.3104

Caro, D.H. (2009). Socio-economic status and academic achievement trajectories from childhood to adolescence. *Canadian Journal of Education, 32*(3), 558-590.

Clarke, B. A., Sullivan, P., & McDonough, A. (2002). Measuring and describing learning: The Early Numeracy Research Project. In A. Cockburn & E. Nardi (Eds.), *PME 26: Proceedings of the 26th annual conference (181-185).* Norwich, UK: PME.

Clarke, B., Clarke, D.M., & Cheeseman, J. (2006). The mathematical knowledge and understanding young children bring to school. *Mathematics Education Research Journal*, *18*(1), 78-102.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., Montgomery, P., Roche, A., Sullivan, P., Clarke, B., & Rowley, G. (2002). *ENRP final report*. Melbourne: ACU.

Gervasoni, A., Parish, L., Hadden, T., Turkenburg, K., Bevan, K., Livesey, C., & Croswell, M. (2011). Insights about children's understanding of 2-digit and 3-digit numbers. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the 23rd biennial conference of the AAMT and the 34th annual conference of the Mathematics Education Research Group of Australasia, Vol 1, pp. 315-323). Alice Springs: MERGA/AAMT.

Gervasoni, A., & Perry, B. (2013). Children's mathematical knowledge prior to starting school. In V. SteinleL. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, pp. 338-345). Melbourne: MERGA.

Gervasoni, A., & Perry, B. (2015). Children's mathematical knowledge prior to starting school and implications for transition. In B. Perry, A. MacDonald, & A. Gervasoni (Eds.), *Mathematics and transition to school: International perspectives* (pp. 51-70). Dordrecht, The Netherlands: Springer.

Ginsburg, H.P., & Seo, K-H. (2000). Preschoolers' mathematical reading. *Teaching Children Mathematics, 7*(4), 226-229.

Gould, P. (2012). What number knowledge do children have when starting Kindergarten in NSW? *Australasian Journal of Early Childhood, 37*(3), 105-110.

Hunting, R., Bobis, J., Doig, B., English, L., Mousley, J., Mulligan, J., Papic, M., Pearn, C., Perry, B., Robbins, J., Wright, R., & Young-Loveridge, J. (2012). *Mathematical thinking of preschool children in rural and regional Australia: Research and practice.* Melbourne: Australian Council for Educational Research.

NSW Department of Education and Communities (2013). *Best Start Kindergarten assessment.* Retrieved from: http://www.curriculumsupport.education.nsw.gov.au/beststart/assess.htm

Papic, M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of pre-schoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237-268.

Papic, M., Mulligan, J., Highfield, K., McKay-Tempest, J., & Garrett, D. (2015). The impact of a patterns and early algebra program on children's learning in transition to school in Australian Indigenous communities. In B. Perry, A. MacDonald, & A. Gervasoni (Eds.), *Mathematics and transition to school: International perspectives* (pp. 217-236). Dordrecht, The Netherlands: Springer.

Perry, B., & Gervasoni, A., (2012). *Let's Count educators' handbook.* Sydney: The Smith Family.

Rimm-Kaufman, S.E., Pianta, R.C., Cox, M.J., & Bradley, R.H. (2003). Teacher-rated family involvement and children's social and academic outcomes in Kindergarten. *Early Education & Development, 14*(2), 179-198.

The Smith Family (2013). *Who we are*. Retrieved from: http://www.thesmithfamily.com.au/

Comparing the Development of Australian and German 7-Year-Old and 8-Year-Old's Counting and Whole Number Learning

Ann Gervasoni
Monash University
<ann.gervasoni@monash.edu>

Andrea Peter-Koop
Bielefeld University
<andrea.peter-koop@uni-bielefeld.de>

This paper compares the counting and whole number knowledge and skills of primary school children in Australia and Germany at the end of Grade 1 and Grade 2. Children's learning was assessed using the Early Numeracy Interview and associated Growth Point Framework. The findings highlight substantial differences between the two groups that vary for the four whole number content domains that have been investigated. These variations are likely due to different curriculum emphases in the two countries.

Introduction

Understanding the mathematical knowledge and capabilities of young children is essential for designing high quality curriculum and teaching methods that enable all children to thrive mathematically at school. Many studies demonstrate that young children learn and use informal mathematical ideas as part of their everyday lives, but countries differ in how they approach more formal whole number learning with young children. A key question is whether these differences matter. Selter, Walther, Wessel, and Wendt (2012) found that at the end of Grade 4 in Australia there were more children in the lowest (9.7% vs. 5.8 %) and highest mathematical competence levels (9.8% vs. 5.2%) than in Germany, while the mean scores in arithmetic (and overall) did not significantly differ between the two countries (p. 103). This suggests that while overall the outcomes of mathematics education in Australia and Germany may be quite similar, the experience and outcomes for children at the higher and lower ends of the competency spectrum may be quite different. In order to explore further the impact of different curriculum and teaching approaches on children's whole number learning, the authors compared the development of 7 to 8-year-old Australian and German children. The children's whole number learning was measured using the task-based *Early Numeracy Interview* and associated Growth Point Framework (Clarke et al., 2002) that was first developed in Australia and then translated into German (Peter-Koop, Wollring, Spindeler & Grüßing, 2007).

Gaining Insights About Young Children's Mathematics Knowledge Using the Early Numeracy Interview

It is well established that teachers need access to high quality information about their students' mathematical knowledge in order to plan effective instruction and to monitor their progress. It is also known that formal written tests are limiting in providing this information about young children as they do not provide information about the strategies that children choose and apply when solving computation problems. For these reasons, the *Early Numeracy Interview* (Clarke et al., 2002; Peter-Koop et al., 2007) was designed especially for young children, is task-based and interactive, derived from extensive research, and enables young children's mathematical learning to be measured in multiple domains. This assessment instrument was originally developed as part of the Early Numeracy Research Project (ENRP) (Clarke et al., 2002; Department of Education,

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 261–268. Sunshine Coast: MERGA.

Employment and Training, 2001). The principles underpinning the construction of tasks and the associated mathematics Growth Point Framework were to:

- describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers;
- reflect the findings of relevant international and local research in mathematics (e.g., Fuson, 1992; Gould, 2000; Mulligan, 1998; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000);
- reflect, where possible, the structure of mathematics;
- allow the mathematical knowledge of individuals and groups to be described; and
- enable a consideration of children who may be mathematically vulnerable (Gervasoni & Lindenskov, 2011; Peter-Koop & Grüßing, 2014).

The development of the interview and Growth Point Framework has been widely reported and is explained in detail in Clarke et al. (2002). The assessment includes four whole number domains (Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies), three measurement domains (Time, Length, and Mass); and two geometry domains (Properties of Shape and Visualisation). Children's growth point data for the four whole number domains are explored in this paper.

The Australian and German Primary School Systems

Children begin school in Australia as a whole cohort in February, after the summer holidays (typical ages are from 4 years 6 months – 5 years 6 months). Australian children are encouraged to complete 15 hours of pre-school in the year before they begin school. This is subsidised by the government. Formal mathematics education begins when children begin school.

In Germany children begin school at 6 years as a whole cohort at the start of the school year in August and after the summer holidays. Most children (over 90%) attend kindergarten prior to school enrolment for at least one year, but more typically for 3 years (between the ages of 3 and 6). Kindergarten education does not follow a mathematics curriculum and is not compulsory. However, kindergarten curricula increasingly acknowledge the importance of early numeracy learning for later success in school mathematics and most kindergarten children would experience activities that involve counting, cardinal, and ordinal numbers as they evolve in every-day situations and in their play. In some cases there is even early support with respect to their mathematics learning prior to school.

Whole Number Learning in Australian and German Mathematics Curricula

The primary school mathematics curriculum in Australia is set by each State and Territory, but follows the framework provided by the Federal Government in consultation with the States. The Australian Curriculum: Mathematics (ACARA, 2013) focuses on the domains of number and algebra, geometry and measurement, and probability and statistics. The curriculum also incorporates four proficiencies: understanding, fluency, problem solving, and reasoning. There is a variety of textbooks used in primary schools, but it is also common for teachers not to use a textbook at all, but rather devise their own tasks or draw on a variety of resources, including textbooks.

Like Australia, the German mathematics curriculum is set by each State following the "National Standards" (KMK, 2005); that is, the curriculum guidelines agreed to by all States. While there is a clear focus on arithmetic in Grades 1 and 2, other content areas include space and shape, measurement, pattern and structure as well as chance and data. Like Australia, the National Standards and respectively the state-based curricula incorporate cross-content proficiencies: communicating and reasoning, problem solving and modelling as well as using representations. In Germany the vast majority of primary mathematics teachers would use one of the major textbooks available for each grade level.

Curricula and Approaches for Teaching Whole Number Concepts and Arithmetic

Teachers in Australia use a variety of teaching approaches for whole number concepts and arithmetic. One common approach is using problems connected to everyday experiences. It is also common for teachers to encourage the use of manipulatives and pictures for modelling a problem to assist children to find a solution. The use of tokens, blocks, and counting frames are customary. Children are encouraged to work in pairs or small groups to discuss their strategies and solutions. Many teachers use a framework, such as the ENRP Growth Point Framework, to evaluate the development of children's whole number learning and arithmetic strategies, and plan experiences that enable children to replace counting-based arithmetic strategies with basic and derived strategies such as building to ten, doubles and commutativity. Initially children work with whole numbers in the range of 1-20 and then expand to increasingly greater number ranges. At this point Multi-base Arithmetic Blocks (MAB) are often used to model the problems and support children's calculation strategies. The Grade 1 Australian Curriculum Mathematics emphasises counting to 100 by 1s, 2s, 5s and 10s, building concepts for numbers to 100, using partitioning to count collections to 100, and representing and solving addition and subtraction problems using a range of strategies including counting-on and partitioning. In Grade 2 the curriculum emphasises investigating number sequences from any starting point, building concepts for numbers to 1000, arranging collections up to 1000 in hundreds, tens and ones to facilitate efficient counting, solving simple addition and subtraction problems using a range of efficient mental and written strategies, recognising and representing multiplication as repeated addition, groups and arrays, and division as grouping into equal sets, and solving simple problems using these representations.

The vast majority of German primary mathematics teachers use a mathematics textbook. In Grade 1 the focus is on whole number arithmetic with numbers up to 20. Counting activities, comparing sets, getting to know and learning to write the numerals from 0 to 9 as well as matching numerals to sets is the focus of the first 4 to 5 months of school. After that, firstly addition and then subtraction is introduced with the aim to help children understand the underlying concepts and to increasingly develop and use heuristic strategies based on derived-facts to replace initial counting-based arithmetic strategies. In most classrooms manipulatives such as the arithmetic rack would be used to model addition and subtraction strategies based on derived facts. In Grade 2 the focus is on addition and subtraction strategies with 2-digit numbers as well as the introduction of multiplicative concepts. This includes understanding and automatising the multiplication facts, as well as understanding division as the counterpart of multiplication and associated with distribution and sharing. Children are invited to share their computation strategies with a partner or small group and discuss multiple strategies for how to solve problems such as 57 – 29. These strategies are also discussed in class and applied to similar problems, emphasising advantages and disadvantages of these different strategies. With respect to the multiplication facts, while teachers would adopt/use different strategies to introduce these (either by tables or by taking a rather holistic discovery based approach), they would spend extensive time and effort on the automisation process.

Methodology

In order to compare the whole number learning of the Australian and German children, *Early Numeracy Interview* data were compared for children who had completed Grade 1 and Grade 2 and who were present for both interviews. This was after the second and third year at school for the Australians and at the end of the first and second years at school for the German children. The 637 Australian children attended school in the States of Victoria and Western Australia, attended schools in low SES communities, were present for both interviews, and were assessed in 2010 and 2011 after the summer holidays (at the beginning of the school year), as was the standard practice. It is noted that, after the summer holidays, it is typical for some children, who may have learnt procedurally, to have some lower growth points than at the end of the school year, but also for some children to reach higher growth points. The 334 German students attended schools in a region in the northwest of Germany and included children from low SES communities to suburbs with predominantly middle class families. The children were assessed before the summer holidays (at the end of Grade 1) in 2013. The Australian students were part of the *Bridging the Numeracy Gap in Low SES and Aboriginal Communities* longitudinal study (Gervasoni, Parish & Upton et al., 2010), and the German students were part of a longitudinal study on children's mathematical development from one year prior to school until the end of Grade 2 (first results of this study are reported in Peter-Koop & Kollhoff, 2015). We do not claim that the two cohorts are closely matched due to the different countries, cultural backgrounds, school starting ages, and curricula. Rather, the selection of participants is pragmatic for enabling the research questions to be investigated. Generalisability of results is not claimed. Importantly, children in both cohorts were about the same age, were assessed using the same instrument, and the results were analysed using the same Growth Point Framework. The research questions guiding the data analysis and comparison are:

1. Does the data show relevant differences between the performances of the Australian and German children in the four whole number domains with respect to each year level and over time?
2. Do any observed differences reflect the different curricula or approaches to teaching whole number concepts and arithmetic in Australia and Germany?

Data Collection and Analysis

The whole number tasks in the Early Numeracy Interview (ENI) take between 20-30 minutes per child and for the studies described in this paper were administered by classroom teachers in Australia and by pre-service teachers in Germany, who all followed a detailed script. The classroom teachers and pre-service teachers were competent with using the interview and had participated in associated professional learning. Throughout the assessment interview process the interviewer continued with the next tasks in a domain for as long as a child was successful, according to the script. The processes for validating the growth points, the interview items and the comparative achievement of students are described in full in Clarke et al. (2002).

A critical role for the interviewer during the assessment was to listen and observe the children, noting their solutions, strategies and explanations for completing each task. These responses were noted in detail on a record sheet and next independently coded to:

- determine whether or not a response was correct;
- identify the strategy used to find a solution; and

- identify the growth point reached by a child overall in each domain.

The assessment data for both groups were analysed to determine children's whole number growth points, according to the ENRP framework (Clarke et al., 2002). The growth point data was then entered into an SPSS database for analysis. Of particular interest for this paper is comparing the distribution of growth points for the Australian and German children to identify any differences.

Results

Figures 1 and 2 show the children's growth point distributions for the four whole number domains across two years of schooling. The data was collected after the first and second years of school for the Germans and after the second and third year of school for the Australians.The children from both countries were approximately 7 years old for the first assessment and 8 years old for the second assessment. For the purpose of discussing the comparative data, and although the Australians were assessed at the start of Grade 2 and Grade 3, we refer to these assessment periods below as 'end' of Grade 1 and Grade 2.

Counting Knowledge and Skills

Figure 1 shows that both groups clearly develop their counting knowledge from Grade 1 to Grade 2. Although the spread of growth points is similar, the median growth point (GP) for the Germans at the end of Grade 1 is GP2 (count 20 items) but for the Australians is GP4 (skip count by 2s, 5s and 10s from zero). One year later the median growth point is GP4 for both groups, and both distributions are similar, except that 19% of the German children reached GP6. We wonder whether the German children's increase in counting knowledge was influenced by the curriculum focus on automatising the multiplication facts that are quite frequently presented in tables that emphasise the counting sequence.

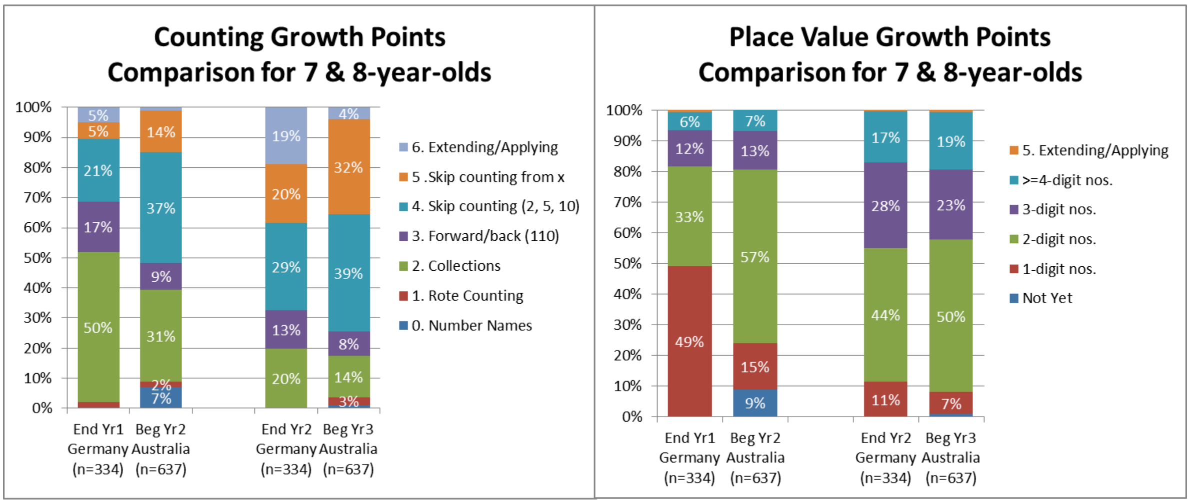


Figure 1. Counting and place value growth point distributions for German and Australian 7 year-old and 8 year-old children.

In order to gain insight about the longitudinal development of children's whole number knowledge, we traced children's knowledge in the Counting Domain for the two preceding years (i.e., immediately before starting Grade 1 (6-year-olds) and one year prior to that (5-year-olds; for details see Peter-Koop, Kollhoff, Gervasoni, & Parish, 2015). The growth point distributions for the 5 year-old children are fairly similar, with the major difference being the number of children able to count 20 teddies (GP2) or count forwards and backwards past 109 (GP3). One year later, as six year-olds, most German children increased one growth point, but a large group of Australian students increased two growth points (typically from GP0 to GP2 or GP2 to GP4). Nearly half of the German 6 year-old children who were attending kindergarten were not yet able to count 20 objects. This type of counting activity is a significant focus of the Australian primary school curriculum, but was not a focus in German kindergartens. Figure 1 highlights that the ability of German children to count 20 items changed dramatically after they began school.

Place Value Knowledge and Skills

While there is a large difference between the two groups concerning the percentage of students who understand 2-digit numbers (GP2) at the end of Grade 1, the spread of knowledge in both groups is almost the same after Grade 2. It is interesting to note that the curriculum in Germany in Grade 2 is limited to numbers up to 100, however 45% of the German children can deal with 3- and 4-digit numbers without that being taught explicitly in school mathematics. In contrast, the curriculum in Australia focuses on numbers to 1000 (GP3) and only 42% of children understand 3- and 4-digit numbers. This suggests that the German curriculum in this domain may be more suitably focused for Grade 1 and Grade 2.

Addition and Subtraction Strategies

The development of Australian and German children’s addition and subtraction strategies appears to follow a different trajectory from Grade 1. By the end of Grade 1 nearly half of the Germans have replaced counting-based strategies with basic and derived strategies, compared with only one-quarter of the Australians. The predominant strategy (72 %) for the Australian children at the ‘end’ of Grade 2 is GP2 (count on) while 80% of German children use more advanced basic and derived strategies (GP5). The Grade 2 German curriculum focuses directly on the development of heuristic strategies and this appears to be reflected in the data.

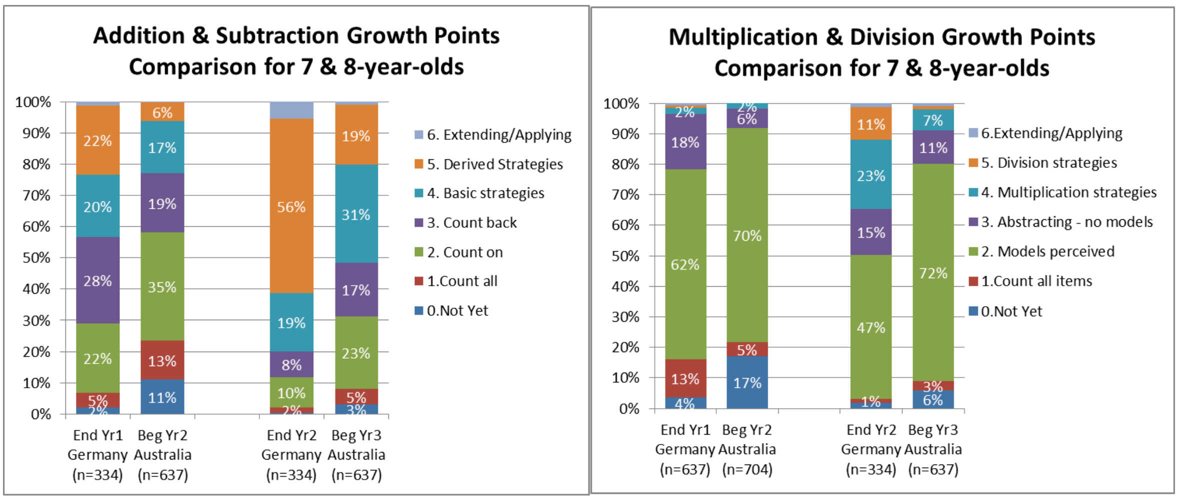


Figure 2. Addition and subtraction strategies and multiplication and division strategies growth point distributions for German and Australian 7-year-old and 8-year-old children.

The Grade 2 Australian curriculum also focuses on children solving simple problems using efficient mental and written strategies, but the heuristic strategies are not as clearly described. Of note is that the Grade 1 Australian curriculum focuses on representing and solving simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts. This focus on representation and counting-on is in stark contrast to the German situation that emphasises heuristic strategies based on derived-facts to replace children's initial counting-based arithmetic strategies.

Multiplication and Division Strategies

At the end of Grade 1, the German and Australian growth point distributions for Multiplication and Division Strategies are quite similar except for the larger group of German children able to use the abstract strategy (GP3) in multiplicative situations. This confirms the trend that German children appear to develop more advanced arithmetic strategies in addition, subtraction, multiplication, and division strategies, while the Australian students reach higher growth points in Counting and Place Value at the end of Grade 1.

After Grade 2, 50% of the German children can solve multiplication and division problems without using any manipulatives (GP 3 and higher) compared with 20% of the Australian children. It is also interesting to note that there are hardly any German children on GP0 and GP1, compared with nearly 10% of the Australian children.

Discussion and Conclusion

The comparisons between the counting and whole number knowledge and skills of German and Australian children highlight some interesting differences. While it is important to note that the Australian children by the end of Grade 2 have spent an additional year at school and certainly show more elaborate competencies in the domains Counting and Place Value at the end of Grade 1, the German children catch up by the end of Grade 2.

While the German children in Grade 1 were more advanced in the Addition and Subtraction Strategies domain, with few differences between groups in Multiplication and Division, their competencies in these two domains significantly increase by the end of Grade 2. Of some concern is that almost 50% of Australian students are still using counting-based strategies (GP1-GP3) at the end of Grade 2 in Addition and Subtraction, compared with about 10% of German children. Typically, this persistent use of counting-based strategies is one criterion for identifying children who are mathematically vulnerable and who may benefit from an intervention program (Gervasoni, 2004).

We hypothesise that the noted differences between the German and Australian children's learning can partially be explained by different emphases in the two curricula at these levels, characterised by a strong focus on the learning and teaching of heuristic computation strategies in Germany, but of counting and place value concepts for numbers up to 1000 in Australia. Further, in Australia, children are more likely to be given manipulatives to help them model calculation strategies. Perhaps this reduces children's opportunities to replace counting strategies with more abstract heuristic strategies. It is likely that the Australian and German children's differing opportunities to formally explore whole number arithmetic at school matters, in at least the short term. It is also possible that the greater Australian curriculum focus on counting and place value for numbers to 1000 by the end of Grade 2 is misplaced. Indeed, the German children perform just as well in these domains, without this higher curriculum expectation.

The findings raise some interesting questions about the influence of curriculum documents and their emphases. Although comparisons between these Australian and

German children are mitigated by differences in cultural backgrounds, country, SES status, and school starting ages, some important differences have emerged. These are most likely explained by differences in curriculum emphases and possibly teaching strategies. Of interest is whether the noted trends and differences persist or diminish over time. This is a profitable area for further research.

References

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2013). *The Australian curriculum: mathematics v2.4.* Retrieved 17 March 2015 from http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., Montgomery, P., Roche, A., Sullivan, P., Clarke, B., & Rowley, G. (2002). *ENRP Final Report*. Melbourne: Australian Catholic University.

Department of Education, Employment and Training. (2001). *Early numeracy interview booklet*. Melbourne: Department of Education, Employment and Training.

Fuson, K. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243-275). New York: Macmillan.

Gervasoni, A. & Lindenskov, L. (2011). Students with ‘Special Rights’ for Mathematics Education. In B. Atweh, M. Graven, W. Secada, P. (Eds.), *Mapping Equity and Quality in Mathematics Education* (pp. 307-323). New York: Springer.

Gervasoni, A., Parish, L., Upton, C., Hadden, T., Turkenburg, K., Bevan, K., Livesey, C., Thompson, D., Croswell, M., & Southwell, J. (2010). Bridging the Numeracy Gap for Students in Low SES Communities: The Power of a Whole School Approach. In Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 202–209). Fremantle: MERGA.

Gervasoni, A. (2004). Extending mathematical understanding: Intervention program specialist teacher manual. Ballarat: Ballarat Heritage Services.

Gould, P. (2000). Count Me In Too: Creating a choir in the swamp. In Improving numerary learning: *What does the research tell us?* (Proceedings of the ACER Research Conference 2000, pp. 23–26). Melbourne: Australian Council for Educational Research.

KMM, Kultusministerkonferenz (2005). *Bildungsstandards im Fach Mathematik für den Primarbereich*. München: Luchterhand.

Mulligan, J. (1998). A research-based framework for assessing early multiplication and division. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, Vol. 2, pp. 404-411*)*. Brisbane: MERGA.

Peter-Koop, A. & Grüßing, M. (2014). Early enhancement of kindergarten children potentially at risk in learning school mathematics – Design and Findings of an intervention study. In U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel & R. Vogel (Eds.), *Early mathematics learning. Selected Papers oft he POEM 2012 Conference* (S. 307-321). New York: Springer.

Peter-Koop, A. & Kollhoff, S. (2015). Transition to school. Prior to school mathematics skills and knowledge of low-achieving children at the end of grade 1. In B. Perry, A. MacDonald, & A. Gervasoni (Eds.), *Mathematics and transition to school. International perspectives* (pp. 65-83). New York: Springer.

Peter-Koop, A., Kollhoff, S., Gervasoni, A. & Parish, L. (2015). *Comparing the development of Australian and German children’s whole number knowledge*. Paper accepted for ICMI-Study 23 Primary Mathematics Study on Whole Numbers“ to be held in Macau, China, June 3–7, 2015.

Peter-Koop, A., Wollring, B., Grüßing, M., & Spindeler, B. (2007). *Das ElementarMathematische Basis-Interview. Zahlen und Operationen* (1st Ed.). Offenburg: Mildenberger.

Selter, C., Walther, G., Wessel, J., & Wendt, H. (2012). Mathematische Kompetenzen im internationalen Vergleich: Testkonzeption und Ergebnisse. In W. Bos, H.Wendt, O. Köller, & C. Selter (Eds.). *TIMSS 2011.* Münster: Waxmann.

Steffe, L., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application.* New York: Praeger.

Wright, R., Martland, J., & Stafford, A. (2000). *Early numeracy: Assessment for teaching and intervention*. London: Paul Chapman Publishing.

Learning at the Boundaries

Merrilyn Goos
The University of Queensland
<m.goos@uq.edu.au>

This paper reports on a project that aims to foster interdisciplinary collaboration between mathematicians and mathematics educators in pre-service teacher education. The project involves 23 investigators from six universities. Interviews were conducted with selected project participants to identify conditions that enable or hinder collaboration, and to identify learning mechanisms at the boundaries between disciplinary communities. A hybrid narrative constructed from the interviews is used to illustrate *transformation* as a learning mechanism that leads to new practices.

Introduction

In Australia, as in many other countries, pre-service teacher education programs are structured so that future teachers of mathematics and science typically learn the *content* they will teach by taking courses in the university's schools of mathematics and science, while they learn *how to teach this content* by taking content-specific pedagogy courses in the school of education. Such program structures provide few opportunities to interweave content and pedagogy in ways that help develop professional knowledge for teaching. A suite of Australian government funded projects is addressing this problem by developing and disseminating new interdisciplinary approaches to mathematics and science pre-service teacher education. This paper reports on preliminary findings from one of the projects – *Inspiring Mathematics and Science in Teacher Education* (IMSITE). The overarching aims of the project are to: (1) foster genuine, lasting collaboration between the mathematicians, scientists, and mathematics and science teacher educators who prepare future teachers and (2) identify and institutionalise new ways of integrating the content expertise of mathematicians and scientists and the pedagogical expertise of mathematics and science teacher educators. The first aim provides the focus for this paper, which explores the potential for learning at the boundaries between disciplinary communities of mathematicians and mathematics educators.

Project Context and Overview

The three-year (2014-2016) IMSITE project is being undertaken by 23 investigators in six universities who are collaborating to develop, test, and evaluate the following approaches:

(a) recruitment and retention strategies that promote teaching careers to undergraduate mathematics and science students;

(b) innovative curriculum arrangements that combine authentic content and progressive pedagogy to construct powerful professional knowledge for teaching;

(c) continual professional learning that builds long term relationships with teacher education graduates, enabling them continually to renew their professional and pedagogical knowledge of mathematics and science.

Three universities are located in state capital cities and three in regional cities. Each university's project team comprises at least one discipline professional (mathematician, scientist) and one education professional.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins*
(Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia),
pp. 269–276. Sunshine Coast: MERGA.

A feature of the IMSITE project approach is its emphasis on diversity. It is not the intention to promote a single model of pre-service teacher education that privileges one structure for degree programs, one way of combining content and pedagogy, or one form of collaboration between discipline and education professionals. In the project's first year, each participating university committed to implement at least one strategy that had already been piloted or tentatively formulated before the project began (see Table 1 for examples). In the second year, the core group of six universities is engaging with a new group of universities to adapt and transfer strategies to new institutional contexts. The third year will be taken up with preparation of case studies of implementation, analysis of survey and interview data collected from project participants, and development of implementation guides to support engagement and transfer of project outcomes to other contexts.

Table 1
Example Teacher Education Strategies Implemented in Year 1

Priority	Strategies
(a) Recruitment and retention	Design courses that provide a taste of education studies to mathematics, science, and engineering undergraduates.
(b) Innovative curriculum arrangements	Design courses that integrate mathematics content and pedagogy, co-taught by a mathematician and a mathematics teacher educator.
(c) Continuing professional learning	Conduct a pre-service teacher education alumni conference to connect current students, graduates, teachers, teacher educators, and mathematicians.

One of the intended outcomes of the project is the development of diverse models of pre-service teacher education that are adaptable to different institutional contexts. This could be viewed as the *product-oriented* outcome of the project. However, an equally important *process-oriented* outcome is concerned with identification of principles for fostering new forms of collaboration between discipline professionals (mathematicians and scientists) and education professionals (mathematics and science teacher educators). The conceptual framework for this latter aspect of the project draws on Wenger's (1998) social theory of learning, and in particular the notions of communities of practice and boundary practices, to understand how the perspectives of mathematicians, scientists, and teacher educators in these fields can be coordinated and connected. At the time the project began, there were few known instances of productive collaboration in the design and delivery of pre-service mathematics and science teacher education programs in Australia, even though it has been argued that both discipline professionals and education professionals have an important role to play in the preparation of teachers (Hodgson, 2001).

The IMSITE project aims to promote strategic change in teaching and learning in the Australian higher education sector. However, the project has also been designed to contribute to a long-term research program that conceptualises learning from a sociocultural perspective (see Goos, 2014). The research program has investigated the learning of school students and teachers (Goos, 2004; Goos & Bennison, 2008), and it is now being extended to explore opportunities to learn through the exchange of expertise across disciplinary boundaries in mathematics education.

This paper is concerned with interactions between the mathematicians and mathematics educators in the project team. Aligned with the first aim of the project – fostering interdisciplinary collaboration – the paper addresses the following research questions:

1. What conditions enable or hinder sustained interdisciplinary collaboration?
2. What learning mechanisms are emerging at the boundaries between communities?

Learning Within, and Between, Communities of Practice

Wenger (1998) argued that learning involves participating "in the *practices* of social communities and constructing *identities* in relation to those communities" (p. 4, original emphasis). He identified *practice* as contributing to the coherence of a community, and described three dimensions of communities of practice: mutual engagement of participants, negotiation of a joint enterprise that coordinates participants' complementary expertise, and development of a shared repertoire of resources for making meaning.

Mathematicians and mathematics educators are members of related, but distinct, communities of professional practice, and it is a fundamental premise of the IMSITE project that connecting the communities is essential to achieving a seamless, meaningful, and rigorous academic preparation for pre-service teachers of mathematics. Wenger (1998) wrote of *boundary encounters* as potential ways of connecting communities. Boundary encounters are events that give people a sense of how meaning is negotiated within another practice. They often involve only one-way connections between practices, such as one-on-one conversations between members of two communities. However, a two-way connection can be established when delegations comprising several participants from each community are involved in an encounter. Wenger suggested that if "a boundary encounter – especially of the delegation variety – becomes established and provides an ongoing forum for mutual engagement, then a practice is likely to start emerging" (p. 114). Such *boundary practices* then become a longer-term way of connecting communities in order to coordinate perspectives and resolve problems.

There is an emerging body of research on learning mechanisms involved in interdisciplinary work on shared problems. This type of work is becoming increasingly important because of growing specialisation within domains of expertise that requires people to collaborate across boundaries between disciplines and institutions. Akkerman and Bakker's (2011) review of this research literature emphasised that boundaries are markers of "sociocultural difference leading to discontinuity in action or interaction" (p. 133). Boundaries are thus dynamic constructs that can shape new practices through revealing and legitimating difference, translating between different worldviews, and confronting shared problems. As a consequence, boundaries carry potential for learning.

Akkerman and Bakker (2011) identified four potential mechanisms for learning at the boundaries between domains. The first is *identification*, which occurs when the distinctiveness of established practices is challenged or threatened because people find themselves participating in multiple overlapping communities. Identification processes reconstruct the boundaries between practices by delineating more clearly how the practices differ: discontinuities are not necessarily overcome. A second learning mechanism involves *coordination* of practices or perspectives via dialogue in order to accomplish the work of translation between two worlds. The aim is to overcome the boundary by facilitating a smooth movement between communities or sites. *Reflection* is nominated as a third learning mechanism that is often evident in studies involving an intervention of some kind. Boundary crossing – moving between different sites – can promote reflection on differences between practices, thus enriching one's ways of looking at the world. The fourth learning mechanism is described as *transformation*, which, like reflection, is found in studies investigating effects of an intervention. Akkerman and Bakker state that transformation is a learning mechanism that can lead to a profound change in practice, "potentially even the creation of a new, in-between practice, sometimes called a boundary practice" (p. 146). They go on to label processes of transformation as including:

- Confrontation – encountering a discontinuity that forces reconsideration of current practices;
- Recognising a shared problem space – in response to the confrontation;
- Hybridisation – combining practices from different contexts;
- Crystallisation – developing new routines that become embedded in practices;
- Maintaining the uniqueness of intersecting practices – so that fusion of practices does not fully dissolve the boundary;
- Continuous joint work at the boundary – necessary for negotiation of meaning in the context of institutional structures that work against collaboration and boundary crossing.

Akkerman and Bakker note that, although transformation is rare and difficult to achieve, it carries promise of sustainable impact. They also propose that identification and reflection, both of which involve recognising and explicating different perspectives, are necessary pre-conditions for transformation to occur.

While boundary practices might evolve spontaneously, they can also be facilitated by *brokering*. Wenger (1998) explained that the job of brokering is complex because it requires the ability to "cause learning by introducing into a practice elements of another" (p. 109). Bouwma-Gearhart, Perry, and Presley (2012) identified brokering as one of the key interdisciplinary strategies for improving pre-service teacher education in the STEM disciplines in US research universities. They found that successful brokers connect the disciplinary paradigms; they are able to speak the specialised languages of mathematics and science, as well as translate the language and concepts of education research into forms that STEM academics can understand and use. Brokers have the ability to understand and coordinate the expertise that academics from all disciplines can contribute to the task of improving pre-service teacher education.

Research Methods

The IMSITE project is jointly led by a mathematician and a mathematics educator (the author of this paper) from one of the participating universities. In the first year of the project, interviews were conducted with the lead investigators based in the other five universities. In Universities A and B, the lead investigators were a mathematician and a mathematics educator, who were interviewed together. In Universities C and D, the lead investigator was a mathematician, and in University E a mathematics educator. The interview for University A was conducted by the two project co-leaders; other interviews were conducted by the lead mathematics educator only. The timing of interviews was arranged to take advantage of events that participants were scheduled to attend. These included the 2014 MERGA conference (June), a project dissemination forum (September), and the *Connections and Continuity* conference organised by the Australian Association of Mathematics Teachers and the Australian Council of Deans of Science to explore the transition in the study of mathematics from school to university (December). Table 2 summarises information about the interview timing and participants.

Interviews were semi-structured to allow for consistency in the topics of inquiry and flexibility in the depth and sequencing of questions. Question prompts included:

- To what extent is there interdisciplinary collaboration between mathematicians and mathematics teacher educators in your university?

- Can you describe any barriers to, and enablers of, such collaboration?
- What types of exchanges and activities that bring together mathematicians and mathematics educators do you consider to be most successful?
- Do you know of any people who act as brokers of interdisciplinary collaboration? What brokering activities do they successfully use? What are their characteristics that make them effective brokers?

Table 2
Interview Timing and Participants

Date	University	Mathematician	Mathematics Educator
September	A	*	*
December	B	*	*
December	C	*	
December	D	*	
June	E		*

Interviews lasted from 20-40 minutes; they were audio-recorded and later transcribed. Analysis of the interviews was guided by the two research questions listed earlier. To answer question (1), regarding enabling/hindering conditions, a content analysis of transcripts identified relevant excerpts and developed a minimal set of categories that allowed similarities and differences in the responses to be highlighted. This part of the analysis was therefore inductive, in moving from data towards principles for developing interdisciplinary collaboration. To answer question (2), regarding emergence of learning mechanisms at the boundary between disciplinary communities, the transcripts were scrutinised for evidence of the mechanisms theorised by Akkerman and Bakker (2011). Supplementary data to address question (2) were drawn from reports presented at a project team meeting in June 2014.

Towards an Understanding of Interdisciplinary Collaboration

What Conditions Enable or Hinder Sustained Interdisciplinary Collaboration?

All participants referred to *personal qualities*, including open mindedness, trust, mutual respect, shared beliefs and values, as being crucial to *enabling* interdisciplinary collaboration. Such qualities allow for productive disagreements and challenges:

> I like the fact that you [mathematician] are challenging what I say, my views of the world. I really value that. Obviously, there's trust there because, I guess, if there wasn't trust I wouldn't be happy. [University B, mathematics educator]

One interviewee (a mathematics educator) identified the importance of having confidence in one's own disciplinary knowledge of mathematics while at the same time being willing to admit ignorance:

> I'm sure that sometimes education people might feel a bit inferior to … mathematicians when they talk to them. Possibly vice versa as well, when they're talking about pedagogy and they [mathematicians] think "I don't know anything about that, that's strange language". So I guess there's that fear of looking like a fool in front of the other, which you've kind of got to get over at some point somehow. [University E, mathematics educator]

A second condition, explicitly mentioned by interviewees from three universities, was identification of a *common or shared problem*. In one case the problem became shared when the mathematician and mathematics educator realised that they could help each other solve problems that were initially unrelated:

> A lot of the stories that X [mathematics educator] told me about what she was facing in terms of challenges with her maths students or the people training to be maths teachers caught my attention; stories of students who weren't capable enough when they were out in the classroom as pre-service teachers. So at that point I knew that I had to put in some effort in terms of meeting X's needs. At the same time X was able to put in effort in meeting my needs because we were having challenges in our first year maths classes around tutorial engagement and that sort of thing. X was able to offer some as a sort of mentoring type of role in an action research project where she was the facilitator. [University A, mathematician]

In other cases, a shared problem was identified when participants recognised that they taught the same pre-service secondary students – "You teach the students maths and I teach them education, we should at least be sharing what we know about the students" (University B, mathematics educator).

A striking *hindrance* to interdisciplinary collaboration, mentioned by interviewees from four universities, was the *physical separation* of the buildings where mathematicians and mathematics educators worked. In one university these disciplines were located on separate campuses, and at the other universities the disciplines were typically on opposite sides of the same campus:

> We are at polar ends of the campus. There's a big gully in between and there is a bridge. So we've got our metaphorical bridge. We alternate weekly meetings between the math and stat side and the education side. So we're walking over to the other side or the other side is coming to us. [University C, mathematician]

A further structural hindrance, identified by interviewees in four universities, was embodied by *workload formulas* or *financial models* that did not recognise or reward interdisciplinary collaboration:

> It's very difficult to get things like what we do [design and teach with a mathematics educator a course on mathematical knowledge for teachers] to be recognised in workload models. We do a lot of things under the radar but we don't actually get acknowledged on our workload. So in a sense we're doing extra stuff. [University A, mathematician]

Despite respectful relationships having been established between the mathematician-mathematics educator pairs who participated in the project, interviewees in three universities referred to entrenched *cultural differences* between the disciplines in their institutions as hindrances to broader collaboration. More often than not, interviewees expressed frustration with the culture of their own discipline:

> It annoyed me when I heard colleagues of mine complain about the other side, the people across the creek. When it came to the science pre-service teachers or the maths pre-service teachers, whatever problems they had, my colleagues blamed the other side. [University A, mathematics educator]

> I think my colleagues are free to let me do whatever I want to do, provide that it doesn't impact on their day-to-day workload and the way they approach what they look to do. So they're very supportive … "but we don't actually care what you're doing". [University B, mathematician]

What Learning Mechanisms are Emerging at the Boundaries between Communities?

Glimpses of some of the learning mechanisms identified by Akkerman and Bakker (2011) emerged during the interviews. The following brief narrative presents a hybrid case constructed from all the interviews. The purpose is not to draw conclusions about boundary practices in any one university, but to illustrate what *transformation* can look like as a mechanism for learning at the boundary between disciplines. (Quotes have been selected from interviews. Names are pseudonyms.)

A mathematician (Carol) is working with a mathematics educator (Tess). Before the IMSITE project began they got to know each other via an externally funded teaching and learning project. Carol was then allocated to the teaching of a first year mathematics subject for pre-service teacher education students. She was surprised by students' apparent lack of mathematical knowledge after having completed 12 years of schooling:

> I was lamenting, "Oh my goodness me, I can't believe they don't know any maths", like they know less that I had anticipated for someone who had come through the Australian schooling system. [Carol, mathematician]

This experience represents a *confrontation*, a kind of discontinuity between the two worlds of school mathematics and university mathematics that prompted Carol to reconsider her current practice as a teacher of university mathematics. Recognising this confrontation led both to explore each other's worlds:

> I learned a lot about how education works and Tess learned a lot about how we function. We broke down some of the scepticism that both sides can have. [Carol, mathematician]

Carol discussed her observations with Tess, who was sympathetic and interested in exploring the differences between teaching mathematics and education in a university environment. Tess remembered "noticing that my pre-service teachers, their content knowledge was not strong", and she pointed out to Carol the areas that she wanted her to focus on in the first year mathematics course. Carol acknowledged that "I was teaching her [Tess's] students at the time", and both thus recognised a *shared problem space* in which both were contributing to the mathematical preparation of future teachers.

Given this problem space, Carol and Tess are working towards a *hybridisation* of practices from their respective disciplinary contexts. The hybrid result is a new mathematics content subject that is jointly planned and taught, as Tess explained:

> We're in the class together, one of us leads and the other acts as a sort of sounding board. We planned the weeks so certain weeks are Carol's weeks and certain weeks are my weeks. [Tess, mathematics educator]

There are encouraging signs that this new hybrid practice will become *crystallised*, or embedded into institutional structures. The teacher education program is under review, and the Heads of Mathematics and Education have invited Carol and Tess to design two new mathematics-specific pedagogy subjects for the revised program. The subjects will be *owned* by Education, with an income sharing arrangement to recognise the teaching contribution from Mathematics.

Despite the success in creating a new hybrid practice, Carol and Tess also *maintain the uniqueness of their established practices* as a mathematician and mathematics educator. Carol acknowledged their complementary expertise when teaching the mathematics subject together:

> We go to class and there are times when she says to me "That's all yours because it's beyond what I understand" and that's fine. Likewise she'll come in and talk about the greats of education and I'm just going blank, no idea. As an educator it comes out very strongly that she's very well practised. [Carol, mathematician]

The collaboration is sustained by *continuous joint work at the boundary* between the two practices. This includes weekly project meetings, attending and teaching into each other's tutorials in mathematics and mathematics education subjects, joint supervision of Honours students, and jointly conducted professional development for practising teachers.

Concluding Comments

Theorising interdisciplinary collaboration in terms of communities and boundary practices makes it possible to conceptualise the boundaries between disciplines as sociocultural differences that are generative of new practices – and, therefore, new learning. This paper has begun to consider what that learning looks like, and what conditions favour or hinder it. Akkerman and Bakker's (2011) classification of learning mechanisms at the boundary, while not a fixed model, does illuminate possibilities that are emerging in the IMSITE project and that could inform the development of future collaborations in other universities. Their review, together with the interview data from the project, also highlights some challenges for sustaining collaboration. One of these is the ambiguous nature of boundaries and the implications for people who work there, especially those who act as brokers between disciplines. As Akkerman and Bakker point out, brokers can feel like they belong to *both* one world *and* the other, or to *neither* one world *nor* the other. This was a challenge articulated by one of the mathematicians who participated in the IMSITE interviews:

> I'm seeing myself more and more in between maths and education, caught a little bit in no man's land so I don't belong to either. I'm not unhappy with that because it's been quite an interesting and exciting mind-opening experience, but I do see that the expertise I'm gaining from being involved in the IMSITE project is not necessarily going to get my career furthered in terms of being a mathematician. [University D, mathematician]

The IMSITE project is providing valuable evidence of learning at the boundary between communities of mathematicians and mathematics educators. It will be important for both communities to support the brokers and boundary crossers who work in this ambiguous space and to acknowledge their innovative role in fostering new practices.

Acknowledgment

This project is funded by the Australian Government Office for Learning and Teaching (grant #MS13-3174). The views expressed in this publication do not necessarily reflect the views of the Australian Government Office for Learning and Teaching.

References

Akkerman, S., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research*, *81*, 132-169.

Bouwma-Gearhart, J., Perry, K., & Presley, J.B. (2012). *Improving postsecondary STEM education: Strategies for successful collaboration and brokering across disciplinary paradigms.* APLU/SMTI Paper 4. Washington, DC: Association of Public and Land-grant Universities. Retrieved from https://www.aplu.org/document.doc?id=4100

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, *35*, 258–291.

Goos, M. (2014). Creating opportunities to learn in mathematics education: A sociocultural perspective. *Mathematics Education Research Journal*, *26*, 439-457.

Goos, M., & Bennison, A. (2008). Developing a communal identity as beginning teachers of mathematics: emergence of an online community of practice. *Journal of Mathematics Teacher Education*, *11*, 41–60.

Hodgson, B. (2001). The mathematical education of school teachers: Role and responsibilities of university mathematicians. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI Study* (pp. 501-518). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Wenger, E. (1998) *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

The Practice of 'Middle Leading' in Mathematics Education

Peter Grootenboer
Griffith University
<p.grootenboer@griffith.edu.au>

Christine Edwards-Groves
Charles Sturt University
<cgroves@csu.edu.au>

Karin Rönnerman
University of Gothenburg, Sweden
<karin.ronnerman@ped.gu.se>

While principals and systemic leaders have a significant role to play in leading, supporting and structuring mathematics education, their influence tends to be indirect and general. However, middle leaders such as curriculum leaders, senior teachers, and faculty heads, exercise their leadership much closer to the classroom, and as such they can have a more direct influence on the quality of teaching and learning in schools. To improve mathematics learning outcomes of student, it is crucial that educational leading is practiced by those with the greatest capacity to bring about positive practical and sustainable change – middle leaders. These school-based curriculum leaders can promote this development by engaging in forms of *Critical Participatory Action Research* that allows them to improve the quality of teaching and learning through an evidence-driven, site-based, collaborative approach.

It has been well known for a long time that leadership is critical for educational reform and this is no less the case in promoting educational development in mathematics education (Sexton & Downtown, 2014). In general, the literature related to leadership focuses on the role and practices of principals and school heads, and indeed their participation is crucial, but it is always at some distance from the classroom. These leaders have the capacity to open a space for pedagogical development and to support innovation, but they are often limited in their capacity to actually make a difference in the classroom. Lingard, Hayes, Mills and Christie (2003) found that the "principal effects on student outcomes were small and indirect" (p. 51), and, "teachers have the greatest impact upon student learning of all 'educational variables'. The effect of principals' practices on student learning are, in contrast, heavily mediated and limited" (p. 148). However, unlike principals, middle leaders are positioned much "closer" to the classroom and the practices that "happen" there and so their potential for impacting student learning is apparent.

While learning occurs in a range of sites within and outside the school, formal education through schooling is primarily focussed on the classroom. The classroom is where all the intentions and requirements of the curriculum meet learners through the practices of the teachers (Edwards-Groves, 2003). It is also the place where the effects of decisions made by principals, and educational managers and bureaucrats, have to be interpreted and enacted to promote learning (Grootenboer & Marshman, in press). It is not surprising then, that a number of studies have highlighted significant role of the teacher in the effectiveness of education (e.g., Lingard et al., 2003). In general, it is the teacher that has to interpret and put into practice the educational policies, programs and procedures in the classroom to facilitate rich student learning. It is the teacher that is the interface between the mathematics curriculum and learners, and so all the educational decisions made 'before or above' to the classroom site, they are always mediated through the teacher (Edwards-Groves, 2003; Grootenboer & Edwards-Groves, 2014).

With this in mind, it is clear that middle leaders are critical in the development of quality educational outcomes because they exercise their leading in and around classrooms. Middle leaders are those who have an acknowledged position of leadership in

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 277–284. Sunshine Coast: MERGA.

their school, but also have a significant teaching role (e.g., senior teacher, Head of Mathematics Department) (Grootenboer, Edwards-Groves & Rönnerman, 2014). In general, they can be viewed as those whose leading practices operate between the Principal or the Head, and the teaching staff – in the middle! It is these people - the middle leaders, who can have the greatest impact on teacher learning and development (Edwards-Groves & Rönnerman, 2013) and more directly impact classroom practices. As such they can be 'instructional' and 'curriculum' leaders who can focus on the core business of schooling – learning and teaching.

The concept of *middle leading* has significance in three ways:

1. ***Positionally*** – middle leading is structurally and relationally practised 'between' the school senior management and the teaching staff. They are not in a peculiar space of their own, but rather than are practising members of both groups.
2. ***Philosophically*** – middle leading is practised from the centre or alongside colleagues. In this sense, middle leaders are not the 'heroic crusader' leading from the front, but rather alongside and in collaboration with their colleagues.
3. ***In practice*** – middle leading is understood and developed as a practice. To this end, the focus is on the sayings, doings, and relatings of leading rather than the characteristics and qualities of middle leadership. (Grootenboer, Edwards-Groves & Rönnerman, 2014, p. 18)

Thus, we see middle leaders as critical educators in the improvement of mathematics learning and teaching.

Leading Mathematics Learning and Teaching

To improve the mathematical learning outcomes for students, the main focus is usually on improving pedagogy. While there are a number of important factors that impact on the mathematical achievement of students, the most amenable to influence and development from a school perspective is the teaching. And, as was noted previously, the teacher is the single most significant player in influencing student learning (Lingard, et al, 2003). Therefore, given the critical role of quality teaching, the focus for improved student learning in mathematics has to be on professional development for teachers. Here we want to argue that to be both effective and sustainable, teacher learning has to be fundamentally site-based (Grootenboer & Edwards-Groves, 2014). While there is a place for externally run and organised courses and programs, primarily professional development needs to be undertaken at a local level. Indeed, the effectiveness of teacher development courses run outside of the school site is determined by the capacity of those involved to take the learning back and apply it in their particular school. Also, pedagogical development needs to be responsive to the particular learning needs of the school site (Edwards-Groves & Grootenboer, under review). Student identities and learning contexts vary greatly from site to site, and so notion of 'best practice' can only have meaning at a very general level (Kemmis, McTaggart & Nixon, 2014) and might therefore, be talked about as 'good enough practice' in the meaning of letting context and site matter (Groundwater-Smith, Smith, Mockler, Ponte & Rönnerman, 2012).

For example, the mathematical pedagogy that might be needed with students in the Torres Strait Islands would be quite different from students in an urban school which would be different again from Aboriginal learners in schools in central Australia. Finally, professional development should be collaborative and critically reflective. Lingard, et al. (2003) commented:

> … productive leadership encourages intellectual debates and discussions about the purposes, nature and content of a quality education; promotes critical reflection on practices; sponsors action research within the school; and seeks to ensure that this intellectual work connects with the concerns of teachers, students, parents and the broader educational community. Such leadership also ensures that teachers, and others working within schools, are provided with the support structures necessary to engage in intellectual discussions about their work, to reflect on the reform processes within their schools, as well as their pedagogical and assessment practices. (p. 20)

Considering these points, it seemed appropriate to focus on developing pedagogical capacity within schools and mathematics classrooms that would be localised and sustainable. To this end, equipping and supporting middle leaders to be curriculum leaders within their own school sites is an important and effective way to improve pedagogy, which in turn facilitates better learning outcomes in mathematics. Furthermore, *critical participatory action research* processes are an effective way to structure pedagogical development that was responsive to the needs and conditions of the school and classroom.

Critical Participatory Action Research (CPAR)

In educational contexts, we believe that there is an imperative to actively pursue the re-emphasis of *educational research* that places the interests of students, teachers and societies at the centre of the research process/project. CPAR is one way to promote this agenda. In this vein, mathematics education research is about transforming and developing mathematics learning practices in schools and classrooms.

Action research in a variety of forms has been employed for many years to facilitate and structure school development. Most commonly action research has been associated with, and seen as synonymous with, the 'action research cycle' (see Figure 1).

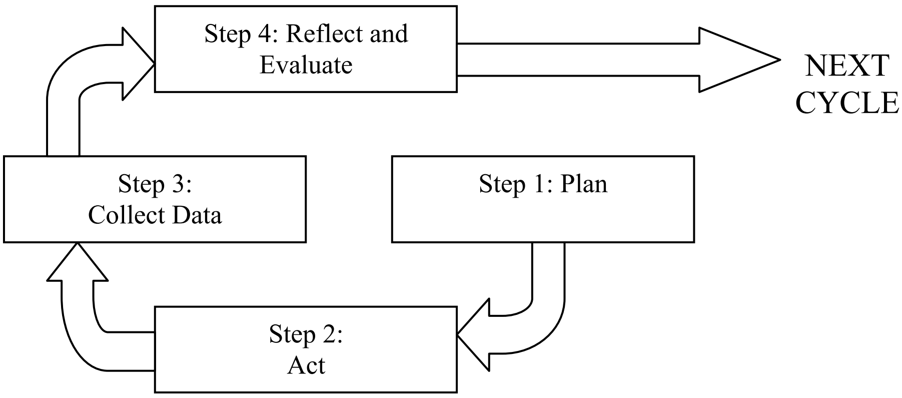


Figure 1: The Action Research Cycle

While the 'cycles' are useful, we see CPAR as more than just a cyclic process. CPAR fundamentally involves participants changing a social practice (e.g., mathematics teaching), and, changing what people *think* and *say*, what they *do*, and how they *relate* to others in that practice. To allow this to happen it is of importance that teachers get time and resources to meet in democratic dialogues where they can share knowledge and experiences related to their mathematics teaching practices in that site (Rönnerman & Salo, 2014).

The Critical Dimension

The *critical* nature of CPAR stems from its essential drive to question the moral and ethical nature of our practices. Specifically, this involves asking whether current educational practices and our educational institutions are:

- **Rational –** or are the practices irrational, unreasonable, incomprehensible, incoherent;
- **Sustainable –** or are the practices unsustainable, ineffective, unproductive, non-renewable; and,
- **Just –** or are the practices unjust, adversely affecting relationships, serving the interests of some at the expense of others, causing unreasonable conflict or suffering? (adapted from Kemmis, McTaggart & Nixon, 2014)

These are not just theoretical or esoteric questions, but rather they provide thoughtful prompts for evaluating whether educational practices are viable and responsive to the needs and circumstances of those involved at the time. For example, in mathematics education we should ask whether our current practices are irrational. Is it rational, reasonable and coherent to have many students completing their mathematics education seeing mathematics as irrelevant, boring and useless? We should also ask whether our current practices are unsustainable. Is it sustainable for the nation to, each year, produce many less mathematics graduates than is needed? And finally, we should ask whether our current mathematics education practices are unjust? Is it just that particular groups of students (e.g., students in remote schools) have lower mathematical outcomes than their urban peers? As these examples illustrate, the critical questions are relevant at a broad level, but also at a local site-based level where mathematics learning and teaching actually occurs.

Participation

Participation in CPAR is about developing a "communicative space" (Habermas, 1987) and requires consideration of who is involved, affected and included. Creating conditions for members to participate freely in this space - within what is described as a public sphere – makes communicative action possible. People who come together around issues of genuine concern about their circumstances and strive for intersubjective agreement about the language and ideas they use, mutual understanding of one another's perspectives, and unforced consensus about what to do (Kemmis, McTaggart & Nixon, 2014). In a school-based CPAR project this would obviously include the mathematics teachers and the school leadership, but fundamentally it also involves the students and often they are not considered as participants. We are not suggesting that all are participants in the same way or to the same degree, but nevertheless the students should be included because they are the prime focus of the mathematics education programs. Indeed, it would be irrational, ineffective and unjust to ignore the students in a mathematics education development project.

Action Research

As is clear from the preceding sections, action research is concerned with the development of social practices – in this case, practices of mathematics teaching and learning. To this end, the *purpose*, the *site* and the *focus* of CPAR are the practices of learners and teachers, and, the associated practice architectures (practice arrangements or conditions which enable or constrain practices). This is consistent with the premise stated earlier that sees the interests of students, teachers and their communities at the centre of the research process. Here, in the context of discussing mathematics curriculum leadership in schools, we are also concerned with the practices of middle leaders, and how their leading practices enable and constrain mathematics education practices in their particular sites.

This is a form of critical hermeneutic research that aims at *understanding* (rather than simply describing and explaining) and *transforming* a situation (so that it is not irrational, unsustainable or unjust). It tends to be interpretive and qualitative in nature with a practical intent (educating practitioners so they can act rightly – *as a form of praxis*).

Mathematics Leading Through Site-based CPAR

As has been noted previously, effective professional development is grounded in the particular arrangements of the site, and the people who are learning and teaching in the school. Therefore, programs and activities that focus on development need to begin with an understanding of the site, and this involves data gathering. Teaching and learning practices are enabled and constrained by the practice architectures, and so any mathematics education development will have to be cognisant of these local arrangements as well as the practices themselves. Evidence-informed site-based pedagogical development will lead to teaching that is responsive to the actualities of the learner's mathematical education and the conditions within which they undertake their learning. To this end, CPAR is useful.

> In CPAR we do not aim to produce generalisations about the 'one best way' to do things. In fact, we don't want to find the best way to do things anywhere *except* here – where we are, in our situation. (Kemmis, McTaggart & Nixon, 2014, p. 69)

> "… productive leadership encourages intellectual debates and discussions about the purposes, nature and content of a quality education; promotes critical reflection on practices; sponsors action research within the school; and seeks to ensure that this intellectual work connects with the concerns of teachers, students, parents and the broader educational community. Such leadership also ensures that teachers, and others working within schools, are provided with the support structures necessary to engage in intellectual discussions about their work, to reflect on the reform processes within their schools, as well as their pedagogical and assessment practices." (Lingard, et al., 2003, p. 20)

To illustrate, below we recount how mathematics middle leaders in one secondary school used a form of CPAR to promote deeper learning and engagement with their students in Years 8 to 10[1]. The middle leader's role was to participate, facilitate, support and resource the action research-based development.

Case Study[2]: Urban Secondary College[3] Mathematics Department

Urban Secondary College (USC) is a large metropolitan high school and broadly their goal was to improve students' mathematical learning outcomes by improving mathematical pedagogy. The mathematics department teachers worked in three smaller groups that focussed respectively on the Year 8, Year 9 and Year 10 classes, and each group was led

---

[1] At this time, these were the first 3 years of secondary school.

2 The first named author worked with these middle leaders as a 'critical friend'. Therefore, what is reported here is not so much a research report, but an account of what occurred.

3 Pseudonyms have been used throughout this paper

by one of the mathematics faculty middle leaders. Generally, the mathematics classes at USC had been fairly traditional in nature involving teacher exposition and textbook work, and through this project the goal was to engage in different forms of pedagogy in order to promote deeper mathematical thinking and conceptual understanding.

The middle leaders in the mathematics department (the three middle leaders noted above and the Head of Department) had been successful mathematics teachers for a number of years, but their challenge was to facilitate engaging mathematical pedagogy across all the mathematics classrooms, including those that were taught by non-mathematics specialists (e.g., a physical education teacher who may have just one mathematics class). Indeed, the middle leaders realised that bringing about pedagogical change in mathematics required a cultural change in the department and this was accepted as being a long-term and on-going project. The consensus of the middle leaders and the department, after engaging in some focussed professional learning on engaging mathematical pedagogies, was that 'hands-on' discovery learning activities were appropriate. The middle leader's first response was to change the focus of their fortnightly department meetings from management and administration to pedagogy[4], hence providing curriculum leadership.

The three Year level groups met fortnightly and developed one 'discovery' type activity for the ensuing unit of work, and each teacher committed to using it and collected some evidence from their students related to the activity. Furthermore, they agreed to visit and observe in each other's class when this activity was being employed. Although these 2 developments may seem fairly small, they were not insignificant for those involved, and they marked a beginning to some cultural changes in the department (i.e., opening up their classrooms to colleagues) and some pedagogical reform (i.e., investigative approaches to learning mathematics). As curriculum leaders, the middle leaders engaged in the same pedagogical and cultural change as the staff, they usually invited others into their classroom first in order to build a climate of trust and collegiality.

To illustrate, in the Year 8 classes the students initially investigated the sum of interior angles in a polygon. In this lesson the students were involved in drawing polygons, marking and cutting off the 'corners', and rearranging the pieces to uncover the relationship between the number of sides of the polygon and the sum of the interior angles. During the lesson, visiting teachers observed students using a range of methods to investigate the relationship and generalise a rule. Historically, the students would simply have had the rule presented to them, however, it was noted that because students were given the time to develop their own conceptual understandings, they became confident in investigating more complex shapes, and in the process they developed more robust problem solving skills and dispositions.

It is important to note that when the teachers visited one another's classrooms, the observations were not so much of the teacher per se, but rather of the students' learning and their engagement with the particular activity. They paid particular attention to the nature of student participation, the learning behaviours they employed, and the questions or comments that they offered. These notes, along with the work samples of these students, provided useful evidence regarding what actually happened in the classroom, and the teachers used this to reflect on the activity, the pedagogical approach, and the mathematical learning practices of the students. Each teacher would reflect on their own practice in the light of the data collected, and then they met as a group and through

[4] Administrative matters were then largely managed through email and the school intranet

dialogue they reflected collaboratively. After these reflections, the teachers then went on to plan their next common lesson, incorporating their understandings from the previous cycle, and thus the next action research cycle began.

Towards the end of the year the teachers again met in their groups, and as a whole department. At this time they looked back over their development throughout the year, using their data and meeting notes as references for what they undertaken. At this time they were able to identify significant changes in their mathematical pedagogy, and although this looked different for each of the individual teachers (i.e. individual praxis), there was clearly a shared approach to teaching that was more responsive to the students' needs (i.e., collective praxis), and a different *department culture*. Furthermore, they were able to specifically identify pedagogies that were more successful in engaging the students and facilitating their learning in mathematics.

Discussion, Conclusions and Implications

Throughout this paper we have highlight two key aspects of leading in mathematics education and pedagogical reform for improved learning outcomes in mathematics – middle leading and CPAR. We believe that these are both important because learning and teaching occurs in actual sites, and therefore, it must be responsive to the particularities of that site. This is no less the case in mathematics education, where generalised notions of 'best practice' are seemingly well established and difficult to change, and yet we know that for many they complete their mathematics education with debilitating and restrictive mathematical identities. To this end, we argue that there is not a single *best practice* per se for mathematics education that can be successfully implemented across all school sites, but rather what is needed is pedagogical leadership and development that is responsive to the specific mathematical learning needs within each school and classroom site.

In the case recounted above, the initial impetus for the change emerged from a critical evaluation of their current practices and students' learning outcomes in mathematics. While not overtly addressing the questions noted previously about their practices being irrational, unsustainable and unjust (although these could have been productively employed to structure their department discussions), they did want to address issues related to the reasonableness and effectiveness of their mathematics education. Specifically, they were concerned that the students' were becoming disengaged and disenfranchised with mathematics, and this was occurring in the very place they wanted to promote engagement and appreciation of the subject – their mathematics classrooms. Their practices in the past had been largely built on an unquestioning acceptance and use of traditional 'best practices' of mathematics teaching, and through their CPAR facilitated by the middle leaders, they brought about changes to their practices.

As we noted at the beginning of this paper, improved educational outcomes in mathematics requires the support, involvement and commitment of educational leaders (Sexton & Downtown, 2014). This leadership is needed at all levels from government and system 'down', and particularly includes school principals. However, if actual classroom practice is to be developed then the critical leadership that is required – curriculum leadership, needs to be exercised 'closer' to the site where teaching and learning is actioned. To this end, middle leaders, as in those who have formal leading positions but also have a teaching role, are the leaders with the capacity and position to most directly influence pedagogy and in turn learning among both teachers and students. They can focus on the key educational site – the classroom where teachers, students and mathematical ideas meet.

Effective middle leading is not simple, and it involves a range of roles including administrator, manager, and teacher, but the critical one is curriculum leader. As a curriculum leader the middle leader is focussed on improving the learning outcomes of the students, and this primarily is done through staff and pedagogical development (Sexton & Downtown, 2014). Furthermore, the middle leader has to nurture a sense of understanding of the students including their educational needs and their broader life worlds, and to facilitate connection with the community. This becomes particularly important when students don't come with the cultural capital necessary for success in school mathematics, and disproportionately these students come from disadvantaged communities.

Pedagogical leadership provided by senior teachers, faculty heads and the like, can ensure that teaching is responsive to the learners needs, thus avoiding the homogenizing effect of a standard approach. While the mathematics curriculum may be standardised across Australia, the way that curriculum is taken-up and presented in the classroom can and should vary through a diverse range of teaching approaches appropriate for the learners in that site. In this way mathematics education may become more rational, sustainable and just.

References

Groundwater-Smith, S., Smith, J., Mockler, N., Ponte, P., & Rönnerman, K. (2012). *Facilitating practitioner research: Developing transformational partnerships.* London: Routledge.

Edwards-Groves, C. (2003). *On task: Focused literacy learning*. Sydney: Primary English Teachers Association (PETA).

Edwards-Groves, C., & Grootenboer, P. (under review). Praxis and the theory of practice architectures: Resources for re-envisioning English Education. *Australian Journal of Language and Literacy.*

Edwards-Groves, C., & Rönnerman, K. (2013). Generating leading practices through professional learning. *Professional Development in Education, 39*(1), 122-140.

Grootenboer, P., & Marshman, M. (in press). *Mathematics, affect and learning: Middle years students' beliefs and attitudes about mathematics education*. Singapore: Springer

Grootenboer, P., & Edwards-Groves, C. (2014). Mathematics teaching as praxis. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics education Research Group of Australasia, pp. 271-278). Sydney: MERGA.

Grootenboer, P., Edwards-Groves, & Rönnerman, K. (2014). Leading practice development: Voices from the middle. *Professional Development in Education*. doi: 10.1080/19415257.2014.924985

Habermas, J. (1987). *The theory of communicative action, (Volume two): Lifeworld and system: A critique of functionalist reason* (trans: McCarthy, T.). Cambridge, Mass: MIT Press.

Hayes, D., Mills, M. Christie, P., & Lingard, B. (2006). *Teachers and schooling: Making a difference*. Crows Nest, NSW: Allen and Unwin.

Kemmis, S., McTaggart, R., & Nixon, R. (2014). *The action research planner: Doing critical participatory action research*. Singapore: Springer

Lingard, B., Hayes, D., Mills, M., & Christie, P., (2003). *Leading learning: Making hope practical in schools*. Maidenhead, UK: Open University Press.

Rönnerman, K., & Salo, P. (Eds.) (2014). *Lost in practice: Transforming Nordic educational action research*. Rotterdam: Sense.

Sexton, M., & Downtown, A. (2014). School mathematics leaders' perceptions of successes and challenges of their leadership role within a mathematics improvement project. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics education Research Group of Australasia, pp. 581-588). Sydney: MERGA.

Teaching Computation in Primary School without Traditional Written Algorithms

Judy Hartnett
Mathematics Education Consultant
<judyhartnett@bigpond.com>

Concerns regarding the dominance of the traditional written algorithms in schools have been raised by many mathematics educators, yet the teaching of these procedures remains a dominant focus in in primary schools. This paper reports on a project in one school where the staff agreed to put the teaching of the traditional written algorithm aside, replaced with computational strategies. The results reinforce a belief that I have held for many years that the traditional algorithms should be removed from the primary mathematics curriculum.

Background

Computation involving the four operations (addition, subtraction, multiplication, and division) is a major content area in primary school mathematics. Curriculum documents advise teachers to take an approach focusing more on strategies and less on traditional written algorithms. For example, the current Australian Curriculum: Mathematics Version 7.3 (ACARA, 2015) states that students “apply a range of strategies for computation and understand the connections between operations” (p. 5). Despite mathematics education research stating concerns about overdependence on procedural thinking (e.g., Hiebert & Lefevre, 1986), and those stating the benefits of computational strategies as leading to deeper understanding of the structure and properties of numbers (e.g., Plunkett, 1979; Reys, 1984; Thompson, 1999), the development of number sense (e.g., Sowder, 1988), the development of problem solving and thinking skills (Callingham, 2005; Plunkett, 1979), and better alignment of school mathematics to the mathematics used beyond the classroom (e.g., Australian Education Council, 1991; Callingham & Watson, 2008; Hedren, 1999; Northcote & McIntosh, 1999), the teaching of computation in primary classrooms is still dominated by the traditional written algorithms.

The dominance of the traditional written algorithms in schools can be traced back to times before calculation machines had been invented and schools needed to prepare students for jobs where they would need to manually add long columns of figures with accuracy. To enable others to check the calculations a standard method was preferred. Today we have several electronic calculation methods with calculators, spreadsheets, and other applications readily available in all classrooms, as well as in the world beyond the classroom. Back in 1999, Northcote and McIntosh conducted a study into how adults completed computations. In a twenty-four hour period only 11.1% of the calculations involved any written component and 6.8% used a calculator. Today, I would predict that the percentage of adults who used a calculator would be much higher given the availability of these devices, especially on mobile phones. These researchers also found that in 60% of the computations situations only required an estimate for the calculation task. The need for traditional written algorithms in the world beyond school today is limited if not non-existent. However, the need to think and reason mathematically is high. The place of traditional written algorithms as a dominant aspect of primary school programs deserves to be seriously questioned. However, teachers and parents maintain a belief that the place of

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 285–292. Sunshine Coast: MERGA.

algorithms in primary school is deserved and that to teach mathematics *properly* its inclusion is important.

The Project

I was asked to begin a teacher professional development project with the staff of one primary school north of Brisbane in January 2012. My brief was to work with a team of teachers to develop their mathematics pedagogy The school had conceived and been using a professional development system that involved the use of *experts* working with teams of teachers across year levels using classroom demonstrations and reflection, followed by the teachers supporting their year level peers toward whole school implementation. I was employed to be the mathematics *expert*. The project's overall aim was to improve student learning outcomes. Student data was consulted which included their current NAPLAN data. The school was performing below cohort, below state, and below national average scale scores (see Figure 1). While the data was not excessively below average, the school and system wanted to see it improve. The school staff also articulated a desire for the students to be more confident in their approach to mathematics and to be able to reason mathematically and problem solve, which are proficiency strands in the current Australian Curriculum (ACARA, 2015).

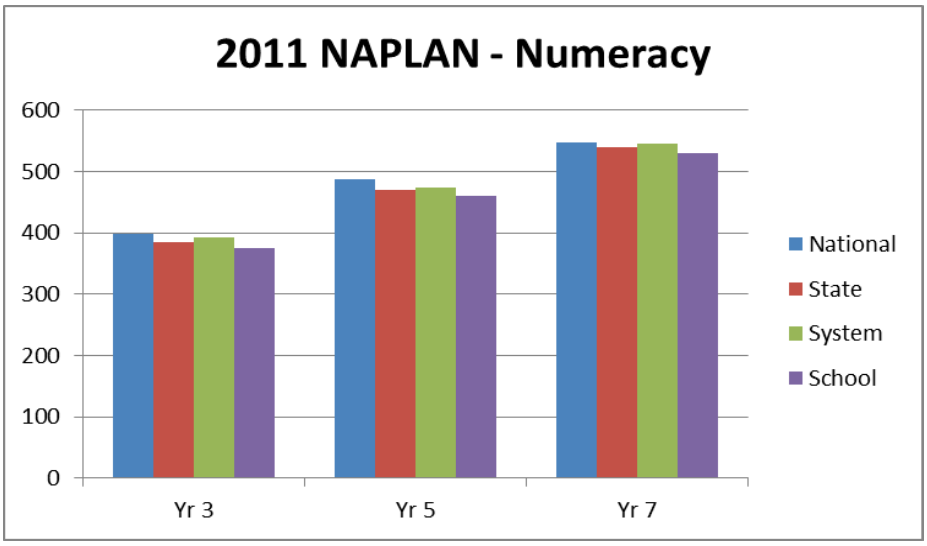
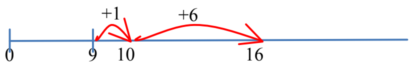


Figure 1. 2011 NAPLAN data for the project school

After discussion with the school administration we decided to focus on encouraging the teachers to challenge the students to think and reason mathematically. We discussed research including work done on student thinking related to the use of computation strategies rather than traditional written algorithms (Hartnett, 2008). The school administration was interested in challenging the teachers to approach the teaching of mathematics in a more investigative way. Given the dominant place of the traditional written algorithm and its procedural focus it was agreed to work with the *Maths team* (at least one teacher from each year level) to develop their own understandings of strategies that could be used instead of algorithms to challenge traditional views of the teaching of mathematics, while offering a professional development pathway that would support the teachers to work differently. The Maths team teachers would then mentor their peer teachers, working with same year level, to change the focus of computation instruction to strategies. The staff, supported by the school administration, agreed to stop teaching the traditional algorithms completely and instead to encourage students use strategies for computation. The school adopted a computation strategy categorisation framework as the organiser of the content to be learnt (based on Hartnett, 2007; see Table 1) and the project began in January 2012.

Table 1.
Categorisation of Computation Strategies used in the Project (based on Hartnett 2007)

Strategy Categories	Examples (addition 27+19)
Break Up 1 Number	27+10=37; 37+3=40; 40+6=46
Break Up 2 Numbers	20+10=30; 7+9=16; 30+16=46
Change 1 Number and Fix	27+20=47; 47–1=46
Change 2 Numbers and Fix	30+20=50; 50–3–1=46
Change 2 Numbers	26+20=46 (27–1 + 19+1)
Count on to Subtract (e.g. 16–9)	

This categorisation framework was chosen for consistency of strategy names across the four operations. The category names described the action of the strategies in language students could understand. In a previous study where this framework was used, students started to use the strategy category labels even though they had initially been designed to assist teachers with their planning for the development of the strategies (Hartnett, 2008). The plan was for the students to make a simple choice between whether they would *break up* numbers, or whether they would *change* one or both of the numbers and decide on the *fix*, if needed. The thinking and number sense required to use the strategies was an identified deficiency with the students at this school.

The project began with Year 3 to Year 7 teachers focussing on introducing the strategies for addition to their students. Teachers in Prep to Year 2 focussed on developing number sense and operation concepts as well as working on basic fact development. The Maths team worked to develop a whole school plan for developing the strategies during the first year of the project. A program of professional development and mentoring was actioned and teachers began to work with their students to develop the strategies and related number sense. Support was provided to the Maths team, as needed, as they worked with their year level peers to introduce the strategies to their classes. Because all of the strategies were new to the staff and the students, most of the first year was spent focussing on strategies for basic addition and multiplication facts and the development of the strategies for addition. The Maths team teachers worked ahead of their peers trying strategies with other operations, as appropriate, and developing lessons and activities to support student understandings and sharing these with their peers.

Initially, students and parents reactions (reflected on through students sharing perspectives from home as well as teachers interacting with parents formally and informally) indicated that the algorithms were viewed as having higher importance than the strategies. It was this perception that influenced the decision in this project to not teach the algorithms at all so as to raise the status of the strategies. Students were not banned from using the algorithms but were encouraged to use strategies and to show their thinking in the way they recorded their responses. This was especially important the older students who were quite familiar with algorithms, but teachers gently shifted the focus from the procedural algorithms to conceptual understanding of numbers and operations.

Observations

The project is ongoing and data presented is observational. The data is anecdotal and qualitative in nature. It is presented as a commentary of the process so far: outlining factors influencing the project, problems encountered, and reflections by the education advisor *expert* supporting the school (the author), teachers, and parents. In the first year of the project, qualitative data was collected where all students in Years 1 to 7 completed a range of computations showing their thinking or working out. Each response was coded for accuracy and strategies used. This data collection has not been repeated as yet. It is planned to conduct this data collection at the end of this year to capture change in the cohort that was in Year 3 at the start of the project. This cohort has not had the traditional written algorithms taught to them at all, unless they have come from a different school. The data would not be able to be used for student comparison but for overall change in the range of strategies used.

At the beginning of the project, students in the upper grades were reluctant to *let go* of the traditional written algorithms they had learned to use already. This was understandable but interesting in terms of their reasoning. When questioned students had difficulty articulating why they preferred the algorithms or why they were not keen on learning other ways to approach the operations. One possible reason was that they were successful with the algorithms and predicted they would not be as successful with something that was new and different. This seemed to be the case with students identified by their teachers as *good at maths*. These students may have decided that it was better to not try than to try and be unsuccessful.

The project included parent information sessions to share the school's direction with the wider community. During these sessions I found there was a need to make a distinction between the use of strategies for computation as an end user beyond school, and as part of a learning program in school. While many parents recognised and acknowledged that some of the computation strategies presented were ones they used in their everyday lives, there were other strategies that they would not choose to use. At school the students were being exposed to a wide range of possible strategies as a learning activity to encourage them to develop their number sense, reasoning, and operation sense as well as the strategies. As the students developed their understandings it was predicted that they too would choose they found personally effective and which made sense to them from the strategies studied. The use of calculation technologies was also discussed as a practical means to finding answers as an end user and that during the learning process the focus was on development of number sense and operation sense that could inform the choice of computational method. This distinction was discussed with teachers in the Maths team during professional development sessions as well.

NAPLAN Data

One set of quantitative data that has been analysed to identify the impact of the project has been the school NAPLAN data. While it is recognised that only a small proportion of the questions on the Yr 3/5/7 tests each year can be directly linked to computation or potential use of computation strategies, the overall aim of the project was to assist teachers to use pedagogy that would improve the students' understanding about maths and their ability to think mathematically so the data could be used to reflect progress on this overall aim. The pre-project NAPLAN data from 2011, the year before the project, is summarised in Figure 1. After three years working with the staff and developing a relationship with them and the students at the school we are starting to see changes. The NAPLAN data below shows the Year 5 and Year 7 school data above system, state and national average scale scores for the first time. The Year 3 data has improved but not passed the other scores (Figure 2). Figure 3 shows the Year 5 data and Figure 4 shows the Year 7 data.

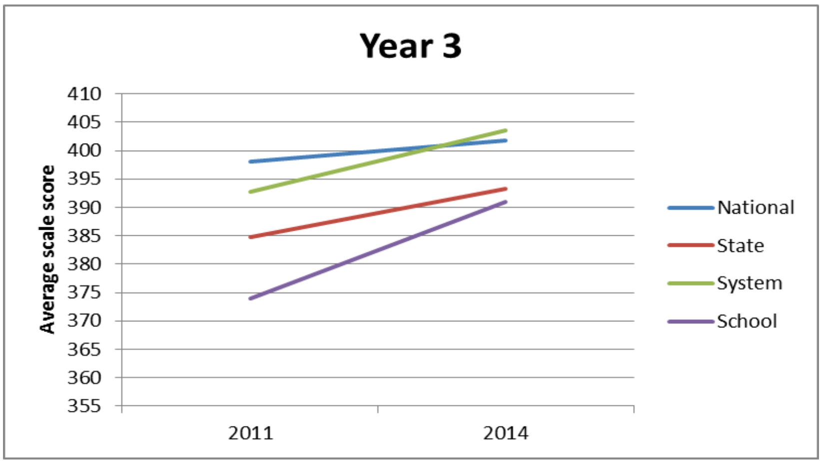


Figure 2. NAPLAN data Year 3 2011 (before the project) and 2014 (current data)

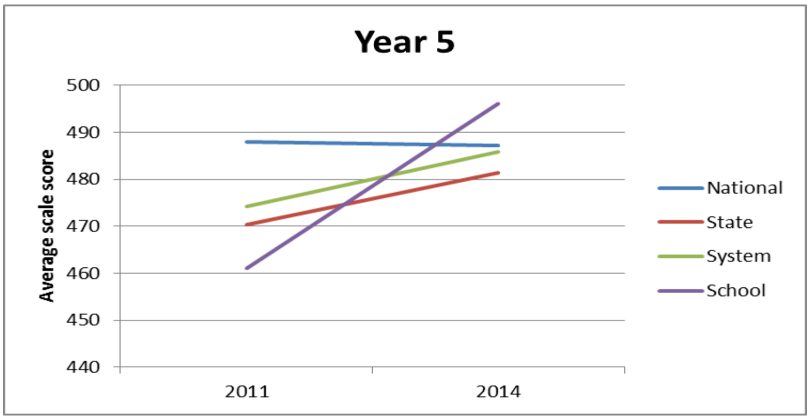


Figure 3. NAPLAN data Year 5 2011 (before the project) and 2014 (current data)

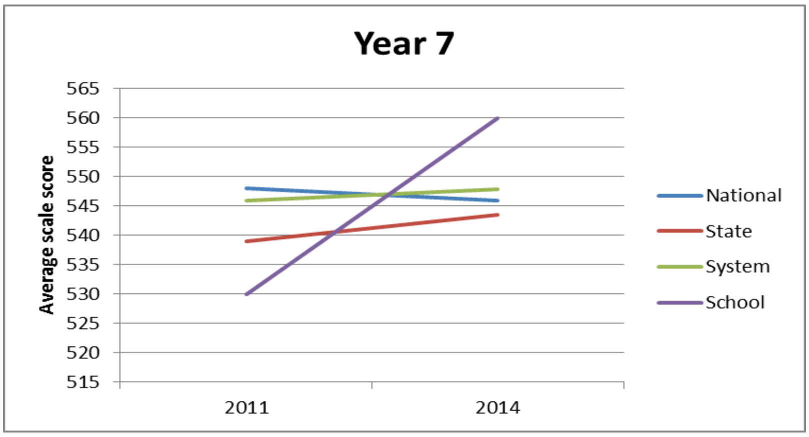


Figure 4. NAPLAN data Year 7 2011 (before the project) and 2014 (current data)

Teacher Reflections

The reflections below provide some further anecdotal data as the project progresses.

After a whole day professional development session with the Maths team where we discussed the computation strategies for each operation, one teacher commented that she could see the reasoning behind the use of strategies instead of the algorithms, but that she still believed there was a place for the traditional methods in primary classrooms. I returned to the school the following week to be greeted by the same teacher who asked me to disregard her previous comments and that she "was now convinced". She had started working with her Year 5 class on a *Break U*p strategy for multiplication where the multiplication was represented using an area model. She reported how the students "loved the strategy" and how it "made so much sense to them" and to her. Her other observation was how confident the students were as they approached multiplication problems; something she had not experienced teaching traditional multiplication algorithms to students this age. (Education Advisor leading the project)

When I began teaching here at [school name] I wasn't sure about how not using an *algorithm* would work. At the end of my first twelve months I was delighted with what I had learned and the progress my class had made in *thinking* about what they were learning and doing in Maths. As I became more confident with the teaching strategies, I was able to clearly see how beneficial it was to teach the children a range of skills, which not only made sense, but also enabled them to solve problems using a variety of strategies, which enhanced their understanding of what they were actually doing. **(**A teacher who came to the school in 2012**)**

I came to [school name] with no concept of these strategies and at first I found it hard to comprehend and tended to stick with the algorithm concept. After teaching these strategies, I found that the students and I really began to improve our mathematical thinking. I was never the strongest in Maths but now I have learnt many new strategies to work with numbers and no longer need to write down algorithms. The improvements I have made using these strategies has given me the confidence and enthusiasm to teach the children and never use the old methods again. (Yr 5 early career teacher who came to the school in 2012)

Parent Reflections

My daughter seemed to have lost confidence in her ability with maths as she moved from Year 3 into Year 4 in 2013. Her Year 5 teacher last year used the strategies and as the year went on her number knowledge grew. At home we noticed she was engaging in conversation involving maths and she was using strategies in everyday situations, like with her pocket money. We noticed that her

> confidence grew and are now quite confident she be more comfortable in high school next year with a better attitude to maths. (Parent of a current Yr 6 student)
>
> My son is in Yr 5 this year. At the start of this project, my husband was very resistant to the strategies focus. As Jack has become more proficient he has been able to explain to his Dad how the strategies work. His Dad is now seeing Jack learning rather than just doing it quickly and getting answers. When Jack makes mistakes he can look back and understand what he did. He has confidence and considers himself good at maths. Anyone who can convince my husband he was wrong must be doing something right. (Staff member and parent)

Conclusions

The project is ongoing. Having the opportunity to work in one school on a long-term project has been a factor in the success so far. Being able to build rapport with the staff and students as an *expert* builds their trust in me to lead them through the process. This school entrusted me to lead them to make the decision about this project. It is to their credit that the results are showing improvement in what they set out to achieve–improvement in the students' ability to think mathematically and to be confident users of mathematics and to improve the teachers' pedagogy in relation to mathematics. By changing a very traditional aspect of the school program, we sent a message to the staff and school community that we wanted to do things differently. I had held a belief for many years that changing students perceptions of mathematics as a subject, as well as changing their ability to think and reason mathematically, could be achieved by starting with a change to the focus for computation. This project has allowed me to test this theory.

We have shown that students can be successful in mathematics without the traditional written algorithm as part of the school mathematics program. The traditional algorithms are procedures that can assist students to get answers to computations but by using strategies and number sense instead students gain more than just answers.

References

Australian Curriculum Assessment and Reporting Authority [ACARA]. (2015). *The Australian Curriculum: Mathematics.* Retrieved 13 March 2015 from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Australian Education Council. (1991). *A national statement on mathematics for Australian schools*. Carlton, Victoria: Curriculum Corporation.

Callingham, R. (2005). A whole school approach to developing mental computation strategies. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education* (Vol. 2., pp. 201-208). Melbourne: PME.

Callingham, R. A., & Watson, J. M. (2008). *Research in mental computation: Multiple perspectives*. Teneriffe, Qld: Post Pressed.

Hartnett, J. E. (2007). Categorisation of mental strategies to support teaching and to encourage classroom dialogue. In J. Watson and K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia. (pp. 345-352). Hobart: MERGA.

Hartnett, J. E. (2008). Capturing students thinking about strategies used to solve mental computations by giving students access to a pedagogical framework. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia). Brisbane: MERGA.

Hedrén, R. (1999). The teaching of traditional standard algorithms for the four arithmetic operations versus the use of pupil's own methods. In I. Schwank (Ed.), *Proceedings of the first conference of the European Society for Research in Mathematics*. Forschungsinstitut fuer mathematikdidktik: Osnabrueck.

Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case for mathematics*. Hillsdale, NJ: Lawrence Erlbaum.

Northcote, N., & McIntosh, A. (1999). What mathematics do adults really do in everyday life? *Australian Primary Mathematics Classr*oom, *4*(1), 19–21.

Plunkett, S. (1979). Decomposition and all that rot. *Mathematics in Schools*, *8*(3), 2-5.

Reys, R. E. (1984). Mental computation and estimation: Past, present and future. *The Elementary School Journal*, *84*(5), 547-557.

Sowder, J. (1988). Mental computation and number comparison: their role in the development of number sense and computational estimation. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades*. Hillsdale: Lawrence Erllbaum Associates.

Thompson, I. (1999). Getting your head around mental calculation. In I.Thompson (Ed.), *Issues in teaching numeracy in primary school* (pp. 145-156). Buckingham: The Open University Press.

Calculating for probability: "He koretake te rima" (Five is useless)

Ngārewa Hāwera
University of Waikato
<ngarewa@waikato.ac.nz>

Merilyn Taylor
University of Waikato
<meta@waikato.ac.nz>

In Māori medium schools, research that investigates children's mathematical computation with number and connections they might make to mathematical ideas in other strands is limited. This paper seeks to share ideas elicited in a task-based observation and interview with one child about the number ideas she utilises to solve a problem requiring probabilistic thinking. The explanations provided by the child demonstrate how early number and spatial patterns can impact on computation, ease of determining possible outcomes and assigning a numerical probability measure to an event.

Crites (1994) states that mathematical literacy is crucial for citizenship in today's society. An important component of mathematical literacy is number sense and the ability to apply it in a range of contexts is essential for coping confidently with the demands of an information-laden society.

Quantifying the probability of an event occurring is linked inextricably with number. Difficulties arise when measuring the chances of an event occurring for learners with limited number knowledge including that of fractional number (Langrell & Mooney, 2005). Reasoning in number and probability is vital for life beyond school (Gal, 2005; Jorgensen & Dole, 2011). For example, making decisions about whether or not a rain jacket is needed or where to invest for retirement are based on probability.

Since the introduction of the Numeracy Development Projects into New Zealand schools in 2000 there has been a major focus on the development of number strategies and knowledge for children learning mathematics. The national implementation of these projects meant that children beginning their formal mathematics education were expected to develop a strong base in numeracy as a foundation for learning a broad range of ideas in mathematics and statistics. Developing number sense and understanding connections between numbers, how they might be manipulated when calculating and noticing patterns with numbers is fundamental to effective mathematical thinking (Jorgensen & Dole, 2011).

Mulligan & Mitchelmore (2013) state that children's development in mathematics is heavily dependent on their awareness of pattern and structure "…mathematical pattern involves any predictable regularity involving number, space, or measure" (p.30). Mason, Stephens & Watson (2009) argue that children who have sound understanding about structure in mathematics not only recognise key ideas about properties in a relationship but they also have an awareness of how numbers might be manipulated.

Numeracy involves computation, interpretation, and making appropriate decisions with and about numbers to support mathematics learning. Learners need to be able to count, quantify, compute and manipulate numbers (Ministry of Education, 2007). By Year 8 Te Marautanga o Aotearoa (the national curriculum document for Māori medium schools in New Zealand) outlines the expectation that children should be working fluently with whole number and fractional number in a range of mathematical contexts including probability (Ministry of Education, 2008). Children are expected to have developed clear conceptual understandings of proportion and be able to utilise that knowledge to determine the likelihood of an event, for example, the probability of getting a 7 when throwing two die and adding the numbers together. Representing, interpreting and evaluating data to make

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 293–300. Sunshine Coast: MERGA.

informed decisions is dependent on having a sound understanding of whole number and fractional number, including percentages (Gal, 2004; 2005).

To develop probabilistic thinking learners will need to understand that the kind of thinking required with probability is different to that typically addressed in school mathematics (Langrell & Mooney, 2005). While exploring probability contexts requires the use of mathematics, this area of study is based on chance (Neill, 2010). Learners have to come to appreciate the idea that it is not possible to determine an individual outcome, but it is possible to predict the frequency of an outcome (NCTM, 2000). The reasoning that is required for such thinking develops over time and may be understood through a framework offered by Jones, Langrell, Thornton & Mogill (1997). The four levels of reasoning involve:

1. Subjective reasoning
2. Transition between subjective and naïve quantitative reasoning
3. The use of informal quantitative reasoning
4. The incorporation of numerical reasoning

The reasoning constructs may not be uniform and children may not follow an ordered progression of learning. Learners do however require frequent experience with actual experiments to develop their probabilistic thinking.

Determining the sample space is fundamental to aspects of probabilistic reasoning and requires the coordination of different cognitive skills. Children need to recognise that there may be different ways of obtaining an outcome. Research suggests that children should be presented with opportunities to actively participate and use physical material for exploring and investigating probability situations (Jorgensen & Dole, 2011; Neill, 2010). Being able to systematically and exhaustively generate possible outcomes is important to help consider the value of experimental probability and its relationship to theoretical probability (Barnes, 1998; Gal, 2004).

According to New Zealand curriculum documents assessment of children's learning in mathematics is to be based on multiple sources of evidence gathered over time (Ministry of Education, 2009). It is crucial that children are presented with opportunities to communicate their mathematical thinking, reasoning and solutions in a variety of ways (Hunter, 2009; Hunter, 2006). Making time for listening to children share their thinking can support assessment of learners' development in mathematics (Higgins & Weist, 2006; Reinhart, 2000). This practice includes listening to their thinking about probability (Barnes, 1998; Neill, 2010).

The ability to represent mathematical ideas using words, symbols or pictures supports children to communicate their thinking. Using different representations can encourage flexible thinking and provide teachers with artifacts constructed by learners, to support teacher judgments about children's learning (Suh, Johnston, Jamieson & Mills, 2008). Representations also serve as tools for justifying and making sense of mathematical ideas while supporting learners to construct knowledge (NCTM, 2000).

Children rate highly those opportunities for mathematics learning that incorporate physical movement and situations that include concrete materials (Attard, 2012). Ensuring that a task is accessible to everyone and that it facilitates reasoning and communicating, while incorporating multiple approaches, can also ensure that it is worthwhile (Breyfogle & Williams, 2008).

The purpose of this paper is to illustrate how one child's thinking in number supported her learning and understanding in a probability context.

Method

Data was collected as part of a larger study in a Year 7-8 class in a Māori medium setting (80-100% teaching and learning in the Māori language). The classroom teacher had base line information that indicated the class had gaps in knowledge with regard to the statistics component of assessment for Whanaketanga Pāngarau: He Aratohu mā te kaiako (Ministry of Education, 2010). She explained that due to a strong focus on number in recent years the children had limited exposure in their formal mathematics education programme to the development of probability ideas. She wanted to focus on helping children to work out the possible outcomes of an event and communicate their process and solutions appropriately and effectively.

This paper concerns one child's ideas for solving a probability task. The child had earlier completed a similar probability task involving the addition of numbers when two dice are thrown and finding the probability of different outcomes. The child was familiar with the researchers who had been into the classroom to observe and listen to them sharing ideas while completing the probability addition task.

This task (shared with the children in the Māori language) was about saving dolphins (six for each player) that had stranded themselves on some sandbanks (labelled 0-5 on same sheet for both players) and needed to be "saved". When players took turns to throw 2 dice and subtracted the numbers to find the difference, they could "save" one of their dolphins if it was on that numbered sandbank. Players have to decide before they start the game where to locate their six stranded dolphins (counters). The "winner" is the one who saves their six dolphins first.

The child for this case study was asked to play the game with one of the researchers in a space away from the other children. This situation was designed so that the researchers could record, listen and probe the child's thinking.

Results

Key ideas and recordings that emerged from this particular 12 year-old child Marino, about finding all possible outcomes and their significance for determining the likelihood of particular events are noted below.

1) Recognition of Spatial Patterns and Using that Knowledge

Upon throwing two dice and finding the difference between the numbers, Marino recorded the results in a tally chart. She noticed that some numbers appeared more frequently than others. When asked to explain why that might be occurring, Marino then recorded what she considered to be all possible ways of obtaining numbers zero to five as seen in *Figure 1.*

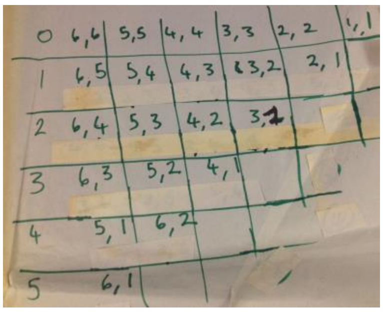


Figure 1. Marino's recording of different ways of getting outcomes 0-5

When asked how many possible outcomes there were in her diagram she said there were 21 because “5 and 6 =11 and 10 and 11 = make 21”. She explained that the above recording resembled a triangle where the first 4 columns (beginning from the right hand side) had a pattern of 1, 2, 3 and 4 sets of numbers. She likened that pattern of numbers to the first 4 rows of another drawing (Figure 2) where the circles represented that same pattern of numbers. The spatial representation of the circles in each row i.e. 1, 2, 3, and 4 automatically indicated to her a total of 10. Continuation of the spatial pattern meant to her that the next two rows would equal 11. Therefore the total number of items had to be 21 because she stated that was a pattern that she had learned when she was much “younger”.

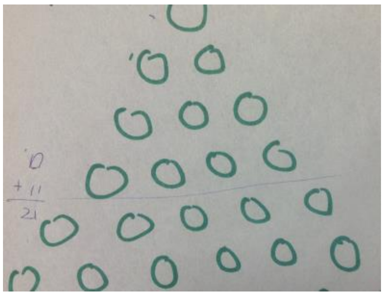


Figure 2. Drawing of circles to support addition of possible outcomes shown in Figure 1

2) Adding up all the Possible Outcomes

After being prompted to consider more possible combinations when throwing two dice, Marino then added to her recordings shown in Figure 1.

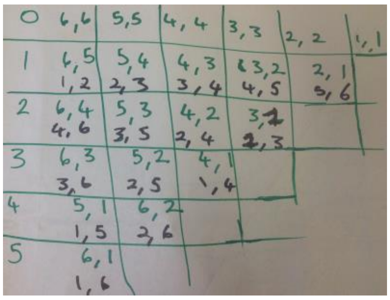


Figure 3. Total number of combinations noted by Marino when throwing two dice and subtracting the numbers

When asked how many outcomes there were when looking at her recording, Marino recognised and utilised another pattern. She started from the right hand side of Figure 3 and counted the number of combinations in each column and stated: “1, 3, 5, 7, 9, 11 is 36”. Only when asked to explain her mental addition strategy to a very puzzled researcher, did she record “1, 3, 5, 7, 9, 11” on the sheet as shown in Figure 4. She then explained that “11 is 10 +1” (as shown to the right of 11), “…then you have 9+1 (left hand side column)) is 10 (right hand side column), then 7+3 is 10 and 5 is 36 altogether”.

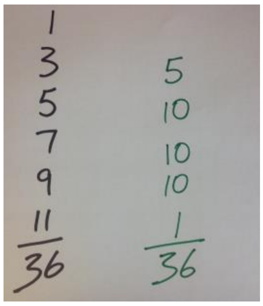


Figure 4. Numbers recorded by Marino when adding total number of combinations shown in Figure 3.

3) Alternative Presentation of Findings

When asked if she could present her findings in another way Marino was able to show quickly in an array the outcomes when throwing 2 dice and finding the difference as shown in the Figure 5.

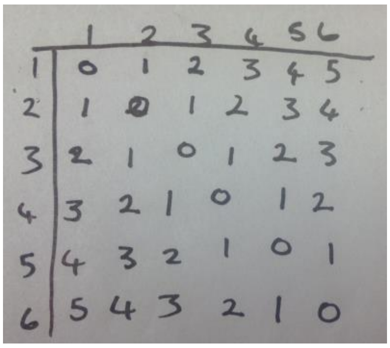


Figure 5. Array showing outcomes when throwing two dice and subtracting the numbers.

To find the "chances of getting a zero" Marino counted the number of zeros on the array and said six. She stated that there were 36 possible outcomes altogether on the array when throwing two dice and subtracting the numbers and the fraction for the chances of getting a zero was 6/36. When asked if 6/36 could be stated as another fraction, Marino said that it was the same a 1/6.

> "He ōrite ngā nama e rua nā te mea e ono ngā ono i roto i te 36"
> (The two numbers are the same because there are six sixes in 36).

> Marino stated "He koretake te rima…Ko te tahi te nama pai. Ko te tekau o te toru tekau mā ono ka puta mai te tahi".
> (…Five is useless…One is a good number. One will appear ten out of thirty-six times).

4) Making Connections between Fractions and Percentages

When asked what one sixth would be as a percentage, Marino showed in Figure 6 that it was about 16.5%. Marino explained that 1/3 of 100 is 33.5 If dividing that by 2 you get about 16.5. Therefore 1/6 is the same as 16.5 because a half of one-third is a sixth.

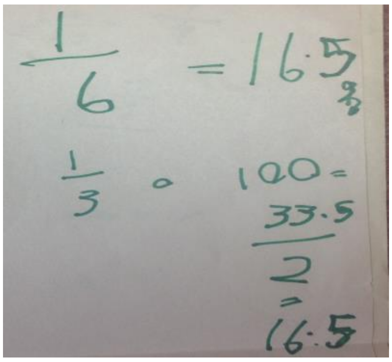


Figure 6. Marino's recording of the link between fractions and percentages.

Discussion

This student was able to recognise patterns and use her knowledge and strategy development in number to support her to calculate all possible outcomes and the chances of an event occurring in probability. For example, she was able to calculate how many combinations there were in the recording (Figure 1), based on connections she made with a spatial pattern. Her immediate recognition of the recorded sets of numbers in a triangle (Figure 1) and knowledge that the first four rows of the pattern (Figure 2) would equal 10 with the next two rows equalling eleven, made it easy for her to add the two totals together to quickly determine that there were 21 possible outcomes. This thinking indicated that the spatial pattern with perceived links to number was a mathematical idea that she had met in the past and had become an example of "predictable regularity" (Mulligan & Mitchelmore, 2013). She was familiar with the spatial pattern and its associated number sequence and knew from past experience that it would be 'true'.

The ease of calculation to 21 enabled Marino to then respond to prompting about the possibility of determining other combinations when throwing two dice. She was able to move forward and reason that there were other possible outcomes for this task and therefore reconsider her original total of outcomes. Accurate working out of the sample space is a crucial aspect of probabilistic thinking (Jorgensen & Dole, 2011).

Failing to understand all the possibilities that a particular context offers reinforces misconceptions about all possible outcomes. There was a need to support Marino to think more deeply about other combinations when throwing two dice so that she could appreciate that probability idea. A fundamental premise of supporting learners to develop probabilistic thinking is helping them to recognise and address misconceptions that might be held (Barnes, 1998; Neill, 2010).

Students with sound number sense can see numbers in a range of combinations and groupings (Jorgensen & Dole, 2011). When it came to calculating the total number of outcomes (36) Marino demonstrated use of a "Make 10" number strategy that she had learned earlier in her mathematics education. This part whole strategy is one that is promoted early for children to learn in their formal mathematics education in New Zealand. Her obvious awareness of the predictable regularity of the tens pattern and how numbers can be restructured to create it (Mason, Stephens & Watson, 2009; Mulligan & Mitchelmore, 2013), assisted her to apply that knowledge in a probability context and not be hampered by the mechanics of calculation.

The systematic recorded representation that Marino presented in Figure 5 showed the arithmetical difference that results when throwing two dice. The picture meant that she could easily count or quantify the likelihood of any number in the zero to five range appearing in this context. She was able to link these numbers to the 36 possible outcomes and make a fraction. Children need to draw on fractional number knowledge when the context demands, so that the probability ideas can be realised.

"Six out of 36" is a number idea that children have to take further when developing ideas about probability. They need to make meaning of such fractions according to the probability context that has been presented. While Marino abandoned the 'dolphin scenario', the context of throwing two dice and subtracting the numbers remained. The 'best' theoretical outcome still had to be ascertained by comparing the various numerical probabilities of each. The uncertainty of particular outcomes needed to be quantified if the focus of the learning was to be about developing probabilistic numerical reasoning (Jones, Langrell, Thornton & Mogill, 1997).

Assigning a numerical probability measure to an event can be demonstrated in a number of ways. Marino showed that she was able to make connections between fractions and percentages and understood key ideas of equivalence. She understood the relative size of fractions; that a third is equivalent to two sixths and is therefore approximately 33.5%. She understood that a sixth as a percentage could be found by dividing 33.5% by two. The ability to make rapid connections between common fractions (1/3 = 2/6) and then between fractions and percentages allowed Marino to express the probability of an event occurring in a variety of ways as expected of Year 7-8 children in New Zealand (Ministry of Education, 2010).

Conclusion

Research suggests the importance of children having sound number ideas if they are to explore and develop appropriate quantitative probabilistic thinking. The task-based interview has provided some specific examples of instances where number sense proved critical for determining the chance of an event occurring. The ease of accessing and understanding the probability ideas was enhanced by a facility with number. Working with a task that was easily accessible, that encouraged the use of concrete materials and inherently provided opportunities for reasoning and communicating, supported engagement with the probability concepts. Being able to record and express significant ideas in a variety of ways indicated a security with two related but distinctly different ways of thinking. Despite limited formal development in probabilistic thinking, early development with numerical and spatial patterns provided a platform to support investigation of probability ideas. A limitation is that this paper examines just one rich example of how a child's robust in number can support learning and understanding in probability. There would be merit in examining further task-based observation and interview data with a wider sample of learners to make a stronger argument for the significance of number understanding in a probability context.

References

Attard, C. (2012). Engagement with mathematics: What does it mean and what does it look like? *Australian Primary Mathematics Classroom, 17*(1), 9-12.

Barnes, M. (1998). Dealing with misconceptions about probability. *Australian Mathematics Teacher*, 54(1), 17-20.

Breyfogle, L., & Williams, L. (2008). Designing and implementing worthwhile tasks. *Teaching Children Mathematics, 15*(5), 276-280.

Clarke, D., & Roche, A. (2010). The power of a single game to address a range of important ideas in fraction learning. *Australian Primary Mathematics Classroom, 15*(3), 18-23.

Crites, T. (1994). Using lotteries to improve students' number sense and understanding of probability. *School Science and Mathematics, 94*(4), 203-207.

Gal, I. (2004). *A brief look at statistical literacy*. Retrieved from
http://ced-mxteachers-news site.wikispaces.com/file/view/iddo+gal+statistical+literacy.pdf

Gal, I. (2005). Towards "Probability Literacy" for all citizens: Building block as and instrumental dilemmas. In G. A. Jones (ed.), *Exploring Probability in School*, (pp39-64). New York: Springer.

Higgins, H. J., & Weist, L. R. (2006). Individual interviews as insight into children's computational thinking. *Australian Primary Mathematics Classroom, 11*(1), 35-40.

Hunter, J. (2009). Developing a productive discourse community in the mathematics classroom. *The New Zealand Mathematics Magazine*, 46(1), 1-12.

Hunter, R. (2006). Structuring the talk towards mathematical inquiry. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces*: (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia, Canberra Vol.1, pp. 309-317). Canberra: MERGA.

Jones, G. A., Langrell, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics*, 32(2), 101-125.

Jorgensen, R., & Dole, S. (2011). *Teaching mathematics in primary schools*. Crows Nest, NSW: Allen & Unwin.

Langrell, C. W., & Mooney, E. S. (2005). Characteristics of elementary school students' probabilistic thinking. In G. A. Jones (Ed.), *Exploring Probability in School. Challenges for Teaching and Learning* (pp.95-120). New York: Springer.

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, 21(2), 10-32...

Ministry of Education, (2010). *Whanaketanga pāngarau: He aratohu mā te pouako*. Wellington: author.

Ministry of Education, (2009). *Mathematics Standards for years 1-8*. Wellington: Author.

Ministry of Education, (2008). *Te Marautanga o Aotearoa*. Wellington: Author.

Ministry of Education (2007). *The New Zealand Curriculum*. Wellington: Author

Mulligan, J., & Mitchelmore, M. C. (2013). *Early awareness of mathematical pattern and structure*. Retrieved fromhttp://download.springer.com/static/pdf/498/chp%253A10.1007%252F978-94-007-6440-8_3.pdf?auth66=1418265982_0478a0315c1ef13173d35fc9d13a3a05&ext=.pdf

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for school Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Neill, A. (2010). There probably is probability. In R. Averill & R. Harvey (Eds.), *Teaching primary school mathematics and statistics: Evidence-based practice*. (pp.153-164). Wellington: NZCER.

Reinhart, S. C. (2000). Never say anything a kid can say. *Mathematics Teaching in the Middle School*, 5(8), 478-483.

Suh, J.M., Johnston, C., Jamieson, S., & Mills, M. (2008). Promoting decimal number sense and representational fluency. *Mathematics Teaching in the Middle School*, 14(1), 44-50.

Students' Relationships with Mathematics: Affect and Identity

Naomi Ingram
University of Otago
<naomi.ingram@otago.ac.nz>

In this paper, an examination of students' relationships with mathematics is informed by affective research into internal mathematical structures and identity research into students' narratives. By analysing the perceptions of a class of 31 adolescents, five interacting elements emerged: students' views, feelings, mathematical knowledge, identities, and habits of engagement. These elements contributed to the context within which students engaged in mathematics and resulted in their unique learning experiences. This framework has potential for researching aspects of students' mathematical journeys and can be used by teachers to get to know individual students' unique connection to the subject of mathematics.

Introduction

A secondary school mathematics classroom is a physical space shared by a teacher and a group of students who have a set of shared norms. They generally work on the same mathematical tasks. Despite these similarities, students engage in mathematics in different ways. Some relish the experience, investigating and discussing further possibilities. Some, bored and restless, follow the necessary steps to get the task over with as quickly as they can. Some steel themselves to have a go, checking the answer frequently and feel lucky if they get it correct. Others avoid the situation by chatting socially or sharpening their pencil.

Students engage in mathematics in different ways because they have unique relationships with the subject. A *student's relationship with mathematics* is defined in this paper as the dynamic connections between the student and the subject of mathematics. This concept has strong links to notions of *mathematical self* or *self-identity* found in affective and identity research. This literature informed the examination of a group of students' relationships with mathematics. This paper reports specifically on these relationships as one aspect of a larger, longitudinal study (Ingram, 2011). The elements of these relationships are specified in this paper and the potential for using this framework in research and practice is explored.

Affect

Learning mathematics is an emotional practice that generates a range of affective responses. *Affect* describes the experience of feelings and emotions (McLeod, 1992). Research into affect in mathematics education explores these as well as other elements in the affective domain such as motivation, anxiety, engagement, attitudes, identity, and beliefs. These elements interact in complex ways and holistically researching across elements is valuable (Grootenboer, 2003).

One aspect of affective research in mathematics education is the conceptualisation of individuals having stable internal structures that relate to mathematics. These have been variously described as a *global affective structure* (DeBellis & Goldin, 2006), *self-system,* (Malmivuori, 2006), *mathematical disposition* (Op 't Eynde, De Corte, & Verschaffel, 2002), or *identity* (Op 't Eynde, De Corte, & Verschaffel, 2006). These structures generally contain the following elements:

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 301–308. Sunshine Coast: MERGA.

- *Beliefs about mathematics* which incorporate students' personal, internal and shared subjective conceptions about mathematics, mathematics teaching and learning, about themselves in relation to mathematics, and about the context (Malmivuori, 2006; Op 't Eynde et al., 2006);
- Related *goals and needs* related to autonomy, competency, and social belonging (Hannula, 2006);
- Other global affects such as *values and attitudes* (DeBellis & Goldin, 2006);
- *Mathematical content knowledge* such as the facts, symbols, concepts, and rules that constitute mathematics (Malmivuori, 2006). Strategies for accessing and using knowledge to solve problems (Op 't Eynde et al., 2006);
- *Meta-knowledge*, which involves knowledge about meta-cognitive functioning and knowledge about affect and its use (Malmivuori, 2006);
- *Habitual affective pathways* and behaviours in mathematics, including affective skills (DeBellis & Goldin, 2006).

These structures develop from students' previous experiences with mathematics in social environments (Malmivuori, 2006). They form part of the context within which students learn mathematics. When learning, students interpret the mathematical situation according to their internal structure. As a result, they experience a wide range of unique affective responses, which can be unstable, hot emotions, with accompanying physiological arousal such as anxiety or joy, or they can be less hot responses such as boredom or interest. These provide information for the individual about their progress towards their needs and related goals and may disrupt or distract the learning process and affect the level of capability while performing mathematics. This information activates self-appraisals, which thus determine how a student approaches the mathematical task, depending on their current level of awareness, control, and regulation capacities. These processes result in unique performances and new learning experiences. Students' interpretations of these experiences reinforce or, if sufficiently powerful or repeated often enough, alter these structures.

This research generally views students' learning as a product of individual cognitive processes and students are usually researched outside of a classroom context in problem solving situations, rather than within the social context of the mathematical classroom. Furthermore, there are few examples in the affective literature of students' perspectives of how their affect and learning are associated.

There has been some recognition of learning as a social process and connections made between affect and identity. Op 't Eynde et al. (2006) see learning as taking place through engagement in the language, rules, and practices that govern activities in the community of the mathematics classroom. They connect affect and identity:

> [Students'] understanding of and behaviour in the mathematics classroom is a function of the interplay between who they are (their identity), and the specific classroom context. Who they are, what they value, what matters to them in what way in this situation is revealed to them through their emotions" (p. 194).

The elements of a student's internal structure related to mathematics need to be viewed as both collectively and individually constituted through participation in the shared practices of the mathematics classroom. To understand better how students' learn mathematics, there seems to be potential in better understanding connections between the notions of a student having stable internal structures relating to mathematics and ideas of mathematical identity. It is these connections that are now explored.

Identity

Identity is variously seen in mathematics education research as how an individual names themselves and how they are looked on by others (Grootenboer, Smith, & Lowrie, 2006), self-concept (McFeetors & Mason, 2005), a performance (Darragh, 2014), or a narrative about a person (Kaasila, Hannula, Laine, & Pehkonen, 2005). Many researchers in mathematics education (e.g., Boaler, 2000; Op 't Eynde & Hannula, 2006) are informed by Wenger (1998) who defined identity as a constant becoming of who one is in a particular social context.

Sfard and Prusak (2005a, 2005b) take a dynamic view of identity powered by their investigation into the differences in mathematical learning processes between immigrant students from the Soviet Union and native Israelis. They dispute any process of defining identity as *who one is*, just as they reject notions of *God-given* personality, ethnicity, and nature; essentialist visions of identity, which "seem to be saying that there is a thing beyond one's actions that stays the same when the actions occur" (Sfard & Prusak, 2005b, p. 15). They developed a narrative approach to identity and see identity formation to be a form of communicational practice. In their view, identities are the stories that surround a person. "No, no mistake here: We did not say that identities were finding their expression in stories – we said they were stories" (Sfard & Prusak, 2005b, p. 14). Specifically, Sfard and Prusak (2005a, 2005b) and later Sfard (2008), equated identities to be those stories surrounding a person which are:

- Reifying – the transformation of an action into a state which suggests repetitious behaviour through the use of the verbs be, have, can, and the adverbs always, never, usually.
- Endorsable – the identified person (the person the story is about) endorses that the story reflects the actual or expected state of affairs.
- Significant – if any change in it is likely to affect the storyteller's feelings about the identified person particularly with regard to membership of a community.

A person has a number of stories told about them by multiple narrators, including themselves. Stories consist of a person's self-dialogue (thinking), spoken-out-loud stories about themselves or other people, stories told about them by other people, interactions with other people, and reactions to events. There are also those stories told about that person by other narrators. Identities, according to Sfard and Prusak (2005a, 2005b) also included extra-discursive (or mind-independent) stories, such as examination results, certificates, and report grades, referred to as institutional narratives.

Sfard and Prusak (2005a, 2005b) divide a person's multiple identities into two sets of identities. Actual identities are attempts to overcome the fluidity of change by freezing the picture (Sfard & Prusak, 2005a, 2005b). These stories are factual assertions about a person, and can be identified by the use of *I am* or *he is* sentences told in the present tense, such as *I am bad at maths* or *He is a good mathematician*. Designated identities – *I should be* stories – have the potential to become part of one's actual identity, and influence one's actions to a great extent. Sfard and Prusak (2005a, 2005b) usefully link affect, learning, and identity because they suggest there is likely to be a sense of unhappiness in a person when there is a perceived and persistent gap between a student's actual and designated identities.

In the affective research, students are conceptualised as having internal structures that connect themselves and mathematics. Viewing identity as a narrative does not discount this view. Students' designated identities are similar to the affective notions of self-directive constructions (Malmivuori, 2006) and needs (Hannula, 2006). Hannula (2006) described a students' needs as relatively stable and there was stability evident in the students' sets of designated identities in Sfard and Prusak's research (2005a, 2005b) because of their cultural basis. This view of identity as a narrative adds the social to the elements in the internal structure and adds to understanding about how students' internal structures change. Using the phrase *internal structur*e from the affective literature now seems too static to describe this very dynamic process. *Students' relationships with mathematics*' seems a better fit.

Learning is seen here as engagement in practices of the mathematics classroom and in other communities of practice. The students negotiate the meanings constructed from their interpretations of their learning experiences and these meanings either reinforce or alter the elements of their relationship with mathematics. A student's relationship with mathematics is therefore understood in this paper to have both individual and shared elements that are constantly changing. It is these elements that this research seeks to identify. Specifically, this research seeks to investigate the nature of students' relationships with mathematics and how these relationships are associated with mathematical learning.

Methodology

The 31 participants attended a co-educational school in New Zealand. They were from the same class so the social norms and views of the class as a whole could be examined as well as the affect and identities of the individual students. Students in Year 10 (aged 14-15 years) were researched because understanding adolescents' relationships with mathematics is vital because they are on the "brink of deciding whether or not to pursue mathematical studies" (Nardi & Steward, 2003, p. 346).

The methodology of this research was informed by the affective research into students' internal structures and Sfard and Prusak's (2005a, 2005b) narrative view of identity. Sfard and Prusak (2005a, 2005b) operationalised the notion of identity by gathering evidence of students' spoken identities. Their research is based around what students say, rather than on the researcher or teacher's perceptions of what is going on in the classroom.

A qualitative framework was employed in this research. The data collected included observations of mathematics and English classes, interviews, metaphors for mathematics, drawings of mathematicians, personal journey graphs, questionnaires, exercise books, assessment results, reports, prizes, and attendance. The teachers were interviewed. Informed by Evans (2000), affective indicators were sought such as verbal expressions of feelings, the use of metaphors, negative or positive self-talk, body language, avoidance, and resistance. Other data collected were students' reflections on their experiences, their views of mathematics, and the language they used to describe mathematics. The students' identity stories were collected mainly through the interviews. Decision-making permeated the process of data collection and analysis.

The data was analysed using a grounded theory approach of constant comparison to seek, refine, and understand the interrelationship of the emerging elements of a students' relationship with mathematics. A data analysis software package NVivo (QSR International, 2006), helped to manage the large data set and aid the analysis.

Results

The students described relationships with mathematics that had five elements:

1. *Views of mathematics:* Subjective conceptions students hold to be true about mathematics. The students had views about the nature, uniqueness, importance, and difficulty of mathematics and perceptions of how boring the subject was.
2. *Macro-feelings:* Coined by the students, *macro-feelings* are a student's overall feelings about the subject of mathematics. These feelings contributed to the context within which they engaged in a specific mathematical activity. When a student had negative macro-feelings for the subject of mathematics, they were more likely to have negative *micro-feelings;* the feelings they experience during each mathematical situation.
3. *Identities:* The students each had a unique set of identities related to their view of their mathematical ability. They had designated identities – overall expectations about mathematics, which included commonly held expectations of class placement, individual expectations related to class positioning and how they expected the subject to contribute to their future life. They also had actual identities – perceptions of how good they were at mathematics, which developed through their interactions with others and through their experiences of success and failure when they engaged in the mathematics.
4. *Mathematical Knowledge:* The students had different levels of mathematical knowledge, which students talked about in relation to their knowledge of facts and mathematical rules that they knew *off by heart*.
5. *Habits of engagement:* The students engaged in mathematics in habitual ways that developed over time. Among were the students' pathways of engagement – the ways they usually engaged in the mathematical tasks.

The elements of students' relationships with mathematics were both shared by the classroom community and unique to the individual. For example, the class shared common views about their expectations of their teachers, yet individual students had unique macro-feelings about mathematics and unique perceptions of their own mathematical ability. The elements also interacted in complex ways. The students' macro-feelings about the subject of mathematics were associated with their views of mathematics and were situated in the gap between their actual and designated identities. The students' mathematical knowledge was closely linked to their views of the nature of mathematics. The ways the students habitually engaged in mathematics were associated with their macro-feelings, their views of mathematics, and their identities.

Figure 1 summarises the process of change in students' relationships with mathematics. Their relationship with mathematics contributed to the context within which they engaged in the task. Students' views of mathematics led them to judge the task's importance and difficulty. Their identities led them to have expectations of success. The ways they habitually engaged in mathematics, interacting with the other elements, affected their engagement in the task. Macro-feelings contributed to the micro-feelings they experienced during the task. Furthermore, when the students engaged in a mathematical task, they were each situated in a unique *context of the moment.* Even when they were experiencing the same classroom conditions – the same teacher, at the same time of day – the students each interpreted the context in a unique way. Students' engagement in the mathematical task was therefore determined by the complex negotiation between elements of their relationship with mathematics and the context of the moment.

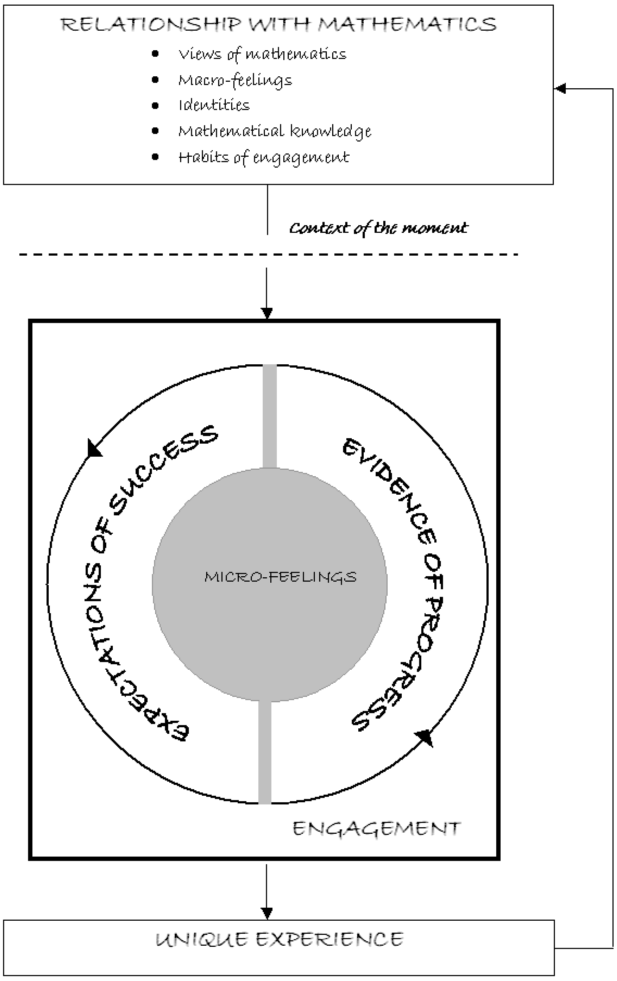


Figure 1. Students' relationships with mathematics

During students' engagement in the task, they collected evidence of their progress. They experienced micro-feelings as they interpreted whether or not their progress met their expectations of success. In Figure 1, the students' expectations and evidence of progress are represented within a circle to show that they surround a student's micro-feelings, and the arrows around this circle show that students' progress can alter expectations of success or vice versa. The way students engaged in the task contributed to their individual experiences and performances, with the way they interpreted these experiences, in turn, reinforcing or altering components of their relationship. These elements, described above, emerged from examining students' perspective of their mathematical learning, yet there are some similarities between these and the components of a student's internal structure, described in the affective research. Both include elements relating to knowledge, beliefs, affect, expectations, and habits. Both include aspects of change and stability. The students' views about school mathematics are similar to the beliefs about mathematics that other researchers found, but these categories emerged from the students' perspective, rather than in response to prompts in a questionnaire. Similar to Op 't Eynde et al.'s (2006) conception of a belief, students' views of mathematics were socially constructed and situated in the context of the mathematics classroom, and dynamic.

Mathematical knowledge is generally defined as the facts, symbols, concepts, and rules that constitute the contents of mathematics as a subject field, as perceived by the community of mathematicians (Op 't Eynde et al., 2002). When students in the current research talked about mathematical knowledge, they usually meant the rules they had been taught by their teacher. The students' knowledge was co-created by the community of the classroom, and may be different to how mathematicians might conceive of mathematics. As discussed by Schoenfeld (1992), the students' conception of knowledge was related to the way the students were taught mathematics – as a series of rules, given with specific examples, and reinforced by practice of that rule from the textbook.

A student's macro-feelings are similar to DeBellis and Goldin's (2006) conceptualisation of *global affect* and McLeod's (1992) notion of a student's *attitude* to mathematics: stable over time compared to transitory emotions. The students' macro-feelings in this research were relatively stable compared to their micro-feelings. The students also described, in some detail, the pathways they usually took when attempting a mathematical task (i.e., their pathways of engagement), a term adapted from Goldin's (2004) use of the term *affective pathway*s to describe individual's dynamic problem solving processes at a task level. The students used it in a more macro sense to describe their habitual pathways of engagement, as described in Ingram (2013).

Conclusions and Implications

Combining the affective concept of students having internal mathematical structures with identity research into narratives informed this examination of students' relationships with mathematics. Students' relationships with mathematics were found to have five elements: views of mathematics, macro-feelings, identities, mathematical knowledge, and habits of engagement. These elements provided part of the context within which the students' engaged in mathematics and contributed to their unique learning experiences. The students' interpretation of these learning experiences reinforced or changed the elements of their relationship with mathematics.

This paper has captured the relationships with mathematics of students in one class. These relationships are connected with that particular context, although there were similarities with students in other classrooms, both in New Zealand (Averill, 2009) and internationally (Boaler, 2000). Despite this, the potential of defining the elements in a student's relationship with mathematics has begun to be realised. The framework of elements was used to analyse the 31 students over two years of their mathematical journeys as they continued to participate, or not to participate, in mathematics (Ingram, 2011). It was used to provide a context for a closer examination of students' engagement (Ingram, 2013), the influence of the parents and teachers, and to explore the tensions between social and mathematical identities (Ingram, 2008). The elements of students' relationships with mathematics have also been communicated to both in-service and pre-service mathematics teachers in New Zealand to provide understanding of how affect and identity play a part in students' learning and to provide a framework for getting to know their students.

References

Averill, R. (2009). "Enjoy your job and enjoy our company": Students talk about mathematics teachers. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice* (Vol. 1, pp. 53-66). Wellington: NZCER.

Boaler, J. (2000). Introduction: Intricacies of knowledge, practice, and theory. In J. Boaler (Ed.), *Multiple perspectives of mathematics teaching and learning*. Westport, CT: Ablex Publishing.

Darragh, L. (2014). Recognising ‘good at mathematics’: using a performative lens for identity. *Mathematics Education Research Journal*, 1-20.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representative perspective. *Educational Studies in Mathematics*, *63*, 131-147.

Evans, J. (2000). *Adults' mathematical thinking and emotions*. New York: RoutledgeFalmer.

Goldin, G. A. (2004). Characteristics of affect as a representational system in RF01: Affect in mathematics education - exploring theoretical frameworks. Paper presented at the 28th Conference of the International Group for the Psychology of Mathematics Education, Bergen, Norway.

Grootenboer, P. (2003). *Preservice primary teachers' affective development in mathematics*. (doctoral dissertation), University of Waikato, Hamilton, New Zealand.

Grootenboer, P., Smith, T., & Lowrie, T. (2006). Researching identity in mathematics education: The lay of the land. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, Cultures and Learning Spaces* (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia, Vol. 2, pp. 612-615). Adelaide: MERGA.

Hannula, M. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics*, *63*, 165-178.

Ingram, N. (2008). Who a student sits near to in maths: Tension between social and mathematical identities. In M. Goos, R. Brown, & K. Makar (Eds.) *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia). Brisbane: MERGA.

Ingram, N. (2011). *Affect and identity: The mathematical journeys of adolescents*. (Doctoral dissertation, University of Otago, New Zealand).

Ingram, N. (2013). Mathematical engagement skills. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics Education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne: MERGA.

Kaasila, R., Hannula, M., Laine, A., & Pehkonen, E. (2005). Autobiographical narratives, identity and view of mathematics. CERME 4. Retrieved 7 May 2011, from http://ermeweb.free.fr/CERME4/CERM4_WG2.pdf

Malmivuori, M. (2006). Affect and self-regulation. *Educational Studies in Mathematics*, *63*, 149-164.

McFeetors, J., & Mason, R. (2005). Voice and success in non-academic mathematics courses: (re)forming identity. *For the Learning of Mathematics*, *25*(3), 16-23.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: NCTM and Macmillan.

Nardi, E., & Steward, S. (2003). Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal*, *29*(3), 345-367.

Op 't Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. In G. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education*? (pp. 13-37). The Netherlands: Kluwer Academic Publishers.

Op 't Eynde, P., De Corte, E., & Verschaffel, L. (2006). “Accepting emotional complexity”: A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics*, *63*, 193-207.

Op 't Eynde, P., & Hannula, M. (2006). The case study of Frank. *Educational Studies in Mathematics*, *63*, 123-129.

QSR International. (2006). *NVivo 7*. QSR International Pty Ltd.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). NY: MacMillan.

Sfard, A. (2008). *Thinking as communicating: Human development, and the growth of discourses, and mathematizing*. NY: Cambridge University Press.

Sfard, A., & Prusak, A. (2005a). Identity that makes a difference: Substantial learning as closing the gap between actual and designated identities. Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Melbourne.

Sfard, A., & Prusak, A. (2005b). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

Using Alternative Multiplication Algorithms to 'Offload' Cognition

Dan Jazby
University of Melbourne
<dan.jazby@unimelb.edu.au>

Cath Pearn
University of Melbourne
<cpearn@unimelb.edu.au>

When viewed through a lens of embedded cognition, algorithms may enable aspects of the cognitive work of multi-digit multiplication to be 'offloaded' to the environmental structure created by an algorithm. This study analyses four multiplication algorithms by viewing different algorithms as enabling cognitive work to be distributed across environmental and mental resources to varying degrees. This produces a plausible framework which could allow further analysis designed to guide the pedagogical use of alternative algorithms.

Many students struggle to learn the traditional written algorithms introduced in primary school (Pearn, 2009). Heirdsfield (2004) states: "vertical algorithms dictate a rigid procedure, and do not lend themselves to encouraging students to manipulate numbers flexibly" (p.8). However, Westwood (2000) suggests that:

> children should have no problem mastering these procedures [algorithms] if they are linked as closely as possible with the more informal methods … that are typically used by children … difficulties arise if the processes are taught without reference to children's prior learning or way of recording (p. 47).

While many teacher publications (such as Randolph & Sherman, 2001) have advocated that alternative algorithms may be beneficial for students, there is less written regarding exactly how and why an alternative algorithm may help students (Carroll & Porter, 1998). This paper uses a theoretical framework based on embedded cognition to try to assess how different algorithms may impact on the cognitive demands of multi-digit multiplication. This analysis may provide some explanation as to why some students develop a preference for an alternative algorithm, enable such preferences to be used to diagnose student understanding and suggest a framework which could demonstrate when teaching an alternative algorithm may or may not be justified. This paper focuses on alternative multiplication algorithms that can be used instead of 'traditional' long multiplication for multi-digit problems.

Student use of Alternative Algorithms for Multiplication

'Alternative' algorithms are defined in contrast to the 'traditional' algorithm which is sometimes referred to as long multiplication (West, 2011). Many articles which explain alternatives to long multiplication are aimed at helping teachers learn how to use these alternatives so that:

> Students' individual needs and styles are the focus of lessons on alternative algorithms. Using these options, students develop their own understanding of, and skills in, arithmetic operations, enhancing their decision-making and critical thinking skills (Randolph & Sherman, 2001, p. 484).

Advocates of alternative algorithms tend to argue that they may be of benefit to students because knowing alternative methods improves general understanding. It has been noted that some students develop a preference for some algorithms. Lattice multiplication is one such alternative algorithm – an example is presented in Figure 1 and explained in detail in Results. When Carroll and Porter (1998) described the method they noted that, "although the reasons are not obvious to us, this method has proved to be very popular with students" (p. 111). They also note that low-achieving students tend to like this

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 309–316. Sunshine Coast: MERGA.

method, perhaps because of its structure. If low achieving students prefer an algorithm such as the lattice method, then perhaps there is something about either the lattice method or student understanding which explains this preference. Also, if students' general understanding of mathematics is enhanced by use of alternative algorithms, some investigation of the mechanisms which underpin this seem warranted. This study uses a framework of embedded cognition to attempt to make sense of students' use of alternative algorithms. The analysis seeks to provide a lens which enables alternative algorithm use to be interrogated in more detail. If alternative algorithms can cater to students' individual needs, as Randolph and Sherman (2001) state, then one should ask which student needs an alternative algorithms addresses. Embedded cognition, described in the next section, provides a method for analysing how cognitive work can be distributed across mental and environmental resources. It is hypothesised that different distributions of this cognitive work by different algorithms may change the mental demands placed on students as they solve a problem.

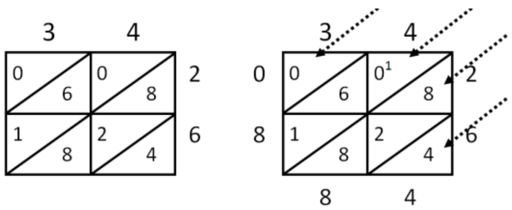


Figure 1. 34 × 26 using the lattice algorithm

An Embedded View of Cognition

Embedded cognition posits that cognition is embedded in an environment. This means that people use environmental structures to 'offload' cognition so that cognition is distributed across both environmental and mental resources (Kirlik, 2007). It is theorised that the environment provides a direct model of a problem and that a person can use and create environmental structures which offload cognition, using a combination of mental and environmental resources to perform tasks which, traditionally, have been seen to occur purely mentally.

Kirlik's (2007) model of embedded cognition has been developed in the research field of Human Factors and Ergonomics. It draws on situated theories of learning, such as those developed by Greeno (1998), to describe the interaction between workers and their task environments – mainly in Kirlik's main field of interest, aviation. While situated learning theorists often focus on how learning is situated within complex social entities (Greeno, 1998), Kirlik's focus examines how cognition is situated within a physical task environment. This paper seeks to assess whether the view that cognition can be 'offloaded' to physical environmental structures could provide a productive lens to understand how children develop preferences for particular algorithms. Kirlik's view of embedded cognition would suggest that, when children use different algorithms, they may be able to manipulate or utilise environmental structures, in such a way as to 'offload' cognition and reduce demands made on mental resources. Information can be stored and updated in the environment. A calculation can be separated into simpler calculations. Each of the simple calculations can be performed mentally, then results can be stored in the environment and combined with the results of other calculations at a later time, so that running totals of simple calculations do not have to be maintained mentally. By writing or typing information into an algorithm's predetermined structure, not only can information be stored, but cognitive processes can be ordered and coordinated. Just as Kirlik (2007) has argued that professionals in aviation are able to improve their task performance by effectively distributing 'cognitive work' across both mental and environmental resources, algorithms may facilitate more effective computation via a similar process of offloading.

What 'Cognitive Work' do Algorithms Perform?

For the purposes of this study, multiplication algorithms are being viewed as cognitive aids which enable a multiplication problem to be broken up into a series of less cognitively demanding subroutines. The authors distinguish between two phases in multiplication algorithms – a multiplication phase and an addition phase. For example, when calculating 34 × 26, most algorithms enable 34 × 26 to be calculated by separating 34 × 26 into a series of simpler calculations (e.g. 6 × 4, 6 × 3, 2 × 4, 2 × 3). Kilian, Cahill, Ryan, Sutherland, and Taccetta (1980) found that 32% of student errors occurred using the traditional algorithm related to miscalculation in this multiplication stage.

During these calculations, the place value position of the numbers being multiplied may be suspended. When calculating 34 × 26, 30 must be multiplied by 6. Algorithms which suspend place value enable this calculation to be carried out as 3 × 6. If an algorithm suspends place value, then successful use of the algorithm requires some cognitive work which recognises that this 3 × 6 is in fact 3 tens × 6 (and is therefore 18 tens rather than 18). Kilian et al. (1980) found that 18% of student errors involving the traditional algorithm related to place value.

As algorithms break 34 × 26 into a series of simpler calculations another cognitive demand of using the algorithm entails ensuring that all of these simpler calculations are performed. The term 'completeness' is used to refer to the cognitive task of ensuring that all relevant simpler calculations have been performed. Kilian et al. (1980) found that 7% of students' errors using the traditional algorithm involved missing one of these simpler calculations.

In the addition phase, the products of the simpler calculations performed in the multiplication phase must be totalled correctly. This is often performed using a traditional addition algorithm which adds like place value parts. Kilian et al. (1980) found that, when the traditional algorithm was used, errors in addition calculations were low (9%), but 'carrying' mistakes accounted for approximately a quarter of all errors (24%). One of the algorithms analysed does not use a traditional addition algorithm. Instead, the products of the simpler multiplication steps are ordered without explicit reference to place value. See the line algorithm in Results for a detailed description of this kind of addition phase.

Method

West (2011) provided a description of nine alternative multiplication algorithms. In this paper we will discuss three algorithms. These three algorithms (Line, Lattice and Area multiplication algorithms) were selected because they make use of structures in their physical layout. These alternative algorithms are compared to the partial product algorithm commonly taught in Australian schools. Each algorithm will be compared to each other when used to solve the same problem. A 2-digit multiplied by 2-digit number problem was selected (34 × 26) with the numbers being chosen using a random number generator.

During the multiplication stage of each algorithm, initial analysis will involve determining which simpler calculations this multiplication problem – 34 × 26 – is broken into. The next analysis involves identifying whether any of these calculations need to be performed mentally or can be offloaded to the structure of the algorithm.

Place value in the multiplicative stage of each algorithm has been categorised as either 'suspended' or 'not suspended'. If an algorithm suspends place value, then some cognitive work must be performed in order to reinstate place value position after simplified calculations have occurred. Algorithms which suspend place value have then been categorised according to whether reinstating place value position needs to be performed mentally or can be offloaded to the structure of the algorithm.

Completeness has been analysed in terms of whether this needs to be maintained mentally or whether this cognitive task can be offloaded to the structure of the algorithm.

There are two classifications that have been used in relation to the addition phase of each algorithm. Addition phases either employ the traditional addition algorithm or they employ a non-traditional addition algorithm which does not add products in place value parts (described in the next section).

Results

In the following section, each alternative algorithm is analysed separately before these separate analyses are compared in Tables 1 and 2.

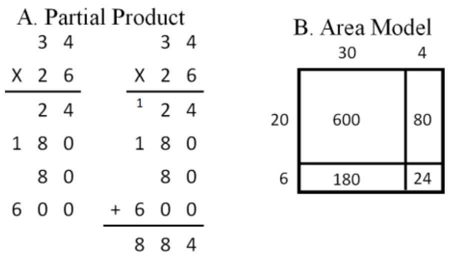


Figure 2. 34 × 26 using the partial product and area model algorithms

Partial Product Algorithm

Part A of Figure 2 shows the partial product algorithm broken down into two phases – the multiplicative phase and the additive phase. In the multiplicative phase, four single-digit numbers must be multiplied separately. These calculations must be performed mentally. Place value is suspended when these calculations are performed. When recording each partial product, place value must be tracked, mentally, by the student. In particular, when '3' is multiplied by '2', students must mentally track that this is really 3 tens multiplied by 2 tens, and thus, the product is 6 hundreds. A significant amount of mental cognition must be employed to track that the product of digits in the second column need to be recorded in the third column. Hence, this algorithm has been classified as requiring mental cognition to maintain place value. Completeness is also only partially helped by the structure of the algorithm. The common approach is to start with the right most digit of the bottom row number and multiply this by the digits of the number on the top row starting from left to right. However, there is no element of the structure of this algorithm which enables students to visually recognise if they have missed a step. Hence, this aspect of the cognitive work of using the algorithm has been coded as requiring mental cognition.

In the addition phase, the column structure of the algorithm allows each place value part to be totalled separately. When the second column results in a total greater than 9, 'carrying' is used. Then students must recognise the need to move into the next column, so that 18 tens is renamed as 1 hundred and 8 tens.

Area Model Algorithm

Part B of Figure 2 shows the area model algorithm. An area model is used to model the problem and the resulting rectangle is partitioned into four separate areas. Place value parts are used to partition the rectangle so that the side of the rectangle which is 34 long is separated into two segments which are 30 and 4 long respectively. The area of each of the four partitions is arrived at by multiplying 4 × 6, 6 × 30, 20 × 4 and 20 × 30. While this entails multiplication of more than a single-digit number, the 2-digit numbers do not contain a non-zero digit in the ones-digit position, which reduces the difficulty of the calculation. This enables calculations to be performed mentally. When this algorithm is used, place-value positions in the multiplicative phase are maintained throughout. Furthermore, the structure of algorithm provides a visual representation of the magnitude of the products of multiplication – the 30 × 20 partition looks considerably bigger than the 20 × 4 partition. As partial products are recorded in each of the four partitions of the original rectangle, completeness is offloaded to the structure of the algorithm. In the addition phase, a traditional addition algorithm is used which has the same processes as a partial product algorithm.

Line Algorithm

Figure 3 shows the line algorithm. The horizontal lines represent 34 as there are 3 lines grouped at the top and 4 lines grouped at the bottom. The vertical lines represent 26 (2 lines to the left and 6 lines to the right). Diagonal lines (dotted in Figure 3) are used to create three areas. The number of line intersections in each area are then counted and totalled. In the top left region, there are 6 line intersections, 26 intersections in the middle region and 24 in the lower right region.

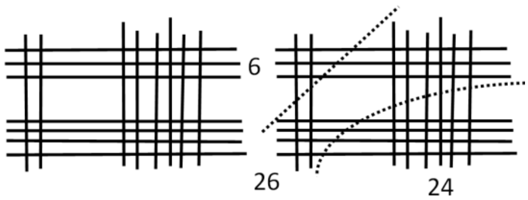


Figure 3. 34 × 26 using the line algorithm

This algorithm allows the problem to be solved without using mental multiplication calculations. A student could use a ‘count all’ strategy to total the number of line intersections. Hence, the structure of the algorithm enables 34 to be multiplied by 26 without having to mentally perform any multiplicative calculations. Place value is suspended and does not need to be tracked mentally. Completeness is supported by the structure as long as all line intersections are counted.

In the addition phase, the three products (6, 26 and 24) need to be combined. Starting with the right-most total (24), digits not in the right-most position are added to the number on the left. Thus, the ‘2’ digit in 24 is added to 26 to get 28, the ‘2’ digit in 28 is added to 6 to get 8. This produces the number 884 as the final product. In this phase as well, the student does need to keep track of place value parts because of the structure of the algorithm. If place value is maintained in this algorithm, then the three products derived from this algorithm are 600, 260 and 24, and the addition phase of the algorithm combines these. However, the algorithm enables these products to be combined without having to mentally reinstate place value.

The Lattice Algorithm

Figure 1 shows the multiplication and addition phases of the lattice algorithm. Like the partition product algorithm, four pairs of single-digit numbers are multiplied. The products of these calculations are recorded differently. When 2 is multiplied by 6 it is recorded in the grid where the tens-digit is recorded above the diagonal line and the ones-digit is recorded below (e.g. 2/4 for the total twenty-four). If there a product is less than 10 (e.g. in the case of 2 × 4), a zero is recorded above the diagonal line (e.g. 0/8).

Calculations during the multiplication phase are performed mentally and place value is suspended. The lattice structure of this algorithm maintains the place value position of these digits so that they do not have to be maintained mentally. This structure ensures completeness without requiring any mental effort on the part of the student as all parts of the grid need to be filled.

In the addition phase, the dotted arrows on Figure 1 indicate which numbers need to be added together. Following the diagonal lines of the lattice, there are four ‘diagonals’ that need to be totalled. When the second diagonal results in a total over 9 (8 + 2 + 8 = 18), the tens-digit is ‘carried’ to the next diagonal. Each diagonal maintains place value in a similar fashion to columns in standard addition algorithms, although zero digits are not needed to communicate place value position.

Table 1:
Calculations and place value in the multiplication phase of each algorithm

	Calculation		Place Value (PV)		
			Suspend	Maintain PV position of digits	
Algorithm	Mental	Structure	PV	Mental	Structure
Partial Product	Y	N	Y	Y	N
Area Model	Y	N	N	N/A	N/A
Line	N	Y	Y	N	Y
Lattice	Y	N	Y	N	Y

Comparing Algorithms

Tables 1 and 2 compare the four algorithms analysed. Each of the four multiplication algorithms analysed enable different aspects of the cognitive work of solving a multi-digit multiplication problem to be distributed differently between the environment and an individual’s mental resources. Multiplication calculations must be performed mentally with all algorithms except the line algorithm. Only the partial product algorithm requires students to maintain place value mentally – the structure of both the line and lattice algorithms allows this cognitive task to be offloaded while the area model does not suspend place value.

Table 2 shows that all three of the alternative algorithms enabled completeness to be offloaded to the environmental structure of each algorithm and the line algorithm used a non-traditional approach to addition which enabled successful addition to take place without mentally maintaining place value position of digits.

Table 2:
Ensuring all multiplication calculations are made and summary of the addition phase

	Completeness		Addition phase	
Algorithm	Mental	Structure	Traditional PV part addition algorithm	Addition algorithm without PV
Partial Product	Y	N	Y	N
Area Model	N	Y	Y	N
Line	N	Y	N	Y
Lattice	N	Y	Y	N

Discussion

Rather than catering to "students' individual needs and styles" (Randolph & Sherman, 2001) alternative algorithms – in this analysis – are posited to have a specific impact on students' mental workload. The analysis presented suggests that this view is theoretically plausible and could be used to guide further investigation. The lattice algorithm, for example, may enable successful calculation of multi-digit multiplication without the need to mentally attend to the place value position of component calculations and completeness. While Carroll and Porter (1998) could not identify any obvious reason why 'low achieving' students would develop a preference for the lattice algorithm, viewing students' use of algorithms as embedded cognition provides a theoretical explanation: if the lattice algorithm offloads aspects of the cognitive work of solving the problem into an environmental structure, students can successfully perform multi-digit multiplication with decreased cognitive load.

If the use of algorithms – like the work of professionals in working environments (Kirlik, 2007) – is embedded, and students can use different algorithms to distribute cognitive work across both mental and environmental resources differently, then one may ask whether all algorithms are created equal; should all algorithms be taught to students; and what would warrant the use of a particular algorithm? Viewing algorithms through the lens of embedded cognition generates hypotheses which can be tested. If 'low achieving' students develop a preference for the lattice algorithm (Carroll & Porter, 1998), an embedded analysis of the algorithm would suggest that these students may also have less understanding of place value than students who use partial product algorithms. Students who prefer the line algorithm may also lack effective mental strategies for multiplication. Future research may be able to test whether such preferences for alternative algorithms correlate with specific mathematical difficulties of students.

Through the lens of embedded cognition, the traditional partial product algorithm enables the least amount of offloading of cognitive tasks to the environment. Completeness and place value must be maintained mentally with limited structural support. Kilian et al. (1980) found that 56% of students' errors using a traditional algorithm related to these procedural aspects (rather than calculation errors). While the partial product algorithm enables a multi-digit problem to be broken down into component calculations, there are many mental demands made when using the algorithm beyond calculations. Further research may be able to ascertain whether alternative algorithms could be used in a sequenced fashion, to enable the mental demands of the partial product algorithm to be approached gradually – in that the line algorithm could be used to introduce multi-digit multiplication algorithms (with a relatively small mental workload) before transition to the area model algorithm followed by the partial product algorithm. Theoretically, the analysis presented in this paper would suggest that this sequence may represent a viable scaffolding of the mental demands required to use the partial product algorithm. While further research is required to test such a sequence, theoretically it may help students avoid the procedural errors Kilian et al. documented with the partial product algorithm.

Conclusion

An embedded view of cognition has been applied to four multiplication algorithms to assess how the cognitive demands of solving 34 × 26 can be distributed across both environmental and mental resources. Results suggest that algorithms differ mainly in relation to how place value and ensuring all calculations are made ('completeness') are supported by algorithms' structures. This provides a plausible explanation as to why some students may develop a preference for particular algorithms as each algorithm requires different aspect of the cognitive work of solving 34 × 26 to be completed mentally. Thus, alternative algorithms may not be just a matter of individual style but may be used in specific ways to enable successful task performance. A traditional partial product algorithm offloaded the least amount of cognitive work to the environment and made the highest demands on mental resources of the algorithms analysed. Hence, further research – guided by a model of embedded cognition – may be able to identify how using alternative algorithms may address specific student errors relating to using traditional algorithms.

References

Carroll, W. M., & Porter, D. (1998). Alternative algorithms for whole-number operations. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics* (pp. 106-114). Reston, VA: NCTM.

Greeno, J. G. (1998). The situativity of knowing, learning, and research. *American psychologist, 53*(1), 5-26.

Heirdsfield, A.M. (2004). Enhancing mental computation teaching and learning in Year 3. In *Proceedings Australian Association for Research in Education*, Melbourne.

Kilian, L., Cahill, E., Ryan, C., Sutherland, D., & Taccetta, D. (1980). Errors that are common in multiplication. *The Arithmetic Teacher*, 22-25.

Kirlik, A. (2007). Ecological resources for modeling interactive behavior and embedded cognition. In W. Gray (Ed.), *Integrated models of cognitive systems*. New York: Oxford University Press.

Pearn, C. (2009). Highlighting the similarities and differences of the mathematical knowledge and strategies of Year 4 students. In R. Hunter, B. Bicknell, & T. Burgess (Eds) *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 443-451)Wellington: MERGA.

Randolph, T. D., & Sherman, H. J. (2001). Alternative algorithms: Increasing options, reducing errors. *Teaching Children Mathematics, 7*(8), 480-484.

West, L. (2011). *An introduction to various multiplication strategies.* (Masters Thesis), University of Nebraska-Lincoln, Bellevue, NE.

Westwood, P. (2000). *Numeracy and learning difficulties: Approaches to teaching and assessment.* Camberwell: ACER.

Successful Mathematics Lessons in Remote Communities: A Case Study of Balargo

Robyn Jorgensen
University of Canberra
<r.jorgensen@canberra.edu.au>

This paper describes the lesson practices at one very remote school that has been highly successful in numeracy. Drawing on a significant body of diverse research that promotes quality teaching and learning, this case study describes the features of the practice that have been implemented across the school. Teachers' voices provide both justification for the adopted practices and insights into why the practices have been effective within the context of the school. Finally, consideration is given to the on-going sustainability of changed practices within the school.

This paper draws on the exemplary work of one site, Balargo, in very remote Australia in bringing about success for First Australians – both Aboriginal and Torres Strait Islander students – in numeracy. The paper is structured differently from the traditional research paper as it is a case study (Stake & Savolainen, 1995) and so seeks to describe the practices of this school. As with the larger project, the case study approach adopted in the project sees the research team enter the school context to document the practices of the school. Interviews are conducted with key personal (leadership team, teachers, other staff), along with lesson observations and document analysis. A synergy between the interviews and observations is sought so as to build rigour and triangulation among the data sources. A story is developed by the research team at the completion of a site visit and then negotiated with the school until a final story is approved and shared via a public space. The intent of the overall project is to develop (and share) case studies of exemplary practice in numeracy education and to celebrate the success of quality teaching in remote areas.

Contextual Statement of the School

Balargo School (a pseudonym for the school in accordance with the University's ethics guidelines) is located in a remote, isolated region located near the sea and the area abounds with much flora and fauna, including crocodiles. There are five communities in the region that are made up of both Aboriginal and Torres Strait Islander people. Culture is very strong in the region with people retaining many of the ancestral traditions in daily living. Being close to the coast, fishing is a major recreational activity as is hunting. The area is popular destination for 4WD adventurers in the dry season with 80% of tourists arriving in the two months either side of the mid-year school holidays. The communities are connected by bitumen roads, but once out of the region, the roads are dirt, and accessible by 4WD only. The largest community has a supermarket, bakery, hospital, police station, hotel, and a range of service providers.

In 2008, Balargo decided to build a culture of learning through a focus on literacy and numeracy. Through the use of explicit teaching methods, students have come to understand the goals of teaching and the approaches being taken. This understanding is believed to support a re-engagement of students in learning by allowing them to experience success. The approach also aims for students to develop understanding of the purposes of schooling leading to consequent improvement in attendance. Since introducing this focus, Balargo has progressively built on developing teacher skills in explicit teaching as a pedagogical

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 317–324. Sunshine Coast: MERGA.

framework, refining and expanding methods to improve the mathematics teaching practices across the school. The impact of this approach is now filtering into the high school as the students advance, identifying themselves as successful learners and entering the high school ready for learning. The school is trialling many strategies to bring about consistency across the three campuses and to build sustainability of the successful practices that have been developed by Balargo. There is a strong cohort of regular attenders, particularly at the main campus, with many of the students achieving 80% or more attendance.

Balargo operates as an urban school and prides itself in this. Being remote is not seen as a valid excuse for lower standards. There are high expectations of learners, teachers, leaders, and community. Teachers are expected to provide a quality education for the students, and the school actively seeks to build a strong learning culture. Building this culture has taken time and has been sustained over the past two successive principals. Many of the teachers and leadership team remain in the school for extended periods of time. A number of teachers have been at Balargo for more than 10 years. This has built a very stable staff, particularly for a very remote site.

Defining Success

Balargo has achieved consistent success in NAPLAN for many years. Since implementing the changed practices at the school, Balargo has increased success in NAPLAN from the lower band to Bands 3 and 4. Now that there is fluency across many aspects of mathematics, the school is focusing on taking student achievement into the higher bands. Balargo also uses a range of assessment tools to monitor student achievement and growth. These data are used to inform teaching and also to track success at the school level. Teachers meet with their Head of Curriculum to discuss data against the agreed goals for the class, to discuss plans for numeracy (and literacy) for the whole class based on the data, and to develop individual plans for particular students who require differentiation. In the following sections, descriptions are provided with regard to the principles and strategies used by Balargo to bring about success in numeracy learning.

Principles for Learning

In the next two sections, I outline the principles that have underpinned the approach adopted by Balargo, and then the specific strategies that have employed to build quality numeracy lessons.

Prioritising Numeracy Learning

In many remote schools, there has been a priority for literacy learning, where literacy blocks have been established to take up the first session of the day, and where approaches such as the Accelerated Literacy Program (AL) has been forthright in promoting uninterrupted learning for the first session of the day. The research behind AL has shown that it is critical for learning to occur in uninterrupted blocks and in the early part of the day. There have been many schools in the broader study that dedicate two hours to literacy and then one hour to numeracy. However, Balargo has dedicated two hours each day to each of literacy AND numeracy. In its early days of reforming Balargo, the leadership team had taken a very strong view that literacy and numeracy were the core business of the school so that a significant component of the day was dedicated to the teaching of these core areas. The final session of the day is dedicated to other curriculum areas. But, it is also noted that numeracy lessons can also be the practical application of mathematical ideas, concepts and processes to other curriculum areas (such as science and social science).

Two-Hour Numeracy Block

The Numeracy Block is conducted in the second block of the day – from 11.40am - 1.40pm – and is divided into four main activities (see Table 1). The activities can vary in structure, form and length depending on the teacher, the student needs, and the topics. So while the block appears to be long, it is divided into smaller, distinct phases so as to maintain student engagement.

High Expectations

Drawing on the work of Sarra and his leadership for learning model, the school has adopted the notion of high expectations for Indigenous learners. Sarra (2012) has consistently argued that teachers and schools must have high expectations of learners (teachers as well as students). The school adopts the national curriculum and the outcomes for the given year level so that students are expected to meet the national outcomes – so that there are no lower expectations for learning because of the backgrounds of the students.

Being Explicit

Balargo has adopted a school-wide structure to mathematics lessons. Teachers follow an explicit teaching model where, through careful and explicit scaffolding, students are able to complete tasks independently and build a strong sense of success and pride in their accomplishments. The Explicit Teaching model (Archer & Hughes, 2011) adopted at the school is the "I do, we do, you do" model. This model has been implemented at the school for seven years and underpins the approaches in all curriculum areas, including mathematics. In this model, the teacher also makes explicit the learning intent for a lesson.

Teacher: By telling the students what they are expected to learn, then they know the focus and point of the lesson. I often take the intent from the curriculum and then work on it so that it is meaningful for the kids.

Teachers also make the criteria by which they will judge the success of learning. This is written alongside the learning intent so that the students can see not only what they are expected to learn, but also how they will know if they have been successful.

Teacher: The success criteria is important as it helps the students know how they have to show me they understand what it is we are learning in maths.

Whole School Approach

All teachers at Balargo are expected to adopt the same consistent model. All mathematics lessons are the same format. The rationale:

Teacher: The kids need to have the same model so that they can come into class each day and know what is happening, what to expect. You know, their lives are often chaotic, so they need consistency at school. They can walk into the classroom and know exactly what to expect. Then they can get on with the task of learning.

The whole school approach is proactively supported by the leadership model that has been enacted across the school. This ensures that there is support and compliance with the vision of Balargo, particularly in terms of curriculum and pedagogy.

Table 1
Numeracy Block Activities

Phase of Lesson	Description of Practice
Consolidation	The teacher revises many of the concepts that are foundational to mathematics. These are concepts that are usually assumed to be known by students but it is recognised that this may not be the case. Topics such as time, calendars, conversions, fractions etc. are revised in this session. The rationale for this session is that the foundational mathematical concepts need to be built into long-term memory so that students then understand basic mathematics concepts and can build automaticity with these both. The Consolidation phase of the lessons adopt a '*recite, recall, apply'* approach where students will often engage with group chanting of mathematics facts (or readings), then the teacher will ask various questions to elicit students' knowledge and understandings; and then the concepts are applied to problems. The pacing of this aspect of the lesson is brisk. All students are expected to respond to questions, so strategies are used for students to display their work (e.g., individual recording on boards that are displayed; ladders are used for counting work), which allows teachers to assess students' understanding immediately and to give feedback or address problems as they appear.
Mental Maths	Mental maths is a strong feature of the maths block. Teachers scaffold students (in the first four days of the week) so that students are able to complete the exercises around various concepts. This builds success and confidence, and prepares them for the Friday test. The practice activities prior to the Friday tests help teachers identify areas where students may need further support in order to comprehend the item and be able to respond correctly. Students are expected to achieve at least 80% in the mental maths quizzes. Their data are displayed on data walls.
Digital Maths	Students access Mathletics and other digital programs (such as apps) to support a range of their mathematics skills. The digital environment appeals to the students and they actively engage with this medium. Prior to the Mathletics activity (usually 2-3 times a week), students are scaffolded in the concepts in which they will be engaged online. The digital learning also supports independence and engagement.
Explicit Teaching	Depending on the year level, various (commercial) programs have been implemented at the school. There has been an explicit alignment of programs used at the school with the Australian National Curriculum (and state documents) so that teachers are confident that they are delivering learning experiences that align with National Guidelines. In the early years of schooling, the school has recently adopted a commercial program as a model and aligned this with the National Curriculum and C2C learning outcomes. In the middle-to-upper years, the school has adopted the Queensland curriculum (C2C) as the basis for this component of teaching. The school has sought to ensure that the students are exposed to curriculum that aligns with national expectations for all Australian students.

Strong and Supportive Leadership

There is a strong leadership team who provide a well-articulated vision of the school and have developed support structures to enable the vision to be realised. An important factor enabling the school to achieve its current level of success has been the successive leadership of two principals who have built and developed the whole of school vision and practices. The leadership team explicitly articulate that there is a vision and detail the practices with which staff need to comply. This is not negotiable and compliance with the vision and associated practices are made transparent to the staff. All staff work on the common approach across the school. Initially commenced at the primary campuses, it is now becoming embedded at the secondary campus. The leadership team is structured to both support teachers to build their skills and knowledges around the approaches adopted at Balargo as well as to maintain a high standard of professional practice across the school.

Each campus has a Head of Campus who works with the Principal as part of the Executive Leadership team. Four curriculum leaders (Head of Curriculum) are also employed across the school. The Curriculum Heads assume responsibility for the leadership at the grassroots level of the classroom and work closely with the daily practice of teachers – supporting teachers with the development of their teaching skills, classroom management, lesson planning and data collection/analysis.

Practices

In this section, I outline some of the specific strategies that teachers have adopted in the classrooms. These align with many of the principles outlined in the preceding sections.

Hands-On Activities

Balargo has adopted practices that focus on the use of hands-on activities in mathematics lessons. There is a strong belief that students learn best through hands-on activities so there is an emphasis on providing a range of activities to engage the learners. This is particularly the case in the early years.

> Teacher: The students here are very tactile learners so we try to make learning maths very hands-on for them. It helps engage them with learning.

Language

As the students come from an English-as-Second language background, there is a strong emphasis on linking mathematical language with Standard Australian English language and the mathematical concepts so that the students can make sense of the concepts and interactions in the classroom. Teachers focus on many aspects of language and have many and rich resources displayed around the classroom. The environmental prints in the rooms also support students with various mathematical terms and concepts, and are displayed to deliberately prompt the students.

Recite, Recall, Apply

A key strategy used by the teachers is the ‘recite, recall, apply’ strategy. A key objective for using this strategy on a daily basis as part of the consolidation phase of the lessons is to build long-term memory. Teachers reinforce the need for students to learn many concepts in mathematics so that these are lodged in long-term memory providing the base for success in later years. The focus of the fast pacing of this aspect of the lesson is

also to cue students into concepts that could be covered in later segments of the numeracy block. This helps to refresh students learning as they build confidence through experiencing success.
Recite: This part of the strategy can include a number of processes used by the teachers. Students may use group reading of information that is provided on the Interactive Whiteboard (IWB), or they may sing songs. This aspect of the lesson is undertaken as a group.
Recall: The recall component of the strategy is a fast paced questioning by the teacher where simple recall questions are presented so that the general facts are reinforced.
Apply: Apply questions are posed so that students either demonstrate that they can apply the knowledge or successfully address questions that are different in structure but require the use of the same knowledge or concept.
Feedback: Students use resources to provide feedback to teachers. Whiteboards were used on which students wrote their responses and teachers could scan the responses to assess for learning.
Praise: When praise was offered, the behaviour being praised was named.

Seeking and Providing Feedback

Formative assessment of student understanding is a feature of all lessons. All classrooms have adopted a range of processes where students are able to provide individual responses to teacher questions. These are typically displayed in some form:

- Individual whiteboards where the student writes his/her answer and then shows these to the teachers.
- Ladders that are laminated and so can be written on where students can display number (or other) sequences
- Other resources appropriate for the year level of the students

The teacher is able to scan the classroom and assess students' level of understanding. Individual students may display mis/understandings and teachers are then able to work with individual students. The whole class may also show that they have mis/understood a key concept and then the teacher can make an informed decision as to where/how to move the lesson based on the feedback the students have provided to them.

When teachers provide feedback to students, they are detailed in their responses – whether for mathematical understandings or processes, or for behaviours. The feedback is very specific so students are aware of why they are being praised: "I love how Daniel didn't go 'I don't know' – he thought about it and then worked out how many sides were in the pentagon."

Grouping Students

The organisation of Balargo is aligned with the learning needs of the students and attendance is acknowledged as a key factor in achievement. Year level classes are based on attendance, behaviour and achievement. Many of the students who attend regularly are working at minimum benchmark or above so these students are clustered into a class set where the teachers are able to pitch learning to meet their needs. Students whose attendance is less regular, and often have gaps in their mathematics learning, are placed in classes where there is a stronger emphasis on differentiation, while those whose attendance is quite poor are in a class where there is an emphasis on Individual Learning Plans (IEPs) that meet the needs of the individual learners.

Environmental Footprints

Classrooms are rich with resources on the walls. Teachers provide a stimulating and rich learning environment with resources to support students in their learning. The intent of the resources is to support students to become independent learners and rely on the resources (rather than the teacher). To transition students to the use of the resources, explicit teaching is undertaken to alert students to how they might use the resources in the classroom. These can include the resources on the walls, and other resources that teachers may have made available to the students. These include *placemats* that contain a range of mathematical information (calendars, multiplication facts, units and conversions of various measures, basic geometric shapes, and solids, etc.). In the various components of lessons, teachers make explicit reference to where students might seek support.

Included in the environmental footprint is the display of student data. Students and families are able to see not only achievement but also are also able to track growth over the year. Students can readily see their progress and achievements – again reflecting the explicitness and transparency of practices valued at the school.

Building and Maintaining the School Model

In order to develop and maintain the whole school approach aligned with the vision for the school, and the specific teaching strategies, there is a heavy emphasis on professional learning within the schools. A strong focus on building a common culture across the school where all teachers adopt the same teaching practices in their classrooms is part of the school practices. Many of the teachers coming into Balargo are often recent graduates and are offered very strong professional development from their induction into the school (and remote education), and throughout their time at the school.

Initial induction into the model for mathematics teaching comes through the induction offered by State Authority's conference where teachers new to remote education receive a weeklong induction program into remote teaching (and living). This is followed by inductions into policies that are operationalised in the region and impact on the teaching at the school; and then a final induction offered by the school in the pupil-free days prior to the commencement of the school year. Teachers are provided with the first unit of work for the year when they commence at the school so that they can focus on teaching (rather than planning). This helps with the transition into the school model. Furthermore, the school has a comprehensive program offered each Tuesday in after-school meetings/forums. These focus on various practices that the school is adopting to improve the teaching and culture of the school. The school is developing a series of Standards of Practice (SoPs) that outline various practices that the College adopts across the three campuses. The SoPs are designed to cement practices at the school after current staff leave; that is, sustainability of practice.

At the Tuesday meetings, only two meetings per term focus on the operations of the school. Other meetings focus on professional learning of the staff. Teachers are expected to attend the meetings. In these sessions, the professional learning may be based on a particular SoP, a focus that the College leadership has nominated, a visiting professional, or a program that the College may have bought into. Collectively the diversity of professional learning not only builds the skills of individual teachers, but also builds a strong culture of learning at the school. It also creates consistency of practice across the three campuses.

Individual teachers are supported by the Heads of Curriculum and the Executive Leadership team. It is usual practice for school leaders to conduct regular walk throughs of classrooms so that teachers are inducted into a culture of open classrooms where leaders can drop in and observe teachers at work, and provide constructive feedback in both structured and ad hoc formats. This helps teachers to build confidence in their teaching and allows the leadership team to ensure that the model of teaching is enacted.

Conclusion

This case study has been intentionally descriptive drawing on data collected and synthesised from the school. The descriptions provided here give a summary of key principles and strategies used at the school to build success in learning mathematics. There are numerous strategies being used, many of which intersect with others. Collectively, these provide a rich tapestry of practice at the school. It has been built and sustained over seven years, and is now embedded at the school. The school is refining and building the practices with constant monitoring to assess the effectiveness of changes. The intent is to enable students to experience success and to gain high levels of achievement (as measured through a range of tools and resources).

References

Archer, L. & Hughes, A. (2011). *Explicit instruction: effective and efficient teaching.* Thousand Oaks: Guildford.

Sarra, C. (2012). Reflections of an Aboriginal school principal on leading change in an Aboriginal school: Australia. In T. Wigley, P. Thomson and R. Lingard (Eds.) *Changing schools: Alternative ways to make a world of difference*. (pp. 61-70) Routledge: New York.

Stake, R.E. & Savolainen, R. (1995) *The art of case study research*. Thousand Oaks: Sage Publications

Differentiated Success: Combining Theories to Explain Learning

Robyn Jorgensen
University of Canberra
<r.jorgensen@canberra.edu.au>

Kevin Larkin
Griffith University
<k.larkin@griffith.edu.au>

This paper explores the value of different paradigms to explain dispositions towards mathematics among primary school students from different social backgrounds. As part of a larger project designed to elicit students' thinking and attitudes towards mathematics, we seek to develop an explanatory model for the socially-differentiated outcomes in students' responses. The three paradigms – psychology, sociology and post-modernism – form the basis of the paper where the data we collected from three geographically close but socially different schools were analysed.

This paper is an exploratory theoretical paper. We have intentionally sought to unite three disparate paradigms to explain outcomes in a larger project. The fundamental premise underpinning this paper is that one theory is inadequate in explaining students' differentiated discussions about their experiences and dispositions towards mathematics in primary mathematics classrooms. This approach, of using two or more different theories to try to explain phenomenon, is not new and has been used by other researchers (see Williams, 2012) in mathematics education. The project sought to develop a tool that would allow students to provide honest feedback about their experiences and feelings towards mathematics. What emerged from the data were distinct patterns in responses that aligned with the socio-economic backgrounds, as indicated by ICSEA scores presented in Table One, of the students. To this end, focusing on the individual was limiting since it failed to recognise the structuring practices of mathematics classrooms and the habitus with which students entered these classrooms. Similarly, focusing on the social backgrounds of the students limited the richness in the responses offered by the students in terms of how they were actively constructing themselves as learners.

To frame this paper, we draw on Bourdieu (1997) who explains that educators need to understand the processes around the conversion of social and cultural backgrounds into school success. The responses offered by the students in this study, were highly varied and have consequences both for their relationship with school mathematics now, and also for future academic success in secondary school mathematics and beyond. This view is argued thus:

> To fully understand how students from different social backgrounds relate to the world of culture, and more precisely, to the institution of schooling, we need to recapture the logic through which the conversion of social heritage into scholastic heritage operates in different class situations (Bourdieu, Passeron, & de saint Martin, 1994, p. 53).

The notion of social heritage thus becomes a central variable in coming to understand differential success in school mathematics. In terms of this project, and for school mathematics in general, we suggest that it is salient to consider the social backgrounds of learners and how this is implicated in the differential outcomes for learners. Using a Bourdieuian framework, the lack of success for some social groups becomes a non-random event as success or otherwise is partially a product of institutionalised practices of which participants may be totally ignorant. When taking a Bourdieuian perspective, success in school mathematics is less to do with innate ability and more to do with the synergistic relationships between the culture of school mathematics and that which the learner brings

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 325–332. Sunshine Coast: MERGA.

to the school context (Jorgensen, 2010). The greater the synergy between the habitus of the student and school mathematics, then there is greater probability of success. In Bourdieuian terms, the habitus thus becomes a form of capital that can be exchanged within the field of school mathematics for forms of recognition and validation that convert to symbolic forms of power. Thus, what becomes important for both psychological and sociological theories, is the ways in which learners internalise practices within school mathematics in relation to their positioning within those practices. For some students, the social and cultural habitus with which they enter mathematics classrooms aligns strongly with the practices and discourses within those classrooms. For these students, it is highly likely that they will see themselves as 'good' learners of mathematics. In contrast the reverse is the case for students whose habitus does not align with the practices and discourses valued within the field.

It is not our intent in this paper to provide a synopsis of the various paradigms as this would restrict the discussion of the data in terms of theory building. However, we will provide a brief discussion regarding the major shifts and foci within the divergent fields to illuminate key moves in contemporary thinking about the impact of individual construction of mathematical identities in terms of access (and marginalisation) in school mathematics.

Table 1
Key Paradigms in Mathematics Education

	Psychologistic	**Sociological**	**Post-Modernist**
Key terms	Affect, dispositions, learning, individualistic	Social groups Differences, equity	Identity formation Intersubjectivity
Explanatory concept	Individualistic	Habitus	Identity
Theorists	Hannulu	Bourdieu, Jorgensen	Foucault, Walshaw

An insight provided by Lewis (2013), with regard to subjective dispositions aligning with the psychologistic paradigm, suggests that "motivation and emotion may be more central to an understanding of the phenomenon of disaffection than that of a quantitative study of attitude" (p.70). Similarly Brown, Brown, and Biddy (2012) argued that there were psychological internalisations for students selecting to opt out of further study in mathematics.

> The analysis supports findings that perceived difficulty and lack of confidence are important reasons for students not continuing with mathematics, and that perceived dislike and boredom, and lack of relevance, are also factors. There is a close relationship between reasons for non-participation and predicted grade, and a weaker relation to gender. An analysis of the effects of schools, demonstrates that enjoyment is the main factor differentiating schools with high and low participation indices. (p.3)

In contrast to the embodiment and internalisation of dispositions towards mathematics as an individual phenomenon, others have suggested that the practices of school mathematics may create opportunities to overtly and/or covertly marginalise particular groups of students (See Jorgensen, Gates, & Roper, 2014).

Another school of thought with implications for mathematics education is post-modernism. Walshaw (2011) describes this position as

> Multiple factors have brought about postmodernism. They include political and social crises of legitimation, and the resulting changing nature of economies and social structures in Western societies. These changes place complex and sometimes conflicting demands on people in ways that they are barely able to understand or predict. The effects of these processes for mathematics education are unsettling. Conceptual tools and frameworks from postmodern thinking help us to develop an understanding of those effects. They help us to understand ideas that are central to mathematics education from beyond the standard categories of thought. In particular, they help us to understand cognition and subjectivity. (p. 7)

Each of the three theoretical paradigms briefly discussed has a unique contribution to make regarding mathematics education. For us, coming to understand the constructions that students from diverse backgrounds are making of themselves and mathematics needs to be understood from an interdisciplinary approach. It is limiting to see construction as individualistic as this view fails to recognise the structuring practices of mathematics; conversely, failing to recognise the agentic power of each individual limits the understanding of how students can rise above restrictive practices in mathematics classrooms.

Approach

The approach adopted in this project was adapted from Noyes' (2004) study where a 'big brother' technique was employed. Students were able to withdraw from the classroom and speak (confidentially) into an iPad recording their thoughts and feelings towards mathematics. The approach was designed to elicit responses from students that may be more valid given that participation was optional and confidential. The recordings were directly between the students and the researchers. We have outlined the approach in more detail in other papers, also discussing strengths and limitations of the approach (see Larkin & Jorgensen, 2014; 2015). As the project has evolved, we also modified the approach to maximise student confidentiality in the iPad diary process.

Data from three primary schools (two from Qld and one from NSW), each representing very different social strata, are included in this paper. The schools were included by purposive selection so that an exploration of social differences could be undertaken. Due to the sample size, statistical significance cannot be established; however, the sample is large enough to allow exploration of the tool for accessing students' perceptions of school mathematics, and for the development of theory. A synopsis of the schools is provided below in Table Two. Data are taken from the My School site for each school. The data are from the 2013 data set which represents the periods within which the data were collected.

Table 2
Key Characteristics of the Three Schools

	School A	**School B**	**School C**
Type of school	State school	State school	Independent girls school, High fees
Year Levels	P-6	P-6	P-12
ICSEA score (2013)	1055	970	1135
Enrolments	922	268	1154
Location	QLD	NSW	QLD

The two state schools performed relatively similar to each other on NAPLAN with no remarkable differences. The Independent girls' school consistently scored better, or significantly better, than the national average in numeracy across all years for the past three reported periods (2011-2013) on NAPLAN. With the ICSEA score representing 1000 as the national mean for a measure of social dis/advantage, each 100 points represents one standard deviation from the mean. Our schools are at least 80 points different from each other and thus are relatively disparate in terms of social advantage.

The data were analysed using Leximancer – a software package that undertakes thematic analysis of the frequency of words as well as establishing relationships between terms used by participants. Leximancer allows researchers to see visually, the trends and themes that appear in the data set/s. The interview data for the three schools were run through the Leximancer program and key themes emerged for each school. The visual reporting shows relative frequency by the size of each theme, and then relationships between themes through connecting lines and overlap of themes. From the Leximancer analysis it was clear that the responses of the students were quite different in their relative frequency in referring to various (key) aspects of mathematics. School C students were more engaged with mathematics (in terms of their articulation around concepts) and had a much stronger sense of themselves as learners of mathematics. In contrast, the students at School A were less likely to enjoy mathematics and focused more on low level mathematics (such as operations) in their articulations. The students at School B were more likely to talk about mathematics being fun and focused on number work. Figure 1 provides a pictorial representation of words used frequently by students in School A.

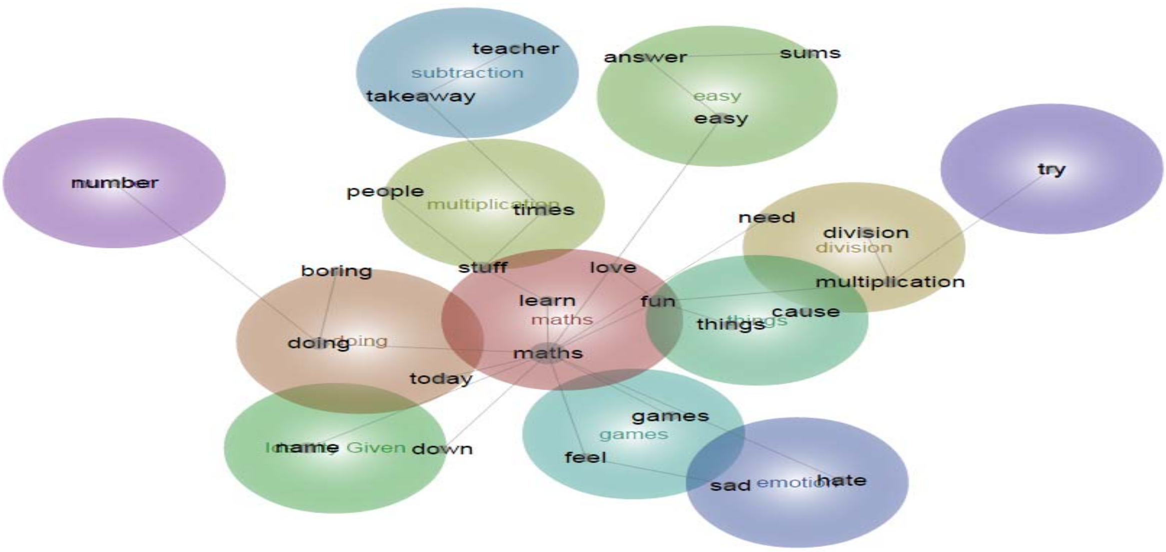


Figure 1. Visual Output from Leximancer – School A

The visual output from Leximancer shows the relationships between concepts for a given unit of analysis (in this example, School A). The distance between concepts gives some sense of the relationships (close or far) between concepts and the lines show the direct relationships between various concepts. This is provided for illustrative purposes to show the ways in which Leximancer draws relationships between concepts. Each school map was quite different in terms of the major concepts and relationships but cannot be included due to the limits of a conference paper. Suffice to say, at this point, there are marked differences between the schools' maps. To provide some rigour to the differences between the schools, a further analysis can be undertaken through tabular representations of the counts associated with concepts. While the program does not differentiate among the use of concepts, for our purposes it was illuminating to see the concepts to which the students referred. This summary is provided in Table 3 below.

Table 3
Frequency Counts for Key Terms Combined and then Individually by School

Entire Cohort			School A			School B			School C		
Word	Count	Rel %	Word	Count	Rel %	Word	Count	Rel %	Word	Count	Rel %
Maths	607	100	Maths	170	100	Maths	262	100	Maths	175	100
Fun	163	27	Fun	59	35	Fun	53	20	Fun	51	29
Feel	96	16	**Easy**	50	30	Feel	51	19	**Teacher**	40	23
Teacher	95	16	Times*	39	23	**Teacher**	39	15	Groups	36	21
Easy	91	15	Division	37	22	Times*	33	11	Feel	26	15
Times*	81	13	Feel	19	11	Numbers	27	10	Fraction	24	14
Groups	70	12	Boring	18	11	Groups	25	10	**Easy**	19	11
Division	57	9	Hate	17	10	Difficult	22	8	Love	18	10
Difficult	48	8	Love	16	9	**Easy**	22	8	Probability	15	9
Numbers	46	8	**Teachers**	16	9	Division	18	7	Diagrams	12	6
Love	43	7	Sad	11	7	Pods	12	5	Chunking	11	6
Fractions	41	7	Numbers	10	6				Difficult	11	6

What this thematic analysis shows us is a number of key differences between the schools. This can be seen, for example, in the differences between the frequency of the concepts *teachers* and *easy*. There is also a notable difference in the emotive words used by the students across the schools. For example, the students in School A referred to maths being easy and the role of the teachers was profiled quite low in the relative comments made by the participants. Conversely, the students in the other schools referred to the teacher more often than their School A peers and there was less reference to the ease of mathematics. Similarly, the table alerts us to differences in the comments being made about mathematics, in terms of content as well as emotional/affective reactions to mathematics. For example the students at School A referred to maths using terms such as *boring*, *hate love*, *sad* while the students at School B only used the term *difficult* and school C students only used *love*. These differences are further expanded in the detailed transcripts of the students.

Student Comments

In this section, we provide more detailed comments as to the responses offered by the students from the different schools. The student comments provide insights into their thinking about what is mathematics, but also their relationship with mathematics, teachers and learning. With the limitations on a brief conference paper, we again use these for illustrative purposes to build our theory.

School A student comments

These comments provide insights into the students' thinking about mathematics. There is a marked difference in the ways that pedagogy is described and their relationship to mathematics.

> I don't like doing maths because I don't get trading and borrowing because it's hard and I don't get how you trade and how you borrow. Thank you. (Gr 3)
>
> I learnt in math today. I learnt how to do dividing and stuff. Let's see, what I don't like about maths. I hate math, I don't really like it. It's not fun. What I like about math is stuff, just stuff and all that because sometimes math can be easy and all that. I don't feel happy when I do maths because it's really hard. What I find difficult in maths. A lot of things basically. So bye-bye, I'm out.

School B Student Comments

The examples from School B also provide illustrations of the students' reactions to mathematics.

> In maths today I learnt about square numbers and I'm sort of finding them out but I don't know what I'm going to use them for but they're got to teach what they've got to teach. And I think we should do maths groups. (Gr 2/3)
>
> I do like maths, a little bit, so I'm like in the middle. A lot of the maths we do is pretty hard for me. Because I just find things hard like most kids, I still try my hardest and people think I'm dumb and the teacher knows that I struggle so she will help me sometimes. Today we did a really, really hard thing. I got it but the teacher said I could stop and do another maths thing because it was hard for me, so I'll show you what it was. (Gr 5/6)

School C Student Comments

The comments below indicate how the students positioned themselves as learners of mathematics and gave insights in the pedagogies being used (groups) and strategies being taught to the students (chunking).

> I think maths is pretty good. Sometimes I like it and sometimes I get a bit bored doing it. Sometimes I feel pretty confident with some things, sometimes I get a bit stuck with other things. I found my favourite strategy is the chunking strategy. I find it very easy and that's why I love it so much. I use it all the time because sometimes I get stuck with sums and I sometimes really don't know what the answer is so I use the chunking strategy. … Thankyou. (Gr 3)
>
> I like maths because we do fun activities to do with the topic. The topics are always fun, like fractions. I like fractions because you can show them in many different ways like in numbers and pictures. It's also really fun because you get to work in groups. That's a bonus because you get to work with your friends. (Gr 6)

As indicated these data are provided for illustrative purposes and have been selected to show some of the differences observed across the schools. The most surprising outcome was the very strong positive dispositions that were evident among the students from School C towards mathematics. It is this difference we seek to explore in the remainder of the paper.

Discussion

The data presented through the iPad diaries alerted us to two key phenomena that we now discuss. First we saw that the students are School C were more likely than their peers at Schools A and B to have strong mathematical identity and more likely to describe mathematics using a mathematically-rich vocabulary e.g. chunking or strategies. The students at Schools A and B were more likely than the students at School C to describe negative feelings towards mathematics, indicate negative identities towards mathematics, and provide low level descriptions of mathematical content. From a psychologistic perspective, it can be argued that the motivations and affective domains for the students were potentially empowering or disempowering in terms of mathematical success. Having favourable dispositions towards mathematics is likely to facilitate the attainment of successful learning outcomes. What can be seen from both the frequency data (Table 3), and reinforced in the quotes from the students is their relationship with mathematics knowledge, not only in terms of the content covered but also in the amount of discussion of mathematics concepts. It is clear from the data in Table 3, that mathematics discipline knowledge for the students at School C is more frequently reported than for the students at Schools A and School B. For the students at Schools A and B, their reporting was more focused on internalisation of dispositions and feelings towards mathematics than was the case for students at School C.

What is also of value to our discussion is a different reading of these data. From a Bourdieuian perspective, it is apparent that the students from School C have dispositions of themselves, and towards mathematics, that are likely to result in improved outcomes when compared to their peers in the other schools. This is not just an individualistic construction since, as Bourdieu has suggested, the social and cultural habitus of the students at the all-girls school (who are likely to be from middle to upper class families) is one that aligns with mathematics and hence, becomes reified through success in mathematics – however defined (either as a disposition or mathematically). The girls at School C have been exposed to practices that they articulate as being strong mathematical, and that have helped them to build a habitus that is empowering in terms of future mathematical studies. The girls have been able to build scholastic capital that is not as apparent, nor as strong, in the students from Schools A and B. Further, from a postmodernist perspective, we can see how the practices position students in particular ways and that these offer various subject positions for learners – some who see themselves as productive learners of mathematics, while others have become positioned as marginalised learners.

What we see as important in the discussion in this paper is that one theory may limit how we come to understand students' experiences of mathematics. Relying on one theory may offer some explanation of these data but is also limiting. What struck us when analysing the data across the three schools were the marked differences in the students comments. Clearly the students at School C have a strong sense of themselves as learners of mathematics. Relying solely on a psychologistic perspective would only allow an understanding of mathematics as an individualistic construction; however by incorporating a sociological perspective (particularly that of a critical sociology), we are better able to understand the structuring of these differences and how they may result in differential access to mathematics learning. Combining the various theories enables a much richer perspective on understanding the ways in which the students come to see themselves in relation to mathematics.

Limitations

While this paper is theoretical in its approach and primarily sought to develop an explanatory approach to the differences in the data collected, we acknowledge that the small sample (3 schools) limits the claims we can make as a much larger sample would help to establish the validity of our analysis. We also acknowledge that some of the differences expressed by students could be due to the teaching practices at the schools, rather than social background *per se*. While this may be a methodological limitation, we also contend that the outcomes are noteworthy. The social stratification that is evident in these students' responses reinforce both psychological (embodied) and sociological (social) theories of learning and access. We also acknowledge that there are limitations of solely relying on the Leximancer word count as exploring the comments that surround those concepts is equally as important. Leximancer does, however, provide a very useful tool for beginning explorations into the differences and similarities among cohorts.

References

Angus, D., Rintal, S., & Wiles, J. (2013). Making sense of big text: A visual-fist approach for analysing text data using Leximancer and Discursis. *International Journal of Social Research Methodology, 16*(3), 261-267.

Australian Curriculum Assessment and Reporting Association (2015). *My school*. Accessed Feb 14, 2015.

Bourdieu, P. (1977). *Outline of a theory of practice*. Cambridge: Cambridge University Press.

Brown, M., Brown, P., & Bibby, T. (2012) "I would rather die": reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education, 10*(1), 3-18.

Hannulu, M. (2012). Exploring new dimensions of mathematics-related affect: embodied and social theories. *Research in Mathematics Education, 14*(2), 137-161.

Jorgensen, R., Gates, P., & Roper, V. (2014). Structural exclusion through school mathematics: Using Bourdieu to understand mathematics as a social practice, *Educational Studies in Mathematics, 87*(2), 221-239.

Jorgensen (Zevenbergen), R. (2010). Language, culture and learning mathematics: A Bourdieuian analysis of Indigenous learning. In C. Wyatt-Smith, J. Elkins, & S. Gunn (Eds.), *Multiple Perspectives on Difficulties in Learning Literacy and Numeracy* (pp. 315-329). Dordrecht: Springer.

Larkin, K., & Jorgensen, R. (2014). Using video diaries to record student attitudes and emotions towards mathematics in year three and year six students. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 373-380). Sydney: MERGA.

Larkin, K., & Jorgensen, R. (2015). 'I hate maths: Why do we need to do maths?' Using iPad video diaries to investigate attitudes and emotions towards mathematics in Year 3 and Year 6 students. *International Journal of Science and Mathematics Education*, 1-20. doi: 10.1007/s10763-015-9621-x

Lewis, G. (2013). Emotion and disaffection in school mathematics. *Research in Mathematics Education, 15*(1), 70-86.

Noyes, A. (2004). Video diary: A method for exploring learning dispositions. *Cambridge Journal of Education, 34*(2), 193-209. doi: http://dx.doi.org/10.1080/0305764041000170056l

Walshaw, M. (2011). Positive possibilities of rethinking (urban) mathematics education within a postmodern frame. *Journal of Urban Mathematics Education. 4*(2), 7-14.

Williams, J. (2012). Use and exchange value in mathematics education: Contemporary CHAT meets Bourdieu's sociology. *Educational Studies in Mathematics*, *80*(1-2), 57-82.

.

The Mathematics Instructional Leader: What a Difference Crucial Conversations Make

Janeen Lamb
Australian Catholic University
<janeen.lamb@acu.edu.au>

Carmel Diezmann
Australian Catholic University
<carmel.diezmann@acu.edu.au>

Jillian Fox
Australian Catholic University
<jillian.fox@acu.edu.au>

The aim of this study was to investigate the role of the instructional leader when introducing digital technology into the mathematics teaching in the Australian Curriculum. The research reported here involves the principal and five teachers from one school and is part of a larger study. Results indicated that principal-led 'crucial conversations' supported educational change that comprised not only curriculum change but also a transition from a pedagogy that draws on technology to a pedagogy in which technology is embedded.

The accountability of primary principals as instructional leaders has increased markedly with substantial challenges in Mathematics Education and the Digital Revolution. Accountability challenges in mathematics education relate to national numeracy testing (NAPLAN) and the new Australian curriculum, which includes mathematics as a learning area, and numeracy across the curriculum as a general capability (Australian Curriculum and Reporting Authority (ACARA, n.d.). The cross curricula focus of numeracy is defined in terms of its social utility (Australian Association of Mathematics Teachers, 1997): "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (p.15).

The digital revolution involves the integration of digital resources into the curriculum (e.g., interactive multimedia resources; audio, photo and video resources; interactive assessment resources; digitised collections of curriculum resources). The Australian government is supporting the integration of digital resources into the curriculum, spending $32.4 million targeting digital resources, professional learning, and infrastructure upgrades (DEEWR, 2012) and using *Scootle*, a new social network, to assist teachers to "learn, teach and collaborate using digital resources" (Education Services Australia, n.d.). With the rollout of the National Broadband Network (NBN), even greater use of digital resources and improvements in student outcomes is expected (Department of Broadband, Communications and Digital Economy, 2013).

In view of the challenges presented by the new Australian curriculum and the digital revolution, the Chief Scientist has argued strongly for inspired school leadership to build teacher capacity and improve students' outcomes in mathematics: "Inspiring leaders will encourage innovation and support teachers as they develop particular ways to deliver the curriculum" (Chubb, 2012, p. 9).

Literature Review

Principals and other school instructional leaders are expected to be agents of change, guiding and engaging with teachers to respond to new educational directions. This role is very challenging in the case of updated reconceptualised curriculum and the digital

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 333–340. Sunshine Coast: MERGA.

revolution, because major changes need to be made regarding *what* is taught (new curriculum), *how* it is taught (digital resources), and *when* the change needs to be implemented (with pressure on schools in early NBN rollout areas). Furthermore, instructional leaders need to be able to persuade teachers of *why* there is a need for change, because these leaders are highly dependent on teachers to implement changes. Similarly, teachers are highly dependent on instructional leaders because they are struggling to keep pace with educational change (Hargreaves & Shirley, 2009), changes in professional practice requires considerable teacher learning (Cobb & Jackson, 2011), and the changes needed to ensure academic rigour is maintained or enhanced (Jackson, Shahan, Gibbons, & Cobb, 2012). Of the numerous models available to support transition to the purposeful use of technology in education, the model selected to guide this study is the Substitution Augmentation Modification Redefinition (SAMR) Model (Puentedura, 2015). This model is appropriate because it describes four levels of increasing sophistication in the use of digital technology (see Figure 1).

4. REDEFINITION where technology allows for the creation of new tasks, previously inconceivable
3. MODIFICATION where technology allows for significant task redesign
2. AUGMENTATION where technology is used as a substitute with function change
1. SUBSTITUTION where technology is used as a substitute without function change

Figure 1. A model of increasing digital use (Puentedura, 2015).

Supporting teachers presents a significant challenge for principals and other instructional leaders because teachers need to adapt to curriculum changes while integrating digital resources into their teaching practices. Millet and Bibby (2004) provide insight into how principals can successfully undertake this leadership task by considering all school staff as members of a professional learning community (PLC). Within their school PLC, principals need to foster a collaborative learning culture where collaborative relationships, shared vision and shared values promote the active development of practices to enhance student learning (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006). The principal also needs to understand a teacher's capacity to change by examining the context and culture of the teacher's 'situation', or working environment (Millet & Bibby, 2004). In addition, the principal needs to be able to have 'crucial conversations' (Patterson, Grenny, McMillan, & Switzler 2012) where topics of change and how to go about that change can be had in a safe environment. These conversations enable the PLC to stay focused on the goal at hand that leads to action and results rather than being distracted by argument and side taking. The recommended initial steps for action in a PLC include: clear identification of the goal with ownership of that goal being accepted by all; a combined 'head and heart' approach where teachers become passionate about the educational change and work logically towards achieving it; and alertness to when conversations change from positive and goal focused to *crucial* where important differences in opinion need to be heard so that a balanced approach to goal setting and achievement is possible. Achieving a task of this magnitude is difficult when educational change comprises not only curriculum change but a reconceptualised view of technology and pedagogy. In order to better understand how this change occurs in schools, the following research question was posed:

How does one primary school principal, as the mathematics instructional leader, support teachers to integrate digital resources into the mathematics curriculum and across the curricula via numeracy?

The Study

The results reported in this paper are part of a larger study. Participatory action research was used in this study because it empowers participants to engage in cyclic iterations of planning, action and observation, and reflection to improve professional practice (Carr & Kemmis, 1986). The study included two complete cycles of participatory action research. The key participant reported in this paper is a primary school principal with a special interest in the use of digital technology in the teaching of mathematics and numeracy across the curriculum. Additional participants were the curriculum leader and three Year 3 teachers. One Year 3 teacher had been teaching for four years and the other two teachers were in their first year of teaching.

Data Collection

The data collection comprised artefacts, observations, interviews, and field notes. The artefacts were school planning documents, teacher work and student work produced during this project. For each cycle, one lesson per class was observed by four school participants, the teacher delivering the lesson (lessons were recorded on video) and one of the researchers. The researchers kept field notes documenting any critical incidents or issues that arose. Interviews were conducted with the teachers after each lesson during the two cycles.

Data Analysis

Data were analysed to construct a rich narrative account of how the principal inspired and supported the integration of digital resources in the teaching of mathematics and numeracy across the curriculum. All data was subject to *content analysis,* seeking evidence of the eight characteristics of a professional learning community, such as shared values, shared vision, and collective responsibility (Stoll et al., 2006). This data was used to create a chronological account of the principal's journey, with critical incidents identified. In addition*, a quality analysis* was undertaken on excerpts of classroom lessons in which digital resources were employed. This quality analysis complemented the content analysis of the lessons.

Study Progression

Phase A: Introduction to project. Step 1 involved a half-day professional development session with the primary school principal participating in this study. This session outlined how digital technology could enhance current pedagogical practices.

Phase B: Audit and preparation. Steps 2 and 3 involved a situational analysis of the current integration of digital resources. Step 2 was an audit of school planning documents. Step 3 consisted of an interview with the Principal.

Phase C: Implementation. Step 4 was two iterative cycles of planning, action and reflection by the teachers, principal and researchers and focused on the incorporation of digital resources into classroom teaching.

Results and Discussion

Phase A: Introduction to the Project

Step 1: Half day professional development. During this session, the principal was introduced to ways to extend the use of concrete materials to include digital technologies as a first step in educational change. The principal identified that the Year 3 teachers were using digital technology to investigate the use of water around the school saying:

> They're [teachers and students] going for a walk around the school to identify uses of water, all around the school, where the water is coming from, what they're using it for. So they'll take their iPad and take photos and download those photos and put them in a disk.

This comment identifies good use of the camera function within the iPad but does not provide any functional change beyond having photographs of the various water points around the school. Discussion between researchers and the principals then focused on ways to allow for functional change that could lead to task redesign incorporating a problematic situation that would engage student interest.

Phase B: Audit and Preparation

Step 2: Audit of school planning documents. The principal provided Year 3 planning documents prior to the situational analysis interview. These included planning documents from the Year 3 Mathematics unit as well as from the Literacy, Science, History/Geography and Religion units in which numeracy could be embedded. These documents allowed the researchers to consider not only the mathematics but instances of numeracy being taught across the curriculum and how digital technologies were being incorporated to support mathematics learning and numeracy development.

The Mathematics unit had a focus on the teaching of subtraction. This unit used some concrete materials but mainly considered the steps in the calculation of abstract numbers without context. It appeared that no attention was given to either numeracy or the use of digital technologies. The Literacy unit did not identify any aspects of numeracy within its content. The only digital technology identified in planning was the use of a data projector. The Science unit included numeracy content targeting measurement in planning, but did not identify any digital technology to be used. The History/Geography unit did consider numeracy in the display and interpretation of data. Digital technology was also identified in planning as the use of a DVD to show historical footage. The Religion unit did not include any numeracy content although the use of a data projector to display *YouTube* clips was detailed in the unit plan.

The unit plans prepared by these three Year 3 teachers included some evidence of planned teaching of numeracy across the curriculum. Using Puentedura's (2015) SAMR Model to guide analysis of the use of technology, it was clear that the planned technology use was restricted to the first level of technology use, *Substitution*, where technology was used as a substitute without any functional change. It is worthy of note that even at the

Substitution level, there was limited evidence technology use in the mathematics unit or other units.

Step 3: Situational Analysis. The principal indicated that the focus of professional learning within the PLC was primarily on literacy, numeracy and higher order thinking skills. She also noted that teachers had made the shift from outcomes-based assessment to criterion-referenced assessment. The principal stated that she expected teachers to understand the difference between "gimmicks and what is actually going to enhance teaching and learning" and noted that "that they [teachers] are discerning enough to know what makes a difference to a lesson". When questioned further about how she encouraged teachers to integrate digital technologies in their teaching, she replied:

> I have had stronger conversations when individuals [interested in the use of digital technology] have brought in their learning plans but I want teachers to feel safe before I ask them to take that next challenge. When we got the iPads two years ago I said, right, we are not going to get any until we have done some work, some work around the General Capabilities and then make a decision on, do we even want iPads...We visited two other schools that had iPads and I wanted us to have that discussion together from a position of information. I then organised some professional learning both from the technical position of how to use your iPad as well as how to integrate it [into the curriculum]. That all happened before we made a decision to buy them.

This discussion suggests that the principal did not make a unilateral decision. She encouraged the exchange of differing views, thereby making it safe for teachers to express their opinion. It was the teachers' decision to adopt the use of iPads into their classrooms and as a result they have taken ownership of this change thereby making a commitment to effective iPad use.

Phase C: Implementation

Cycle 1, Planning, Action, and Reflection. The planning of lessons in the first cycle of the project was initially prepared by each classroom teacher and then shared at a meeting with all teachers, the curriculum leader and the principal. This meeting provided feedback to each of the teachers and gave each of the five school members' ownership of the final lessons.

During the *action* step a researcher attended lessons, observed and recorded them. She was accompanied by the principal, curriculum leader and the other two teachers. One of these three lessons will be discussed here as an example typical of the teachers' professional practice. The lesson called "Time Travellers" made use of a PowerPoint slideshow of children from the class meeting the school bus in the morning. The problem situation was created by asking students to indicate on their individual laminated clocks what time each member of the class should join the bus. The PowerPoint slideshow had a slide for each child and asked students to draw the time on their clock. After discussion, the teacher wrote on the whiteboard the correct time before proceeding. Students found this lesson very engaging as they each eagerly waited for their photo to appear.

Following the lesson, the five staff and the researcher adjourned to the staff room where the researcher asked the five members of the team to *reflect* on the lesson. The buzz from them was on the level of student engagement and how the use of their photographs via the data projector had made this possible. The use of technology meant that each student was 'hooked' and actively wanted to participate because it involved them. The principal moved beyond this discussion to consider additional aspects of the pedagogy saying:

> With the clocks everyone felt really confident to just go. They knew the red [large hand on the clock], the blue [small hand on the clock], the minute, the hour…the ones who needed to keep using this could. It is having the resource there to help everyone. Like everyone had a clock right there on their desk, they could use.

After this positive discussion about the successful lesson, the researcher moved the discussion to the role that technology had played in the lesson. It was evident that the PowerPoint slideshow with student photographs had achieved the specific purpose of student engagement. But the teachers were challenged because technology was being used as a mere substitute for more traditional pedagogical practices without it changing the function of the task. Discussion on task modification where the use of technology allows for task redesign resulted. The principal took the lead, looking for clarification allowing the other teachers to ask questions, and supporting a discussion on how to approach this type of planning. The researcher outlined the importance of task redesign to take advantage of technology. She also pointed out the planning task should ensure that the academic rigour of the task is not lost when technology is used; rather it should add a dimension not possible without it. At the end of the discussion, it was agreed that all five school members would work on the one lesson together with a view to making the use of technology more purposeful as a pedagogical tool to enhance student learning.

Cycle 2, Planning, Action, and Reflection. The principal, curriculum leader and teachers collaboratively planned a probability lesson that promoted the use of accurate change vocabulary to describe the probability of chance events. Each pair of students had a laptop and was directed to the Math and Teaching Technology Innovation (2014) website to source virtual manipulatives. On this site students, created their own spinner combinations where they were asked to make spinners to represent probabilities of landing on one colour that were: certain, impossible, unlikely, and had an even chance.

The *action* again involved all five school participants and a researcher in the classroom participating in or observing the lesson. Again, the students were highly engaged with the use of technology but many spontaneous opportunities for learning were not followed up. Firstly, students had to construct a spinner that had an equal chance of landing on one of two colours. They were then directed to spin the spinner 10 times and discuss the results. Similarities and differences were discussed. The teacher then asked the student to spin the spinner 100 times and compare their results. The spinner tool allowed for immediate feedback with a graph showing the distribution of each colour. Again, class discussion resulted where the teacher was able to hook up different computers to show the class the differing results. This discussion of why some graphs looked different introduced the use of experimental probability and various unique combinations of spinners were created. The Year 3 students were then directed to create a spinner that had five colours on it, but there was a 50% chance of the spinner landing on one colour. The rigour of the tasks was maintained throughout the lesson and the teacher was able to orchestrate the discussion when she showed different student solutions. At the conclusion of the lesson, the students were able to confidently construct digital spinners that addressed the chance elements posed by their teacher. As they had been able to access quickly the results through the use of the virtual manipulatives, they were able to predict outcomes of various spins.

The *reflection* on this lesson was led by the principal who led the planning relating to the intentional use of technology in the lesson. When asked by a researcher if she thought the use of technology was extending the pedagogy, she answered:

> I would say it was quite a different purpose. The last time we [the technologies] were more about the notion of display where we could have just used an overhead projector or written it up on the board. But we were trying to do something different, to enhance the learning, the mathematics itself [with technology].

When the classroom teachers were asked to comment, they could see the benefits of this approach with one saying:

> My students could easily see what the outcome would be if they made it four sections or five sections being easily able to change and then spin the spinner 10 or 100 times and get an immediate response...Children were able to share their results or repeat their experiment to see if they got the same result as their classmates. It all happened so fast!

The principal agreed saying:

> It would have not have been possible to have that lesson drawing a spinner and doing (sic) the experiment 10 or 100 times…you couldn't see the fear of making a mistake on their faces today because you could just push the button and it would all disappear and they could do it again.

The collaborative nature of this planning exercise seemed to have all teachers including the principal positive about the potential of this type of task. They had learnt together as a team to tackle the integration of technology into a mathematics lesson and were rewarded by the students' engagement and achievement.

Conclusion

Instructional leaders in the primary school are confronted with educational change demands that extend beyond changes to curriculum to one that requires them to lead a transition in pedagogy from one where technology was not prioritised to one where technology use was optimised thereby creating redefined or novel 21st century pedagogical approaches. The achievement of this change requires inspiring leaders to promote and support innovation in the delivery of curriculum (Chubb, 2012). But innovation is hard to grapple with as teachers try to see what the innovation might look like and why it would be more effective than what they currently do. One model supporting teachers with the purposeful employment of technology into pedagogy is the SAMR Model (Puentedura, 2015). This model helps teachers see how technology can be used beyond the simple substitution of existing pedagogical practices with technology to one where tasks are redesigned and new tasks are created capitalising on the affordances of technology, such as the rapid spinning of 100 spinners and the recording of this experiment. But guidance from the SAMR Model is not sufficient. The principal needs to stimulate and support the professional learning community in this endeavour (Lamb, 2010).

The principal as instructional leader was very aware of the pressures for curriculum change in line with the new Australian Curriculum: Mathematics and numeracy across the curriculum. She is also confronting the digital revolution and the need for her teachers to keep pace with this change. Her approach to confronting these changes is reflective of a leader who understands the need to foster a collaborative learning culture where collaborative relationships, shared vision, and shared values were promoted (Stoll et al., 2006). Her approach was reflective of collaborative conversations that allowed difficult points to be discussed in a safe environment.

The principal in this study worked alongside her teachers in the PLC. She engaged fully in the project, proposing, analysing and owning the changes in pedagogy as much as the teachers. She created an environment for all to feel safe as she had *crucial conversations* with two first year teachers, one in her fourth year out and her curriculum leader. Importantly, she joined with them in this professional journey celebrating the growth in their professional knowledge of how to commence the incorporation of technology into the teaching of mathematics and numeracy across the curriculum. The journey has just commenced with the PLC planning and practising the new pedagogies which over time should become part of their professional repertoire with the embedding of technology becoming second nature. The challenge will be to overcome the obstacles at each level of the SAMR Model to ultimately effect a transformation in pedagogy made possible by technology.

Acknowledgement

We thank the study participants for their efforts and commitment to this project and extend our gratitude to Brisbane Catholic Education for project funding.

References

Australian Association of Mathematics Teachers (1997). *Numeracy = everyone's business.* Report of the Numeracy Education Strategy Development Conference. Adelaide: AAMT.

Australian Curriculum and Reporting Authority (ACARA) (n.d.). Mathematics: Foundation to Year 10 curriculum. Retrieved from *http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10*

Carr, W., & Kemmis, S. (1986). Becoming critical: Education, knowledge, and action research. London: Falmer.

Chubb, I. (2012). *Mathematics, Engineering & Science in the national interest*. Retrieved from http://www.chiefscientist.gov.au/wp-content/uploads/Office-of-the-Chief-Scientist-MES-Report-8-May-2012.pdf

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development, 13*(1), 6-33.

Department of Broadband, Communications and Digital Economy (2013). *Advancing Australia as a Digital Economy: An update to the national digital economy strategy.* Retrieved from http://www.nbn.gov.au/files/2011/06/Advancing-Australia-as-a-Digital-Economy-BOOK-WEB.pdf

Department of Education, Employment and Workplace Relations (2012). *Supporting the Australian Curriculum Online.* Retrieved from http://deewr.gov.au/supporting-australian-curriculum-online

Education Services Australia (2013). *Scootle.* Retrieved from http://www.esa.edu.au/projects/scootle

Hargreaves, A., & Shirley, D. (2009). The persistence of presentism. *Teachers College Record*, 1-15. Retrieved from http://www.tcrecord.org/PrintContent.asp?ContentID=15438

Jackson, K., Shahan, E., Gibbons, L., & Cobb, P. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School*, *18*(1), 24-29.

Lamb, J. (2010). Leading mathematics reform and the lost opportunity. *Mathematics Teacher Education and Development* [Special Issue], *12*(2), 32-46.

Millett, A. & Bibby, T. (2004). The context for change. In A. Millett, M. Brown, & M. Askew (Eds.), *Primary mathematics and the developing professional* (pp. 1-17). Netherlands: Kluwer Academic.

Patterson, K., Grenny, J., McMillan, R., & Switzler, A. (2012). Crucial conversations: Tools for talking when stakes are high. New York: McGraw Hill.

Puentedura, R. (2015). *Dr R Puentedura Weblog.* Retrieved from http://www.hippasus.com/rrpweblog/

Math and Teaching Technology Innovation (2014). Retrieved from http://nlvm.usu.edu/en/nav/frames_asid_186_g_2_t_5.html?open=activities&from=category_g_2_t_5.html

Stoll, L., Bolam, R., McMahon, A., Wallace, M., & Thomas, S. (2006). Professional learning communities: A review of the literature. *Journal of Educational Change*, *7*, 221-258.

The Search for Fidelity in Geometry Apps: An Exercise in Futility?

Kevin Larkin
Griffith University
<k.larkin@griffith.edu.au>

As the use of mathematics apps in classrooms becomes more prevalent, robust research into their effectiveness is required to inform best practice regarding their use. This is particularly the case for Geometry apps where accurate and dynamic representations are critical in enhancing mathematical learning. This paper provides findings from an initial critique of 53 Geometry apps. Early findings indicate that the majority of these apps were limited in their ability to assist students in developing Geometrical conceptual understanding; however, all is not lost as a number of apps were highly appropriate.

This paper briefly synthesises the research literature concerning the use of digital manipulatives and then outlines the qualitative component of a broader three-step methodology for critiquing the appropriateness of Geometry apps. Early findings of the research indicate that the majority of the iPad Geometry apps reviewed would do little to assist students in developing Geometry concepts. Research into apps is needed as, although there has been some research into the mathematical effectiveness of apps (See Attard & Curry, 2012; Larkin, 2013) there has been little to no specific research into their usefulness in developing Geometry concepts. In addition, much of the current research into apps, with some exceptions (Larkin, 2014, 2015; Moyer-Packenham, et al., 2015) has been largely descriptive. An initial review of apps (Larkin, 2013) uncovered very few Geometry specific apps; however, the app market has since burgeoned with the creation of a range of Geometry specific apps. For the purpose of this paper, Geometry apps are those that include content from the Geometry sub-strands of the Australian Mathematics Curriculum. As was indicated in Larkin (2014), determining the quality of an app is difficult not only because of the lack of research, but also because the information that is available at the iTunes Appstore is written by the app developers to sell their app and thus not reliable. The problem of determining app quality in relation to Geometry is additionally complex as these apps require the creation of mathematically accurate external representations. Earlier research (Larkin, 2013) suggests that accuracy in representations was not commonly evident; consequently, a new methodology for evaluating Geometry apps was designed. This paper outlines how the constructs of pedagogical, mathematical and cognitive fidelity were used to evaluate 53 Geometry specific apps. The goals of this paper are two-fold. Firstly, to articulate a component of a broader methodology for reviewing the apps such that other researchers can use the methodology; and secondly, as reviewing apps is a time consuming process, an outcome of the research was the creation of a web-based database, available to teachers, of Geometry apps. This research recognises that the selection of appropriate Geometry apps needs to be based on a deeper understanding of the strengths and weaknesses of the apps, and what makes them pedagogically, mathematically and cognitively reliable (Bos, 2009).

Literature Review

It is taken as given in this paper that manipulatives (concrete and digital) support mathematical learning (e.g., Burns & Hamm, 2011; Carbonneau, Marley, & Selig, 2013; Moyer-Packenham, et al., 2015; Özel, Özel, & Cifuentes, 2014). This affords space to

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 341–348. Sunshine Coast: MERGA.

address more fully the research on three aspects of fidelity required in Geometry apps; namely, pedagogical, mathematical, and cognitive fidelity (Dick, 2008).

Pedagogical, Mathematical and Cognitive Fidelity

Pedagogical fidelity is defined by Dick (2008) as the degree to which a student can use a tool to further their learning. Zbiek, Heid, Blume and Dick, (2007) suggest that pedagogical fidelity also refers to "the extent to which teachers (as well as students) believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teacher's practice" (p.1187). Dick (2008) suggests that a pedagogically faithful tool will likely be described by students in terms of how it allowed them to interact with mathematics (e.g., "I created this triangle" etc.) rather than simply as a description of procedures for use (e.g. "I set the preferences to the fastest level" etc.). Therefore, to be an effective pedagogical tool, an app must support any action by the student that will lead to conceptual understanding of the underpinning mathematical principle.

The second aspect of fidelity to consider is mathematical fidelity. Zbiek et al. (2007) defines it as the "faithfulness of the tool in reflecting the mathematical properties, conventions, and behaviors (as would be understood or expected by the mathematical community)" (p.1173). Thus, mathematical fidelity is present when the activity of a student "is believable, is concrete, and relates to how mathematics is a functional part of life" (Bos, 2011, p. 171); and when they add strength to an understanding of mathematics as a language of patterns and order. Dick (2008) cautions that the drive for user friendliness can sometimes run contrary to faithfulness to an accurate mathematical structure. This is particularly worrisome as most apps are designed for (a) market reasons and (b) by non-educators (Larkin, 2013). Keeping the notion of mathematical fidelity at the forefront of decisions when selecting apps reminds teachers to avoid apps that do not deliver accuracy in terms of mathematical content or constructs e.g. correct scaling may not be evident in transformations.

The final aspect of fidelity is cognitive fidelity, which refers to "the faithfulness of the tool in reflecting the learner's thought processes or strategic choices while engaged in mathematical activity" (Zbiek et al., 2007, p.1173). Cognitive fidelity can be viewed largely in terms of the external representations provided by the tool. Zbiek et al. further note that "if the external representations afforded by a cognitive tool are meant to provide a glimpse into the mental representations of the learner then the cognitive fidelity of the tool reflects the faithfulness of the match between the two" (p.1176). This notion of cognitive fidelity is critical in Geometry apps which are likely to utilise many external representations. The digital nature of "app objects" potentially results in high levels of cognitive fidelity, for example, 3D objects can be pulled apart and put back to together, and in so doing, reinforce the link between 3D objects and their 2D representations (i.e. nets); however, we will see that such potential is often unrealised in Geometry apps.

Although an understanding of the three types of fidelity can assist teachers in making decisions about whether or not to use an app, I have argued above that an issue for teachers is the time required to determine app quality via the three fidelities or other evaluative measures. In addition, although it might be expected that many of the findings on the use of virtual manipulatives would reflect the experience of using mathematics apps, rigourous quantitative research into mathematics apps is still in its infancy (Larkin, 2015; Moyer-Packenham et al., 2015). Therefore research into Geometry apps, which might be best placed to take advantage of the iPad's representational capability, is required.

Methodology

This section outlines the process for initially finding the Geometry apps and then explains how a qualitative review and a descriptive, quantitative measure of fidelity were used to evaluate the apps.

Locating and Scoring the Apps

Evaluation of the apps commenced with a targeted search for mathematics apps at the iTunes Appstore in October, 2014. The following search terms were used: Geometry Elementary Education; Geometry Junior Education; Geometry Primary Education; Symmetry Education; and Transformations Education. Many of the apps appeared in two or more of the searches.

Table 1
Levels of Fidelity in Geometry Apps – Adapted from (Bos, 2009)

Type of Fidelity	**Low Level (1-3)**	**Medium Level (4-7)**	**High Level (8-10)**
Pedagogical (Including Technological) The degree to which the App can be used to further student learning.	App is difficult to work with. Accessing all aspects of the app is difficult. App is not appropriate for the mathematics concepts it uses. Transitions are inconsistent or illogical.	Using App is not initially intuitive; but with practice becomes so. Mathematical activities presented are appropriate but could be developed without app. Transitions evident but only made via trial & error.	Manipulation of App is intuitive & encourages user participation. Little or no training or instructions are required. Transitions are logical & aid sense making.
Mathematical The degree to which the App reflects mathematical properties, conventions and behaviours.	Mathematical concepts are underdeveloped or overly complex. Lack of patterns. Lack of connection to real world mathematics.	Application of mathematics concepts unclear. Patterning is evident but lacks predictability or is unclear. Some connection to real world mathematics.	Mathematics concepts developed are correct & age appropriate. Patterns are accurate & predictable. Clear connection with real world mathematics.
Cognitive The degree to which the App assists the learner's thought processes while engaged in mathematical activity.	No opportunities to explore or test conjectures. Static or inaccurate representations. Patterns do not connect with concept development.	Limited opportunities to explore or test conjectures. Minor errors with representations but still make sense. Patterns connect in a limited way with concept development.	App encourages exploration & testing of conjectures. Representations are accurate & easily manipulated. Patterns clearly aide concept development.

Apps were excluded from the final review according to a variety of criteria whereby only one app in any series was reviewed and apps categorised as Games, Entertainment or Lifestyle; apps where mathematics was part of a larger package of reading, writing, and spelling skills; and apps that required additional costs for access or further online registration were excluded.

As indicated earlier, Dick's (2008) three dimensions of pedagogical, mathematical, and cognitive fidelity have been used by other researchers to determine the quality of mathematics manipulatives (e.g., Bos, 2009; Zbiek et al., 2008). Two methodological innovations in this research are using the measures to evaluate apps; and the use of numerical values to represent the degree to which these three dimensions are present. Bos (2009) went some way towards using the dimensions as an assessment tool categorising software as Low, Medium and High fidelity in each dimension. Table 1 is an adapted version of Bos' work, modified specifically for evaluating Geometry apps. In order to make sophisticated comparisons between the three dimensions of fidelity, the nominal levels of Low, Medium and High have been replaced by an ordinal continuum ranging from 1 (no fidelity) to 10 (very high fidelity) for each of the three dimensions.

An app is considered low level (1-3) if it is generally static and inaccurate mathematically and fails to develop mathematical concepts. It is considered medium level (4-7) if more than one solution is possible and conjectures are possible (but not testable) and transitions between different aspects of the app are possible but unclear. Finally, an app is considered high level (8-10) if it uses accurate representations that are easy to manipulate with transitions between app elements that are logical and consistent, and it affords the formation of multiple, testable conjectures. In this evaluative schema, an individual app could score, for instance, highly on mathematical fidelity, yet poorly on cognitive or pedagogical fidelity.

Findings and Discussion

Prior to a brief discussion on the initial descriptive statistics collected in this research, a comprehensive qualitative evaluation (see Table 2) of the apps is provided. The author's prior research into the use of apps has indicated that this type of qualitative information is very important for teachers in making decisions about whether or not to use an app. The qualitative reviews of each of the 53 apps are available at (*link removed for peer review*). I have included below an example of one of the reviews.

Table 2
Example Qualitative Geometry App Review

App Name	**Content**	**Yr. Level**	**Generic Features of the App**
3D Geometry Basica	Shapes	Years 6-7	This app includes eight common 3D objects. The only action which can be performed on the objects is a simple zoom in or out. Each object includes a mathematical description in mathematics language and includes formulas for Surface Area and Volume.
Reviewer Comments re Mathematical Fidelity: Using the app is intuitive, largely due to the limited options available, and the content is accurate. From a conceptual development perspective the app contains complex formulas for finding SA and Volume in Platonic Solids, spheres and cylinders, but no linkage is established between the SA of an object and its Volume, or between the SA and Volumes of the different objects – E.G. between Pyramid and Octahedron. No connection to the real world. Static representations, no nets, no option for patterning or for testing conjectures. Very limited usefulness and the app does nothing that the actual physical objects couldn't do.			

As outlined in Larkin (2013) initially locating potential useful apps is a complex and time consuming process and therefore the provision of this qualitative review of each app is very useful for teachers. Apps are difficult to find due to the sheer number of apps [approx. 150 000 education apps at the iTunes store (148AppsBiz, 2015)] and this difficulty is compounded by mismatches with naming (name of app at iTunes store is different to name of app on iPad), similar naming (a dozen apps had variations on the word geometry), the rapid turnover of apps at the store, and finally a very poor search engine (apps not sorted according to category or alphabetically). As indicated in the 2013 research, teachers are extremely time poor and thus are likely, if they decide to use apps at all, to be guided by the description at the iTunes store. These are at best "infomercials" and provide misleading details about the app. For all these reasons, educationally robust reviews such as the one provided here are critical if teachers are to be directed to find what amounts to a "needle in a haystack" – i.e., an app that is appropriate for them to use with their students.

Provided in the following paragraphs are findings based on initial descriptive analysis of the data regarding types of app content, levels of quality according to each of the three fidelities, an analysis of the range of scores across the three fidelities, and finally a brief description of seven apps which scored above 6/10 for each fidelities indicating a high level of appropriateness for classroom use. Turning to content analysis first, Table 3 indicates the number of apps that included a range of Australian Curriculum Geometry content.

Table 3
Number of Apps Providing Australian Curriculum Geometry Content #

Sub-Strand / Concepts	No. of Apps	Sub-Strand | Concepts	No. of Apps
Lines (1D)	16*	Slide (Translate)	10
Shapes (2D)	31	Flip (Reflect)	21
Objects (3D)	17	Turn (Rotate)	16
Angles	15	Dilations	6

***NB**: Total app count exceeds 53 as a number of apps include more than one type of content and are therefore counted more than once. # Pythagoras and trigonometry is only introduced in Australian secondary schools and so was beyond the scope of this review.

A number of apps just focussed on one content area (e.g., Simitri – line symmetry); however, many others took a broad brush stroke approach and covered content from two or more areas (e.g. EZ Geometry or Jungle Geometry). This is not always an advantage as broad coverage often meant shallow conceptual development and less usefulness as only one section of the app was appropriate for a particular year level. By far the most popular content area was Shapes and this may be because many of the apps were targeted at very young students (Foundation and Early Years) and also because these apps appear easy to create from a technical perspective. Whilst most common, many of these Shapes apps were very basic and only included naming of the shapes and very simple matching exercises. Many of these activities could more easily be completed using actual shapes. Reflections were the most common of the four major transformations presented in the apps and this may be a consequence of the desire to link the apps to symmetry in nature or the built environment which is more easily represented than rotational symmetry, translations or dilations. Angles and 1D Geometry apps appear were common; however, this is a result of a large number of quiz apps (largely concerning geometric reasoning) rather than the availability of a large number of apps developing understanding of 1D and Angles.

Table 4 provides a breakdown of the number of apps scoring six or more in each of the three respective fidelities. Although this looks like a healthy number of apps (42) scoring at least one six, this is not the case as many of the better apps scored a six or more in two or three categories. Overall, 26 of the 53 apps failed to score a six in any category; the average score of the 53 apps was 12.9/30; and none of the three fidelity categories scored an average of 50%. This is a clear indication that there are a large number of Geometry apps, categorised as educational at the iTunes store, which do not even meet a very low benchmark for appropriateness in classrooms. As might have been anticipated [given the findings of previous research (Larkin, 2014; 2015) which indicated that many apps are instructional and focus on declarative or procedural knowledge], the apps which were of some use tended to score well on the pedagogical fidelity dimension, less well in terms of the quality of the mathematics they contain, and generally poorly in their ability to assist cognitive development. This again mirrors the generally poor level of conceptual knowledge developed by apps in the research noted above.

Table 4
Number of Apps Scoring 6 or More in Respective Fidelities

Type of Fidelity	**Number of Apps (n=53)**	**Percentage* (to nearest 0.1)**	**Average Score / 10**
Pedagogical	21	39.6%	4.9
Mathematical	13	24.5%	4.3
Cognitive	8	15.1%	3.7
Overall Average Score for Apps on the three measures / 30			12.9

Overall, the apps scored more highly in terms of pedagogical fidelity because this is the easiest of the categories for non-mathematical app designers to mimic in their apps. Many of the apps met one of the pedagogical criteria, namely, they were easy to use without instruction, and many of them partially met the criteria of appropriateness of activity without necessarily doing anything more than could be easily replicated with an IWB, physical manipulatives, or even pen and paper. Many of them incorporated multiple choice quizzes (of varying degrees of quality) which may serve some use as revision exercises. This was particularly the case where quizzes drew from a large bank of questions, did not allow multiple guesses, and allowed results to be emailed (e.g. Kids Math-Angle Geometry and Symmetry School Learning).

Mathematical fidelity issues generally related to incorrect naming or classification of shapes and objects, (e.g. diamonds instead of rhombuses, cubes not considered prisms, squares not considered as rectangles, triangles not included as polygons); use of prototypical shapes and standard orientations (only three apps focused on non-prototypical shapes – Cyberchase Quest, Maths Geometry, and Shapes MyBlee); and lack of connection to any notion of real world application of mathematics (minor exceptions to this include Geometry 4 Kids and Simitri).

Of most concern was the low cognitive fidelity of most apps and this is problematic in terms of classroom use as this relegates many of the apps to only being useful as revision activities of for rote learning. The majority of apps did not meet the criteria for supporting cognitive development. Despite being technically capable, most apps only provided static representations and, where dynamic representations were used, they did not mimic the physical activity of, for instance, turning or sliding or flipping but used arrows or numbers to direct the transformations (noteworthy exceptions were Squares and Colors and Shapes MyBlee). In addition, very few apps allowed opportunity for students to create patterns and develop their own conjectures regarding shapes, objects, angles, or transformations. This is a serious shortcoming of the vast majority of the apps.

Despite the comments above, it is not all doom and gloom in “AppLand” as there are some apps that shine in the overall geometric darkness that is the iTunes store (see Table 5). Of the apps reviewed, seven of them (13% of the total apps reviewed) scored six or more out of 10 for each of the three fidelities. These are clearly the apps that teachers should be utilising in their classroom practice. What is interesting here is that apart from the top three, even the better apps were inconsistent in meeting the three fidelity standards as four of the seven scored one six and two of these four scored two sixes.

Table 5
Apps that Scored 6 or More on Each of the Three Fidelities

App Name	Pedagogical	Mathematical	Cognitive	Total
Co-ordinate Geometry	9	8	9	26
Transformations	9	8	9	26
Attribute Blocks	8	8	8	24
Shapes – 3D Geometry	9	6	8	23
Shapes and Colors	7	6	7	20
Pattern Shapes	8	6	6	20
Isometry Manipulative	7	6	6	19

This level of inconsistency mirrors the findings of Moyer-Packenham et al. (2015) in relation to virtual manipulatives. In their research they noted multiple affordances within each virtual manipulative such that one or more of these affordances may be more influential and beneficial for student learning. An example of this in terms of apps is Isometry Manipulative, where one component of the apps is extremely beneficial whilst the second component, if used, is likely to undermine student learning. This inconsistency becomes more apparent as scores further down the total list of scores are examined, for example, Geometry Montessori (9, 6, 5) scored equal to or higher than three of the apps listed in the top seven but was relatively poor in terms of cognitive development. Three other apps scored highly in pedagogical and mathematical fidelity but poorly in terms of cognitive development (GeoEng- 8, 6, 5; Geometry 4 Kids- 8, 6, 3; and Geometry Explore- 6, 6, 4). It is worth noting that only one app (Simitri- 4, 9, 8) scored very lowly in pedagogical fidelity but very highly in mathematics and cognitive fidelity. This indicates that this app should not be used unsupervised by students; however, with correct scaffolding from the teacher, it is very useful for developing mathematical understanding due to its high level mathematical and cognitive fidelity.

It is clearly the case that, other than with the top three apps, teachers need to decide the exact purpose they want to achieve by using an app and then look at the content covered and individual fidelity scores of each app, to find one that meets that specific purpose. In this manner, Geometry Montessori would be most appropriate to use in a revision mode but less so in terms of developing mathematical or conceptual fidelity. The full list of scores is available for teachers at the URL provided earlier in the paper.

Limitations and Next Steps

As was the case in Larkin (2014), a limitation of any study reviewing apps is a consequence of two factors; initially locating (and relocating apps), and the nature of the iTunes App store. Firstly, the sheer number and method of labelling apps (e.g., multiple apps called Geometry [or very similar] or apps containing geometry but not indicated in their name – e.g., Koala Math) means that there may be useful Geometry apps not reviewed. Secondly, the iTunes store is a moveable feast as apps are generated, renamed, relocated, or removed on a daily basis. This research has indicated that, although many Geometry apps are quite poor in terms of their fidelity, it is, to return to the question posed in the title, certainly not a futile exercise to use some of them in primary mathematics classrooms. Due to the shortened nature of MERGA conference papers, only one component of the quantitative measures used in the broader research has been presented to support this claim. A more substantive examination of their quality incorporating three quantitative measures, using modified versions of Haugland's (1999) Software Scale, Bos' (2009) software categorisations and Dick's (2008) three fidelities will be used in future use to more comprehensively determine the quality of Geometry apps in supporting primary students mathematical learning.

References

148AppsBiz. (2015). *App store metrics*. Retrieved from http://148apps.biz/app-store-metrics/

Attard, C., & Curry, C. (2012). Exploring the use of iPads to engage young students with mathematics. In J. Dindyl, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons* (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, Singapore, pp. 75-82). Singapore: MERGA.

Larkin, K. (2014). Ipad Apps that promote mathematical knowledge? Yes, they exist! *Australian Primary Mathematics Classroom, 19*(2), 28-32.

Larkin, K. (2015 in press.). "An App! An App! - My Kingdom for an App": An 18 month quest to determine whether apps support mathematical knowledge building. In T. Lowrie & R. Jorgensen (Eds.), *Digital games and mathematics learning: Potential, promises and pitfalls.* Springer Press.

Bos, B. (2009). Virtual math objects with pedagogical, mathematical, and cognitive fidelity. *Computers in Human Behavior, 25*, 521-528. doi: 10.1016/j.chb.2008.11.002

Burns, B. A., & Hamm, E. M. (2011). A comparison of concrete and virtual manipulative use in third and fourth grade mathematics. *School Science and Mathematics, 111*(6), 256-261. doi: 10.1111/j.1949-8594.2011.00086.x

Carbonneau, K. J., Marley, S. C., & Selig, J. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380-400. doi: 10.1037/a0031084

Dick, T. P. (2008). Fidelity in technological tools for mathematics education. In G. Blume & M. Reid (Eds.), *Research on Technology and the Teaching and Learning of Mathematics: Cases and Perspectives* (Vol 2, pp. 333-339). Information Age Publishing.

Haugland, S. (1999). Computers and young children: The newest software that meets the developmental needs of young children. *Early Childhood Education Journal, 26*(4), 245-254.

Moyer-Packenham, P. S. et al. (2015). Young children's learning performance and efficiency when using virtual manipulative mathematics iPad apps. *Journal of Computers in Mathematics and Science Teaching, 34*(1), 41-69.

Özel, S., Özel, Z., & Cifuentes, L. (2014). Effectiveness of an online manipulative tool and students' technology acceptances. *International Journal of Educational Studies in Mathematics, 1* (1), 1-15.

Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 1169-1207). Charlotte, NC: Information Age.

Pre-service teachers and numeracy in and beyond the classroom

Gilah C Leder
Monash University
<Gilah.leder@monash.edu>

Helen J Forgasz
Monash University
<Helen.forgasz@monash.edu>

Natalie Kalkhoven
Monash University
<Natalie.kalkhoven@monash.edu>

Vince Geiger
Australian Catholic University
<Vince.Geiger@acu.edu.au>

Data from a pilot study concerned with pre-service teachers' perceptions of the numeracy demands on Australian teachers are reported. The sample comprised 211 students enrolled in pre-service teacher education courses at a large Australian university. While most recognised the importance of mathematics and its applications in everyday life, less than half considered there were mathematical demands on teachers beyond their teaching domain. NAPLAN-related information data was used to examine the group's ability to access, apply, and interpret the statistical information.

Introduction

In Australia there is no shortage of recent reports concerned with the quality of pre-service teacher education and graduate performance (e.g., Australian Institute for Teaching and School Leadership [AITSL], 2014), the falling interest among students at all levels of education in mathematics (e.g., Wienk, 2014), the scope and quality of the mathematics curriculum (Donnelly & Wiltshire, 2014), and its relevance to the work place and daily life of the country's students and citizens (e.g., The Australian Association of Mathematics Teachers [AAMT] and the Australian Industry Group, 2014). The pilot study reported in this paper was fuelled by the contents and recommendations of several such reports, the challenges they present for those involved in initial teacher education, and the need, ultimately, to develop practical solutions.

Providing a context

The establishment in 2010 of AITSL powered a renewed focus on the requisites for excellence in teaching and school leadership. Expectations for commencing and newly graduated teacher education students have also attracted close and careful scrutiny. With respect to the former, a clear standard has been advocated by state Ministers for Education: that all initial teacher education students will have a level of literacy and numeracy equivalent to the top 30% of the population. At the end of their course, it is mandated, graduate students must "have an understanding of their subject/s, curriculum content and teaching strategies … (and be) able to design lessons that meet the requirements of curriculum, assessment and reporting" (AITSL, n.d., para 2). With respect to numeracy, they are expected both to know and understand appropriate teaching strategies and their applications in teaching areas. Elaborations of this expectation are readily found, for example:

> In the Australian Curriculum, much of the explicit teaching of numeracy skills occurs in Mathematics. Being numerate involves more than the application of routine procedures within the mathematics classroom. Students need to recognise that mathematics is constantly used outside the mathematics classroom and that numerate people apply general mathematical skills in a wide range

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 349–356. Sunshine Coast: MERGA.

> of familiar and unfamiliar situations. (Australian Curriculum and Assessment Reporting Authority [ACARA], n.d., para 1).

That the teaching of numeracy embraces aspects beyond the mathematics classroom is a view echoed by the AAMT:

> To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life. In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. (c. 1998, p. 2).

In brief, all teachers, not only those explicitly involved in teaching mathematics, are deemed responsible for contributing to the numeracy development of their students. It can be inferred from the mantra above that teachers should also be familiar with the quantitative requirements of various work settings, including those relevant and applicable in their own work place. As Steen (2001) maintained, "(n)umeracy and mathematics should be complementary aspects of the school curriculum. Both are necessary for life and work, and each strengthens the other. But they are not the same" (p.15).

The influential Organisation for Economic Co-operation and Development [OECD] (2013) summarised indicative levels of information-processing skills used in nine clusters of occupation. Numeracy was defined as the "ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life" (p. 59). Numeracy skills were said to apply at all levels of occupation and to increase with the demand of reading skills required. The numeracy demands of the school work place were not specifically identified.

In a recent project 12 secondary school mathematics teachers spent time in various Australian businesses to explore in some depth the nature and scope of the quantitative skills needed and used by workers in different settings (The Australian Association of Mathematics Teachers and the Australian Industry Group, 2014). The project "was designed to look at the requirements for mathematical skills and understanding in the modern workplace and to develop a clearer picture of the matches and mismatches between current mathematics (curriculum, teaching methods and resources) and the quantitative skills required" (p. 11). Space constraints prevent inclusion here of a summary of the project's main findings and recommendations. Intriguingly, none of the teachers was allocated the work place seemingly best known to him/herself, that is a school, and given the task to identify and clarify the quantitative skills required in this setting. While much emphasis has been placed on the numeracy skills linked to teaching mathematics or to meet the numeracy demands of other disciplines in the classroom (e.g., Geiger, Forgasz, & Goos, 2015), little attention has been given to the numeracy demands of the school work place *per se*, that is, work not directly linked to classroom teaching but necessary to function as a teaching professional. Pre-service teachers' views about the mathematical/quantitative demands in everyday life and on their role as teachers were examined in the pilot study reported here.

In this paper we focus primarily on pre-service teachers' perceptions of applications which are part of their teaching responsibilities but may be outside their subject teaching areas. A specific example to probe their "ability to access, use, interpret and communicate mathematical information" (OECD, 2013, p. 59) was included in the survey. We presented performance data from The National Assessment Program – Literacy and Numeracy [NAPLAN] and asked respondents to interpret the data. The NAPLAN tests are said to "provide information for students, parents, teachers and principals about student achievement which can be used to inform teaching and learning programs" (Victorian

Curriculum and Assessment Authority, 2015, para. 3). The sharing of test information constructively with both students and parents is evidently an obligation for teachers if an important goal of the testing regime is to be realised. The data are also intended to be used to check the efficacy of instructional programs in place.

The study

Aims

The central aim of the study was to develop an instrument to determine teachers' views of the numeracy demands on Australian teachers and their numeracy capabilities as captured in statements such as that of the AAMT (c. 1998) and the OECD (2013) and, in the first instance, to trial the instrument with a group of pre-service teachers. The research questions of particular interest in this paper are:

1. What are pre-service teachers' views about their proficiency in mathematics?
2. What are pre-service teachers' views about the importance of mathematics for teaching?
3. Do pre-service teachers recognise mathematical demands in everyday life?
4. Do pre-service teachers recognise mathematical demands on teachers in schools apart from what is taught to students?

The instrument

The full instrument included biographical items (e.g., gender, level of schooling able to teach in at the completion of the course, and if relevant – specialisation/teaching methods). Views about, and attitudes towards, mathematics (e.g., importance of mathematics for teachers, levels of confidence, etc.), and the utility of numeracy skills for teaching and for teachers in their workplace, the school, were also tapped.

As well as numerical items gauging basic mathematical skills, numeracy problems were set in the following contexts: everyday life, informed citizenry, and the workplace (the school). Participants were not only asked to provide answers to numeracy items and items involving numerical calculations, but also to indicate their level of self-efficacy in the answer they gave. Most of the numerical items were in multiple-choice format; for others, participants had to provide answers and explain their responses. The numerical items were drawn from publicly available Australian grade 9 NAPLAN tests[1] and from the pool of released PISA[2] items (with permission); a few items were developed by the researchers. The instrument was prepared for online completion using Qualtrics (www.qualtrics.com). Selected items are included in the presentation of the results.

Space constraints prevent a more detailed listing of the contents of the full instrument and limit inclusion of the often informative and thoughtful explanations given.

Data gathering

All pre-service teachers enrolled at one Australian university which offers undergraduate and graduate programs in teacher education were invited to participate in

[1] NAPLAN is the National Assessment Program for Literacy and Numeracy which, since 2008, has been administered to all Australian students in grades 3, 5, 7 and 9.

[2] PISA is administered every three years to samples of 15 year old students in many countries around the world, including Australia.

the pilot study. The university's guidelines for the recruitment of its students for research studies were adopted: advertisements were placed on selected Moodle sites with a link to the online instrument; lecturers in core units of study advertised the study in their classes; and posters and flyers were displayed within the buildings at the university campuses where the students were enrolled. A four week timeframe was allowed for the online instrument to be completed.

The sample – contextual details

The university from which the sample was drawn offers teacher education courses that would qualify teachers for early years [EY] teaching (birth to 8 years of age), primary (elementary) [P] teaching (grades Prep to 6), secondary [S] teaching (grades 7 to 12), as well as two cross-sectorial levels: EY-P (birth to grade 6) and P-S (grades P to 12).

The sample comprised 237 students. Of these 23 (10%) opted out of the survey after answering only the first two or three items. These surveys were excluded from the analyses. Of the remaining 214 respondents who answered all or most of the items, 174 (81%) were female and 40 (19%) were male. Just over half, 119 (56%), were aged under 25. Of the rest, 53 (25%) were aged between 25 and 34, while 42 (20%) indicated they were older than 35. The gender and age distributions are in line with data provided by AITSL (2014) for the 2012 initial teacher education intake. That year females comprised 76%, and 64% of the intake were aged under 25.

Most of the respondents, 164 (78 %) of the 211 who answered the question, had completed their secondary schooling in Australia. Only eight among the respondents nominated mathematics as one of their secondary teaching specialisations.

Results

Pre-service Teachers and Proficiency in Mathematics

The majority of respondents 104 (54%) considered themselves to be good or excellent at mathematics, 75 (39%) self-rated as being average, and 15 (8%) thought they were weak or below average. As reported by Forgasz, Leder, Geiger, and Kalkhoven (submitted), these judgements were supported by the group's more than credible performance in solving the set of numerical items taken from the sources already described above.

Pre-service Teachers' Views about the Importance of Mathematics for Teaching

As reported above, only eight of the group gave mathematics as their specialisation. Yet most respondents considered it important for teachers to be good at mathematics: 147 (76%) agreed, 18 (9%) disagreed, and 29 (15%) were unsure. Just over half (100: 52%) thought they had studied enough mathematics to be a competent teacher, with the remainder almost equally divided between those who thought they had not (45: 23%) and those who were unsure (49: 21%).

Pre-service Teachers and Mathematical Demands in Everyday Life

Many of the respondents (≈200) to items relevant to this issue recognised the importance of mathematics and its applications in everyday life. For example, approximately 90% agreed that "In everyday life, understanding fractions, decimals, and percents is very important in our society", considered that "given the price per square metre, I could estimate the cost of new carpet needed for a room" and that they "could easily extract information from tables, plans, and graphs”. In contrast, only 21% agreed that “mathematics is communicated well in newspapers and the media”, 44% disagreed, and the remaining 35% indicated that they were uncertain.

Pre-service Teachers’ Recognition of Mathematical Demands on Teachers in Schools apart from what is taught to Students

1. *Direct responses to the above item.* Respondents were about equally divided between those (44%) who considered that there were mathematical demands on teachers apart from what is taught to students and those who were unsure (42%); the remaining 14% considered that there were no such demands. Those who acknowledged that there were numeracy demands on all teachers beyond the classroom touched on a number of pertinent areas in which numeracy skills are needed. Representative explanations for the Yes, No, and Unsure responses included:

> Yes: Understanding of statistics for analysis of NAPLAN results, class tests etc.
>
> Yes: Teachers need to possess broader/higher levels of organisational/analytical/linguistic skills which are entailed in mathematical abilities in order to successfully perform bureaucratic/ organisational responsibilities required in school settings.
>
> Yes: Mathematics such as class numbers, number of years teaching, salary…
>
> Unsure: I would say that there are mathematical demands on everybody, to some level, but whether teachers have more than anybody else is questionable.
>
> Unsure: I don't have any experience to decide
>
> No: I am not aware of the external mathematical demands on teachers
>
> No: I don't really think there are

2. *The NAPLAN-information item and pre-service teachers’ responses.* The information below was provided on the survey preceding the NAPLAN questions:

> Here are the NAPLAN Reading and Numeracy results for Year 7 students at one Australian school (Aussie HS) taken from the MySchools website. The school’s results (blue) are shown together with ‘similar schools’ (orange).
>
> The instructions for interpreting the graphs are provided below the graphs.

The NAPLAN data that were provided are shown in Figure 1. Students had to interpret these data to answer the NAPLAN-information item questions. The specific questions asked about the data are used as headings for the relevant results. Because of space constraints, only a limited number of explanations for the answers selected are provided.

a. In which Year did the Aussie HS Students achieve Best in Reading?

The majority of those who answered this question (128: 89.5%) selected 2012 as their answer. A small number answered 2010 (9: 6.3%) or 2011 (6: 4.2%).

b. In which Year did the Aussie HS Students achieve Best in Numeracy?

Most (114: 81%) selected 2012. Of the others, 2 (1.4%), 12 (8.6%), 10 (7.1%), and 2 (1.4%) selected 2008, 2010, 2011, and 2013 respectively.

For both items most participants focussed on the mean score obtained by the school’s students on the test. That the difference in the mean scores between the school and "similar" schools, in favour of the school, was also largest in 2012 rarely featured as a reason for it being selected as the year of the school's best performance.

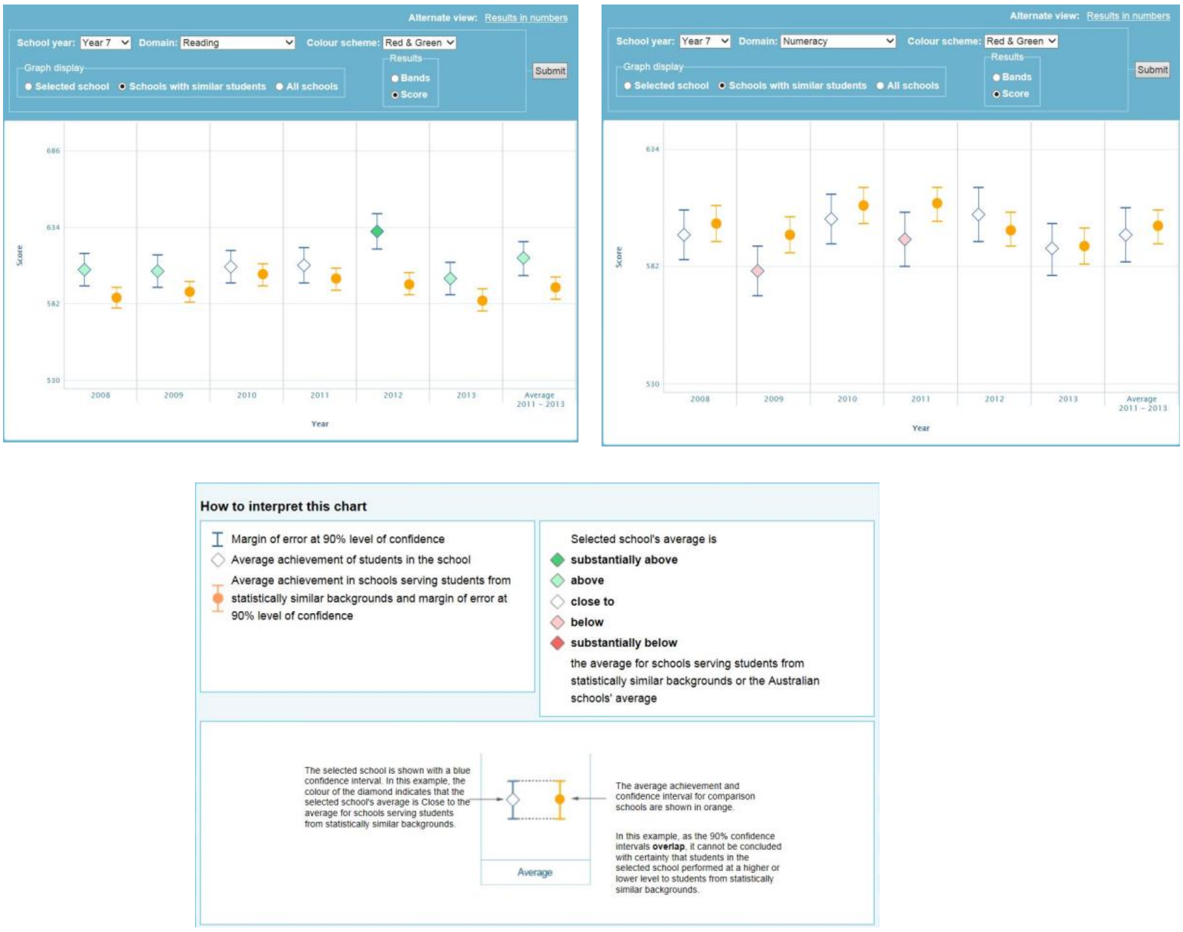


Figure 1. NAPLAN-information item (devised by research team – drawn from publicly available data from MySchools website)

For the Literacy item (a.), those who did not select 2012 generally gave mathematically irrelevant explanations such as "It is shown in the graph" and "The average achievement bubble is at its highest point over the selected year period". These students did not demonstrate the relevant mathematical skills to interpret the graphical representation.

In general, the Numeracy results (b.) elicited more complex explanations, not necessarily demonstrating mastery of the pertinent mathematics, for example:

> The orange bar reached the highest score (when also taking into account the margin of error). (Explanation in support of choosing 2010 as the best year of Numeracy performance)
>
> On the numeracy graph the school diamond was at its highest point in 2012. However the margin of error was higher in this year. It was also only one of 2 years where the school performed above the average. (Explanation in support of choosing 2011 as the best year of Numeracy performance).

c. *Based on the Reading and Numeracy NAPLAN results, what should the Curriculum Co-ordinator be concerned about: Reading, Numeracy, Both Reading and Numeracy, Neither Reading nor Numeracy, or Unsure?*

Each of the options listed was selected by at least some of the respondents. Twelve (8.5%) identified Reading, 98 (69%) nominated Numeracy, Both Reading and Numeracy was selected by 20 (14.1%), seven (4.9%) considered that neither was of concern, and five (3.5%) indicated that they were unsure. A selection of explanations for each of the alternatives presented serves as indicators of the sample's proficiency in interpreting graphs, a skill some 90% had claimed they had mastered.

Not all respondents provided an explanation for their interpretations but, for this item, too, a majority did. A representative sample of responses is shown below.

Reading

It seems that in reading, most students are on average unlike numeracy

Reading has sharply declined from 2012 to 2013 - you would want to know what drove it so sharply down (also what was (it) about their literacy program that caused big spike in 2012) - even though it is still ahead of average or similar schools, the gap is much narrower in 2013 vs. 2012 with other similar schools. / / Numeracy can still be improved (always room for improvement!) however scores are still close to the average

Numeracy

Numeracy is a concern as it has dropped (in) the last two years and is now below the average for similar schools in the area. In Reading the Aussie HS regularly receives higher average student scores than similar schools and the average student scores are fairly consistent. In Numeracy the Aussie HS mostly had lower average scores than similar schools and student scores are also less consistent.

Both Reading and Numeracy

Schools should never just focus on one subject, all subjects are important. If the school must have separate teachers in order to help students understand then so be it (if this were a primary school).

Whilst numeracy needs more help there is still room for improvement for reading

Whilst the numeracy results were lower, neither was consistently above average.

The results in 2012 were improving but slipped backwards in 2013.

Neither Reading nor Numeracy

The coordinator doesn't need a standardised test to dictate where his (sic) students are struggling

All subjects. The question doesn't ask me if it specifically relates to just this data or for just this year (2012).

Unsure

My first instinct said that the co-ordinator needs to focus on reading because the score was so much better for numeracy in 2012. However on the other hand the results for numeracy are generally lower than the national average. So I'm inclined to say both but I'm not entirely sure.

That the one set of information generated subtle differences in interpretation can be seen from the excerpts above. As expected, many of the more thoughtful, elaborated, responses referred to both sets of data. Yet it also appears from the responses that different directions for the school's teaching and learning programs might be inferred from the data.

The findings above suggest that the majority of the pre-service teachers in the study were able to interpret the data presented to them and provide appropriate supporting explanations. What was evident, however, was that most of the explanations were fairly superficial and did not reflect a full appreciation for the arguments that might be needed to convince others of their views.

Final comments

The pilot study yielded fruitful insights into the pre-service teachers' perceptions of the numeracy demands on Australian teachers. Although the sample who participated in the pilot study comprised students from only one university, the group's gender and age distributions were similar to those reported by AITSL (2014) for the 2012 initial teacher education. The data reported here are particularly relevant to pre-service teachers whose specialisation is not in mathematics. As indicated by AAMT (c. 1998, p. 2), "in school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum". That less than half of the sample (44%) agreed that there were mathematical demands on teachers apart from what is taught to students is a matter of concern. Whether those preparing to be teachers of mathematics share this view warrants close scrutiny.

Do Australian teacher education programs present pre-service teachers with examples which illustrate the types of numeracy demands on teachers that they will encounter when they work in schools? The NAPLAN item in our survey draws on fairly sophisticated graphical representations; the interpretation of the data requires a good understanding of the related mathematical concepts. The numeracy demands on teachers in schools are not limited to the interpretation of NAPLAN data. We are left to speculate on what AITSL will produce as the numeracy/mathematics test that pre-service teachers will need to pass prior to graduating from teacher education programs.

References

AAMT. (c.1998). *Policy on numeracy education in schools.* Adelaide, SA: Author. Retrieved from www.aamt.edu.au/content/download/724/19518/file/numpol.pdf

ACARA. (n.d.) *Numeracy across the curriculum*. Retrieved from http://www.australiancurriculum.edu.au/generalcapabilities/numeracy/introduction/numeracy-across-the-curriculum

AITSL. (n.d.). *Graduate teachers*. Retrieved from http://www.aitsl.edu.au/australian-professional-standards-for-teachers/standards/career-stage/graduate

Forgasz, H., Leder, G., Geiger, V., & Kalkhoven, N. (submitted). Pre-service teachers and numeracy readiness.

Geiger, V., Forgasz, H., & Goos, M. (2015). A critical orientation to numeracy across the curriculum. *ZDM The International Journal on Mathematics Education.* [Accepted: 6 November 2014. Published online: 3 December 2014]. DOI 10.1007/s11858-014-0648-1

OECD (2013) OECD Skills Outlook (2013). First results from the Survey of Adult Skills. OECD Publishing. Retrieved from http://dx.doi.org/10.1787/9789264204256-en

Steen, L. (Ed.) (2001). Mathematics and democracy: The case for quantitative literacy. Retrieved from http://www.maa.org/sites/default/files/pdf/QL/MathAndDemocracy.pdf

The Australian Association of Mathematics Teachers and the Australian Industry Group (2014). *Identifying and supporting quantitative skills of 21st century workers: Final report.* Canberra: Commonwealth of Australia. Retrieved from http://www.chiefscientist.gov.au/wp-content/uploads/Quantitative-Skills-of-21st-Century-Workers-Report.pdf

Victorian Curriculum and Assessment Authority, State Government of Victoria (2015). *National Assessment Program - Literacy and Numeracy testing (NAPLAN)*. Retrieved from http://www.vcaa.vic.edu.au/pages/prep10/naplan/index.aspx

Wienk, M. (2014). *Discipline profile of the mathematical sciences 2014*. Melbourne: Australian Mathematical Sciences Institute. Retrieved from http://amsi.org.au/publications/discipline-profile-of-the-mathematical-sciences-2014/

Gender Differences in Mathematics Attitudes in Coeducational and Single Sex Secondary Education

Kester Lee
University of Sydney
<klee7740@uni.sydney.edu.au>

Judy Anderson
University of Sydney
<judy.anderson@sydney.edu.au>

Exploring why more boys than girls continue to study higher levels of mathematics in senior school when there appear to be no gender differences in achievement in earlier years is worthy of investigation. There are potentially many reasons why this occurs including career aspirations, interest, and attitudes. One factor explored in this study was the gender composition of classes in Years 7 to 9. Data were collected from students in a single-sex boy's school, a single-sex girl's school and a coeducational school. Data revealed differences in attitude to mathematics with girls in the single-sex school having the most positive attitudes and girls in the coeducation setting having the least positive attitudes.

At a time when there has been an explosion in the amount of data available to inform research and development, there is an increasing need for well-trained mathematicians and statisticians. However, the numbers of students continuing to study advanced levels of mathematics in senior secondary schooling and at the university level are declining (Office of Chief Scientist, 2012). There is an urgent need to arrest the decline but to do this more information is required about why students are choosing to discontinue their study of mathematics at the earliest opportunity.

It has already been established that many students find mathematics boring and frustrating (Brown, Brown, & Bibby, 2008), and attitudes toward mathematics appear to decline for many students as they progress through school (Watt, 2004). Anxiety and avoidance is a persistent and growing issue in mathematics education (Ashcraft & Moore, 2009). In addition, there appear to be gender differences in relation to attitudes to mathematics, self concept, and career aspirations (Martin, 2003; Watt, 2007). Research has a role to play in developing new understandings about these situations and investigating ways to improve the teaching and learning of mathematics in secondary school contexts.

Literature Review

Gender differences in secondary mathematics are a prominent issue that has been the focus of many studies, with reported differences in mathematics achievement between boys and girls a contentious issue. The literature has not come to a clear consensus; some studies have shown girls outperforming boys (e.g., Stevens, Wang, Olivarez, & Hamman, 2007), while others find boys outperforming girls (e.g., Preckel, Goetz, Pekrun, & Kleine, 2012). Recent research from large-scale studies such as the Trends in International Mathematics and Science Study (TIMSS) has found that "there were no gender differences in 22 of the 42 countries that tested at Year 8, including Australia", and no gender differences were found within any single state or territory, including New South Wales (Thomson, Hillman, & Wernert, 2012, p. 20). While there are studies that undoubtedly do find differences between boys' and girls' achievement in mathematics, it appears that on a national level this is not the case.

However, while studies focusing on gender differences in achievement are inconclusive, there is clearer evidence that positive attitudes, behaviours and participation rates in mathematics generally favour boys. Information from the Board of Studies,

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 357–364. Sunshine Coast: MERGA.

Teaching and Educational Standards NSW (BOSTES) shows that girls are under-represented in advanced mathematics courses. In the NSW Higher School Certificate courses of Extension 1 Mathematics and the higher Extension 2 Mathematics, girls constituted 40.0% and 35.6% of enrolments respectively in 2014 (BOSTES, 2015). Research has also shown that, compared to boys, girls are less likely to choose careers related to mathematics (Watt, 2007), feel less confident and suffer from mathematical anxiety in greater proportions (Ai, 2002; Hannula, 2002; Leedy, LaLonde, & Runk, 2003), have lower self-concept in mathematics (Kyriacou & Goulding, 2006), suffer from gender stereotyping where mathematics is viewed as a male domain among the general public (Leder & Forgasz, 2010) and among parents (Jacobs, Davis-Kean, Bleeker, Eccles, & Malanchuk, 2005), and also have fewer female mathematical role models as examples to emulate or follow (Lee & Anderson, 2014).

The causes of the gender differences in attitudes, behaviours and participation rates are varied, and it is likely that any truly comprehensive explanation would require a complex combination of factors. Gender stereotyping is often cited as a potential cause of these differences, as stereotyping underpins many other background factors such as parental, teacher and peer attitudes, which can in turn have an effect on the attitudes, behaviours and participation rates of boys and girls, and there is some merit to this view (Mael, 1998).

However, recently there has been some research investigating whether single-sex or coeducational schooling is a contributing factor to some of these gender differences. In an Irish study involving four schools, Prendergast and O'Donoghue (2014) found that the type of school had a statistically significant effect (p = .02) on student enjoyment of mathematics. The single-sex male school scored the highest, followed by the single-sex female school. Within the two coeducational schools, males enjoyed mathematics significantly more than females (p = .02). Interestingly, across the study females scored higher than males on diagnostic examinations, indicating that “females outperformed males even though they enjoyed the subject less” (Prendergast & O'Donoghue, 2014, p. 1125). This finding seems to confirm that enjoyment of mathematics is driven by something other than achievement and that the gender composition of classrooms may have some impact.

The Irish finding of girls in single-sex settings having more positive attitudes towards mathematics than girls in coeducational settings is not an isolated occurrence. A Zimbabwean study found that girls’ self-concept was higher in a girls-only school than in a coeducational school, although in this case there were no significant differences in achievement (Tambo, Munakandafa, Matswetu, & Munodawafa, 2011). An Australian study of female engineering students enrolled at the University of Technology in Sydney (UTS) found that female students from single-gender schools outscored their male counterparts on measures of self-perception of mathematical skill and ability (Tully & Jacobs, 2010).

However, a new study is needed to investigate the possibility of gender composition (single-sex or coeducational) in junior secondary mathematics classrooms having an effect on students’ attitudes to mathematics in Australia. A pilot study of three schools was undertaken to investigate the following question:

Does the gender composition of classrooms in Years 7 to 9 influence students’ attitudes towards mathematics?

The Study

Three independent schools in a large metropolitan area took part in the study. School MF was a co-educational school, while School M and School F were a single sex boys' and girls' school respectively. This particular investigation was undertaken as part of a larger study that focuses on interest in mathematics in the lower secondary years. The study involved the completion of a written questionnaire, followed by individual interviews with selected students. All students in Years 7 to 9 completed the written questionnaire, resulting in a total of 1,229 responses. The distribution of participants by school, year and gender is shown in Table 1.

Table 1
Distribution of Participants

School	Year	Males	Females
MF (Co-ed)	7	58	71
	8	53	65
	9	61	45
M (Boys)	7	180	-
	8	186	-
	9	168	-
F (Girls)	7	-	123
	8	-	132
	9	-	87
Total		706	523

The written questionnaire consisted of 5 items measuring the perceived interest of the respondent's female carer, male carer, teacher, friends and classmates, as well as 26 Likert-scale items (adapted from Stevens & Olivarez, 2005), in addition to open-ended questions and other basic demographic information provided by the participants. Eight of the 26 Likert-scale items specifically measured attitudes towards mathematics, and analysis of these data form the basis of the results reported in this paper.

Results and Discussion

Gender differences were examined by comparing means with an independent samples *t*-test utilising *SPSS* software. The Likert scale consisted of five points, with a score of '1' indicative of the respondent strongly disagreeing with the statement and a '5' indicating strong agreement. In accordance with common statistical convention, a *p*-value less than .05 indicates a significance difference, and a *p*-value less than .01 indicates a strong significant difference. Table 2 lists the gender differences across the whole sample for the eight Likert-scale items measuring attitudes towards mathematics. Apart from Item 4, where girls displayed higher levels of anxiety when working on maths, there were no significant differences in attitudes across the whole sample.

Having analysed the sample as a whole, the next step involved the examination of these gender differences in the coeducational School MF, and gender differences between the boys in School M and the girls in School F. This analysis would shed light on the hypothesis that the school setting (single-sex or coeducational) could have some significance for the gender differences. Gender differences in School MF are presented in Table 3.

Table 2
Gender Differences Across the Whole Sample

Item	Means		*p*-value
1. I like maths	M = 3.283	F = 3.303	.741
2. I feel anxious when working on maths	M = 2.372	F = 2.596	.001**
3. Doing maths is one of my favourite activities	M = 2.268	F = 2.231	.569
4. I often find that the things we deal with in maths are really exciting	M = 2.562	F = 2.571	.890
5. I don't enjoy maths	M = 2.672	F = 2.678	.935
6. Maths is fun	M = 2.721	F = 2.759	.595
7. Maths is very stressful for me	M = 2.690	F = 2.787	.163
8. When I'm doing maths I feel pretty happy	M = 2.580	F = 2.583	.967

Table 3
Gender Differences in School MF

Item	Means		*p*-value
1. I like maths	M = 3.253	F = 2.939	.008**
2. I feel anxious when working on maths	M = 2.320	F = 2.702	.002**
3. Doing maths is one of my favourite activities	M = 2.183	F = 1.938	.036*
4. I often find that the things we deal with in maths are really exciting	M = 2.515	F = 2.254	.021*
5. I don't enjoy maths	M = 2.852	F = 3.153	.028*
6. Maths is fun	M = 2.562	F = 2.384	.172
7. Maths is very stressful for me	M = 2.692	F = 3.045	.009**
8. When I'm doing maths I feel pretty happy	M = 2.432	F = 2.213	.064

As can be seen in Table 3, there were significant gender differences in six of the eight Likert-scale items measuring attitudes towards mathematics, and in each case, girls had more negative attitudes than boys. Girls were more likely to feel anxious when working on maths, were more likely to say that they did not enjoy maths and found it stressful, and they were less likely to find maths exciting, likeable, or name it as one of their favourite activities. It is clear that in School MF there was a tendency for boys to have more positive attitudes towards mathematics than girls. The investigation then compared the boys of School M and the girls of School F in Table 4.

It should be noted here that in presenting the data as means, we are ignoring student individual differences (Mael, 1998). For each item, the range was from 'strongly disagree' or '1' to 'strongly agree' or '5' for both males and females for all items. This suggests that in any large group of students, there is the potential for at least some students to have extremely positive or negative beliefs and feelings about mathematics. Another noteworthy point is that overall, the attitudes of students in the study were not as positive as we would have liked. Few students chose 'strongly agree' for the items 'I like maths' or 'Maths is fun'.

Table 4
Gender Differences Between School M and School F

Item	Means		*p*-value
1. I like maths	M = 3.292	F = 3.494	.005**
2. I feel anxious when working on maths	M = 2.389	F = 2.541	.059
3. Doing maths is one of my favourite activities	M = 2.295	F = 2.383	.268
4. I often find that the things we deal with in maths are really exciting	M = 2.577	F = 2.736	.035*
5. I don't enjoy maths	M = 2.614	F = 2.431	.031*
6. Maths is fun	M = 2.772	F = 2.953	.032*
7. Maths is very stressful for me	M = 2.690	F = 2.652	.640
8. When I'm doing maths I feel pretty happy	M = 2.628	F = 2.776	.039*

In Table 4, five out of the eight Likert-scale items have gender differences and in all 5 cases they favour the girls, who have more positive attitudes towards mathematics than the boys. This is a strong reversal to the results of Table 3. There were few gender differences across the whole sample (Table 2) because of the combination of the opposing results of Tables 3 and 4.

The remaining items in the written questionnaire were then analysed to determine if this pattern of gender differences held true for the rest of the questionnaire. In School MF, 18 of the 26 Likert-scale items were found to have significant differences ($p < .05$) between boys and girls and in every case the differences favoured the boys in terms of more positive attitudes towards mathematics. For the single-sex settings, School M and School F, nine of the 26 Likert-scale items were found to have significant differences between boys and girls, and in every case the differences favoured the girls in terms of more positive attitudes towards mathematics.

The sheer clarity of these results was striking and required more comparisons to be made to further establish these findings. When comparing the boys of the coeducational School MF to the boys in single-sex School M, only three statistically significant differences were found in the 26 items, with all three favouring the boys in the single-sex School M. Comparisons of the girls in School MF to the girls in School F predictably favoured School F by an overwhelming margin. Therefore it appears that in order of most positive attitudes to least positive attitudes, the order of cohorts is: single-sex girls, single-sex boys, coeducational boys, coeducational girls. It must be said that the two middle groups of boys are reasonably similar, and the main disparities lie between the first and second cohort, and the third and fourth.

At this stage, it would be disingenuous to attribute these striking gender differences solely to the single-sex or coeducational nature of the schools involved. No two schools are alike and there are doubtless many other factors that may contribute to these disparities. However, all three schools are in a similar metropolitan region, and in the National Assessment Program - Literacy and Numeracy (NAPLAN), which is the national testing scheme in Australia and occurs in the high school years of 7 and 9, numeracy scores were comparable as shown in Table 5 below. Scores have been given within a 10-point range to protect the identities of the schools involved.

Table 5
Numeracy Scores in NAPLAN Testing 2013

School	Year 7	Year 9
MF (coeducational)	570-580	630-640
M (single-sex boys)	620-630	690-700
F (single-sex girls)	620-630	670-680

The single-sex schools were very similar in NAPLAN scores, with the boys' school having a slight edge in performance in Year 9, even though the girls' school generally had more positive attitudes towards mathematics. The coeducational School MF's NAPLAN scores were somewhat lower than either of the single-sex schools, which raises a potential hypothesis for future studies: since these two higher-performing single-sex schools have fewer gender disparities in attitudes toward mathematics (and where disparities exist, they favour the girls), is the gender disparity in attitudes toward mathematics a particular issue for girls in lower-performing schools?

Given that the academic performance in NAPLAN does not strictly predict the findings on attitudes toward mathematics - if it did, one would expect the boys' school to be slightly ahead of the girls' school on attitudes, which was earlier seen not to be the case - it does appear that academic performance is an insufficient explanation in and of itself for the gender disparities. Therefore there is still some credence for the study's original hypothesis that the coeducational or single-sex nature of schooling has some effect on the gender differences in student attitudes towards mathematics.

One final avenue of investigation was to analyse the five perceived interest items where respondents were asked to rate the level of interest in mathematics of their female carer, male carer, teacher, friends, and classmates. If there were significant differences in these items across the schools that matched the pattern of gender disparities, then there is the possibility that it is these differences that could be responsible for the gender disparities, rather than the schooling system.

Table 5
Means of Perceived Interest Items Across Schools

Group	School MF Girls	School MF Boys	School M (Boys)	School F (Girls)
Female carer	2.350	2.114	2.399	2.631
Male carer	3.059	2.820	3.016	3.172
Teacher	3.637	3.730	3.853	3.875
Friends	1.787	2.037	1.863	1.924
Classmates	2.092	2.346	2.284	2.551

At first glance, the means listed in Table 5 have a better correlation with the attitudes displayed, as it follows the stated order of single-sex girls and single-sex boys followed by the coeducational groups. However, upon closer inspection, it is not immediately clear how the perceived interest items could have generated the gender disparity in the coeducational school, as the School MF girls' female and male carers are significantly more interested in mathematics than the School MF boys (p=.037 and p=.047 respectively), despite the more positive attitudes of the School MF boys towards mathematics than the School MF girls. Examination of the remaining three perceived interest items proves equally problematic;

the similarity of teacher interest in School M and School F despite the clear attitudinal differences between these schools discounts the teacher as a potential source of gender disparities, while the higher scores in the friends and classmates items of the School MF boys over the School M boys despite their attitudinal similarities (and slight favouring of School M) discounts these as potential causes.

Conclusions and Implications

In this study, attitudes towards mathematics were clearly divided into three distinct groups. The most positive group was the single-sex girls' school, followed by the single-sex boys' school and the coeducational school. The differences between each of these groups were statistically significant. When the coeducational school was split into two further divisions of girls and boys, it was found that the coeducational boys were similar to (albeit slightly more negative than) the single-sex boys, while the coeducational girls had significantly more negative attitudes than the coeducational boys. When the sample was taken as a whole, boys and girls had very similar attitudes towards mathematics. For the girls involved in this study, students in single-sex settings resulted in much more favourable attitudes towards mathematics than those in coeducational settings.

The potential exists for other factors to have caused these phenomena rather than the gender of the school setting. However, academic achievement in the form of NAPLAN scores, as well as the perceived interest of key people of influence (female and male carers, teachers, friends and classmates) could not accurately explain the gender disparities that were found. The correlations were not strong and suffered from some aberrant cases. Therefore, the potential for school setting to have affected the attitudes towards mathematics of boys and girls cannot be discounted.

Care must be taken when interpreting these results, as a study involving three schools is unsuitable for broad generalisations regarding single sex or coeducational settings. However, these results are in strong agreement with other international studies (Prendergast & O'Donoghue, 2014; Tambo et al., 2011) as well as related studies in Australia (Tully & Jacobs, 2010). The robust sample sizes within each school also lend validity to the findings, even if the number of schools involved was comparatively small.

The suggestion that gender differences in attitudes to mathematics may be more pronounced in coeducational schools than single-sex schools raises the larger issue of gender stereotyping and the possible impacts of school setting. It may be that in a coeducational school, students are more likely to conform to gender stereotypes, whereas in single-sex schools there is more freedom for students to not 'live up to' gendered expectations. This has implications for the way in which educators and other stakeholders might address problems associated with negative attitudes towards mathematics. For example, some coeducational schools have implemented single-sex classrooms for mathematics as a strategy to address boys' underachievement (Jackson, 2002) but it may be a useful strategy to address girls' negative attitudes to mathematics. Further research in this area could provide fruitful for a greater understanding of the challenges and possible solutions of gender differences in attitudes to mathematics.

References

Ai, X. (2002). Gender differences in growth in mathematics achievement: three-level longitudinal and multilevel analyses of individual, home, and school influences. *Mathematical Thinking and Learning, 4*(1), 1-22.

Ashcraft, M. H,. & Moore, A. M. (2009). Mathematics anxiety and the affective drop in performance. *Journal of Psychoeducational Assessment, 27*(3), 197-205.

Board of Studies Teaching and Educational Standards NSW (2015). *Student entries by sex - 2014 Higher School Certificate*. Retrieved 28 February 2015 from http://www.boardofstudies.nsw.edu.au/ebos/static/EN_SX_2014_12.html

Brown, M., Brown, P., & Bibby, T. (2008). “I would rather die”: reasons given by 16 year-olds for not continuing their study of mathematics. *Research in Mathematics Education*, *10*(1), 3-18.

Hannula, M. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics 49*, 25-46.

Jacobs, J. E., Davis-Kean, P. E., Bleeker, M. M., Eccles, J. S., & Malanchuk, O. (2005). “I can do it but I don’t want to.” The impact of parents, interests and activities on gender differences in math. In A. Gallagher & J. Kaufman (Eds.), *Gender difference in mathematics* (pp.246-263). New York: Cambridge University Press.

Jackson, C. (2002). Can single-sex classes in co-educational schools enhance the learning experiences of girls and/or boys? An exploration of pupils’ perceptions. *British Educational Research Journal*, *28*(1), 37-48.

Kyriacou, C., & Goulding, M. (2006). *A systematic review of strategies to raise pupils' motivational effort in Key Stage 4 mathematics.* London: EPPI Centre, Institute of Education.

Leder, G. C., & Forgasz, H. J. (2010). I liked it till Pythagoras: The public’s view of mathematics. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp.328-325). Fremantle, WA: MERGA

Lee, K. J., & Anderson, J. A. (2014). Who is really interested in mathematics? An investigation of lower secondary students' mathematical role models. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 397-404). Sydney: MERGA.

Leedy, M. G., LaLonde, D., & Runk, K. (2003). Gender equity in mathematics: Beliefs of students, parents and teachers. *School Science and Mathematics, 103*(6), 285-292.

Mael, F. A. (1998). Single-sex and co-educational schooling: Relationships to socioemotional and academic development. *Review of Educational Research*, *68*(2), 101-129.

Martin, A. J. (2003). Enhancing the educational outcomes of boys: Findings from the ACT investigation into boys’ education. *Youth Studies Australia*, *22*(4), 27-36.

Office of the Chief Scientist. (2012). *Mathematics, engineering and science in the national interest*. Canberra: Commonwealth of Australia.

Preckel, F., Goetz, T., Pekrun, R., & Kleine, M. (2008). Gender differences in gifted and average-ability students: Comparing girls’ and boys’ achievement, self-concept, interest, and motivation in mathematics. *Gifted Child Quarterly*, *52*(2), 146-159.

Prendergast, M. & O'Donoghue, J. (2014). Influence of gender, single-sex and co-educational schooling on students' enjoyment and achievement in mathematics. *International Journal of Mathematical Education in Science and Technology 45*(8), 1115-1130.

Stevens, T., & Olivárez, A., Jr. (2005). Development and evaluation of the Mathematics Interest Inventory. *Measurement & Evaluation in Counseling & Development*, *38*, 141–152.

Stevens, T., Wang, K. Olivarez Jr., A., & Hamman, D. (2007). Use of self-perspectives and their sources to predict the mathematics enrollment intentions of girls and boys. *Sex Roles*, *56*, 351-363.

Tambo, L. K., Munakandafa, W., Matswetu, V. S., & Munodawafa, V. (2011). Influence of type of school on self perception of mathematical ability and achievement among girls in secondary school in Harare. *Gender and Behaviour 9*(2), 3897-8915.

Thomson, S., Hillman, K., & Wernert, N. (2012). *Monitoring Australian Year 8 student achievement internationally: TIMSS 2011*. Camberwell, Victoria: Australian Council for Educational Research.

Tully, D., & Jacobs, B. (2010). Effects of single-gender mathematics classrooms on self-perception of mathematical ability and post-secondary engineering paths: An Australian case study. *European Journal of Engineering Education 35*(4), 455-467.

Watt, H. M. G. (2004). Development of adolescents’ self-perceptions, values and task perceptions according to gender and domain in 7th- through 11-th grade Australian students. *Child Development*, *75*, 1556-1574.

Watt, H. M. G. (2007). A trickle from the pipeline: Why girls under participate in maths. *Professional Educator, 6*(3), 36-41.

Developing a Theoretical Framework to Assess Taiwanese Primary Students' Geometric Argumentation

Tsu-Nan Lee
The University of Melbourne
<tsunanl@student.unimelb.edu.au>

Geometric competences of students have sparked great concern in Taiwan since the release of last TIMMS assessment. Geometric argumentation is viewed as to play an important role to enhance the competences of geometry and reasoning. This study adopts Toulmin's (2003) model to develop such indicators, including naming, supporting ideas, and transformation reasoning. It is expected that further research will provide empirical evidence in these indicators to apply to topics in mathematics other than geometry.

Introduction

Taiwan has participated in several international assessments since 2000 and although Taiwanese grade 4 students are ranked top 4 in the Trends in International Mathematics and Science Study (TIMMS) in comparison with 62 other countries, results reveal some weaknesses of students in particular areas of mathematics. More specifically, TIMMS results suggest that grade 4 students are weaker in geometry and reasoning when compared with their performances in other mathematical topics (Mullis, Martin, Foy, & Arora, 2012). These learning problems may be caused by the curriculum and teaching instructions. National Council of Teachers of Mathematics (2000) indicates that in many countries the curriculum in geometry overemphasises the naming shapes and ignores the relationships between geometric properties. This may result in Taiwanese students being successful when it comes to memorise each shape's name and its specific geometric property correctly, yet less able to use geometric properties to identify or categorise shapes. Most eastern Asian countries (Taiwan, Hong Kong, Japan, Korea, and China) are ranked in the top 10 in TIMMS, but these countries have each specific teaching instruction (Li & Shimizu, 2009). Taiwanese primary teachers prefer to use the direct instruction (Chiang & Stacey, 2013) instead of cooperative learning in teaching mathematics (Mullis et al., 2012). In mathematics classes, students are used to repeating several tests and are trained to solve various types of problems in order to get higher scores in Taiwan. To score higher points, Taiwanese teachers may focus on students' cognitive skills in problem solving and ignore how students think mathematically. Some Taiwanese educators have noticed the consequences after participating in these international assessments and have put effort into reforming the educational system in Taiwan (Yang & Lin, 2015).

Although the educational reform is a controversial issue in Taiwan, research shows that educators have tried to improve students' weaknesses in learning mathematics (Yang & Lin, 2015). Since 2000, Argumentation in Taiwan has been shown to have several functions that could lead students to improve their competences in problem solving and communication in mathematics (Horng, 2004). The underlying nature of these competences is reasoning and the competence of reasoning is an essential one to support students to learn mathematics (Horng, 2004). Moreover, Lin and Cheng (2003) also state that the core competence of geometry is argumentation and learning geometry is tightly linked to learning argumentation in Taiwan. Therefore, we assume that reinforcing the use of argumentation in mathematics classroom may play an important role for improving students' mathematical competences in terms of reasoning and geometry.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 365–372. Sunshine Coast: MERGA.

Argumentation is one useful tool to help students improve their competences of reasoning and geometry for class teachers. Students' oral or written discourse is able to reflect on what they think and how they solve problems. Vanderhye and Zmijewski Demers (2007) also claim that class teachers are able to utilise students' mathematical conversation to understand their thinking. Although there are some tools that are used to assess students' argumentation (Healy & Hoyles, 1999; Lin & Cheng, 2003), they do not emphasise students' cognitive abilities. In this study, we will develop a theoretical framework to assess students' geometric argumentation from the cognitive perspective. In the following sections, we will identify the definition of geometric argumentation and develop the theoretical framework with indicators in greater detail.

The Definition of Geometric Argumentation

Mathematical argumentation is related to mathematical concepts and reasoning abilities in students' discourse. Durand-Guerrier, Boero, Douek, Epp, and Tanguay (2012) define mathematical argumentation as either a written or oral discourse and the discourse links between premises and a conclusion through reasoning. The processes of reasoning combines mathematical rules with plausible statements and the plausible statements are formed by knowledge that is valid (Durand-Guerrier et al., 2012). Discourse is one kind of communication and students should discuss with each other. According to this definition, Krummheuer (2000, 2007) admits that argumentation is one type of social interaction and students' understanding is constructed through social interaction. Social interaction has been recognised as improving students' learning since students' ideas will be justified and clarified with their classmates (Wood, 1999). From this perspective, although Durand-Guerrier et al. (2012) regard argumentation as a process, we hold a different perspective. In the next section, we will discuss the differences with several reasons.

Students' oral or written discourses can take various types of formats and *proof* is one specific format which has to use deductive reasoning in argumentation (Aberdein, 2005; Durand-Guerrier et al., 2012). Although there are some similarities between argumentation and proof for mathematics educators (Durand-Guerrier et al., 2012), we have to point out that both argumentation and proof can take different *shapes* when considering primary students. We can distinguish between argumentation and proof from three perspectives: types of reasoning; pedagogical meaning; and formats. From the first perspective, while students are able to use inductive, deductive, and abductive reasoning in argumentation (Douek, 1999), students are only allowed to use deductive reasoning in proof (Ayalon & Even, 2008). From a pedagogical perspective, the purpose of argumentation is to cultivate students' thinking and engage students' understanding (National Council of Teachers of Mathematics, 2000; Wood, 1999) while proof aims at, amongst others, providing means for mathematicians to discuss the validity of mathematics results and communicate with each other (Department of Elementary Education, 2008). Finally, while students use their daily language to explain their thoughts in argumentation (Wood, 1999), they are expected to use formal mathematical language to reason and explain their ideas logically (Ayalon & Even, 2008). For these reasons, we conclude that argumentation is different from proof and both argumentation and proof play two different but important roles at the primary level.

In summary, geometric argumentation in this study refers to the activity that occurs when students (in our case mainly primary students) employ geometric concepts or properties to form plausible statements in order to link premises and a conclusion through their daily language. There is a theoretical framework with three indicators that are essential to analyse students' geometric argumentation: naming, supporting ideas, and transformation reasoning. We will explain the theoretical framework and three indicators in greater detail.

Constructing a Theoretical Framework for Assessing Students' Geometric Argumentation

The theoretical framework is originally from the definition of geometric argumentation and there are three indicators that are developed from Toulmin's (2003) model in this framework. Toulmin's model is a structure of argumentation (Aberdein, 2005) and identifies some specific elements in argumentation (Toulmin, 2003). In the following sections, we will introduce the relationships between the definition of geometric argumentation and Toulmin's model, and describe three indicators.

The Relationship between the Definition of Geometric Argumentation and Toulmin's Model to Develop the Theoretical Framework

As stated previously, geometric argumentation refers to an oral or written discourse to link between premises and a conclusion with some geometric properties reasons. The theoretical framework in this study refers to a conceptual structure and the structure means that each element in the structure has some relationships with other elements. These elements are developed by Toulmin's model and their meanings come from the definition of geometric argumentation in this study.

The structure of the theoretical framework is originally from Toulmin's model, but we modify and simplify some elements in mathematics education settings (Krummheuer, 1995). Toulmin's model is a structure of argumentation and is used to argue others' claims (Hitchcock & Verheij, 2006). Since Toulmin's model does not only focus on mathematical communication, the definitions of elements in Toulmin's model are general statements. In Toulmin's model, the meaning of *data* refers to facts and information that students know; *warrants* represent evidence that is used to support their conclusion; *qualifiers* have no clear definition, but the function of qualifiers is to justify whether evidence is correct or not; *backing* is defined as theories that are to challenge evidence which people give; *claims* refer to drawing a conclusion from data, and *rebuttals* are other claims to criticise the conclusion (Toulmin, 2003).

Although Toulmin's original model encompasses six elements; namely, data, claim, warrant, backing, qualifier, and rebuttal (Toulmin, 2003), Krummheuer (1995) states that researchers tend to ignore the elements of rebuttals and qualifiers when they analyse students' argumentation, which may come from the definition of argumentation itself. The function of rebuttals is to justify and clarify students' thinking. Krummheuer (1995, 2000, 2007) regards students' argumentation as a product and *rebuttals* play a role to help students produce an appropriate response. Inglis, Mejia-Ramos, and Simpson (2007) also claim that although *qualifiers* have no psychological value, but can help students think logically. Both rebuttals and qualifiers are to improve students' argumentation, and have pedagogical meanings. Thus, both rebuttals and qualifiers seem to have pedagogical value when it comes to improving students' argumentation and we argue that these two elements should also be considered when modelling argumentation from a mathematics education perspective. Adopting Toulmin's work, Aberdein (2005) defines these four terms clearly: *Data* refers to the information given in a mathematical problem such as mathematical concepts and geometric properties. Students use this information to reach a conclusion. *Claim* means that students use the above-mentioned information to reach a conclusion.

Backing and *warrant* have different meanings when adopted in mathematics education. The meaning of backing refers to a mathematical theory and warrants represent evidence (Aberdein, 2005). Both of them are often called *reasoning* and the process of reasoning itself combines both backing with warrants (Prusak, Hershkowitz, & Schwarz, 2012). Supported by the above-mentioned research, we hypothesise that using argumentation enhances students' geometric concepts and reasoning, but Toulmin's model is limited in that although it introduces a theoretical framework to describe the role of argumentation, it just illustrates a structure of argumentation. For this reason, this study uses the structure to develop some indicators with the definition of geometric argumentation to assess students' geometric argumentation.

We adopt Toulmin's model and identify elements in the model in the definition of mathematical argumentation in this study: premises refer to what Toulmin calls data, mathematical rules refer to backing, plausible statements can be viewed as warrants, and conclusions refer to claims in mathematics. Therefore, within the mathematics education setting, geometric argumentation could be viewed as students' use of geometric properties or geometric concepts to link the relationships between premises and a conclusion through their oral or written explanations. Figure 1 shows the relationships between the definition of geometric argumentation and Toulmin's model.

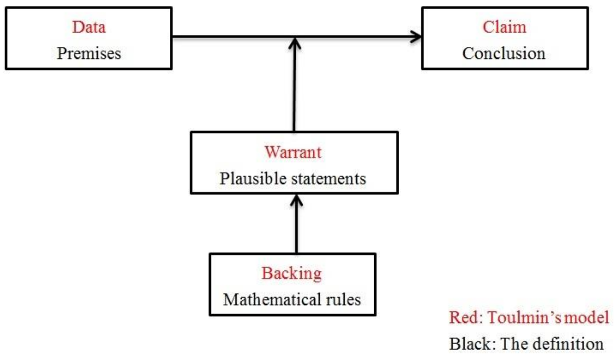


Figure 1. The relationships between the definition of geometric argumentation and Toulmin's (2003) model

Three Theoretical Indicators to Assess Students' Geometric Argumentation

As stated earlier, there are several existing frameworks designed to evaluate students' mathematical argumentation and each framework provides different information to mathematics educators. Healy and Hoyles (1999) investigated how secondary students construct a mathematical proof and analysed the proofs from two different perspectives: by analysing forms of arguments used and by attributing a score for correctness. There are four categories to distinguish students' proof which are the outcome of the two perspectives: "No basis for the construction of a correct proof, No deductions but relevant information presented, Partial proof, and Complete proof" (Healy & Hoyles, 1999, p.19). In a similar study, Lin and his colleagues adapted the categories from Healy and Hoyles to evaluate secondary students' arguments in Taiwan and developed four levels: "intuitive response, improper argument, incomplete argument and acceptable proof" (Lin & Cheng, 2003). In our view, these two frameworks do not seem suitable to be used as is, in a primary school setting. While both frameworks are useful to analyse students' argumentation, they may not reflect on students' learning problem in argumentation from the cognitive perspective, especially in the process of reasoning. Furthermore, both frameworks emphasise the concepts of *proof* for secondary students and may be not useful for primary students' *argumentation* (cf. our earlier discussion about the distinction between proof and argumentation). On another level, students' argumentation analysed by both frameworks is categorical data and categorical data is used to reflect on the types of students' argumentation. The aim of this framework is expected to understand students' geometric argumentation, especially in their cognitive competences, such as reasoning. For these reasons, we will develop three indicators to assess primary students' geometric argumentation in order to solve these problems in this study.

Reasoning is one essential component in argumentation (Mercier, 2011), and the indicators have to reflect students' competence in reasoning. There is one kind of writing style that is called *argumentative writing*, and a core competence of argumentative writing is reasoning (Reznitskaya, Kuo, Glina, & Anderson, 2009). Reasoning is the core competence in both geometric argumentation and argumentative writing. Argumentative writing has two scales to analyse students' works: the analytic and holistic scales (Reznitskaya et al., 2009). The former one lists several indicators and each indicator has different scales, and the later one is to rate students' writing holistically. Reznitskaya et al. (2009) develop five indicators, which are called the analytic scales, to evaluate students' argumentative writing, including *fluency, flexibility, alternative, focus, form*. Both fluency and flexibility are related to how students employ their ideas and alternative means whether students are able to give the opposite perspective to justify their ideas. Focus represents that students are able to utilise their ideas and form means that the structure in students' writing is complete. These five indicators are divided into two parts: content (fluency, flexibility, and alternative) and organisation (focus and form). However, mathematical argumentation is one kind of discourse and has no regular format. Thus, the dimension of organisation can be ignored.

On the other hand, the holistic scales have several points that combine with many criteria at one point. The scales are complex and students who get the same point may have different performances. The scales at each level are related to several factors, such as students' writing structure, supporting evidence, reasoning abilities, and giving opposite evidence. Each of these factors does not have the same criteria to assess. It may be difficult to reflect on the cognitive competences of geometric argumentation in this study. Hence, we adapt the analytic scales to develop three indicators: naming, supporting ideas, and transformation reasoning.

Naming. The indicator of naming relates to students' ability in identify the name of geometric shapes correctly. The indicator has two subscales: premises and conclusions. The subscale of premises means whether students are able to gather correct information from what they were taught and what a task is given. Premises have an important position in argumentation and they will influence other components of Toulmin's model. On the other hand, the subscale of conclusions means whether students are able to make a conclusion correctly. However, students have to do the first step correctly since the first step can be justified and needs to be correct in mathematical argumentation (Aberdein, 2005). For example, students have to choose correct shapes (premises) and name them correctly (conclusions) or they will do invalid reasoning in the sort task of geometric shapes.

Supporting ideas. The indicator of supporting ideas relates to students' ability of employing an appropriate geometric property in order to link premises and a conclusion. The scale in this indicator is affected by the indicator of naming. Even if students give complete and correct geometric properties, but the premises are incorrect, students cannot get any point in this indicator.

Transformation reasoning. As we previously discussed, reasoning plays a significant role in argumentation, thus the need to incorporate this indicator in the assessment of students' argumentation. The term of transformation reasoning has been developed by Simon (1996) as:

> The mental or physical enactment of an operation or set of operations on an object or set of objects that allows one to envision the transformations that these objects undergo and the set of results of these operations. Central to transformational reasoning is the ability to consider, not a static state, but a dynamic process by which a new state or a continuum of states are generated. (p. 201)

According to this definition, transformation reasoning is one kind of reasoning, which may be seen as in between inductive and deductive reasoning. Primary students learn geometric concepts through their visual cues and operations (Duval, 1998) and transformation reasoning is that students use visual cues and operations to reason. Based on this perspective, we define the indicator of transformation reasoning that students are able to provide what they operate, measure, and see and convert their actions into geometric concepts. Therefore, students have to provide their actions (evidence) and connect their actions to geometric concepts (theoretical reason). The indicators of transformation reasoning and supporting ideas influence each other: their actions determine how students employ what geometric property or using which geometric property decides how students confirm their ideas. Figure 2 shows the relationships among three indicators, the definition of geometric argumentation and Toulmin's model.

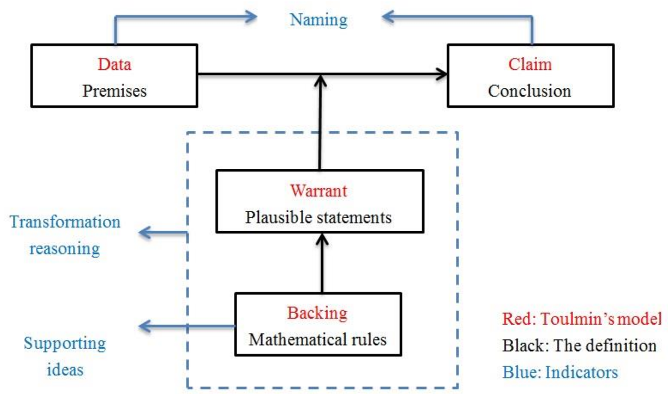


Figure 2 The relationships among the indicators, the definition of geometric argumentation and Toulmin's model

Discussion and Conclusion

Three indicators have been introduced in these sections, but how to use these indicators may be questioned by researchers. However, we do not develop the specific scales to assess students' geometric argumentation in each indicator and here are some reasons: first, there are several geometric activities that are related to geometric argumentation. Each activity has some pedagogical purposes and teachers or researchers should develop scales based on their purpose. Second, the scales are related to the curriculum design. The curriculum in different countries has different perspectives for educating students. While the curriculum in Eastern countries has a content orientation, Western countries adopt the creative approach. Therefore, the criteria of the scales should reflect their curriculum design. For both reasons, we encourage researchers to develop scales to reflect the students' learning problem in geometric argumentation.

The assessment tools have two points that should be of concern: one is that the tools are handy to understand students' cognitive competences in geometric argumentation for the practical purpose, and the other is that the tools have empirical evidence to support their use for the academic purpose. We cannot deny that there exist several scales, schemes, or rubrics to evaluate students' thinking and reasoning in argumentation. The theoretical framework in this study emphasises students' cognitive competences and points out three indicators to assess. Researchers are able to use this framework to understand students' learning problem and whether argumentation can improve students' weakness in mathematics in Taiwan. On the other hand, we still have to be concerned about the academic purpose. Although the indicators may be appropriate to assess students' cognitive competences in geometric argumentation, these indicators lack empirical evidence such as validity and reliability. Both validity and reliability support researchers and class teachers to use assessment tools confidentially. Hence, researchers should put effort into constructing validity and reliability in these indicators and these three indicators have theoretical evidence to put them into practice.

In summary, this study adapts different theoretical perspectives and these perspectives converge into one framework. It may be easy to use these three indicators for researchers, yet they still have empirical data to support. In the future, it still has a long way to go to construct the theoretical framework in geometric argumentation and we expect that the framework can be applied into other mathematical argumentation.

Reference

Aberdein, A. (2005). The uses of argument in mathematics. *Argumentation, 19*(3), 287-301. doi: 10.1007/s10503-005-4417-8

Ayalon, M., & Even, R. (2008). Deductive reasoning: in the eye of the beholder. *Educational Studies in Mathematics, 69*(3), 235-247. doi: 10.1007/s10649-008-9136-2

Chiang, P.-C., & Stacey, K. (2013). Materials use for teaching geometry in Taiwan. In A. M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 161-168). Kiel, Germany: PME.

Department of Elementary Education. (2008). *General guidelines of grades 1-9 curriculum for elementary and junior high school education*. Retrieved from http://140.111.34.54/EJE/content.aspx?site_content_sn=15326.

Douek, N. (1999). Argumentation and conceptualization in context: a case study on sunshadows in primary school [Special issue]. *Educational Studies in Mathematics, 39*(1/3), 89-110. doi: 10.1023/A:1003800814251

Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. S., & Tanguay, D. (2012). Argumentation and proof in the mathematics classroom. In G. Hanna & M. d. Villiers (Eds.), *Proof and proving in mathematics education: the 19th ICMI study* (Vol. 15, pp. 349-368): Dordrecht, Springer.

Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Villani (Eds.), *Perspectives on the teaching of geometry for the 21st century: an ICMI study* (Vol. 5, pp. 37-51). Dordrecht, Boston : Kluwer Academic Publishers.

Healy, L., & Hoyles, C. (1999). *Justifying and proving in school Mathematics: Student Conceptions and School Data*. Colchester, Essex: UK Data Archive Retrieved from http://dx.doi.org/10.5255/UKDA-SN-4004-1.

Hitchcock, D., & Verheij, B. (2006). Introduction. In D. Hitchcock & B. Verheij (Eds.), *Arguing on the Toulmin model. [electronic resource] : new essays in argument analysis and evaluation* (pp. 1-23). Dordrecht: Springer.

Horng, W.-S. (2004). What for proof vis-a-vis education reform issues? *The Journal of National Taiwan Normal University: Science Education, 49*(1), 1-14. doi: 10.6300/JNTNU.2004.49(1).01

Inglis, M., Mejia-Ramos, J. P., & Simpson, A. (2007). Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics, 66*(1), 3-21. doi: 10.1007/s10649-006-9059-8

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: interaction in classroom cultures* (pp. 229-270). Hillsdale, N.J.: L. Erlbaum Associates.

Krummheuer, G. (2000). Mathematics learning in narrative classroom cultures: studies of argumentation in primary mathematics education. *For the Learning of Mathematics, 20*(1), 22-32. doi: 10.2307/40248315

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom. two episodes and related theoretical abductions. *Journal of Mathematical Behavior, 26*(1), 60-82. doi: 10.1016/j.jmathb.2007.02.001

Li, Y., & Shimizu, Y. (2009). Exemplary mathematics instruction and its development in selected education systems in East Asia. *ZDM – The International Journal on Mathematics Education, 41*(3), 257-262. doi: 10.1007/s11858-009-0179-3

Lin, F.-L., & Cheng, Y.-H. (2003, December). *The competence of geometric argument in Taiwan adolescents*. Paper presented at the International Conference on Science & Mathematics Learning, National Taiwan Normal University, Taipei, Taiwan.

Mercier, H. (2011). Reasoning serves argumentation in children. *Cognitive Development, 26*(3), 177-191. doi: 10.1061/j.cogdev.2010.12.001

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 International Results in Mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Prusak, N., Hershkowitz, R., & Schwarz, B. B. (2012). From visual reasoning to logical necessity through argumentative design. *Educational Studies in Mathematics, 79*(1), 19-40. doi: 10.1007/s10649-011-9335-0

Reznitskaya, A., Kuo, L.-J., Glina, M., & Anderson, R. C. (2009). Measuring argumentative reasoning: what's behind the numbers. *Learning and Individual Differences, 19*(2), 219-224. doi: 10.1016/j.lindif.2008.11.001

Simon, M. A. (1996). Beyond inductive and deductive reasoning: the search for a sense of knowing. *Educational Studies in Mathematics, 30*(2), 197-210. doi: 10.1007/BF00302630

Toulmin, S. E. (2003). *The uses of argument* (Updated ed.). Cambridge, U.K ; New York: Cambridge University Press.

Vanderhye, C. M., & Zmijewski Demers, C. M. (December 2007/January 2008). Assessing students' understanding through conversations. *Teaching Children Mathematics, 14*(5), 260-264.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30*(2), 171-191. doi: 10.2307/749609

Yang, K.-L., & Lin, F.-L. (2015). The Effects of PISA in Taiwan: Contemporary Assessment Reform. In K. Stacey & R. Turner (Eds.), *Assessing Mathematical Literacy* (pp. 261-273): Springer International Publishing.

Starting a Conversation about Open Data in Mathematics Education Research

Tracy Logan
University of Canberra
<Tracy.Logan@canberra.edu.au>

This position paper discusses the role of open access research data within mathematics education, a relatively new initiative across the wider research community. International and national policy documents are explored and examples from both the scientific and social science paradigms of mathematical sciences and mathematics education respectively are provided. Within these examples, some of the more well-known concerns associated with making data open and accessible are acknowledged and debated.

This paper is to provide insights into a research mandate that will become increasingly relevant to mathematics education researchers; namely, the obligation to ensure research data and findings are made public. The paper describes the international context, from both policy and practice perspectives, drawing on specific examples from mathematical sciences and mathematics education within Australia and beyond. The intent of the paper is to establish a critical analysis of current practices.

Within Australia, government funding for research is at a crossroads. There is a growing concern that severe cut backs will eventuate over the next few years. For the top scientists and academics this will be problematic as scarce funds will be even harder to secure. For other researchers, it could spell the end of their research programs. Within these politically uncertain times, simmering under the surface is the question, what will research look like in the future? Who and how will research be funded? In conjunction, there is the an increased awareness that more and more research data are being collected and stored, more often than not in digital forms. Universities around Australia (and indeed the world) are increasingly dealing with a data deluge (Borgman, 2012), with the storage, curation and cost issues associated with large data repositories yet to be fully realised. The philosophies behind such repositories are that data are manageable, connected, accessible, and discoverable. In effect, making the data as open as possible for re-use and re-analysis. The paper provides an overview of open research data both internationally and nationally and describes examples from both the scientific paradigm—mathematical sciences; and the social science paradigm—mathematics education. The distinctions are made to highlight the differences between the two paradigms in the advancement of open research data. Some of the concerns regarding social science data being made available via open access are considered.

International and National Research Policy Perspectives

The capacity to retrieve and share research data is not a new phenomenon. In the years 1996-1998, key stakeholders working on the Human Genome Project (HGP) developed the Bermuda Principles. This was a set of principles that stated the sharing of DNA sequencing information developed from the project should be publicly and freely available within 24 hours of being collected. The release of data pre-publication was ground-breaking across most research fields (Contreras, 2011). Indeed, the Bermuda Principles set the scene for other fields of research to consider benefits of releasing data sets, not necessarily pre-publication of results, but certainly in conjunction with publication (see for example the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 373–380. Sunshine Coast: MERGA.

2003 Report *Sharing Data from Large-scale Biological Research Projects: A System of Tripartite Responsibility*, commonly known as the Fort Lauderdale agreement). In 2007, the Organisation for Economic Co-operation and Development (OECD) (2007) developed a report outlining guidelines and principals for accessing and sharing data produced by government-funded research. They argued that:

> access to research data increases the returns from public investment in this area; reinforces open scientific inquiry; encourages diversity of studies and opinion; promotes new areas of work and enables the exploration of topics not envisioned by the initial investigators (p. 3).

It was from this point on that the international research community's awareness was heightened. Within the United Kingdom and United States, research funding bodies such as the Economic and Social Research Council (ESRC, 2010), the Wellcome Trust (2010) (UK) and the National Science Foundation (NSF, 2010) (USA) have documented policies stating data management plans and provisions for the sharing of data must be submitted with grant applications, that these sections are subject to review and will be influential in the decision to award the funding. The European Union (European Commission, 2013) also identified the need for policies on open access data within its major research and innovation program called Horizon 2020. All publications and data generated through this funding must comply with their guidelines for open access.

From the Australian perspective, the Australian Code for the Responsible Conduct of Research (Australian Government, 2007) was published outlining the principles and practices of researchers and institutions when conducting research. Section 2 in this document outlined management of data and primary materials. In summary, it highlighted the need to retain data for verification purposes and appropriate access for the wider research community. Around the same time, changes started appearing in the Australian Research Council's (ARC) Discovery Project funding rules for 2008 (Australian Government, 2006) where a section was added (1.4.5. Dissemination of research outputs, p. 13) regarding the dissemination of data and outputs:

> The ARC therefore encourages researchers to consider the benefits of depositing their data and any publications arising from a research project in an appropriate subject and/or institutional repository wherever such a repository is available to the researcher(s). If a researcher is not intending to deposit the data from a project in a repository within a six-month period, he/she should include the reasons in the project's Final Report.

This general statement has remained relatively consistent throughout the Discovery Project funding rules since 2008 and presently, for the funding rules for 2016 Discovery Projects, the statements read:

> A11.5.1 All ARC-funded research projects must comply with the ARC Open Access Policy on the dissemination of research findings, which is available at www.arc.gov.au. In accordance with this policy, any publications arising from a Project must be deposited into an open access institutional repository within a twelve month period from the date of publication.
>
> A11.5.2 Researchers and institutions have an obligation to care for and maintain research data in accordance with the Australian Code for the Responsible Conduct of Research (2007). The ARC considers data management planning an important part of the responsible conduct of research and strongly encourages the depositing of data arising from a Project in an appropriate publically accessible subject and/or institutional repository. (Australian Government, 2014, p. 19)

The ARC Open Access Policy (Australian Government, 2013a) specifically relates to publications being placed in open access repositories. This is mandatory. However, the interesting change is the separation of publications and data, with researchers being strongly encouraged to deposit data into repositories. This highlights the increased importance placed on the accessibility of research data to the wider community.

In late 2013 (Australian Government, 2013b), the ARC released the Discovery Projects—Instructions to applicants for funding commencing in 2015. This document generally provides advice to applicants on dealing with the relevant systems and explaining what each section of the proposal should contain. For the first time, that document identified that the project description (Part C) is required to have a heading titled *Management of Data*. This stated that all proposals must "outline plans for the management of data produced as a result of the proposed research, including but not limited to storage, access and re-use arrangements" (Australian Government, 2013, p. 15). Through this inclusion, the ARC is effectively making data management and data re-use an assessable component of the proposal, in a similar vein to the UK and USA systems. As Borgman (2012) commented in relation to the NSF policy on data management, " the NSF has accelerated the conversation about data sharing among stakeholders in publicly funded research" (p. 1061). The separation of publications and data in the ARC funding rules and the inclusion of an assessable component related specifically to data management in the proposal emphases the growing awareness from a political perspective that the data generated by public funding is becoming increasingly valuable and needs to be made accessible.

Data Repositories

There are a myriad of data repositories situated globally, with almost every university having some form of searchable digital repository. This does not take into account government funded resources or independent enterprises. Hence, the main priority over the past few years has been the consolidation of, and access, to all the various data repositories. The UK Data Archive (http://www.data-archive.ac.uk/) provides access to social science and humanities data repositories and across Europe and the USA, re3data.org is a registry of data repositories. These registries provide access to a wide variety of data repositories internationally.

Within Australia, since 2004 previous and current federal governments have invested approximately $2.5 billion through the National Collaborative Research Infrastructure Strategy (NCRIS) funding scheme to support the infrastructure required to consolidate and coordinate research across Australia (Lowe, 2015). This has included various aspects of big data collections. Table 1 outlines some of the projects undertaken in relation to the consolidation of data.

This paper will focus on the Australian National Data Service (ANDS) and Research Data Australia as the national registry of research data within Australia.

The main aim of ANDS is to create:

> a cohesive national collection of research resources and a richer data environment that will:
>
> - Make better use of Australia's research outputs
> - Enable Australian researchers to easily publish, discover, access and use data
> - Enable new and more efficient research (ANDS, n.d.).

Among other responsibilities, ANDS developed and currently manages Research Data Australia, a searchable registry of data. This registry provides access to a large number of research data, projects, documents, people, institutions and groups. It has been designed utilising the following categories: Collections; Parties; Activities; and Services. Collections are research datasets or collections of research materials. Parties are researchers or research organisations that create or maintain research data sets or collections. Activities are projects or programs that create research data sets and collections. Services are the services that support the creation and use of research data sets and collections. Entries are categorised accordingly and there are linking nodes among these categories. With regard to access, there are three levels of access identified within Research Data Australia: Open; Conditional; and Restricted. Open access is defined as online data that can be electronically accessed free of charge with no conditions imposed on the user. Conditional access is seen as online or offline data that can be accessed free of charge, providing certain conditions are met (e.g., registration is required to access data online). Restricted access is online or offline data where access to the data is heavily restricted.

Table 1.
A Sample of Projects Undertaken Through NCRIS Funding to Support Data Consolidation

Projects	Description
National Computing Infrastructure and Supercomputing Centre	High-end supercomputing services to researchers.
Research Data Storage Initiative	Supporting national data storage
National eResearch Collaboration Tools and Resources	Desktop-based data analysis and modelling tools for researchers
Australian National Data Service (including Research Data Australia), National Research Network and Australian Access Federation	Building better electronic communication, connectivity and collaboration networks between national and international research institutions
Australian Data Archive and Australian Data Archive Social Science	Collection and preservation of digital research data

Note: Adapted from Lowe (2015).

The information within Research Data Australia is supposed to represent all fields of research within Australia, so in order to understand how mathematics education is situated, a comparison between a scientific paradigm, mathematical sciences and a social science paradigm, mathematics education is presented.

Open Research Data in Two Paradigms

Within mathematics education, and education more generally, there is an increasing awareness of data storage and re-use. However, compared to the mathematical sciences, education appears to be well behind in their understanding of, and participation in, making research data more open. To demonstrate this, a brief comparison is presented between the scientific paradigm and the social science paradigm. A search was conducted of Research Data Australia to determine the number of entries under mathematical sciences and Education. As described above, entries are represented by collections, parties, activities, or services. The entries are also collated under subjects according to the ANZSRC Field of Research (FoR) classification. It was through these subject classifications that the search was initially conducted. It should be noted that if the entry was not attached to a specific FoR, it does not show up in these classifications, but may be identifiable through other keywords searches. As such, subsequent keyword searches were conducted to identify the number of collections, parties, activities, and services related to the keywords. These keyword searches also enabled filtering to identify those entries with open data access.

Scientific Paradigm: Mathematical Sciences

The Mathematical Sciences is the *01* classification under the ANZSRC FoR. It includes research areas such as Applied Mathematics, Statistics, and Pure Mathematics. A search at the two-digit level revealed 12,435 entries linked to this FoR. A keyword search of *mathematical sciences* revealed more than 85,000 entries, as categorised in Table 2.

Table 2.
Number of Entries Identified by Keyword Search of "Mathematical Sciences" and Open Data Licence in Research Data Australia by Category

Category	Mathematical Sciences	Open Data Licence
Collections	57,273	19,999
Parties	2,129	—
Activities	25,495	160
Services	120	—

That is a large number of open data licences, so what does that data actually look like. The data in these fields of research are more often than not quantitative and may contain complex systems of numbers and text and spatial information. Generally, this data relates to environmental, biological, or other physical phenomena as opposed to human subjects. It could be argued that much of this type of data is objective and factually based.

Many areas in these sciences have established data archiving and sharing practices, with some academic journals even making it a condition of publication that data be deposited into a publicly accessible database or provided as appendices for others to access (Borgman, 2012). However, this is not the case for the social sciences.

Social Science Paradigm: Mathematics Education

Education is the *13* classification under the ANZSRC FoR and includes Education Systems, Curriculum and Pedagogy, and Specialist Studies in Education. Under the two-digit code, 280 entries are identified. This is an underwhelming amount and there is a large difference in the number of entries between the two subject codes at this level. A keyword search for *mathematics education* revealed 73 entries as categorised in Table 3. None of the entries provided open data licences; however, almost all of the collections indicated an available data set. It is acknowledged that mathematics education is a much more specialised field compared to the general classification of mathematical sciences; however, even at the two-digit level, the differences are stark.

The data sets linked to those collections were classified as conditional or restricted access, which required contacting the chief investigator or the research group/institution to negotiate terms and conditions of use. For example, the research team at the International Centre for Classroom Research at the University of Melbourne have listed all their data sets from the International Learner Perspective Study. However, access must be negotiated with the Centre.

Without an openly available data set to compare with the mathematical sciences, the following section draws on the literature to better understand what mathematics education data might look like and highlights some of the common issues associated with openly sharing this type of data.

Table 3.
Number of Entries Identified by Keyword Search of "Mathematics Education" and Data Sets in Research Data Australia by Category

Category	Mathematics Education	Data Sets
Collections	45	44
Parties	18	—
Activities	10	—
Services	0	—

Understanding Mathematics Education Data

Mathematics education research data comes in varied forms. Similar to other social science research and depending upon methodology, it can include surveys, interviews, focus groups, tests, classroom observations, policies and other documentation, and various types of digital media such as audio and video recordings. Much of the data collected within mathematics education is rich qualitative data; however, quantitative data is also widely collected. It could be argued that this type of data is subjective insomuch as it specifically relates to human endeavour and behaviour.

There has been much research attention afforded to the storage, archiving and re-use of qualitative data (Bishop, 2012; Cheshire, 2009; Cheshire, Broom, & Emmison, 2009; Corti, 2012; Fielding, 2004; Hammersley, 1997; Mauther & Parry, 2009). Overwhelmingly, the debate revolves around four main areas as identified by Cheshire (2009):

> Broadly, these concerns revolve around issues of research ethics, specifically informed consent and participant confidentiality; data security and access; intellectual property; and the enhanced insight into meaning that is gained from being involved in the data collection enterprise and which is subsequently lost in any secondary analysis. (p. 27)

These four issues will be discussed briefly to highlight the nature of the debate and identify any steps that have been taken to alleviate some of these issues.

Ethics, Security, and Access

The ethical issues with storing and re-using data from human participants tend to focus on the type of informed consent provided at the beginning of data collection and the need to maintain confidentially. Previously, participants were told that after a certain period of time their data would be destroyed and that only members of the research team would have access to it. Hence, the majority of research conducted under those ethics will never be able to be re-used outside of the research team. Those terms have changed and now participants need to be informed about how their data will be kept and that other researchers may have access to the de-identified data. There are real possibilities that participation in research from the Education sector may decline because of these requirements. Certainly when researching sensitive areas, such as different cultures, often the participants only consent because their words, information or data will only be heard or seen by the research team, and often it has taken years of developing trust to get to even that point (Cheshire, 2009). Coinciding with this is the levels of security and access that others have to the data sets. Much of this can be decided upon by the researcher. As was demonstrated in the example above, many of the mathematics education data sets in Research Data Australian are restricted access, meaning that any form of re-use is negotiated with the owner of the data. ANDS recently published a guide to publishing sensitive data (Olesen, 2014). This outlines some of the steps that can be taken to make sensitive data more open and accessible through data repositories.

Intellectual Property

The majority of research projects that actually get funded are a result of the reputation and knowledge and skills of the chief investigator and the research team. Not only does the idea have to be good and the methodology sound, the researchers must be deemed fit to carry out the project. In some circumstances, the collection of the data comes at a personal cost also. Hence, it is little wonder that many researchers covet their data. However, the data itself actually belong to the researcher's institution and upon retirement or leaving, that data remains the property of that institution.

Context

Research conducted with human participants and about the characteristics of those participants is contextually based. Without context, much of the data is sometimes rendered meaningless and often very hard to interpret. Bishop (2012) identified that "for qualitative methodology, a key issue is context, as data are deemed inseparable from the context in which they are generated" (p. 345). In order to store data and make it appropriate for re-use, often very detailed descriptions of the context of data collection will be required along with data collection instruments and the data itself.

Implications Moving Forward

Given the current political climate and the requirement for ARC funded projects to have their data deposited into a repository, conversations need to begin within the mathematics education community about data storage and open data access. The relatively low number of mathematics education entries into Research Data Australia may be indicative of the culture of our research environment, but it may also highlight the difficulty of having a data set that can be easily stored and made accessible. Despite the advances in technology that have allowed such data repositories to exist and function, it could be the case that much of the data collected in mathematics education is done so in non-digital form and hence time, money and equipment are needed to make it repository ready. Alternatively, it could be the case that consent for such storage and access was not sought or not granted by the participants. Regardless of the reasons, research funding is limited and looking into the future, data repositories may be the only viable source of data available to conduct research.

References

Australian Government. (2006). *Australian Research Council: Discovery projects funding rules for funding commencing in 2008*. Retrieved 18 February 2015 from: http://www.arc.gov.au/ncgp/dp/dp_fundingrules.htm

Australian Government. (2007). *Australian Code for the Responsible Conduct of Research*. Retrieved 18 February 2015 from https://www.nhmrc.gov.au/guidelines-publications/r39

Australian Government. (2013a). *ARC open access policy*. Retrieved 18 February 2015 from http://www.arc.gov.au/applicants/open_access.htm

Australian Government. (2013b). *ARC Discovery Projects—Instructions to applicants for funding commencing in 2015*. Retrieved 17 March 2015 from http://www.arc.gov.au/pdf/DP15/DP15_ITA.pdf

Australian Government. (2014). *Australian Research Council: Funding Rules for schemes under the Discovery Program for the years 2015 and 2016—Australian Laureate Fellowships, Discovery Projects, Discovery Early Career Researcher Award and Discovery Indigenous*. Retrieved 18 February 2015 from: http://www.arc.gov.au/ncgp/dp/dp_fundingrules.htm

Australian National Data Service [ANDS] (n.d.). About ANDS. Retrieved 17 March 2015 from http://ands.org.au/about-ands.html

Bishop, L. (2012). Using archived qualitative data for teaching: Practical and ethical considerations, *International Journal of Social Research Methodology*, *15*(4), 341-350. http://dx.doi.org/10.1080/13645579.2012.688335

Borgman, C.L. (2012). The conundrum of sharing research data. *Journal of the American Society for Information Science and Technology*, *63*(6), 1059–1078.

Cheshire, L. (2009). *Archiving qualitative data: Prospects and challenges of data preservation and sharing among Australian qualitative researchers*, Discussion paper of The Australian Qualitative Archive, University of Queensland, Brisbane, Australia. Retrieved 17 March 2015 from http://www.assda.edu.au/forms/AQuAQualitativeArchiving_DiscussionPaper_FinalNov09.pdf

Cheshire, L., Broom, A., & Emmison, M. (2009). Archiving qualitative data in Australia – an introduction. *Australian Journal of Social Issues*, *44*(3), 239-254.

Contreras, J.L. (August, 2010). Bermuda's legacy: Policy, patents and the design of the genome commons. Washington University in St. Louis Legal Studies Research Paper No. 10-09-02. Retrieved 17 March 2015 from: http://ssrn.com/abstract=1667659

Corti, L. (2012). Recent developments in archiving social research. *International Journal of Social Research Methodology*, *15*(4), 281-290. http://dx.doi.org/10.1080/13645579.2012.688310

Economic and Social Research Council. (2010). ESRC Research Data Policy (updated 2013). Retrieved 17 March 2015 from http://www.esrc.ac.uk/_images/Research_Data_Policy_2010_tcm8-4595.pdf

European Commission. (2013). *The EU framework programme for research and innovation: Horizon 2020. Guidelines on open access to scientific publications and research data in Horizon 2020*. Retrieved 17 March 2015 from http://ec.europa.eu/research/swafs/index.cfm?pg=policy&lib=science

Fielding, N. (2004). Getting the most from archived qualitative data: Epistemological, practical and professional obstacles. *International Journal of Social Research Methodology*, *7*(1), 97-104. http://dx.doi.org/10.1080/13645570310001640699

Hammersley, M. (1997). Qualitative data archiving: Some reflections on its prospects and problems. *Sociology*, *31*, 131-142.

Knight, S., Nguyen, H.X., Falkner, N., Bowden, R. & Roughan, M. (2011). The Internet topology zoo. *IEEE Journal on Selected Areas in Communications*, *29*(9), 1765-1775. Retrieved 18 March 2015 from http://www.topology-zoo.org/gallery.html

Lowe, A. (2015). Explainer: the National Collaborative Research Infrastructure Strategy (NCRIS). The Conversation Australia. Retrieved 18 March 2015 from http://theconversation.com/explainer-the-national-collaborative-research-infrastructure-strategy-ncris-38837

Mauther, N.S., & Parry, O. (2009). Qualitative data preservation and sharing in the social sciences: On whose philosophical terms? *Australian Journal of Social Issues*, *44*(3), 291-307.

Olesen, S. (2014). *ANDS guide: Publishing and sharing sensitive data*. Canberra: Australian National Data Service.

Organisation for Economic Co-operation and Development (OECD). (2007). *OECD principles and guidelines for access to research data from public funding*. Paris, OECD. Retrieved 18 February 2015 from http://www.oecd.org/science/scitech/oecdprinciplesandguidelinesforaccesstoresearchdatafrompublicfunding.htm

National Science Foundation. (2010). NSF Data Sharing Policy. Retrieved 17 March 2015 from http://www.nsf.gov/bfa/dias/policy/dmp.jsp

Wellcome Trust. (2010). Policy on data management and sharing. Retrieved 17 March 2015 from http://www.wellcome.ac.uk/About-us/Policy/Policy-and-position-statements/WTX035043.htm

A Snapshot of Young Children's Mathematical Competencies: Results from the Longitudinal Study of Australian Children

Amy MacDonald
Research Institute for Professional Practice, Learning and Education, Charles Sturt University
<amacdonald@csu.edu.au>

Colin Carmichael
Research Institute for Professional Practice, Learning and Education, Charles Sturt University
<ccarmichael@csu.edu.au>

This article presents a snapshot of the mathematical competencies of children aged four to five years in Australian early childhood education settings, as perceived by their educators. Data are presented from a nationally-representative sample of 6511 children participating in the Longitudinal Study of Australian Children (LSAC). The results reveal that children are seen to possess a number of mathematical competencies at 4-5 years, with the majority of children displaying interest in mathematics. Moreover, differences were noted with respect to the different program types in which the children participated. These results are discussed in relation to previous research, and implications for future research, policy and practice are presented.

Children who enter primary school with high levels of mathematical knowledge maintain these high levels of mathematical skill throughout, at least, their primary school education (Baroody, 2000; Klibanoff, 2006). Despite this, early childhood mathematics education remains a developing area of research with work yet to be done in terms of identifying young children's mathematical competencies (Peter-Koop & Scherer, 2012). Doig, McCrae and Rowe (2003) have suggested several reasons for the importance of understanding children's mathematical development in the years prior to school, including the increasing number of children participating in early childhood programs and growing recognition of the importance of mathematics in general. Furthermore, De Lange (2008) has suggested that in the years prior to commencing formal education, young children have a curiosity about scientific phenomena—including mathematics—that, for many, seems to dissipate as they enter and continue formal education.

An opportunity to explore young children's mathematical competencies has been afforded through the Longitudinal Study of Australian Children (LSAC) (Sanson, Nicholson, Ungerer, Zubrick, Wilson et al., 2002). LSAC utilises a cross-sequential design to follow two cohorts of children: a Birth cohort of approximately 5000 children aged between 6 and 12 months; and a Kindergarten cohort of approximately 5000 children aged between 4 years 6 months and 5 years. This study focuses on children from the combined Birth and Kindergarten cohorts of LSAC when they were aged four to five years and in particular the mathematical competencies of those attending a formal early childhood education program. The overarching research question guiding this study is: What are the mathematical competencies of 4-5 year old Australian children who attend formal early childhood education programs? Consideration is also given to the related question: Are there differences in mathematical competencies across prior-to-school and school sectors; and if so, what are they?

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 381–388. Sunshine Coast: MERGA.

Background

In this section we provide a brief review of extant research pertaining to the mathematical skills possessed by young children, and the impact of different early childhood program types on the development of children's mathematical skills.

Young Children's Mathematical Skills

A number of studies have demonstrated that children begin developing mathematical skills from a very young age. In a study of 1003 Norwegian children aged between 30 and 33 months, Reikerås, Løge, and Knivsberg (2012) found that the toddlers showed mathematical competencies in all areas observed (encompassing number and counting, geometry and problem solving). Similarly, Björklund's (2008) study of children aged between 13 and 45 months demonstrated that toddlers interact with concepts of dimensions or proportions, location, extent, succession and numerosity, and use a range of strategies to express their understanding. The seminal Australian study, the *Early Numeracy Research Project* (see for example, Clarke, Clarke, & Cheeseman, 2006) investigated the mathematical knowledge of over 1400 children in their first year of primary school. An important finding from the study was that much of the content which formed the mathematics curriculum for the first year of school was already understood clearly by many children on arrival at primary school (Clarke, Clarke, & Cheeseman, 2006), a finding echoed in several other studies, both in Australia (e.g. Gervasoni & Perry, 2013; MacDonald, 2010) and internationally (e.g. Aubrey, 1993; Wright, 1994).

Of course, there will be substantial variance in the mathematical competencies children develop prior to school (Peter-Koop & Kollhoff, 2015), and both standardised tests and experimental tasks reveal marked individual differences in children's mathematical knowledge by the time children enter preschool (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Given the compelling research pertaining to the relationship between mathematics at the time of school entry and later school achievement (Levine et al., 2010), it is important to ascertain the mathematical competencies of children in the early years in order to understand the foundation on which subsequent mathematics education should build.

Impact of Program Type on Mathematical Opportunities and Skills

In Australia, children aged 4-5 years will typically participate in either *prior-to-school programs* or *school programs*. The prior-to-school programs on offer are many and varied, and differ in the different states and territories. However, the program types can be generalised as including centre-based care (long day care or occasional care), stand-alone preschools, supported play groups, family day care, and early intervention services. School-based programs are similarly complex and diverse. In all states and territories, however, children commence school with a pre-Year 1 program, though it is termed "Kindergarten" in some jurisdictions (e.g. NSW) and a "Preparatory" year in others (e.g. Victoria).

At the time the data in this study were collected (2004-2008), each state and territory was responsible for providing curricula and policy documents for use in the various education sectors. The prior-to-school sector was the least regulated in terms of curricula frameworks. However, a common feature was a lack of explicit focus on the teaching of mathematics in the early childhood sector. On the other hand, mathematics has been a part of the formal primary school curricula from the first year of school, with each state and territory guided by its own mathematics syllabus.

Clearly, the curricula and policy frameworks utilised in the different settings will have some impact upon the extent to which mathematics is an explicit focus of the educational program on offer, and it can reasonably be assumed that explicit teaching of mathematics is likely to occur more frequently in school settings. However, there are other factors beside curricula which will influence children's opportunities to explore mathematics in early years education settings. A study of mathematics in the childcare context by Graham, Nash, and Paul (1997) has shown that children's experiences in childcare vary greatly, with differences in the physical set-up, schedule, age grouping, teacher-student ratio, teaching styles, and beliefs about child development. However, a common feature of the childcare settings investigated was the minimal amount of mathematics instruction in these settings.

Method

Sample

The sample utilized both cohorts (Kindergarten and Birth) of LSAC. Collectively this consisted of 9369 children aged from 4.2 to 5.7 years (M = 4.8 years, SD = 0.2) of whom 51.1% were male. A substantial number of the full sample (n = 2716), however, did not attend a formal early childhood education program. In addition, teachers of 142 children failed to provide data for their students. Consequently the sample on which this analysis is based comprises 6511 children with similar age and gender characteristics as the full sample.

Program Type

These children attended a range of early childhood education programs and these are shown in Table 1, which also reports the mean age of children in each group. As is seen from the table, more than half of the children (53.7%) participated in preschool programs, which operate only during school hours and terms, and where children may attend half-days or limited sessions a week. Almost a quarter (22.8%) attended centre-based programs which operate at least eight hours a day, five days a week and most weeks of the year. Less than one fifth (17.6%) attended pre-Year 1 school programs, which are full-time, school-based programs. A small proportion attended other programs including early intervention programs, or participated in multi-age classrooms. As is also seen, children attending pre-Year 1 school programs were on average 4 months older than those attending preschool and centre-based programs.

Table 1
Participation in Early Childhood Programs (N = 6511)

Program type	Frequency	%	Mean age (months)
Centre based childcare program	1483	22.8	57
Preschool program	3495	53.7	57
Pre-Year 1 school program	1149	17.6	61
Other	124	1.9	59
Not stated	260	4.0	

Indicators of Mathematical Competencies

Analysis of children's mathematical competencies was based on the mathematical skills scale (Social Development Canada, 2005), which was included in the teacher questionnaires. One significant limitation of the LSAC study design is that no opportunity was given to parents and/or other caregivers, or the children themselves, to provide a response to these six items. As such, the data reported in this article are formed on the basis of *early childhood teachers'* judgements of children's competence in relation to the following items:

1. ability to sort and classify;
2. ability to count objects;
3. ability to count to 20;
4. ability to recognise numbers;
5. ability to do simple addition; and
6. interest in numbers.

These were phrased as questions allowing a "Yes" or "No" response, with the final item asked from a negative perspective. Clearly, these items do not address all mathematical competencies a young child may possess and indeed, privilege number concepts above all other mathematical concepts. Nevertheless they do provide insight into some of the mathematical competencies 4- and 5-year-old children possess, as perceived by their educators.

Analysis Plan

Descriptive statistics were used to answer the overarching question in this study. These were estimated, however, through the use of a series of logistic regression models: One for each competency. These models also allowed for the later testing of program type on children's mathematical competency, whilst controlling for the influence of their ages. Given the statistical power associated with the sample size, the statistical significance of regression coefficients was assessed with a Bayesian information criterion (BIC), with values exceeding ten considered to be "very strong" effects (Pampel, 2000, p. 31). In order to account for the complex sampling design used with LSAC, all estimates and their standard errors were calculated using the R-package "Survey" (Lumley, 2012).

Results

The estimated proportions of children, who according to their teachers possessed the given mathematical competencies, are shown in Table 2. As is seen, most children were able to sort and classify, and count objects. Far fewer, however, were able to recognise numbers and undertake simple addition. Variations in these competencies, however, may have been due to differences in age and the program type that children were attending. In order to control for these factors, program-type (a four-level factor) and age (in months) centred on the mean, were regressed onto children's mathematical competencies; a series of dichotomous variables indicating whether the child had or had not met the relevant competency. The results of these models are shown in Table 3, which reports estimates of the influence of age and program type on the probability that a child will meet the given competency. More specifically, these estimates relate directly to the logit transformation (natural logarithm of the odds ratio) of this probability. In each model, the influence of program-type is relative to those children in *centre-based* programs. The specification of these models is shown in Equation 1, where π_i is the probability that a child will meet the relevant mathematical competency.

$$logit(\pi_i) = \beta_0 + \beta_1 Age + \beta_2 PreSchool + \beta_3 PreY1 + \beta_4 Other \text{ ------(1)}$$

Table 2

Proportion of Students Meeting Each Competency

Competency	Overall (%)
Able to sort and classify	96
Able to count objects	94
Able to count to 20	62
Able to recognize numbers	72
Able to do simple addition	32
Uninterested in numbers	2

Table 3

Results of Logistic Regression Models

Competency	Intercept (β_0)	Age (β_1)	Preschool (β_2)	Pre-Year 1 (β_3)	Other (β_4)
Able to sort and classify	2.59	0.01	0.36	-0.03	-0.79
Able to count objects	-1.52	0.07	0.25	0.16	-0.71
Able to count to 20	-5.24	0.10	-0.42	-0.19	-0.49
Able to recognize numbers	-3.84	0.08	-0.12	0.29	-0.12
Able to do simple addition	-6.39	0.09	-0.19	0.67	0.28
Uninterested in numbers	-3.34	-0.01	-0.07	-0.20	0.78

Note: emboldened coefficients report BIC>10.

As is seen from Table 3, age has a significant influence on children's ability to count to 20, recognize number, and to do simple addition. The odds ratios corresponding to each of these effects are 1.10, 1.08, and 1.09 respectively, suggesting that an increase in age of one month relative to the mean age (57.6 months) will produce small, but significant increases in the likelihood of gaining these competencies. When controlling for age, children attending preschools were less likely to be able to count to 20 than children attending centre-based programs. The corresponding odds ratio for this effect is 0.66, suggesting that preschool attendees are two thirds as likely to gain this competency as those attending centre-based programs. In addition, children attending a pre-Year 1 program were more likely to be able to do simple addition than those attending the centre-based programs. The corresponding odds-ratio for this effect is 1.95, suggesting that these children are nearly twice as likely to meet this competency as those attending centre-based programs.

Discussion

As reported in Table 2, the children in this study demonstrated a high level of competence on the majority of the items. This is consistent with international research showing that children develop a range of mathematical understandings in the years prior to starting school (Reikerås et al., 2012; Clarke et al., 2006).

The years subsequent to the collection of the LSAC data has seen the implementation of Australia's first national schooling curriculum, known as the *Australian Curriculum* (incorporating the specific *Australian Curriculum: Mathematics*) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2014). The *Australian Curriculum: Mathematics* has content grouped into three areas: number and algebra; measurement and geometry; and statistics and probability (ACARA, 2014). A mapping exercise has been undertaken to examine the alignment of the competencies demonstrated in this study with the current expectations of children in the early years of primary school. This exercise has revealed some points of concern. For example, the *Australian Curriculum: Mathematics* mandates that simple addition is not taught until Year 1, yet one third of the LSAC children were perceived by their educators to be already doing this, either in their prior-to-school year or first year of school. Sorting and classifying, counting (including to 20) and number recognition are stated as content to be *taught* in the Foundation year; however; the majority of children in this study are already demonstrating competence in these areas. This is consistent with the findings of other Australian studies (Gervasoni & Perry, 2013; MacDonald, 2010), indicating that there is growing evidence that the early years mathematics curriculum is misaligned with children's existing competencies. Of concern is that this lack of challenge might result in children becoming disinterested in mathematics as they progress through the schooling years.

It is important to note that according to their teachers 98% of the LSAC children showed interest in numbers at 4-5 years. This is heartening because studies show a decline in levels of mathematics over the entire school period (e.g. Fredricks & Eccles, 2002). If children engage in meaningful and enjoyable mathematics education in the early childhood years, they are much more likely to appreciate and continue to engage in later mathematics education (Linder, Powers-Costello, & Stegelin, 2011).

Children attending preschools were less likely to be able to count to 20 than children attending centre-based programs. This is somewhat counter to the common perception that preschools provide "higher quality" education programs and hence are more likely to produce better outcomes (Marriner, 2013). It may be the case that preschool programs focus on developing skills beyond simple rote counting, whereas the mathematics in centre-based care is typically limited to activities such as counting and identifying shapes (Cohrssen, Church, Ishimine, & Tayler, 2013).

Children attending pre-Year 1 programs were more likely to be able to do simple addition than those attending the centre-based programs. On the one hand, it could be argued that this makes sense, given an explicit focus on mathematics education (as expressed through formal curricula) in school settings. Of note, though, is the point that simple addition typically does not feature in the formal curriculum for the first year of school; rather, it typically appears as content for teaching in Year 1 (children's second year at school). This suggests that not only is the first year of school curriculum failing to recognise the competencies children bring with them from prior-to-school settings, this lack of recognition is maintained as children progress to their second year of formal schooling.

Limitations and Opportunities for Further Research

The analysis has been undertaken within the limits of the LSAC study design, including its measures. Although the mathematical skills scale may be viewed as limited, and there may be more appropriate measures elsewhere, the analysis could only include data from the existing study. However, this highlights the importance of future research which takes a broader view of mathematical competence and is more inclusive of other conceptual domains in mathematics.

A further limitation is that mathematical competencies were based on educators' judgements only and indeed was restricted to children enrolled in formal early childhood programs. There is much research which indicates powerful mathematical ideas are developed in home and community settings (MacDonald, 2012). Consequently further research in *all* early childhood settings, and using multiple sources, is required.

Conclusion and Implications

This article has presented evidence to suggest that young children are perceived as competent by their educators in several aspects of mathematics, as assessed within the scope of the LSAC data gathering. However, given that data regarding children's mathematical competencies was *only* collected from the age of 4-5 years, this begs the question: What competencies do children possess at younger ages? Consistent with Peter-Koop and Scherer's (2012) call for further research, it seems clear that there is much work yet to be done in identifying the mathematical competencies developed by young children. Much of the extant research and existing assessment tools specifically target preschool-age children (i.e. 4-5 year old children)—as exemplified in the LSAC study—with relatively little research on the mathematical development of younger children (Mousley & Perry, 2009). As Doig et al. (2003) state, it appears that the development of an assessment instrument that gives due emphasis to the full range of young children's mathematics is long overdue. Indeed, this call for further research persists a decade later, with Peter-Koop and Scherer (2012) arguing that research leading to the development of a detailed competency model that goes beyond number and integrates the different content areas of mathematics is still needed.

Acknowledgements

The authors would like to thank all involved in the LSAC study. *Growing Up in Australia* was initiated and funded as part of the Australian Government Stronger Families and Communities Strategy by the Australian Government Department of Housing, Families, Community Services and Indigenous Affairs (FaHCSIA). The study is being undertaken in partnership with the Australian Institute of Family Studies, with advice being provided by a consortium of leading researchers at research institutions and universities throughout Australia. The data collection is undertaken for the Institute by the Australian Bureau of Statistics. All views expressed in this paper are the authors', and do not represent the views of FaHCSIA or the Australian Institute of Family Studies.

References

Aubrey, C. (1993). An investigation of the mathematical knowledge and competencies which young children bring into school. *British Educational Research Journal*, *19*(1), 27-41.

Australian Curriculum Assessment and Reporting Authority (2014). *Australian Curriculum: Mathematics*. Retrieved from http://www.australiancurriculum.edu.au/mathematics/Curriculum/F-10

Baroody, A.J. (2000). Does mathematics for three and four year old children really make sense? *Young Children*, *55*(4), 61-67.

Björklund, C. (2008). Toddlers' opportunities to learn mathematics. *International Journal of Early Childhood*, *40*(1), 81-95.

Clarke, B., Clarke, D., & Cheeseman, J. (2006). The mathematical knowledge and understanding young children bring to school. *Mathematics Education Research Journal*, 18(1), 78-102.

Cohrssen, C., Church, A., Ishimine, K.,, & Tayler, C. (2013). Playing with maths: Facilitating the learning in play-based learning. *Australasian Journal of Early Childhood*, *38*(1), 95-99.

Doig, B., McCrae, B., & Rowe, K.J. (2003). A good start to numeracy. Retreived from http://www.dest.gov.au/archive/schools/publications/2003/goodstart.pdf

De Lange, J. (2008). *Talentenkracht [Curious minds]*. The Netherlands: Freudenthal Institute for Mathematics and Science Education.

Fredricks, J. A.,, & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: growth trajectories in two male-sex-typed domains. *Developmental Psychology*, *38*(4), 516-533.

Gervasoni, A., & Perry, B. (2013). Children's mathematical knowledge prior to starting school. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics Education: Yesterday, Today and Tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne: MERGA.

Graham, T.A., Nash, C., & Paul, K. (1997). Young children's exposure to mathematics: The child care context. *Early Childhood Education Journal*, *25*(1), 31-38.

Klibanoff, R. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology*, *42*(1), 59-69.

Levine, S.C., Suriyakham, L.W., Rowe, M.L., Huttenlocher, J., & Gunderson, E.A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*(5), 1309-1319.

Linder, S.M., Powers-Costello, B., & Stegelin, D.A. (2011). Mathematics in early childhood: Research-based rationale and practical strategies. *Early Childhood Education Journal*, *39*, 29-37.

Lumley, T. (2012). Survey: Analysis of complex survey samples (R package version 3.28 – 2). Retrieved from: http://cran.r-project.org/web/packages/survey/index.html

MacDonald, A. (2010). Young children's measurement knowledge: Understandings about comparison at the commencement of schooling. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp.375-382). Fremantle: MERGA.

MacDonald, A. (2012). Young children's photographs of measurement in the home. *Early Years*, *32*(1), 71-85.

Marriner, C. (2013, August 11). Preschools better for children than day care. *The Sydney Morning Herald*. Retrieved from http://www.smh.com.au/federal-politics/federal-election-2013/preschools-better-for-children-than-day-care-20130810-2roqx.html

Mousley, J., & Perry, P. (2009). Developing mathematics concepts in Australian pre-school settings: The background. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, vol. 1, pp. 646-649). Palmerston North, New Zealand: MERGA.

Pampel, F.C. (2000). *Logistic regression: A primer*. Thousand Oaks, CA: Sage Publications.

Peter-Koop, A., & Kollhoff, S. (2015). Transition to school: Prior to school mathematical skills and knowledge of low-achieving children at the end of Grade 1. In B. Perry, A. MacDonald, & A. Gervasoni (Eds.), *Mathematics and transition to school: International perspectives* (pp. 65-83). Singapore: Springer.

Peter-Koop, A., & Scherer, P. (2012). Early childhood mathematics teaching and learning. *Journal für Mathematik-Didaktik*, *33*, 175-179.

Reikerås, E., Løge, I.K., & Knivsberg, A. (2012). The mathematical competencies of toddlers expressed in their play and daily life activities in Norwegian kindergartens. *International Journal of Early Childhood*, *44*, 91-114.

Sanson, A., Nicholson, J., Ungerer, J., Zubrick, S., Wilson, K., Ainley, J., & Wake, M. (2002). *Introducing the longitudinal study of Australian children* (LSAC Discussion Paper No. 1). Melbourne: Australian Institute of Family Studies.

Social Development Canada (2005). *National longitudinal survey of children and youth cycle 5: Survey instruments 2002/03, Book 2 – Kindergarten teacher and youth questionnaires*. Ottawa, ON: Author.

Wright, R.J. (1994). A study of the numerical development of 5-year-olds and 6-year-olds. *Educational Studies in Mathematics*, *26*, 25-44.

Examining PCK in a Senior Secondary Mathematics Lesson

Nicole Maher
University of Tasmania
<Nicole.Maher@utas.edu.au>

Tracey Muir
University of Tasmania
<TraceyMuir@utas.edu.au>

Helen Chick
University of Tasmania
<Helen.Chick@utas.edu.au>

Teacher knowledge, including Pedagogical Content Knowledge (PCK), continues to be the focus of research, with the general consensus being that PCK impacts upon teaching and learning. Much of the current research has focused on pre-service teachers and practicing primary teachers, with few studies focused on studying senior secondary teachers' PCK. Even rarer are studies which examine PCK from students' perspectives. This study investigates the nature of PCK as experienced in a lesson by a class of senior secondary mathematics students. The findings indicated that there were a number of PCK elements incorporated in the lesson and that these were noticed by the students.

Introduction

Effective mathematics teaching requires knowledge of mathematical content, knowledge of students' thinking, and knowledge of how to represent the content so that it makes sense to others (Hill, Ball, & Schilling, 2008). There has been substantial research into identifying and characterising the constituent parts of teacher knowledge including pedagogical content knowledge (PCK) (e.g., Chick, Baker, Pham, & Cheng, 2006; Krauss et al., 2008). PCK is knowledge about the way subject matter is transformed from the knowledge held by the teacher into the content of instruction. Shulman described PCK as an intricate blend of content and pedagogy that encompasses all that is needed to teach a subject or topic in a way that makes it comprehensible to others (1986).

It is generally accepted within the mathematics education community that PCK impacts upon teaching and learning (e.g., Ball, Lubienski, & Mewborn, 2001; Krauss et al., 2008). Most research into PCK has tended to focus on pre-service and practicing teachers in the context of primary mathematics (e.g., Rowland, Huckstep, & Thwaites, 2005) but comparatively few studies have examined PCK for teaching secondary mathematics (Matthews, 2013). Furthermore there has been little research into how multiple sources of evidence of PCK may inform us about the nature of this aspect of teacher knowledge. This paper focuses on investigating PCK within the context of a senior mathematics classroom by exploring the following research questions: What aspects of PCK does a teacher of senior secondary mathematics demonstrate in a lesson? To what extent are these aspects perceived by students as being helpful to their learning?

Review of Literature

Several frameworks have been developed to conceptualise the multi-faceted nature of mathematics teacher knowledge, including PCK (e.g., Ball, Thames, & Phelps, 2008). The domain map for Mathematics Knowledge for Teaching developed by Ball and colleagues delineates the boundaries of different categories of teacher knowledge and is widely cited in the literature on mathematics teacher knowledge. Some scholars however, have questioned if it is possible, particularly in practice, to precisely demarcate subject matter knowledge and pedagogical content knowledge in the context of teaching (e.g., Marks, 1990).

The framework for analysing PCK in mathematics teaching developed by Chick and her colleagues (e.g., Chick et al., 2006) gives a detailed inventory describing evidence for

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 389–396. Sunshine Coast: MERGA.

identifying key components of PCK within three broad categories. These include “clearly PCK”, “content knowledge in a pedagogy context” and “pedagogical knowledge in a content context” and represent the varying degrees to which content and pedagogy are intertwined without rigid delineation. Space prevents the inclusion of the entire framework but brief descriptions of some key PCK elements are given in Table 1. Some PCK elements relate to teachers’ knowledge of students’ existing conceptions about mathematical concepts, and others relate to knowledge of how to transform mathematics knowledge to facilitate learning (e.g., *Deconstructing Content to Key Components*).

Table 1.
A Framework for Pedagogical Content Knowledge. (Based on the framework in Chick and colleagues, 2006)

PCK Category	Evident when the teacher …
Clearly PCK	
Teaching strategies – general	Discusses or uses general strategies or approaches for teaching a mathematical skill or concept.
Student thinking	Discusses or responds to possible students’ ways of thinking about a concept, or recognises typical levels of understanding.
Student Thinking – Misconceptions	Discusses or addresses typical/likely student misconceptions about mathematics concepts.
Cognitive Demands of Task	Identifies aspects of the task that affects its complexity.
Knowledge of Examples	Uses an example that highlights a concept or procedure
Content Knowledge in a Pedagogical Context	
Deconstructing Content to Key Components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident)
Methods of Solution	Demonstrates a method for solving a mathematical problem
Pedagogical Knowledge in a Content Context	
Assessment Approaches	Discusses or designs tasks, activities or interactions that assess learning outcomes

The framework enables close inspection of teachers’ PCK by applying it to data such as interview transcripts, written responses to items about teaching and learning mathematics content, and actual teaching episodes (Chick et al., 2006). As such, it provides an appropriate theoretical framework for the study discussed in this paper. Overlap among facets is also plausible. For instance, in discussing a method for solving a problem a teacher may show *Procedural Knowledge* as well as demonstrate evidence of having *Deconstructed Content into Key Components* (Chick et al., 2006).

Teachers often use examples in their classroom in order to illustrate key principles. The Chick et al. framework highlights examples as a facet of PCK. Other scholars (e.g., Krauss et al., 2008; Zodik & Zaslavsky, 2008) identify examples as integral to learning and teaching as they represent powerful learning opportunities for students. Examples refer to a particular case from a larger class from which one can reason and generalize (Zodik &

Zaslavsky, 2008). In fact it is not the specific example or even the answer that is the most important but the general principle illuminated by the example (Chick, 2009). In senior secondary mathematics, examples form a large part of classroom instruction; they were a feature of the lesson studied for this paper.

Methodology

This paper uses data from a larger study and explores aspects of PCK from the perspectives of a teacher, his students, and the researcher by examining one lesson in detail. A Grade 11/12 Mathematics Methods class from a large metropolitan secondary college in Tasmania participated. Mathematics Methods is one of the most demanding mathematics courses offered in Tasmanian schools. It is assessed by internal unit tests and a final external examination; the major topics are function study, differential and integral calculus, and statistics. Data presented in this paper focus on two examples involving optimisation, a practical application of differential calculus.

The participants were Mr Jones, a teacher of Mathematics Methods during 2014, and his 16-18-year-old students. Mr Jones has been a teacher of secondary mathematics for 25 years, including seven years at the senior secondary level. Of the 18 students enrolled in Mr Jones' class, 15 (six females and nine males) contributed data by participating in the focus group interview and/or completing a short answer survey. Teacher and student names are pseudonyms in this paper.

Data were collected during one lesson on applications of differential calculus. The 100-minute lesson focused on optimisation. The lesson was observed, video-recorded, and partially transcribed. At the end of the lesson a short-answer survey was completed by participating students, eliciting responses about the types of explanations and strategies that assisted them with their learning. A semi-structured audio-recorded focus group interview was also conducted, with five participants for 15 minutes, where students were asked to comment on aspects of the lesson that were particularly helpful for their learning. Mr Jones also participated in a 20 minute interview after the lesson. These approaches yielded three data sources for examining PCK: the researcher's notes on the lesson and accompanying video, student perspectives, and the teacher interview.

Data from the lesson observation, interviews, and surveys were transcribed and aligned with one of several teaching events in the lesson (e.g., the presentation of a solution to a particular example, the process of differentiating a function). Teacher actions during the class, student comments, and teacher interview comments were examined to see if they matched any of the PCK lesson descriptors (see right hand column of Table 1). The transcripts were read independently by each author, to ensure consistency. The multiple data sources linked to each lesson event were then examined for commonalities in PCK type. Of particular interest was the extent to which the multiple sources of evidence of PCK corroborated each other and what insights this provided about the nature of PCK.

Results and Discussion

This section begins with a brief overview of the lesson, followed by the presentation of results from multiple data sources linked to particular events in the lesson. Results are arranged in sections based on these lesson events. The teaching events described in each section have been classified using the categories from the Chick et al. framework for PCK (see Table 1). In some cases categories are grouped or paired (e.g., knowledge of examples/knowledge of assessment) to reflect situations where different aspects of PCK were clearly intertwined.

Lesson overview

Mr Jones began the lesson by providing an overview of the remaining content to be covered in differential calculus before the end of unit test in the following week. He forewarned the students that it would be “a frantic lesson” as this was the last lesson allocated to applications of differential calculus.

The instructional phase of the lesson focused on two optimisation (also called maximum and minimum) problems: “the bushwalker problem” (see Figure 1) and “the distance problem” (see Figure 2). Optimisation problems are the key focus of applications of differential calculus in the Mathematics Methods course, and involve practical situations in which students are required to minimize or maximize a quantity. The bushwalker and the distance problems involved obtaining a particular function and calculating its minimum value using calculus. Mr Jones demonstrated each example on the whiteboard starting with the bushwalker problem, modelling the written mathematical steps and explaining the process. At the conclusion of the presentation of the two examples students worked on similar problems.

A bushwalker can walk at 5 km/h through clear land and 3 km/h through bushland. If she has to get from point A to point B following a route indicated in the diagram on the right, find the value of x so that the route is covered in the minimum time. (Note: time = $\frac{\text{distance}}{\text{speed}}$)

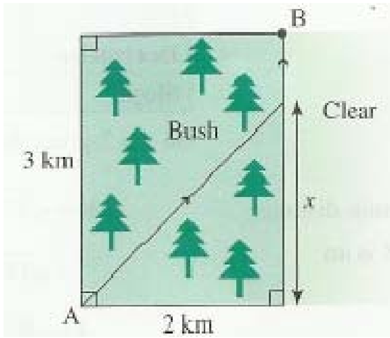


Figure 1. The bushwalker problem (Hodgson, 2013)

Find the minimum distance from the straight line with equation $y = x - 4$ to the point (1,1).

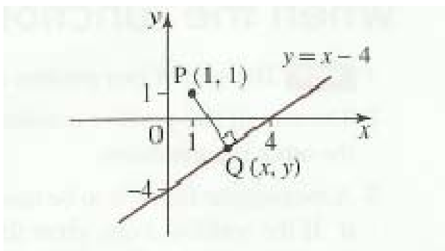


Figure 2. The distance example (Hodgson, 2013)

Lesson Events

Developing the functions. Mr Jones highlighted the development of the functions in each example, which he identified in the lesson as a key challenge in solving optimisation problems (*Cognitive Demand of Task*). He carefully unpacked the examples and drew attention to variables that would be crucial for the development of the appropriate function

(Deconstructing Mathematics into Key Components). For example, in the bushwalker problem, Mr Jones emphasised that distance x (see Figure 1) may not be labelled in an exam situation "and you would need to come up with it yourself; that this is a crucial missing distance" *(Deconstructing Mathematics into Key Components)*. Similarly, for the distance problem Mr Jones emphasised the idea that the point Q(x, y) must be expressed in terms of x only, that is (x, x-4), in order to develop a distance function with respect to x (*Cognitive Demand of Task*). Later in the post-lesson interview Mr Jones commented that "often some kids don't realise when and why they need to express one variable in terms of another even if it seems quite obvious" (*Knowledge of Students Thinking – Misconceptions*). While the students did not comment specifically on obtaining the functions, there was some evidence that they valued the way Mr Jones emphasised critical aspects of the examples: "It was helpful the way he used the board and some diagrams to show how to do certain things" (James; survey). Similarly, Alan commented during the focus-group interview: "His diagrams were, like, clearly set out … to show the different things; it makes it clearer in your mind".

Selection of examples. Mr Jones introduced the lesson by explaining to students why he had specifically chosen the bushwalker and the distance problems (*Knowledge of Examples*)

> We've spent a lot of time on area and volume problems but I don't want you to think that "that's it" for applications of calculus. It's probably the focus of my two problems today is to show you some of those other applications. All the function unknown ones we've done so far have been area and volume ones but there are other types that could pop up in the exam. (in-class comment)

He also highlighted the distance problem as a typical question for the non-calculator section of the examination, given that its "nice neat" answer could be obtained without the aid of the CAS calculator (*Knowledge of Examples/Knowledge of Assessment*).

> Remember how I said that "function unknown" problems are more calculator than non-calculator? Well, have we needed our calculator for this one yet? No we haven't, so that's why I wanted to do this one today, because it's the classic example of one that could be in the non-calculator section of the exam … because a lot of the other ones we've done have applied to realistic situations which don't end up being nice neat figures like the square root of eight, they could end up being something like 2.9564323… (in-class comment)

Mr Jones elaborated further on his choice of examples in the post-lesson interview.

> I wanted to give them an example of one that didn't require use of the calculator at all, because the nature of our course is that there is a calculator and a non-calculator section of the exam. So that was an important example because, I mean, I don't want to get too caught up in the exam, but in reality I have to be faithful to anticipating what sort of questions come up. (interview)

The interview data provided some further insight into Mr Jones' enacted *Knowledge of Examples/Knowledge of Assessment* in terms of the impact a high stakes examination has on teaching decisions, including the selection of examples. In one of his interview responses, Mr Jones expressed a tension between teaching the mathematics per se and teaching to the examination. The source of this tension was not discussed in the interview. While several students commented on the usefulness of the chosen examples, only one response mentioned the examination explicitly.

> The most helpful thing was when we went through the different types of questions on minimums and maximums. It helped me learn what types of questions I can expect on exams and tests. (Elizabeth; survey)

Highlighting a general method of solution. A common teaching strategy often used by Mr Jones was to encourage students to recognise the key processes involved in solving optimisation problems by directing their thinking through questioning (*Teaching Strategies/Deconstructing Knowledge into Key Components*).

> Mr Jones: Because we are looking for a minimum, what are we going to have to find eventually somewhere in this question? [class response: the derivative]. Yes, and then we make the derivative equal to? [class response: zero]. Good and then solve for? [Class response: x]. Good. And that should be an automatic reaction when we see the word "minimum" or "maximum"; [it] should be our trigger to say "right, that's our process". (*Teaching Strategies/Deconstructing Content into Key Components/Method of Solution).*

Further evidence of the teacher's knowledge of deconstructing ideas was apparent during the post lesson interview, particularly in relation to Mr Jones' emphasis on identifying the key steps involved in solving optimisation problems (*Method of Solution/Deconstructing Content to Key Components*).

> Mr Jones: This year I've probably identified key words in the question and making sure that they understand what the process is. When you are teaching a topic like that, these are the key steps you've got to do. So give them the framework I suppose and hopefully they can apply that framework to understanding other situations. This year I've really concentrated on that.

One student's response appeared to align closely with Mr Jones' focus on deconstructing the key components of the problem to provide a general method of solution: "The examples on the board helped me recognise when to do what (e.g., "when to look for a minimum or maximum and making $d'(x) = 0$"). (Lucy; survey). Other students tended to comment on specific aspects of the examples, such as the use of the distance formula in the distance example: "The example on the board finding minimum/maximum distance between points using the distance formula".

Close examination of evidence of PCK observed by the researcher and discussed by Mr Jones suggested that the deconstruction of the mathematics was limited to standard differentiation approaches. For example, a visual representation of the minimum values for each example could have been obtained using the CAS calculator, even though the examples had been selected specifically to be solved without the aid of technology. It would have been interesting to find out Mr Jones' reasons for omitting the graphs of the functions in each example, however this was not sought during data collection.

Algebraic skills. Mr Jones guided the class through the differentiation of the respective functions for each example step-by-step (*Procedural Knowledge*). Again he involved the students by asking strategic questions as demonstrated in the following lesson transcript based on the distance problem (*Procedural Knowledge/Teaching Strategies*). Note that Mr Jones referred to the surd and power forms of the distance function as $d(x) = \sqrt{2x^2 - 12x + 26}$ and $d(x) = (2x^2 - 12x + 26)^{\frac{1}{2}}$ respectively.

> Mr Jones: Before I can get the derivative of this function [points to the surd form of $d(x)$]. What form do I need to put it in Jessie? It's in surd form at the moment.
>
> Jessie: Power form.
>
> Mr Jones: That's right power form [rewrites the function $d(x)$ in power form]. Ok so to find the derivative $d'(x)$, what comes out the front Angela?
>
> Angela: Umm a half
>
> Mr Jones: That's right a half and then we multiply by what Ryan?
>
> Ryan: Oh umm the derivative of the bracket.

Mr Jones: Yes. The derivative of the bracket which is (4x – 12) and then multiplied by … what's the last bit Harry?

Harry: Umm the brackets to the power of negative a half.

Mr Jones: Yes good [completes the differentiation process to yield:

$d'(x) = \frac{1}{2}$ x (4x-12) x$(2x^2 - 12x + 26)^{-\frac{1}{2}}$]. Are we all right with that,? There's your process. OK, so we've got 4x – 12 in the numerator, and in the denominator we've got the 2. Remember that your negative-a-half [points to the expression $(2x^2 - 12x + 26)^{-\frac{1}{2}}$] moves to the denominator so we have $d'(x) = \frac{(4x-12)}{2\sqrt{2x^2-12x+26}}$. Now tell me if I've done too many steps at once there? OK, so the (4x – 2) and the 2 have stayed where they were and the bracket to the negative-a-half has gone underneath. Then I've just changed it from the power of a half to the square root.

There was also evidence that the students' noticed and valued Mr Jones explicit questioning about mathematical processes as suggested in Christopher's survey response: "The whiteboard examples were the most helpful. He engaged everyone in the class and you had to pay attention as he asked people for different values and numbers" (*Teaching Strategies*).

A similarly explicit approach was used to solve the bushwalker example. During the post-lesson interview Mr Jones also commented on his ongoing focus on consolidating students' algebraic skills (*Procedural Knowledge/Teaching Strategy*).

Even at this (almost) final lesson on differential calculus there might be gaps in their basic skills… Umm you might have noticed I asked a number of times what do we do when we have a square root, they know by now, they've got to convert it to a power, drum, drum, drum. Umm I don't know if that's effective or not but yeah. (Mr Jones; interview).

During the focus-group interview several students highlighted Mr Jones' step-by-step procedures as being particularly helpful (*Procedural Knowledge*).

Angela: When he did the steps on the board I could just look back to see how to do it.

Tom: Yes, helpful for differentiating square roots with more than one thing in it, like when there was x squared plus 2x and then the square root of all that and you had to differentiate it.

Danny: If you've copied one of his [Mr Jones'] examples down and you're at home and you get sort of one like it you can sort of match things up. You try and follow the same procedure with the different numbers and that can help you through.

Three students also commented that the step-by-step explanations helped them to learn to solve the "harder" optimisation problems: "His [Mr Jones'] step-by-step examples were very useful for the harder questions. It helped me to learn how to do the harder function unknown questions" (Dylan, survey). Similarly Emma commented that "The worked examples on the board with consistent pausing to further explain steps helped me gain an understanding of the work" (survey).

Conclusions

The results depict aspects of a senior secondary mathematics teacher's PCK based on evidence from three main data sources. The Chick et al. (2006) PCK framework provided a set of filters through which to examine elements of PCK that were observed or noticed and discussed by the teacher and students. The most prominent PCK categories identified in the data from the lesson were Teaching Strategies, Student Thinking, Cognitive Demand of Task, Knowledge of Examples, Method of Solution, Procedural Knowledge, and Deconstructing Content into Key Components (see Table 1). Broadly speaking, some categories tended to relate to an awareness of students' thinking about mathematical skills and concepts, and others focused on the mathematics itself and how it is transformed to make it comprehensible to others. Furthermore, these categories were often inextricably linked. For example, Mr Jones demonstrated both Procedural Knowledge and Knowledge of Student Thinking – Misconceptions during the differentiation of the distance function d(x) in that he was attentive to potential difficulties students may experience at each stage of the solution process.

The multiple sources of data tended to corroborate each other where particular lesson events were observed by the researcher and also discussed by Mr Jones and the students. For example, Mr Jones' step-by-step approach to solving the problems was particularly noticed and valued by the students. Similarly the use of questions to guide students' thinking about skills or ideas during the process of solving the problems was clearly observed by the researcher and mentioned by some students. Although the study is limited in that it was one account of a senior secondary mathematics lesson, it does provide insight into the nature of Mr Jones' PCK and its impact on students. Further studies that investigate PCK across different senior secondary mathematics topics and with different senior mathematics teachers would also add to the limited research in this area, and reveal if there are common aspects of PCK evident in teachers' work at this level.

References

Ball, D., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (Vol. 4, pp. 433-456). New York: Macmillan.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education*, *59*(5), 389-407.

Chick, H. (2009). Choice and use of examples as a window on mathematical knowledge for teaching. *For the Learning of Mathematics*, *29*(3), 26-30.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 297-304). Prague: PME.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372-400. doi: 10.2307/40539304

Hodgson, B. (2013). *Maths quest 12 mathematical methods CAS* (2nd ed.). Milton, Qld: John Wiley & Sons Australia.

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology*, *100*(3), 716-725.

Marks, R. (1990). Pedagogical content knowledge: from a mathematical case to a modified conception. *Journal of Teacher Education*, *41*(3), 3-11.

Matthews, M. E. (2013). The influence of the pedagogical content knowledge framework on research in mathematics education: A review across grade bands. J*ournal of Education*, *193*(3), 29-37.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The Knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255-281.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14. doi: 10.2307/1175860

Zodik, I., & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics*, *69*(2), 165-182. doi: 10.1007/s10649-008-9140-6

Teacher's Scaffolding over the Year to Develop Norms of Mathematical Inquiry in a Primary Classroom

Katie Makar
The University of Queensland
<k.makar@uq.edu.au>

Arthur Bakker
Utrecht University
<A.Bakker4@uu.nl>

Dani Ben-Zvi
Haifa University
<dbenzvi@univ.haifa.ac.il>

Developing mathematical inquiry practices requires that teachers are explicit about classroom norms that support these practices. In this study, we asked: *How can a teacher scaffold the development of norms and practices in mathematical inquiry over time?* Analysis of classroom video over a year showed that the teacher constantly diagnosed classroom norms and responsively used strategies to improve them. By the end of the year, there was evidence of inquiry norms and practices independent of the teacher's presence.

In contrast to mathematics classrooms that focus on memorisation and reproduction of procedures, inquiry-based classrooms value building a climate of intellectual challenge (Goos, 2004). "Rather than rely on the teacher as an unquestioned authority, students in [inquiry-based] classrooms are expected to propose and defend mathematical ideas and conjectures and to respond thoughtfully to the mathematical arguments of their peers" (p. 259). In this study, mathematical inquiry is taken to be the process of addressing ill-structured tasks using mathematical evidence. Ill-structured tasks are those in which the problem statement and/or solution pathway contain ambiguities that need to be negotiated (Reitman, 1965). For example, children may address questions such as "What makes the best map?" (Fry, 2013) where the word "best" requires negotiation.

Although an inquiry approach has been promoted by the mathematics education community, it has been slow to take hold. A number of challenges exist for both teachers and students in addressing ill-structured questions. The challenge addressed in this paper is that of developing classroom norms for inquiry. The goal is to understand how a teacher experienced in teaching mathematical inquiry managed to use scaffolding to establish and develop classroom norms of mathematical inquiry with her students over a year. In order to gain insight into this scaffolding process, our research question was: *How can a teacher scaffold the development of norms and practices in mathematical inquiry over time?*

Scaffolding Norms and Practices of Mathematical Inquiry

Scaffolding is traditionally described as a support provided by a teacher or more able peer to assist a student to solve a problem that they would not normally be able to solve on their own. Work by Smit, van Eerde, and Bakker (2013) goes beyond the conventional one-on-one teacher-student interaction to investigate scaffolding within a whole class context. A focus on scaffolding classroom talk becomes a key component for classroom-based inquiry because of its emphasis on collaboration (Quintana et al., 2004) and the need for making thinking visible (Linn, 2000). Researchers have cautioned against using the scaffolding metaphor as if it were "a technique that can be applied in every situation in the same way" (van de Pol, Volman, & Beishuizen, 2010, p. 272). Scaffolding has often been studied as in the moment interactions between teacher and pupil/student (Anghileri, 2006; van de Pol et al., 2010). However, for enacting and studying the scaffolding of longer-term

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 397–404. Sunshine Coast: MERGA.

processes such as establishing of norms, there is a need to understand scaffolding that would account for this time dimension (cf. Mercer, 2008).

Goos (2004) argues that norms of participation in a mathematical inquiry classroom are based on assumptions that mathematical thinking is an act of sense-making and that mathematical inquiry relies on habits of reflection and self-monitoring. However, students struggle with having a "taken-as-shared sense of *when* it is appropriate to contribute" and "the actual *process* by which students contribute" (Yackel & Cobb, 1996, p. 461, emphasis in original). The teacher and students' practices are therefore inter-linked. Under Goos' assumptions, the teacher could model for students what these practices look like and encourage them to take some ownership for developing mathematical solutions. In doing so, students need to be willing to take risks, for example by sharing incomplete ideas. If the teacher consistently judged student contributions as correct or incorrect, students would be less likely to contribute partial or emerging ideas. To encourage risk-taking, the teacher could withhold judgement on students' suggestions and elicit comments and critique from peers. Goos suggested that students would then more likely begin to offer conjectures and critique without teacher prompting. Other practices from literature on classroom talk and collaboration include the need for active listening, explaining and justifying to peers and building on others' ideas (McCrone, 2005; Mercer, 1996). The literature also points to the continuous work required by the teacher to establish classroom norms (Yackel & Cobb, 1996); in this paper we explore how these norms can be scaffolded over a school year.

Methodology

Participants and Data

The data presented in this paper come from a three year project on argumentation in primary mathematics and statistics. This paper focuses on data from one teacher Kaye Bluett (pseudonym) and her 26 Year 4 students (9-10 years old). The students represented a range of performance levels, with several students receiving additional learning support. Each student had a laptop or iPad with these devices used in many of the lessons. Kaye was an experienced teacher who taught mathematical inquiry for a number of years.

The focus of data was on the collective development (Towers, Martin & Heater, 2013) of norms and practices of inquiry and consisted primarily of semi-structured interviews with the teacher and videotaped lessons over eight months. In the interviews, Kaye was asked about her intentions for developing students' norms and practices, to reflect on progress and to discuss plans for the following term. Video data were collected in four units designed or modified by the teacher:

Term 1: Problem solving. Students solved closed multi-step problems, individually first, shared in pairs, then discussed solutions in class. (Three lessons videotaped, March)

Term 2: What is the best route for a 'walking school bus'? Students collected and analysed data on how far they lived from school, then designed a route for a walking school bus (www.walkingschoolbus.org). (Three lessons videotaped, June.)

Term 3: How far does a paper airplane fly? Students built a paper airplane from instructions and worked in groups to determine how far their planes typically flew. (Six lessons were videotaped, August.)

Term 4: How long does it take to read a book? Students designed a method to collect data on the time it took to read part of a book and then inferred how long it would typically take to read an entire book. (Five lessons videotaped, November.)

Data Analysis

Data went through non-linear phases of analysis adapted from Powell, Franscisco and Maher (2003), although described here sequentially. In the *first phase*, logs were created of each audio and video file to catalogue content. Timestamps and/or screen shots assisted with visualisation of class discussion and small group work. Sections with potentially rich excerpts were flagged, with emphasis placed on identifying excerpts in which the teacher explicitly scaffolded classroom norms and practices, students attempted to apply these practices, or missed opportunities were noted. In the *second phase*, flagged episodes were transcribed and annotated to note how the particular episode illustrated an example or outcome of scaffolding. In the *third phase*, each episode was reviewed again in reverse order (last video first and first video last) to seek traces of how later examples of classroom discussion practices were developed. This was important to seek the beginnings of and follow up on the results of teacher's intentions, diagnoses and specific actions to promote inquiry norms. Insights were tagged with phrases to assist with identifying emerging threads through the data. The audio files and videos were reviewed again in their entirety to seek further examples that may not have been obvious previously. *Finally,* a few episodes were selected to succinctly illustrate norms being developed or practiced.

Results

We start with a classroom episode in which students were functioning well in their developing mathematical inquiry practices, independently from the teacher. Following, the development of these practices was tracked from the start of the year to observe how the teacher slowly scaffolded this development through particular teaching strategies.

Term 4: Established Norms

A key goal of scaffolding is to be able to hand over responsibility to learners. We begin with an episode near the end of the school year when students were finalising their solutions to answer the inquiry question, *What is the typical time it takes for a Year 4 student to read a book?* The children had learned formal and informal statistical terms such as centre, typical, spread, shape, gap, and "clump" (but not their calculations)—notions that go beyond what is usually taught in Australian Year 4 classrooms. Students prepared a draft poster of their inquiry solution to give to another group to critique. The teacher rotated between groups but had not yet arrived at this group, so one may consider these students as working independently. In the episode below, Wes and Shane offered feedback to Jake, Jonah and Emma.

Wes:	For your table I was maybe wondering like you could write like, be a bit more specific, like time to read a chapter and then like … [calculating] total time reading the book in minutes or something. Because I don't really know what you're talking about.
Jake:	I don't even get it (what we wrote)! Total time? What's the total time? (Mocking themselves for not showing this information on their poster.)
Wes:	… On your diagram here I really like how you made your answers [data] into colours and put it on [a graph], it really is easier [to read] now. … Um, what's like the pattern in your data? [Jake: um. (thinking)] Like range, spread.
Jonah:	There. (Points at graph)
Shane:	Put some borders in between the – (he can't think of the word "clump")
Wes	And you've got it really nicely set out.

Shane: Yeah it's really nice, but put … barriers where most of the data is … because I can't see where it's bunched. (24:17, Classroom video 24 November 2014)

Wes and Shane provided the second group with feedback on how they could improve the presentation of their solution to the inquiry. Wes' language was tentative and respectful in telling the group that there wasn't enough detail to "know what you're talking about". The boys took the risk to challenge the detail shown by the second group, implying that it was a normal practice. Jake's jovial response suggested he did not find Wes' feedback as a personal criticism; and queried how his group could have missed such important information (how they calculated how long it took to read a book). The feedback given by Wes and Shane was non-trivial and genuinely provided the second group with ways to improve their final results. They also gave positive feedback on what the second group had done well, recognising the importance of both kinds of feedback. The students used terms they had learnt to describe distributions (range, spread, clump) and ways to show an interval to estimate the answer (borders or barriers around "where most of the data is" to show "where it's bunched").

These episodes showed one group's exchange as they worked independently to provide genuinely useful feedback to one another. We argue that this exchange provides evidence that the group was tacitly demonstrating a number of inquiry norms such as active listening, justifying and explaining to peers, sharing incomplete ideas, building on the ideas of others, and questioning and challenging ideas. There was no teacher present during this exchange, suggesting that these students were practicing these skills independently (a wireless mic was next to them with the researcher filming from several feet away; the students were used to her presence). We are not claiming that all students demonstrated this level of exchange, and clearly Emma's voice was absent from this particular exchange. However, nearly all groups were observed to be functioning in a similar fashion independently, with the teacher rotating between groups.

Term 1: How the Scaffolding Process Started

The children did not arrive at the start of the year with fluency with these norms. In order to answer our research question, we return to the first term (March 2014). Given our focus on long-term scaffolding, we highlight the teacher's strategies to foster inquiry norms among the children. In an interview before the first lesson, Kaye explained her intentions for beginning to develop the classroom norms and practices.

> I guess it is with classroom culture – it's got to be a model. It's got to be having ways, being creative in ways to ensure everyone is working collaboratively, that everyone is having their say, that everybody's opinions are feeling valued. … Having those strategies when things are not working … [means] you can come and specifically target those elements. (10:35, Interview 7 March 2014)

One strategy that Kaye used to develop classroom norms was to create posters that would help build a language around practices she was expecting. For example one laid out her expectations for quality "classroom talk" with roles for the listener ("Active listeners reflect on others' ideas"), speaker ("clear audible speakers") and group members ("active contributors"). Another poster reminded students that there was more than one way to solve a problem; everyone was expected to think; all ideas were valued; and ideas could be questioned or challenged respectfully. From these models, the teacher regularly acknowledged positive behaviours when she saw them. This simple act was intended to reinforce these behaviours as valued so that students would adopt them independently. In the following episode, Kaye Bluett explained to students what it meant to be an "Active Listener". She elaborated on the responsibilities of both the speaker and the listener from the classroom posters. By providing them with language for their behaviours, "Active Listeners" could become part of the classroom discourse to articulate these expectations.

Mrs Bluett: When we have quality classroom talk in this room, it needs all of you to be *active listeners*. And I'm looking around the room and I'm seeing some really active listeners. They're giving me that body language, they're looking at me and you're taking in what I'm saying. … Now today I had two or three groups on the floor working brilliantly at classroom talk. So congratulations to those six people. In fact something Bill did, (to Bill) you might want to share with everybody, what did you and your partner do for classroom talk?

Bill: Sat beside each other, and um, gives you more of an idea of the story.

Mrs Bluett: Yes, so they were sharing. And … instead of Bill sitting facing his partner, he and his partner actually sat side-by-side. … So when Jonah was reading, Bill could hear what he was saying but he could also *see* what he was saying. So he was actively listening, he was giving himself every opportunity to question what Jonah was saying and [be] an active contributor. (0:41, Classroom video 12 March)

Using students as examples to reinforce what she valued was a regular occurrence in Mrs Bluett's class. The teacher frequently used phrases such as "I like the way that students are…" as a way to scaffold the norms she was expecting. In doing so, she was diagnosing and responding to students in the moment. To further empower students towards an ownership of these behaviours (shifting them towards independence) she had the students themselves explain what they did. Once Bill told the class what he and Jonah were doing, Mrs Bluett re-expressed Bill's words to develop and improve students' language of the norms. Thus re-stating is another strategy for scaffolding by diagnosing and responding to students' expressions of their behaviours. Her exemplification of Bill and Jonah went beyond their visible actions to highlight implications of these actions; it was not the specific actions she wanted students to copy, but the intentions of the actions.

Following this exchange, students moved into pairs to share their work on a problem posed to the class. Kaye felt that developing initial norms of working, speaking and listening collaboratively would be more effective with a series of problems that were shorter in duration and less open-ended. They were told to share their individual solution to their assigned partner and were expected to practice active listening. Most groups had difficulty; for example, groups sat face-to-face with students reporting "at" one another, but not listening. The teacher rotated between groups and stopped the class occasionally as they were working to remind them what they were to be doing and to include a mild level of accountability. Norms require long-term development with ongoing support; therefore, students initially needed to be reminded regularly of expectations.

In their first attempt, students demonstrated the challenges of developing "active listening"; their attempts did not yet try to make meaning of the speaker's intent, as would be expected in an inquiry classroom. However, these initial attempts were important. First, they legitimised the practice of listening to peers, which is not typically a norm in mathematics classrooms. Second, they provided a starting point from which the teacher could diagnose and respond to their progress over time. As expected, norms do not develop quickly but require a concerted effort by the teacher throughout the year. The explicit nature of the teacher's talk and her actions in Term 1 were important as students: were introduced to *what* norms were expected using explicit frameworks, discussed *how* they were to engage in norms through teacher modelling and co-construction, and *developed* these norms through practice and valuing by the teacher. These strategies, only some of which are mentioned here, did not end with the close of Term 1, however they sometimes became tacit as they moved from ideas into practices.

Term 2: Intermediate phase of scaffolding inquiry norms

Developing norms in a classroom which practices mathematical inquiry takes a substantial commitment from the teacher to scaffold students (Yackel & Cobb, 1996). A focus was on giving students repeated experiences during the year to practice norms. Maintaining and developing norms is a different pedagogical skill than initiating them. Kaye now wanted students to begin to practise more advanced skills such as building on others' ideas and taking intellectual risks by sharing incomplete ideas, which require students to be already somewhat proficient at active listening. In one lesson, students were making sense of data each had generated about themselves about how far they lived from school. The teacher took the class outside to create a physical dot plot of their collective data. Kaye had a number of skills and concepts she hoped to develop in this lesson, including extending students' initial strategy of organising the data into two groups and further improving their inquiry norms. In the excerpt below, we focus on scaffolding Kaye used to promote her students to adopt talk around generating ideas. Students were seated in two groups organised by Chloe: students who live less than 5 km from school and students who live more than 5 km from school.

Mrs Bluett: What is the typical distance that students [in this class] live from school? If I look at the way that Chloe has organised our data can we answer that question now?

Chloe: Yes. (pause) Not exactly.

Mrs Bluett: If I ask people how far they live from school, what would I expect the answer to be from what we've just seen here? Jinny? (24:08, Classroom video 17 Jun 2014)

In an inquiry classroom, ideas are public rather than personally owned. Mrs Bluett acknowledged Chloe's contribution as a starting point and used the inquiry question to prompt students to challenge and/or build on Chloe's initial idea. Chloe herself then cast doubt on her idea. Rather than answer, Mrs. Bluett's sought others to respond.

Jinny: Maybe we should put the groups into 0.1 or 1 [km] sets. If we mix them altogether [in just two groups] it will be harder to organise.
Mrs Bluett: All right at the moment, can I say that students in [this class] typically live less than five kilometres from the school? [Jinny: No.] (To the class) Could I say that?
(Mostly students initially respond yes, but then expressed some disagreement)
Chloe: Yes you could!
Mrs Bluett: …*Why* could I say it? (25:31)

Jinny suggested that it may be better to organise them into smaller categories by tenths or whole kilometre distances rather than two large groups. The teacher rephrased the overarching question, prompting students to decide if the current arrangement in two groups would allow them to answer this question or if Jinny's suggestion should be considered. There was an expectation that students justify their answers, so rather than just acknowledge Jinny and Chloe's responses, she pushed the class to explain the reasoning.

Chris: Because there's more people in this [<5km] group.

Mrs Bluett: Because there's a whole lot of people here. … So I *can* say students [in our class] typically live less than five kms from the school. But Jinny is saying that I can make my answer *better*. Jinny wants us to make the answer better by doing *what*? (26:06)

The teacher had created an environment where students were encouraged to think and reason without worrying whether their answer was correct or complete. This invitation to "think aloud" as ideas are formed can encourage students to take risks to share their incomplete ideas. By acknowledging that Chloe's idea was an improvement over unorganised data, Mrs Bluett affirmed that this did answer the question; but suggested that an answer to the question was not enough. She returned to Jinny's contribution to model what it meant to build on others' ideas and pushed students to explain why Jinny's statement improved on the current solution of putting students into two groups.

Whole class discussions like this were not happenstance. Mrs Bluett explicitly worked to extend students' developing practices to become norms by (1) giving them opportunities in context to use the practices which they had adopted, such as active listening, explaining and justifying to peers and expecting there to be more than one way to do a problem; and (2) modelling, co-constructing and reinforcing what appeared to be more advanced practices, such as sharing incomplete ideas, respectfully challenging suggestions and building on the ideas of others, all of which required greater intellectual risk-taking.

The scaffolding the teacher undertook to move them forward required her to reflect on the progress they had made, then develop focused strategies to improve students' work. Kaye discussed this lesson in an interview at the end of the unit:

> They all had how far they live from school and we were all in a big group, "Okay, how are we going to make sense of this data? … [And then] somebody came up with, "We could split them into kilometre groups and we could go from 1 to 11." … I need to see more of this risk-taking. I need to see more of sharing of ideas and then building and working it through. (5:02) … [Next semester I plan] just pulling back a little bit, so that we can start letting them perhaps meet a few of the challenges and hit a few more walls. (12:25, Interview 27 June 2014)

In the interview, Kaye noted that students still needed work in intellectual risk-taking. She saw the need to elicit more independence from students in the inquiry by "pulling back a little bit", allowing them to "hit a few more walls".

Summary

At the end of the year, Kaye reflected on the progress students had made in response to the scaffolding during the year.

> The whole purpose of what we've been building on all year has been you know, taking kids right from that very first stage of having no real notion of what it means to talk with each other and through all the different inquiries we've been doing, to bring us to this. (0:14) … That constant scaffold to try and, making sure that they're on the right page and to try and move forward. … It doesn't matter what we're doing. ... I think it's just a culture that's developed (12:41, Interview 3 Dec 2014)

In order to build towards the independence observed in the excerpt at the beginning of the paper, there was a long road of explicit scaffolding undertaken by the teacher. Her commitment of a "constant scaffold" regardless of what they were doing (i.e., daily and across subject areas) was critical for developing mathematical inquiry norms in her classroom.

Discussion

In this paper we addressed the question: *How can a teacher scaffold the development of norms and practices in mathematical inquiry over time?* In answer to this question we first illustrated students' inquiry practices and norms late in the year, and then analysed the scaffolding process from Term 1 onwards. In line with our view on scaffolding as entailing frequent diagnoses, responsive actions and gradual handover to independence, we focused our analysis on the teacher's diagnoses and intentions of how to respond to what she had observed. These strategies included the use of posters with expectations, frequent reminders of the norms, positive feedback, and many opportunities to enact practices.

The analysis suggests that the meaningful and long-term scaffolding process helped to foster the practices the teacher chose to develop. The teacher's scaffolding responded to

Yackel and Cobb's (1996) challenge of the taken-as-shared mechanics of norms: both *how* to respond and *when* it is appropriate to do so. Kaye Bluett's strategies were meaningful rather than formulaic in that it was not the strategies themselves, but the norms she chose to develop which was the focus of her energies. In term 1, for example, instead of telling students to engage in active listening, she aimed to persuade them of its utility by introducing the reasoning behind active listening and creating contexts in which active listening made sense. Students' successes were validated, which encouraged their adoption. Later in the year, students showed greater comfort with these practices which facilitated the introduction and development of more advanced norms. The time between the excerpts in term 1 and term 4 was nearly nine months, a non-trivial amount of time.

As a case study, our analysis is considered a proof of principle: It is possible to achieve this, and our results illustrate some key ingredients of this teacher's approach. However, case studies have limited generalisability. Working across a diversity of classrooms or on larger scale would therefore be an important topic for future research.

Acknowledgement

This research was funded by the Australian Research Council (DP120100690). The authors wish to thank Sue for her insights in teaching and Janine for her timely support during analysis of the data.

References

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, *9*(1), 33–52.

Fry, K. (2013). Students 'holding' the moment: learning mathematics in an inquiry mathematics classroom. *Mathematics education: Yesterday, today and tomorrow (*Proceedings of the 36th Annual Conference of the Mathematics Education Research Group of Australasia, Melbourne, Vol. 1, pp. 306-313). MERGA.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for research in mathematics education*, *35*(4), 258-291.

Linn, M. C. (2000). Designing the knowledge integration environment. *International Journal of Science Education*, *22*(8), 781-796.

McCrone, S. S. (2005). The Development of Mathematical Discussions: An Investigation in a Fifth-Grade Classroom. *Mathematical Thinking and Learning*, *7*(2), 111-133.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and instruction*, *6*(4), 359-377.

Mercer, N. (2008). The seeds of time: Why classroom dialogue needs a temporal analysis. *Journal of the Learning Sciences*, *17*(1), 33-59.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, *22*(4), 405-435.

Quintana, C., Reiser, B. J., Davis, E. A., Krajcik, J., Fretz, E., Duncan, R. G, Kyza, E., Edelson, D., & Soloway, E. (2004). A scaffolding design framework for software to support science inquiry. *The Journal of the Learning Sciences*, *13*(3), 337-386.

Reitman, W. R. (1965). *Cognition and thought: an information processing approach*. Oxford UK: Wiley.

Smit, J., & van Eerde, D. (2013). What counts as evidence for the long-term realisation of whole-class scaffolding? *Learning, Culture and Social Interaction*, *2*(1), 22-31.

Smit, J., AA van Eerde, H., & Bakker, A. (2013). A conceptualisation of whole-class scaffolding. *British Educational Research Journal*, *39*(5), 817-834.

Towers, J., Martin, L. C., & Heater, B. (2013). Teaching and learning mathematics in the collective. *The Journal of Mathematical Behavior*, *32*(3), 424-433.

van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher–student interaction: A decade of research. *Educational Psychology Review*, *22*(3), 271-296.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458-77.

Middle Years Students Influencing Local Policy

Margaret Marshman
University of the Sunshine Coast
<mmarshma@usc.edu.au>

Middle Years students often do not see mathematics as useful. Authentic and real mathematics tasks and artefacts are frequently advocated as arresting this situation. However, often such experiences are contrived and lack authenticity. This paper reports on how a group of Middle Years students used mathematics and technology to engage in a real community issue, of the need for a teenage safe space, to inform local Council policy. Data were collected for this case study via journal observations and reflections, semi-structured interviews, samples of students' work and videos of students working. The data were analysed by identifying the main themes that were connected with designing and locating the space and focused on the stages of the statistical investigation cycle used. How this impacted students' beliefs about the usefulness and value of mathematics is discussed.

Science and mathematics knowledge and skills are important for our increasingly technological and information-rich society. However, many middle years students opt out of mathematics and science as soon as they can. The review of literature by Tytler, Osborne, Williams, Tytler, Cripps, & Clark (2008) found substantial evidence that before students are 14 years old, they decide that they do not wish to pursue mathematics and science careers and are reluctant to change their minds. A new pedagogy is required to change students' perceptions about the value and relevance of mathematics and science.

Background

To cater for and nurture academic excellence in Middle Years students there is a need to design curriculum that involves them in authentic, meaningful activities with a real purpose. At the core is the view that to meet the intellectual needs of students in the 21st century they need to be given opportunities not only to *consume* knowledge but also to *produce* new knowledge that will benefit others.

The theoretical framework of this paper is located in the Knowledge Producing Schools (KPS) (Bigum & Rowan, 2009) pedagogical approach, in combination with the statistical investigation cycle (Wild & Pfannkuch, 1999). KPS is a variant of project-based learning where students work in teams to formulate and solve a problem or issue in their local community that is important to them. They are encouraged to formulate and model solutions to the problems on which they are working. Typically this might involve improving something in their local environment, solving a local problem, or designing and delivering a product for their local community that makes a difference. These activities: "produce some kind of product – be it a discussion, a story, a plan, a project or a product – that can be externally validated and which thus forms a bridge between school and not-school." (McGrath & Rowan, 2012, p.69)

In the KPS approach, the classrooms/schools become the organising base for learning but are not the only sites at which knowledge work takes place. The teacher is no longer the ultimate authority on the knowledge produced. Students draw on relevant expertise (not necessarily school personal) as required by the problem. The outputs are prepared for community groups who value what is produced and students develop new and interesting relationships with the local community and a broad set of experts. In this way, work results in 'products' that approximate, as closely as possible, *expert* productions in approach and

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins*
(Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia),
pp. 405–412. Sunshine Coast: MERGA.

quality. For KPS to be successfully managed, the teacher must be willing and able to establish partnerships beyond the school. They need to be able to source the necessary expertise to ensure intellectual rigor and genuine feedback. The teacher also needs the courage to take a risk and step back and allow the students and community experts to drive the process. Student engagement has been shown to increase when the students determine how they will achieve the goal (McGrath & Rowan, 2012).

The KPS framework proposes the following: students are positioned as the producers rather than consumers of knowledge and produce products with a genuine purpose and value beyond school assessment regimes; students are actively engaged and have a real world audience; all students and all forms of knowledge are valued; the audience facilitates a connection to the broader community which is involved in the actual learning process; and that the experience creates positive relationships between diverse children and knowledge, and between diverse children and the community. (McGrath & Rowan, 2012).

The statistical investigation cycle of problem, plan, data, analysis and conclusion (Wild & Pfannkuch, 1999) is used by statisticians and has been adapted for use by school students by Census@School in New Zealand (Figure 1). The first step in the cycle is to define the *problem* which includes understanding the context and how to approach the question. The *plan* stage includes: deciding what and how to measure; the design of the study; how to record and collect data. The *data* are then collected, managed and cleaned before the *analysis* where the data is sorted, tables and graphs are created, patterns are identified and a hypothesis is generated. The final stage is the *conclusion* which includes interpreting the findings to draw conclusions, communicating these appropriately and perhaps generating new ideas. This may also mean that it is necessary to go through the cycle again.

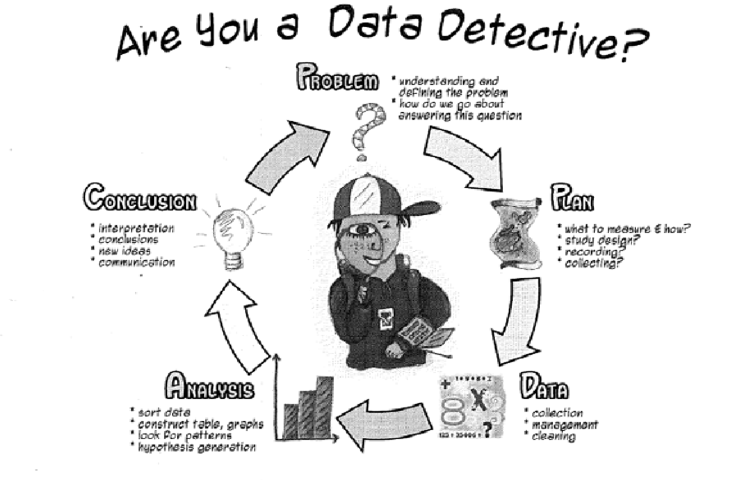


Figure 1. The statistical investigative cycle (census@school, n.d.)

Combining the KPS framework with the statistical investigation cycle provides a new mathematical pedagogical approach that suited the work undertaken with Middle Years students in this study.

The Study

This study brought together a group of twenty-seven Middle Years students (from Years Five to Nine) who were identified as Gifted and Talented, on the basis of their general intellectual ability, specific academic aptitude, creative or productive thinking, leadership, or visual or performing arts skills. This provided a diverse group of participants with different talents and expertise, and a range of ages. The students called themselves Project Beyond Limits (PBL). Generally the students met weekly for about four hours, half in school time and half after school. They worked regularly with a teacher from the school and the researcher, a mathematician/mathematics educator, with input from the Council's Community Youth Development Officers (Youth Officers), a third year design student and the Youth Activity Space (YAS) Project Manager from the Council.

The Project

The brief for the PBL was to plan and design a project that would reflect their collective talents. The students decided that they wanted to create a *teenage-safe space* that was teenage safe, and also family friendly, would include multicultural artwork that represented the community, and had a landscaping and design element. They wanted seating and barbeques to encourage a wide range of people to use the park as a meeting place and to make a visible difference in their local community.

To bring this project to life the students approached the local Council to seek permission to build their teenage safe space. It was at this point that the project lifted from being real-life, to being authentic, as before Council was a plan to build a Youth Activity Space.

Methodology

Researching the work of the PBL adopted a case study design. Data were collected by:

- regular journal observations and analytical, critical reflections throughout the project by the researcher, who attended most sessions, and included discussions with the teacher;
- audiotaped interviews with students and the Community Youth Development Officer and the project manager;
- copies and photographs of student work;
- videos of some sessions while students were planning their strategy to petition Council.

The data collected in this project were analysed by identifying the main themes and issues that emerged, connected with designing and locating the space, looking for the 'working mathematically' moments and in particular the use of the statistical investigation cycle (Wild & Pfannkuch, 1999). The research question was, 'How did these students work mathematically as they solved their community problem to develop a *teenage-safe space*?'

Results and Discussion

This section focuses on the how the students used mathematics to communicate to the Council their preferences for the location and function of the YAS. Results are presented as stages in the statistical investigation cycle: problem, plan, data, analysis and conclusion.

Problem

Having identified two possible locations the Council needed to engage in public consultation. The chosen location was close to the community business centre and was perceived to be a safe public area due to the high frequency of passing adult pedestrians.

The business leaders were concerned about losing a car park and believed that it would attract an undesirable element to the area. Some staff from the school were concerned that being close to the school it would encourage truanting. However, staff who lived in the area believed the development of the site as a *teenage-safe space* would lead to a positive outcome for local young people. This location then became the students' *problem*. How could they plan a design concept, collect and present data that demonstrated the chosen location could enable students to feel safe and welcome?

Plan

For the *planning* stage, the Youth Officers worked with the students to decide how they would determine what the local young people wanted. The usual Council methods included: talking to young people in parks and shopping centres consulting with specific youth organisations, programs and services; conducting online surveys and discussion forums via Facebook and blogs; and marketing with posters, flyers and school newsletters. The students were concerned that they had no knowledge of, or input into, any of these consultation processes previously so they believed that these processes excluded a large proportion of 'everyday' young people. Consequently, they decided to personally hand a survey to every student in the two schools (later changed to every student in Year Five to Year Twelve). The students believed that this would give a much better representation of the views of young people aged up to seventeen or eighteen. This then was how they would *collect* their data.

The Council's main focus for the space was for a skate park as both locations adjoined other parks. However, students did not feel safe in either of these parks so they needed to design their surveys to enlist the support of other young people to ensure they would feel safe in the chosen location. One of the Youth Officers worked with the students to design the survey. (the other was transferred to another project.) The Council had assigned the bulk of the money for a skate park, but this was not the students' top priority. This meant the survey needed to include questions that reflected their aims. This was determining *what to measure*.

It proved challenging *to design* questions that could be easily answered by ticking a box but allowed their aims to be included without the questionnaire containing leading questions and therefore having experimenter bias. Students needed to consider what was included in the other major skate parks in the region that they had visited, for example, amenities, drink fountains, playgrounds, seating, barbeques etc. and how their aim of a teenager-safe space would be included. After writing the survey they waited a week and then answered it themselves. This led to further refinement of the survey as one student said, "I don't know what the question is asking. It made sense last week." Further changes were made when students were asked whether it would be possible to argue for their aims using responses from the survey.

> This provided 'Ah ha' moments for many students about what is involved in writing survey questions and how rewording questions made a difference to the way it is answered. This was highlighted by the comment, "Is it always this hard to write a survey?" (Researcher reflecting on student responses).

The survey was trialled with a couple of classes before the final version was decided upon. The Youth Officer's reflections on this process of designing the survey included:

> My role as development officer was to ensure young people had a voice in the process. … ensure they understood their roles and responsibilities … I knew the type of questions Council would want to know, the data they would be able to crunch … meaningful data, relevant data.

Researcher: But the questions weren't in quite the direction you thought they would go.

The Youth Officer: No … they are the experts in what young people want. … Young people had identified what they wanted in the space … a space inclusive of all young people … Council wanted a recreation space … young people wanted more than that - the social aspect. … I think it just came out.

The questions the students used to draw out their ideas are shown in Figure 2 with the response choices which included the safety issue overtly in question 2, and then covertly in question 6 with the possible inclusion of facilities such as children's play area, fitness equipment, picnic/barbeque/eating area, tables and seating and a stage for entertainment which would also appeal to adults. This inclusion of adults was important as an adult presence would discourage anti-social behaviour. Therefore students could collect data to inform the policy for a teenage-safe space that was inclusive of the whole community.

2. Which of the following helped you choose the location for the Nambour Youth Activity Space (if any)?

- ☐ I like the current location
- ☐ It's easier to get to
- ☐ It's closer to town
- ☐ It's closer to amenities e.g. public toilets
- ☐ It's a safer space
- ☐ It's a family friendly space
- ☐ Privacy
- ☐ More opportunities for youth activities

5. How would you use the Nambour Youth Activity Space?

- ☐ Skateboarding
- ☐ Bike riding
- ☐ Scooting
- ☐ Entertainment
- ☐ Meeting and hanging out with friends and family
- ☐ Picnic / BBQ / eating
- ☐ Sport and recreation activities

6. Other than a skate park, what else would you like included in the Nambour Youth Activity Space?

- ☐ Art wall or spaces
- ☐ Children's play area
- ☐ Drink fountains
- ☐ Fitness equipment
- ☐ Grassed areas and gardens
- ☐ Picnic / BBQ / eating area
- ☐ Public toilets
- ☐ Shade / Shelter areas
- ☐ Stage for entertainment
- ☐ Tables and seating

Figure 2. A sample of questions from the survey.

To achieve the largest response rate, with the *recording* and *collecting,* the surveys were handed out either on year level parades, or in class for the younger students. This was a logistical challenge for all involved and at times it was difficult for the teacher to maintain students' motivation. Groups of students spoke on year level assemblies or to individual classes about the survey, and then hand out and collect the completed surveys. Students impressed on their peers that by completing the survey they were having a voice in the decision-making. Eight hundred and thirty-six students completed the surveys.

Data

Data management included collating and presenting their data. Having prepared in advance Kerry, (Year Nine), led the discussion on how the surveys would be analysed. She discussed which graph would be the most appropriate and whether for some questions the mean, the median or the mode was the most appropriate measure to use. This was an interesting discussion as the students debated which type of graph would give a clearer picture and better support their argument as well as which measure of central tendency to use. The fact that much of the data was not numerical caused confusion for some students who just wanted to 'calculate the average'. This then became an opportunity to build some mathematical understanding about what these measures really meant, why you need more than one and which one to use in which situation.

Some students had drawn graphs using Excel but none had used it to collate data. The CYDO described how Excel was frequently used by the Council to collate surveys from public consultation and helped the students to set up a template to enter their data. Students were given responsibility for entering class sets of data into the template for analysis.

Analysis

To help students understand their data, the teacher gave some stimulus questions for each student to explore the data they had entered. This included: "What was the relevance of questions 2 to 6? Which location do the students prefer? What were the top two reasons for selecting this location? What was the most common way your students would use the … Youth Activity Space? What are the four most important features that are wanted in the new … Youth Activity Space?" This was a valuable activity for all the students who began to understand the reasons for the choice of data and all students had some experience analysing the data. For example, they were actively discouraged from counting the number of zeroes and ones in their spreadsheet and were instructed to use 'Countif' and/or 'Sum'. The students were then given a copy of two previous Council reports, as an indication of what was expected, and asked to allocate themselves to either the report writing or report analysing team. After spending many months waiting for the Council to make a decision on possible locations, the Council now wanted the public consultation to happen fairly quickly. The writing group developed an outline of 'report inclusions' and who was responsible for each section. This was sorting the data in preparation for communicating their findings. The teacher worked with the writing group and the researcher worked with the maths group.

The maths group divided the questions between smaller groups. Each group determined what type of graph would best represent the responses from their question, created it using Excel, analysed the graphs and presented those findings. This usually meant converting data to percentages either using a calculator or formulae in Excel. For these students who had never used Excel in class there was a lot of mathematical exploring and learning happening as they were looking to produce mathematically correct, persuasive results rather than the colourfully presented graphs they had initially wanted to use.

Four students worked on the demographic analysis, Matt (Year Nine), Tim (Year Eight), and Jim and Callum (Year Six). The survey had collected data on age groups, gender, and which suburb the students lived in. Once the graphs were drawn, the researcher asked whether their sample was representative of young people living in the area. Following this discussion the demographic group visited the Australian Bureau of Statistics website to collect data about where young people in the region lived to determine whether the school population was similar to the total population. They calculated the percentage of young people who responded to the survey compared with the total population of young people aged 5 to 19 that lived in each suburb in the region. This provided an "Ah ha" moment for Matt:

> I worked on the demographic analysis. One of the things I learnt was how to work with percentages. How to calculate them from just ordinary numbers.

This is an interesting statement made by a high achieving Year Nine student who has been 'doing' percentages since Year Six. Students find percentages notoriously difficult (Hãwera & Taylor, 2011). This is due in part to the need for the learner to make connections between fractions, decimals, ratios and proportions (Reys, Lindquist, Lambdin, & Smith, 2007) and the need to consider the percentages in the context they are working in (White & Mitchelmore, 2005).

Tim also commented that the demographic analysis had improved his mathematics and computer skills.

> While doing the report for the Council I extended my knowledge of doing both maths and computer skills. PBL also helped me with my team work and my time management.

Jed in Year Six was excited about the usefulness of the mathematics he was doing.

> I enjoyed PBL because we actually got to do something useful in the community instead of just doing maths sitting in the classroom. We could actually go out there and design a skate park, something awesome.

Conclusion

For the conclusion of the statistical investigation cycle the students wrote a report and presented it to the local Council. Students needed to *interpret* their data, draw *conclusions* and *communicate* these findings to the Council. For John it was linking mathematics with the real world has helped to deepen his knowledge of and appreciation for the usefulness of mathematics:

> Being part of Project Beyond Limits has let us work together outside school time. This has extended our skills beyond core subjects using the real world.

Figure 3 shows a small section of how one student (John) communicated his findings on how the students would use the YAC.

This concludes that the students of ... High School and … State School surveyed would prefer to use the new ... Youth Activity Space for meeting and hanging out because almost half of the students selected this as their preference for question 5.”

Figure 3. Part of the report showing how the students would use the YAS. (NSHS and NSS, 2012)

The project manager from the Council was very supportive of the students' efforts writing:

> The students provided much support and information that aided in the successful site relocation … the students and teachers involved played a vital role in the success of this project. The community would not have this outcome without the assistance of the Project Beyond Limits Team.

He believed that it was the students' report that swayed the Council to accept the young people's choice of location:

> [T]he collective information that the students provided in their report was used as additional evidence to further support a report that was compiled internally … the students report strengthened the argument for the endorsement. The critical aspects of the report that were used were the number of people surveyed and the outcomes of the survey, for and against the relocation.

The Council minutes of the meeting indicated that the motion was passed thirteen to one and that one member raised a point of order that was upheld regarding people speaking out of turn (Sunshine Coast Council, 2013). This reflects the fiery nature of the meeting and the conflict between the young people and the Business Alliance.

Conclusions

This paper discussed how a group of students used mathematics to communicate the desires of the young people in the community to the decision makers in Council, influencing local policy. As the students worked through the statistical investigation cycle they identified their problem, planned an approach that included designing a survey, collected data by surveying the students in their schools, collated and analysed the results of the survey, and concluded by writing a report which they presented to Council. The report highlighted the young people's desire for a teenager-safe space that was inclusive of all members of their local community. Utilising a KPS pedagogical approach, the classroom teacher from one of the schools enlisted support from a Council Youth Officer and a mathematician/mathematics educator from the local university to work with these students. Through this project these students saw the value and usefulness of mathematics as they developed their knowledge and skills of a number of mathematical concepts including percentages, mean, median, mode, graphing, writing survey questions, and analysing statistical data. The value of this KPS pedagogy was highlighted by a high achieving Year 9 student who learnt "how to work with percentages. How to calculate them from just ordinary numbers." It had taken this project for him to develop a deep understanding of percentages rather than his previous procedural understanding.

The students were proud of their efforts to make a difference in their local community and the mathematical, statistical, computing and social skills and understandings they learnt along the way. Jack in Year 8 summarised this with:

> During PBL I have learnt many skills like, how to work in a team with many people from different age groups. This has helped learn how to use other people's skills. It is also very good and I learnt many things on the computer, such as graphing and how to analyse certain questions like in the report that we did and it's a very fun experience and I've learnt many things.

Acknowledgements

This research was funded by a University of the Sunshine Coast Research Grant.

References

Bigum, C., & Rowan, L. (2009). Renegotiating Knowledge Relationships in Schools. In S. E. Noffke & B. Somekh (Eds.), *The SAGE handbook of educational action research* (pp. 102-109). Los Angeles: Sage.

Census@School (n.d.) Are you a data detective? Retrieved from www.censusatschool.org.nz

McGrath, C., & Rowan, L. (2012). Things that matter: Student engagement and technologies in Knowledge-Producing Schools. In L. Rowan & C. Bigum (Eds) *Transformative approaches to new technologies and student diversity in futures orientated classrooms*. Dordrecht, The Netherlands: Springer.

Hãwera, N., & Taylor, M. (2011). 'Twenty percent free!' So how much does the original bar weigh? *Australian Primary Mathematics Classroom 16*(4) 3-7.

Nambour State High School and Nambour State School (NSHS and NSS) (2012). *Nambour Youth Activity Space Youth community engagement 2012.*

Reys, R., Lindquist, M. M., Lambdin, D.V., & Smith, N. L. (2007). *Helping children learn mathematics*. John Wiley & Sons: New Jersey.

Sunshine Coast Council (2013). Minutes Ordinary meeting 13 January, 2013. Retrieved from http://www.sunshinecoast.qld.gov.au/addfiles/documents/minutes/Signed%20minutes_OM_310113.pdf

Tytler, R., Osborne, J., Williams, G., Tytler, K., Cripps, & Clark,J. (2008) *Opening up pathways: Engagement in STEM across the Primary-Secondary school transition*. Retrieved from http://www.innovation.gov.au/skills/ResourcesAndPublications/Documents/OpenPathinSciTechMathEnginPrimSecSchTrans.pdf

White, P., & Mitchelmore, M. (2005). Teaching percentage as a multiplicative relationship. In P. Clarkson, A Downton, D. Gron, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds), *Building connections: Theory, research and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, pp. 783–790). Melbourne: MERGA.

Wild, C.J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review 67*(3) 223-265.

Early Years Teachers' Perspectives on Teaching through Multiple Metaphors and Multimodality

Paula Mildenhall
Edith Cowan University
<p.mildenhall@ecu.edu.au>

Recent research findings indicate that using multiple metaphors in multimodal learning experiences are effective teaching approaches in early years mathematics. Using a social semiotic lens this paper reports on eight early years teachers' perceptions of this approach whilst engaging in a small collaborative professional learning group. This group focussed on observing video footage of one teacher implementing multiple metaphors such as the number line and using multimodalities such as gesture, drawings and concrete materials in her classroom. Analysis of the data revealed variations in the teachers' perceptions of this particular teaching approach.

In 2013 the researcher and author of this paper conducted a small scale study to explore a West Australian teacher's use of multiple metaphors in an early years classroom. In that study, with careful scaffolding by the teacher and in paired learning experiences, the teacher facilitated the children to engage with multiple metaphors such as motion on a path (number line) and parts of a whole (ten frames) (Mildenhall, 2014). This study is a sequel to the 2013 research; it involved the researcher gathering perceptions of the 2013 research from the teacher researcher, 6 pre-primary teachers (teaching children aged 5-6) and 2 kindergarten teachers (teaching children aged 4-5), 2 numeracy coordinators and 1 primary principal, who comprised a collaborative research group. The researcher's purpose in this study was to explore how the members of the research group perceived this teaching approach and whether they thought it would be appropriate in their particular context.

Conceptual Framework

The researcher used social semiotic theory as a theoretical lens as this research is focussing on how teachers use these various multimodal representational forms, such as language, gesture, symbols and objects, as semiotic resources from which students could generate meaning (Lemke, 1990). Multimodality can be defined as the modes of learning that are intertwined across sensory modalities (O'Halleron, 2011). Although semiotics was traditionally associated with linguistics (Lemke, 1990), mathematics education has broadened its definition to encompass the complexity and inherent multimodality of the classrooms (Arzarello 2006; Lakoff & Núñez, 2000; O'Halloran, 2005; Radford, 2003). It is now becoming apparent that modalities such as bodily movement and gesture are integral parts of the learning process (Radford, 2003).

Multiple Metaphors

Recently the value of metaphor as an important mathematical learning tool has also been observed (Lakoff & Núñez, 2000). In order to understand what a metaphor is, it is useful to consider the following statement "these metaphors, which map inferential structure of a source domain on to a target domain, allow us to conceive abstract concepts in terms of more concrete concepts" (Núñez, Motz, & Teuscher, 2006, p. 133). Lakoff and Nunez (2000) claimed that there are four main metaphors used when teaching number and arithmetic which "allow us to ground our understanding of arithmetic in our prior

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 413–420. Sunshine Coast: MERGA.

understanding of extremely commonplace physical activities” p. 54. Young children need to be engaged in learning experiences that use metaphors such as the number line as this metaphor does not appear to be intuitive to children (Edmonds-Wathan, 2012).

Using multiple representations and metaphors has been suggested as an effective strategy by Griffin (2004). Griffin stated that by exposing children to multiple representations of a number in one activity children can gradually come to the ways that they are equivalent. Bills (2003) found that often children talked about mathematics using the metaphors they are familiar with and he asserted that children could be impeded if they have not been exposed to multiple metaphors. Ainsworth, Bibby and Wood (2009) do point out that as multi representational environments can be difficult for children and a single representation can result in more successful outcomes. They concede that this success is only possible “if the design of the represented world ensures that this one representation encapsulates all the necessary information” (2009, p. 59). As it is more likely that each mathematical metaphor would have limits and it logically leads to the perspective that there is value in providing students with multiple metaphors in order to develop their mathematics.

Purpose and Research Questions

As noted above the researcher’s purpose in this study was to gather and analyse teachers’ perceptions of the research (including trialling it in their own contexts) conducted by the researcher in 2013. The researcher approached several schools to recruit members for a small evaluation teacher professional learning community. These recorded perceptions would inform mathematics educators if it was possible to replicate the 2013 findings in a different context and inform the researchers’ future research. The research question for this study was:

- What are the early years teachers' perceptions of a teaching approach that focuses on the pre-primary teachers using a multi- semiotic approach?

Methodology

A case study was selected as the methodological approach for this study because it was ideally suited for collecting multiple sources of data in a rich context (Yin, 2009). A case study is a bounded system (Yin, 2009) and the case was the professional learning community. In qualitative research it is important that participant voices are prominent (Hatch, 2002; Patton, 2002). As this study focussed on teacher perceptions it was important to design the study so that the teachers’ voices were heard (Patton, 2002) but acknowledge that these voices were perceptions of the subject matter. The main method for the data collection was a focus group discussion (Kruger, 2009). This method is appropriate for providing insights into the matter under investigation: the teachers’ perceptions of the multi-semiotic approach in regards to their own personal context (Rabiee, 2004). It is important to declare that the researcher had a bias in that she believed in the value of a multi-semiotic approach to teaching.

Research Participants

Purposeful sampling was used to recruit early years teachers (teaching children aged 4 to 6) from schools that were from a variety of socio-economic backgrounds. The invitations, using email, were to government and non-government schools. From the school teachers who replied, stating that they were interested, two were from independent schools and one worked in a government school (the same school as the teacher/researcher). Two of the schools had previously worked with the researcher using multiple metaphors and multimodalities and one interested contact, who was the school numeracy coordinator, had no previous contact with the university.

These teachers then invited other early years teachers from their own schools and together they formed this community. At the first community meeting, members examined the findings from the previous research project "Semiotic resources in the kindergarten classroom" and particularly the highlights video package.

The highlights video package from the 2013 study showed a teacher in a low socioeconomic school using multiple metaphors where she focused on the use of the ten frame and the number line and discrete objects to teach effective counting, more or less than 5, and the addition of two single digit numbers. After this initial meeting the teachers implemented the use of the ten frame and number line in their own classrooms and reflected on this. The group then met regularly to share their reflections on their teaching.

Because the data gathering technique employed in this research study was dependent on a manageable group discussion, the ideal number of participants for the study was considered to be approximately 8-10 (Kruger, 2009). As it happened, the research group comprised of 11 participants, but during the four meetings there were some absences due to illness or other commitments so the attendance at the meetings ranged from 8 participants to 11.

Data Gathering Techniques

Data gathered in this study included: 1) teacher journals and work samples, 2) research field notes from regular meetings, including the research participants' own notes made whilst trialling the suggested approach, and 3) full transcripts from the audio recordings of the focus group meetings. The researcher was the facilitator of the research group and she explicitly assured participants that her aim in the research was to trial and develop this multiple metaphor and multimodal approach in different contexts, and therefore, everyone's opinion was to be respected.

Data Analysis

In line with the research question, the reflections of the participants were focussed on their perceptions of a multi-metaphor and multimodal teaching approach. Using NVivo10, the researcher entered all of the data into the software package. Although the study had a particular focus and was therefore somewhat deductive (Bitektine, 2008), at this stage a grounded theory approach was used to explore what the data revealed about teacher perceptions. The researcher conducted initial coding (Charmaz, 2014; Glaser, 1978), which involved reading the full transcripts from the focus group interviews, teacher journals and student work samples, and looking for the participants' viewpoints from a sentence, a paragraph or a picture (De Wever, Schellens, Valke, & Van Keer, 2006). In this way the researcher aimed to understand and represent the participants stand points.

After this was completed, the researcher commenced the second phase in the coding process: focussed coding (Charmaz, 2014). This involved recoding the initial codes to identify important themes pertaining to the teachers' perception of this particular approach to teaching early years mathematics and these are shown in the findings as focussed code/theme.

Findings and Discussion

The analysis and coding of the comments offered by each of the participants in the study revealed three major themes that were specifically focused on how the teachers perceived a multi-metaphoric and multi-semiotic approach.

Focused Code /Theme 1: Perception that Using a Multi-Metaphoric Approach was Valuable

Four of the teachers chose to incorporate multiple metaphors into their teaching. Three of those teachers, Polly, Diane and Brenda were able to explain how they had used all three metaphors of parts of a whole (the ten frame), points on a line (the number line) and the discrete objects in one activity (Figure 1). These teachers appeared to understand the importance of exploring the concepts deeply and perceived it to be a successful approach. They were able to identify that, just because children could articulate a mathematical idea using one metaphor, this didn’t necessarily mean that they could articulate it in another. The fourth teacher, Toni, incorporated all three initially but this adoption slightly waned as time progressed (the reason for this will be discussed later). The four teachers also had mentoring support in the school, such as the teacher/researcher in the 2013, or a numeracy support teacher who had worked as a research assistant on the 2013 research and this appeared to support the implementation as Toni stated “When I watched the videos because we watched them with Natalie (the mentor) I got a bit excited because I thought I've been doing some of this” (Figure 1).

Two of the strong adopters of the multiple metaphors also began to consider how their teaching could be multi-modal using gestures. Donna stated she used “hands to show bigger than/ smaller than” (Figure 1).

Initial codes	Samples of quotes
Teachers found value in teaching across three metaphors	So we had the sock we had to count with and we used the ten frames and on the number line we used a frog. They had four different items in ice cream containers … and I showed them how to use the other hand to scoop into the cup …We used the ten frame … and they quite liked it. I would just say “so what is 7” and they would say back “it is five and two” and we used the number line.
Mentoring in school supported multi-metaphor approach	I did the same as Julie … I set it up very much the way Julie set it up That's alright I'm Toni, forgot my bits for this. When I watched the videos because we watched them with Natalie (the mentor) I got a bit excited because I thought I've been doing some of this
Children found it a challenge to transfer from one metaphor to another	It’s funny the children that I thought had got it when they'd drawn these beautiful tens frames because they were getting it and then when I was talking about it on the number line they weren't getting it. So he scooped out approximately 12 counters. He then counted 1 to 1 correspondence up to 12 … then asked could he find that same number on a number line … So I had a number line, so he then pointed to the 1 and the 2 on the number line at the start so I praised him for finding the two numbers and then I said that was number 1 and 2 not number 12 which I'm sure was very confusing.
Metaphorical gestures were used in a multi-modal approach	Sweeping my finger along the ten frame to show direction I was more aware of the language I was using, how I used my body language and gestures to communicate Hands to show bigger than/ smaller than

Figure 1. Example of codes in focussed code/theme 1.

Theme 2: Factors that Impeded the Adoption of Using Multiple Metaphors in Early Years Teaching

Five of the teachers, Caris, Elle, Gemma, Lucy and Toni had some reservations about using multiple metaphors. Whereas all of the teachers perceived that the use of these tools separately had some value, and Toni was initially very enthusiastic about multiple metaphors, in their reflections four of the teachers (Caris, Elle, Lucy, and Toni) appeared to view that if children could solve the problem using one metaphor there was no reason to explore the same problem using a different metaphor (three of those teachers came from the school without the strong mentoring support). Interestingly, it did appear that in one class, even when the teacher (Lucy) focused on just using the ten frame, which “really came in handy as a different way of explaining the addition process “(Figure 2), some children chose to seek the number line out and use it as well as the ten frame suggested to them. The biggest factor that appeared to impede the adoption of this approach by three of the teachers was the use of a mathematics textbook scheme in their pre-primary classroom.

Lucy outlined that the textbook led the teachers' pedagogical approaches rather than allowing them to make independent pedagogical decisions (Figure 2). One of the teacher's planning documents indicated that their text book scheme did not incorporate any multi-semiotic approaches and it may have encouraged the pencil and paper activities to be done quite separately from other more concrete activities.

Codes	Samples of quotes
Focus on solving problems	The ten frame really came in handy as a different way of explaining the addition process. In terms of the number line I basically just used that more again as a tool to assist with reversals. So we did it on the number line and they ended up actually not using the number line and they were quite enamoured with using the lines to count.
Textbook approach followed	Yeah yep so some of them are reversing their numbers still and stuff but yeah I mean the tally bit is hard and we were talking about that because with our i maths program we don't really focus on that until term 3/4. I think that I think I mentioned before we do the iMaths program in pre-primary and in particular this term the ten frames have really assisted us because obviously like, you get the text book and we always go through the text book

Figure 2. Example of codes in focussed code/theme 2.

Theme 3: Using Multiple Metaphors to Develop an Awareness of Pattern and Structure of Computational Strategies

Using the resources of the ten frame, the number line and the discrete objects, some of the teachers commented that the children were able to reason mathematically and use the resources to match their thinking. Some students were at the foundational stage of the Australian Curriculum (Australian Curriculum Assessment and Reporting Authority, 2011) and were using counting discrete objects as their only strategy. As children developed, and began to identify the concept that the number could be partitioned, the teachers reported that the children began to use different computational strategies. Toni noticed that her children used "counting on" as the first strategy the children implemented (Figure 3) which corresponded with the literature (Sarama & Clements, 2004).

The pre-primary teachers who did not use this multiple metaphor perspective did not mention this type of interaction with their children. Their main focus was on the concept of "*altogether"* which was limiting children to only think of addition as "counting all". They used a textbook "iMaths" that was aligned to the Australian Curriculum and at the "Foundation" stage there is only a requirement to model addition, which the teachers had implemented. This suggests that following the Australian Curriculum too prescriptively may prevent the children's potential from being realised.

Codes	Samples of quotes
Learning to count effectively	Then they had a strategy if they don't know what it looks like, everybody knows what 1 looks like, so they start at 1 and then they count on and when they get to 8 oh that's what it looks like and then they record the number. Toni: And then what I did with a few kids was get them to put it on the number line 5 and then adding 4 I get this number here. Paula: Oh great yeah so you did actually count on on the number line. Toni: And they were doing the back to zero, and going 1,2,3,4,5;
Developed mental strategies such as counting on, doubles	We added them together - one used the counting on method the other used the number line We'd used this tens frame. I had to demonstrate that you fill the top in first and I said, because we've got two colours red and yellow, and I said you have to use the same colour at the top so I explained that first and it was funny how some children, especially with 6 they want to do 3 and 3 ah, number line and counting on and my way of being able to identify ... count on when she did the others she said I used the counting on strategy, tally marks, drawing a picture and then writing the number sentence
Partitioning	Understand that numbers are made up of 2 or more parts, reinforcing the concept of part, part , whole.

Figure 3. Example of codes in focussed code/theme 3.

Conclusions

It is not possible to generalise from this small scale study. In this study and particular context, three of the teachers were strong adopters of a multiple metaphor approach (using the ten frame and number line in the same learning experience) into their practice and these were the teachers who also had the mentoring support in their individual school. The teachers who only had access to the focus group sessions, without mentoring support in school, had the lowest level of adoption. The latter group of teachers also used a school textbook scheme and this, which did not include a multiple metaphor approach in its texts, appeared to impede the implementation of the approach.

The four teachers who were reluctant to use multiple metaphors did mention that once children could solve the mathematical problems using one metaphor i.e., the ten frame, they did not extend the learning experience by exploring the same mathematical concept using a different metaphor. This approach was the one recommended by the text book that they followed. In a climate where there is not one clear approach to teaching mathematics, it is understandable that teachers rely on published textbooks and their suggested approaches (Shoenfeld, 2004). Future research is now planned by the researcher to create a collaborative research study with a Year 2 teacher to explore how to implement this multiple metaphor and multi-modal approach with slightly older primary school children and this will be reported on at a later date.

References

Ainsworth, S., Bibby, P., & Wood, D. (2009). Examining the effects of different representational systems in learning primary mathematics. *Journal of the Learning Sciences, 11*(1), 25-61.

Arzarello, F. (2006). Semiosis as a multimodal process. *Relime (Revista Latinoamericana de Investigación en Matemática Educativa),*(Special issue), 267-299.

Australian Curriculum Assessment and Reporting Authority. (2011). The Australian curriculum: Mathematics.

Bills, C. (2003). Metaphor in young children's mental calculation. Retrieved 13th March, 2015, from http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG1/TG1_bills_cerme3.pdf

Bitektine, A. (2008). Prospective case study design: Qualitative method for deductive theory testing. *Organisaitonal Research Methods, 11*(1), 160-180.

Charmaz, K. (2014). *Constructing grounded theory*. London, England: Sage publications.

De Wever, B., Schellens, T., Valke, M., & Van Keer, H. (2006). Content analysis schemes to analyze transcripts of online asynchronous discussion board: A review. *Computers & Education, 46*, 6-28.

Edmonds-Wathan. (2012). Spatial Metaphors of the Number Line. In J Dindyal, L. P Cheng & S. F. Ng (Eds.), *Mathematics education: Expanding horizons: Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia*. Singapore: MERGA.

Glaser, B. (1978). *Theoretical sensitivity*. Mill valley,CA: Sociology press.

Griffin, S. (2004). Number worlds: A research based mathematics program for young children. In D. Clements & J. Sarama (Eds.), *Engaging Young Children in mathematics* (pp. 325-342). Mahwah, NJ: Lawrence Erlbaum Associates.

Hatch, J. (2002). *Doing qualitative research in education settings* Albany, NY: Suny Press.

Kruger, R. (2009). *Focus groups: A practical guide for applied research*. London, England: Sage.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic books.

Lemke, J. (1990). *Talking science: Language learning and values*. Norwood, NJ: Ablex Publsihing Corporation.

Lemke, J. (2005). *Multiplying meaning*.

Mildenhall, P. (2014). Number sense development in the primary classroom: How is it communicated? *Australian Primary Mathematics Classroom, 19*(3), 6-11.

Núñez, R., Motz, B., & Teuscher, U. (2006). The psychological reality of the ego and time reference point distincion in metaphorical constructs of time. *Metaphor and Symbol, 21*(3), 133-146.

O'Halleron. (2011). Multimodal discourse analysis. In K. Hyland & B. Paltridge (Eds.), *Companion to Discourse*. London: Continuum.

O'Halloran, K. (2005). *Mathematical discourse: Language, symbolism and visual images*. London, England: Bloomsbury Publishing.

Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, Calif.: Sage Publications.

Rabiee, F. (2004). Focus-group interview and data analysis. *Proceedings of the Nutrition Society, 63*(04), 655 - 660.

Radford, L. (2003). Gestures, Speech, and the Sprouting of Signs: A Semiotic-Cultural Approach to Students' Types of Generalization. *Mathematical Thinking and Learning, 5*(1), 37-70. doi: 10.1207/S15327833MTL0501_02

Sarama, J., & Clements, D. (2004). Building blocks for early childhood mathematics. *Early Childhood Research Quarterly, 19*(1), 181-189.

Shoenfeld, A. (2004). The math wars. *Educational Policy, 18*(1), 253-286.

Yin, R. (2009). *Case study research design and methods*. Thousand Oaks, California: Sage Publications.

Young Indigenous Students' Engagement with Growing Pattern Tasks: A Semiotic Perspective

Jodie Miller
Australian Catholic University
Jodie.Miller@acu.edu.au

The aim of this study was to determine the role of semiotics in assisting young Indigenous students to engage with and identify the general structure of growing patterns. The theoretical perspective of semiotics underpinned the study. Data are drawn from two Year 3 students, including analysis of pretest questions and two conjecture-driven lessons. Results indicate that particular semiotic signs (iconic signs) contribute to how young Indigenous students attend to, and identify the structure of growing patterns.

In Indigenous contexts a deficit perspective with regard to students' capability to learn persists, and impacts on the types of mathematics they experience in their classrooms (Warren, Cooper, & Baturo, 2010). Teachers continue to hold "low expectations for Indigenous students and perceptions that the gap in educational outcomes… [are] somehow normal" (Purdie et al., 2011, p.4). Thus it is conjectured that in these contexts there are limited opportunities for Indigenous students to engage in higher levels of mathematical thinking, such as generalising mathematical structures. The ability to generalise is an important aspect of algebraic thinking, and is the key to success in higher levels of mathematics (Lee, 1996). The development of this ability occurs as early as Year 2, and leads to a deeper understanding of mathematical structures (Blanton & Kaput, 2011; Cooper & Warren, 2011). In addition, the exploration of numberless situations, such as growing patterns provides an opportunity for young students to engage in powerful schemes of thinking (Carpenter, Franke and Levi, 2003).

Past research indicates that young non-Indigenous students are capable of generalising growing patterns (Blanton & Kaput, 2005; Leung, Krauthausen & Rivera, 2012; Warren, 2005), however, little is known about (a) Indigenous students' capability to generalise growing patterns, and (b) what types of tasks help Indigenous students to generalise. The focus of this paper is to explore how pattern task development, and the use of semiotics can enhance and support the engagement of young Indigenous students' in early algebraic thinking. In particular, the research question, how does semiotics assist young Australian Indigenous students to engage with and identify the general structure of growing patterns?

Literature

Growing patterns are characterised by the relationship between elements, which increase or decrease by a constant difference. In developing understanding of a growing pattern structure, students are asked to form the functional relationship between the terms in the pattern and their position. That is, they are asked to reconsider growing patterns as functions (covariational thinking – the generalisable relationship between the term and its position), rather than as a variation of one data set (recursive thinking – relationship between successive terms within the pattern itself) (Warren, 2005).

Findings from past research indicates that the way in which growing patterns are presented and taught potentially limits students' awareness and accessibility to the generalisable structure of the pattern (Küchemann, 2010; Moss & McNab, 2011). Often the growing patterns presented to students are abstract representations, displayed as drawn

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 421–428. Sunshine Coast: MERGA.

geometric shapes (e.g., circles drawn to make a T shape), and consist of the first three or four pattern terms. The position of each term is rarely displayed in the drawn representation. Students engaging with growing patterns in this way often identify the recursive pattern rule, as their attention is not attracted to the two variables or the co-variable relationship between the two variables. Recent studies with young non-Indigenous students have confirmed the benefits of explicitly representing the underlying structure of the pattern and using semiotic signs to draw attention to this structure when generalising (e.g., Warren & Cooper, 2008; Radford, 2006).

There have been few studies that have focused on the act of grasping a generalisation. Grasping a generality is to notice a commonality that holds across all terms (Cooper & Warren, 2011). Radford (2006) asserts that the act of grasping a generalisation rests on perception and interpretation. This is an active process, and is dependent on the use of signs (gesture, speech, concrete objects) that indicate where the perceived object is located. Radford's study (2006) focused on better understanding the role of signs in students' perceptive processes underpinning generalisation of number and geometric patterns.

While the theory of semiotics has been long established, it is only recently that studies in the area of pattern generalisation have considered how semiotics impacts on the learning process (e.g., Radford, 2006; Miller, 2014; Warren & Cooper, 2008). For example, aspects of the various semiotic resources (gestures, language, materials) used by students and teachers when exploring mathematical generalisations (Radford, 2006; Miller, 2014; Warren & Cooper, 2008) have been delineated. The gap still remains in the research with respect to how semiotics assists young Indigenous students to attend to both variables in growing pattern representations. Hence, the theoretical construct underpinning this research was semiotics.

Theoretical Framework

Semiotic signs assist students in developing mathematical understanding (Sabena, Radford, & Bardini, 2005). Semiotics is the study of cultural sign processes, analogy, communication, and symbols. Signs (such as bodily movement, oral language, concrete objects) play the role of making the mathematics apparent (Radford, 2003). As the teaching of mathematics draws on a variety of representations and resources to assist students to engage with mathematical processes, semiotics provides the tools to understand these processes of thought, symbolisation, and communication. Semiotics has a two-fold role in this study. First, it informed the selection of materials used to represent the growing patterns, and second, it provided the lens to interpret the signs within and between all social interactions in the learning experiences. The semiotic terms from Saenz-Ludlow and Zellweger (2012) model, adapted from Peircean theory (Peirce, 1958), are adopted for this study. Figure 1 displays the triadic concept of sign that has been developed with the classifications of sign object, sign vehicle and sign interpretant (Saenz-Ludlow & Zellweger, 2012).

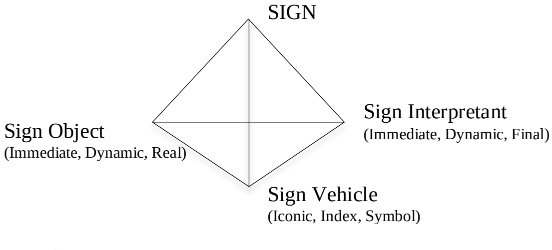


Figure 1. The tridactic concept of sign with terminology adapted by Saenz-Ludlow and Zellweger, 2012.

For the purpose of this paper, the focus is on sign vehicles. Sign vehicles are mediators between the sign objects and sign interpretant (student/teacher), and are then deduced to attempt to build understanding of the overall concept. The sign object can be represented by different sign vehicles that capture particular aspects of the object. Multiple sign vehicles are required, as one sign vehicle cannot encapsulate the entire object. To come to a complete understanding of the concept, students must be exposed to multiple and interrelated sign vehicles. These sign vehicles can be classified as iconic, indexical or symbolic (Saenz-Ludlow & Zellweger, 2012). Iconic signs exhibit a similarity to the subject of discourse (object); the indexical sign, like the pronoun in language, forces the attention to the particular object without describing it; and the symbolic sign signifies the object by means of an association of ideas or habitual connection (Peirce, 1958). These sign vehicles can be both static and dynamic (Radford, 2006; Saenz-Ludlow, 2007).

Research Design

A decolonised approach has been adopted with a focus on valuing, reclaiming, and having a foreground for Indigenous voices (Denzin & Lincoln, 2008). For this particular study, a relationship needed to be cultivated with Indigenous Education Officers (IEO) to assist with knowledge that may not be explicitly recognisable to the researcher. It was thus imperative to create space for critical collaborative dialogue within the study; hence the choice of teaching experiments to collect data. This is because teaching experiments provided the platform to investigate the teaching and learning interactions that support the development of students' ability to generalise in students' own setting. In effect, this brings the researcher and the participants into a shared space, where empowerment can occur (Denzin & Lincoln, 2008).

The larger project was based on two conjecture-driven teaching experiments for the primary purpose of directly experiencing students' mathematical learning and reasoning in relation to their construction of mathematical knowledge. A crucial aspect of the conjecture-driven teaching experiment is the conjecture itself. It needs to be aimed at both theoretical analysis and instructional innovations (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). Conjectures are based on inferences, and within mathematics education, these inferences may pertain to how mathematics is organised, conceptualised, or taught in order to reconceptualise the content and pedagogy (Confrey & Lachance, 2000). Each teaching experiment consisted of three 45-minute mathematics lessons, therefore six lessons in total. The researcher conducted the lessons in the study. Three conjectures were explored in each lesson, a mathematical (e.g., Exploring growing patterns where the structure is multiplicative (e.g., double) assists students to generate the pattern rule), semiotic (e.g., Providing growing patterns where the variables are embedded in the pattern ensures that students attend to both variables*),* and cultural conjecture (e.g., Exploring growing patterns from environmental contexts assists Indigenous students relate growing patterns to their prior experiences). Due to space limitations, this paper only draws from the first teaching experiment (teaching experiment 1), focusing on the semiotic conjectures from two lessons conducted with Year 2/3 Indigenous students.

Data Collection

The research was conducted in one Year 2/3 classroom (7-9 year olds) of an urban Indigenous school in North Queensland. This school was purposively selected because these students had not previously engaged in mathematics lessons focusing on the concept of growing patterns. Additionally, a relationship was already formed with the school community, an important aspect of Indigenous research perspectives, and the researcher had worked in the classes as part of a larger mathematics research project. In total, 18 students and 2 Indigenous Education Officers (IEOs) participated in the study. To explore how students engaged with the growing patterns, data-gathering strategies used in teaching experiment 1 were: (a) A pretest to ascertain what the students knew before the lessons; (b) video-recorded mathematics lessons, and (c) audio-taped interviews with the IEOs. There were two video cameras in each lesson, one focused on the researcher and the other on the students. Data reported in this paper are from two students (S1 - Aboriginal girl & S6 – Aboriginal boy). These students were selected, as they represent cases of mathematical achievement, high (S1) and low (S6) achiever in mathematics, as identified by the classroom teacher and IEO.

Data Analysis

In this study, data analysis was contemporaneous and formative during data collection. It informed each stage of the data collection process and assisted in refining conjectures (Confrey & Lachance, 2000). Pretests were analysed not only to ascertain what students knew, but to also determine the ways in which the students answered the questions. This analysis informed selection of tasks for the first lesson of teaching experiment 1. The analysis of the videotaped lessons formed a major component of the qualitative data analysis. The teaching experiments required two phases of data analysis, ongoing analysis and in-depth analysis. Ongoing analysis occurred at the conclusion of each lesson of the teaching experiment, and informed the next stage of data collection. This assisted with refining conjectures and hypotheses, and the development of tasks for the next lesson (Confrey & Lachance, 2000). Peer debriefing between the researcher, supervisor, teacher, and Indigenous Education Officers was conducted at this point to determine conjectures for the following lesson. Member checks occurred during the teaching experiments with students to ensure that the researcher had correctly interpreted each student's response.

In depth analysis occurred at the conclusion of the data-collection phase. All data were reanalysed using an iterative approach (Srivastava & Hopwood, 2009). Initial video-footage were transcribed to capture students' verbal responses and the semiotic interactions. Data were coded and analysed focusing on semiotic signs (iconic and indexical) of both the student and researcher. This entailed identifying signs that assisted students to engage with the growing pattern structures. Finally, the data were reanalysed and aligned with the cultural perspective provided from the Indigenous Education Officers with regard to the semiotic signs. Their input was audio-recorded and then transcribed in order to capture cultural interactions in the lesson.

Findings

The data from S1 and S6 are presented by structuring the findings according to the order of data collection. First, results from the pretest that served to ascertain what students knew prior to the commencement of the lessons, and second, the conjectures that framed the three lessons in teaching experiment 1 are presented. *Pretest*: The test comprised 10 items. Figure 2 illustrates Student 1 and Student 6 responses to Question 3 (Copy the pattern) and 7 (How many possum eyes will there be if there were 10 possums hanging on the tree?) of Pretest 1, key questions that illustrate the differences in understanding between the two students, and the data that informed the development of Conjecture 1

(Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship).

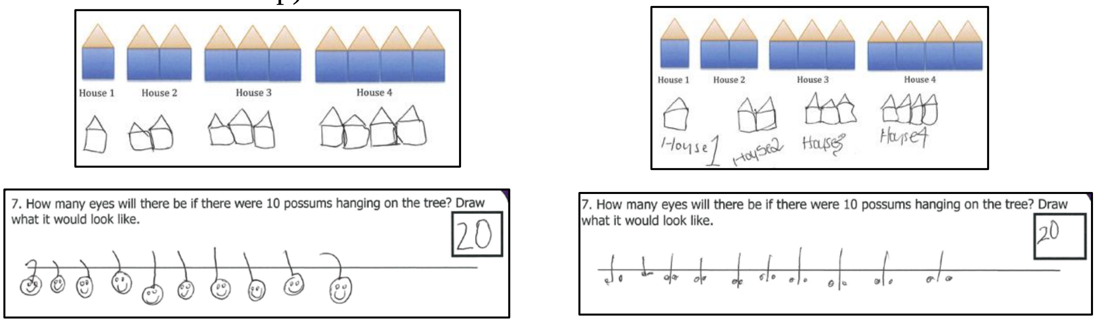
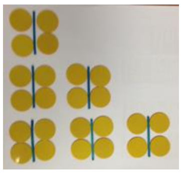
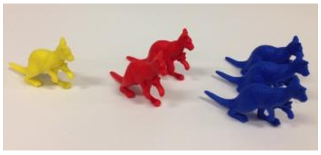


Figure 2. S1 and S6 responses to Question 3 and 7 of Pretest 1.

For Question 3 S1 only copied the houses in the pattern. She did not attend to both variables in the pattern (the houses (pattern term)) and corresponding label (pattern position). By contrast, S6 attended to both variables in the pattern. For Question 7 S1 and S6 started to work with both variables in the possum pattern. They both drew the possum tails and eyes. It appears that having the two variables (the possum tails and possum eyes) embedded in the single pattern structure (a drawing of a possum) assisted S1 and S6 to attend to both variables. At the conclusion of the Pretest, Indigenous Education Officers shared that within their context they could not identify any growing patterns that were appropriate for these young students to engage with. They suggested that it would be best for students to begin exploring patterns from a shared context (e.g., environmental context). They also confirmed that a hands-on approach (using concrete materials) would be appropriate for these students. Figure 3 presents the following patterns used in lesson 1 (butterfly pattern), and lesson 3 (kangaroo pattern).



If I have 20 butterfly bodies how many butterflies would there be?



If I had 20 kangaroo tails how many kangaroo ears would there be?

Figure 3. Growing patterns used in teaching experiment 1.

Lesson 1 Conjecture: Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship. During lesson 1, while both variables were visually explicit (blue matchstick – pattern term, yellow counter – pattern quantity) and embedded in the butterfly pattern, students attended to the iconic signs (matchsticks and counters – iconic signs) separately. When considering a butterfly in the natural environment the body and wings cannot be separated. However, when using the concrete materials, the sign vehicles were easily separated. S1 did not split the two signs; S1 placed one matchstick on her desk and then immediately placed the four counters around that matchstick before constructing the next butterfly. S1 was able to copy and continue the structure identical to that presented by the researcher (see Figure 3). It was for these reasons that S1 was considered to have high structural awareness of the butterfly pattern. S6 attended to the sign vehicles separately. Other students in the class also attended to the pattern in this manner. First, he placed an array of matchsticks on the desk to represent the butterfly bodies, and then added the counters (wings) retrospectively. Thus, when constructing the pattern, S6 attended to the two sign vehicles (the iconic signs) sequentially rather than simultaneously. Whether he recognised the co-variational relationship between the two sign vehicles is difficult to determine. It is because of these actions, separating the sign vehicles, that the pattern for lesson 2 was selected (See Figure 2). Additionally, the IEO stated that the students were confident using the number ladder, and they were able to 'act out the pattern' by standing on the 'feet'.

Lesson 3 Conjecture: Providing growing patterns where only two variables are embedded and cannot be physically separated from each other, assists students to attend to both variables simultaneously. S1 was now attending to both variables when working with the kangaroo pattern. She was able to express further predictions of the pattern using both the tail and the ears to communicate her understanding. S1 explained to the class that if she had 1 million tails she needed to double the number of tails to determine the number of ears. She was also able to determine the number of tails if there were 10 kangaroo ears (five kangaroo tails). As both variables were embedded in the kangaroo pattern, and could not be separated, this assisted S1 to attend to both variables when discussing the pattern. S6 was able to attend to both variables in the kangaroo pattern to assist him explain the relationship between the tails and ears. He was able to predict how many ears there would be if there were 100 kangaroo tails (200), and explained that he was doubling the number of tails to find the number of ears.

Discussion and Conclusion

It has been demonstrated in past research that young non-Indigenous students can engage in covariational thinking (Blanton & Kaput, 2005); however, this current study adds new knowledge to the pattern task types that assist young Indigenous students in 'noticing' the relationship between two variables. Past research has highlighted an issue that arises from covariational thinking is the need to coordinate two data sets, and identify the relationship between these sets (Blanton & Kaput, 2005). Thus, in this present study the growing patterns selected for the tasks were deliberately chosen to ensure that this relationship was transparent. Results from this study provide initial evidence that iconic signs appear to assist students to move quickly from recursive thinking to covariational thinking. This was achieved by using iconic signs to highlight the two variables. Additionally, a recursive approach to solving growing patterns is still a major challenge for both young and older students (Rivera & Becker, 2009; Warren, 2005). The results of this present study suggest that this issue relates to the way the patterns are structured, and can be overcome by using iconic signs to highlight both variables in growing patterns, namely, the pattern number (term) and the pattern quantity.

While it is recognised that signs play a central role in the construction of new knowledge (Peirce, 1958; Saenz-Ludlow, 2007), literature pertaining to how these signs are represented in pattern generalisation tasks is scarce. This present study begins to contribute to this limited research, and suggests that there are two potential ways that sign vehicles can be considered when constructing growing patterns: (a) embedding sign vehicles (possum and kangaroo pattern), and (b) splitting sign vehicles (house and butterfly patterns). When considering growing patterns the sign vehicles represent the two variables within the pattern (i.e., pattern term and pattern quantity). Embedding both sign vehicles in a single hands-on artefact ensures that students attend to both variables of the growing pattern. This aligns with past research, indicating that students were successful generalising patterns where both iconic signs were embedded in the single structure

(Blanton & Kaput, 2005; Leung, Krauthausen & Rivera, 2012; Warren, 2005). Young Indigenous students were supported in making connections with co-variation, as demonstrated by S1 in TE1 when discussing the general rule for the kangaroo pattern. It appeared that the use of *embedded variable patterns* assisted students to attend to both variables: that is, students needed to discuss the pattern attending to the pattern position (tails) and the pattern quantity (ears).

It appears that iconic sign vehicles (e.g., concrete materials) provide opportunities for dynamic interactions between the student and the pattern. Findings from this study further nuance the importance of the role that dynamic signs play when students physically engage with geometric patterns to construct the general rule (Mason, 1996; Saenz-Ludlow, 2007). Through the use of iconic signs (butterfly bodies and butterfly wings), a geometric pattern created with concrete materials provides opportunities for young students to manipulate both variables, as they examine the pattern structure on their way to constructing generalisations (Cooper & Warren, 2008). This approach differs from geometric patterns that are traditionally depicted in textbooks (as students can not physically manipulate textbook pictures), and it is argued that potentially students may not engage with, or interpret these signs (textbook pictures), with the same intensity. It is conjectured that growing pattern task should be set up to have, dynamic iconic signs that represent both variables, so that students can physically engage with these signs.

The contribution of this study is that growing pattern task design should mirror and support students use of semiotics as a thinking tool and as such one needs to consider signs in the representation of patterns. These tasks types have implications for both the teaching and learning of growing pattern generalisations. As two cases were presented, it is acknowledged that there are limitations for the study. Thus further research is needed to consider larger cohorts of both Indigenous and non-Indigenous students, to determine if these pattern tasks assist young students to engage in growing pattern generalisations, and if there is a potential hierarchy to which growing patterns should be introduced to young students. Finally, and most importantly, this study provides a positive story for Indigenous students indicating that they are capable of engaging with early algebraic thinking challenging deficit models of mathematics learning.

Acknowledgement: The research reported in this paper was supported by the ARC (LP100100154). The author expresses her thanks to participating Indigenous Education Officers and students.

References

Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36*, 412–446.

Blanton, M., & Kaput, J. (2011). Functional thinking as a route into algebra in the elementary grades. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives. Advances in mathematics education* (pp. 5–23). Berlin Heidelberg, Germany: Springer-Verlag.

Carpenter, T. P., Franke, M. L., & Levi, L. W. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9–13.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231–265). Mahwah, NJ: Lawrence Erlbaum Associates.

Cooper, T. J., & Warren, E. (2011). Years 2 to 6 students' ability to generalise: Models, representations and theory for teaching and learning. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives. Advances in Mathematics Education* (pp. 187–214). Berlin Heidelberg, Germany: Springer-Verlag.

Denzin, N. K. & Lincoln, Y. S. (2008). Critical methodologies and Indigenous inquiry. In N. K. Denzin, Y. S. Lincoln, & L. T. Smith (Eds.), *Handbook of critical and Indigenous methodologies* (pp. 1–30). London, UK: SAGE Publications.

Küchemann, D. (2010). Using patterns generically to see structure. *Pedagogies: An International Journal*, *5*(3), 233–250.

Lee, L. (1996). An initiation into algebraic culture through generalization activities. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra: Perspectives for research and teaching* (pp. 87–106). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Leung, C., Krathausen, G., & Rivera, F. (2012, July). *First grade students' early patterning competence on figural and numerical sequences : Cross-country comparisons between Hong Kong and the United States*. Paper presented at the 12th International Congress on Mathematical Education, Seoul, South Korea.

Mason, J. (1996). Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra: Perspectives for research and teaching* (pp. 65–86). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Miller, J. (2014). Young Australian Indigenous students' growing pattern generalisations: The role of gesture when generalising. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 461-468). Adelaide: MERGA.

Moss, J., & McNab, S. (2011). An approach to geometric and numeric patterning that fosters second grade students' reasoning and generalising about functions and co-variation. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives. Advances in mathematics education* (pp. 277–301). Berlin Heidelberg, Germany: Springer-Verlag.

Peirce, C. S. (1958). *Collected papers, Vol. I–VIII*. (P. Hashornem, P. Weiss, & A. Burke, Eds.). Cambridge, MA: Harvard University Press.

Purdie, N., Reid, K., Frigo, T., Stone, A., & Kleinhenz, E. (2011). *Literacy and numeracy learning: Lessons from the longitudinal literacy and numeracy study for Indigenous students*. Camberwell, VIC: ACER.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning, 5*(1), 37–70.

Radford, L. (2006). Algebraic thinking and the generalisation of patterns: A semiotic perspective. In S. Alatorre, J. Cortina, M. Saiz, & A. Mendez (Eds.), *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 2–21). Mexico: PME.

Rivera, F. D., & Becker, J. R. (2009). Algebraic reasoning through patterns. *Mathematics Teaching in the Middle School*, *15*(4), 212-221.

Sabena, C., Radford, L., & Bardini, C. (2005). Synchronizing gestures, words and actions in pattern generalizations. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 129–136). Melbourne, VIC.: University of Melbourne.

Saenz-Ludlow, A. (2007). Signs and the process of interpretation: Sign as an object and as a process. *Studies in Philosophy and Education, 26*, 205–223.

Saenz-Ludlow, A., & Zellweger. S. (2012, July). *The teaching-learning of mathematics as a double process of intra- and inter-interpretation: A Peircean perspective.* Paper presented at the 12th International Congress on Mathematical Education. Seoul, South Korea.

Srivastava, P., & Hopwood, N. (2009). A practical iterative framework for qualitative data analysis. *International Journal of Qualitative Methods*, *8*(1), 76–84.

Warren, E. (2005). Young children's ability to generalise the pattern rule for growing patterns. In H. Chick & J. Vincent (Eds.), *Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 305– 312). Melbourne, VIC.: University of Melbourne.

Warren, E., & Cooper T. J. (2008). Generalising the pattern rule for visual growth patterns: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics, 67*, 171–185.

Warren, E., Cooper, T. J., & Baturo, A. R. (2010). Pedagogies to support Indigenous students' mathematics learning in rural and remote Queensland classrooms. In J. Zadja (Ed.), *Globalisation, comparative education and policy research 10: Globalisation, education and social justice* (pp. 161–184). Dordrecht, The Netherlands: Springer.

Professional Knowledge Required when Teaching Mathematics for Numeracy in the Multiplicative Domain

Judith Mills
The University of Waikato
judith@waikato.ac.nz

This paper presents findings as part of a wider study that investigated the professional knowledge of teachers when teaching mathematics for numeracy in the primary school classroom. This paper focuses on teachers in action as they taught two lessons on multiplication. It outlines the specific pedagogical categories the teachers used and the impact their knowledge had on student learning.

Capturing the essence of teaching by studying what it is a teacher does, why they do it, and what effect it might have on student learning is an on-going topic of research and discussion (Barton, 2009). As Barton (2009) explained, we do not currently have the theories, or research, to inform teachers why it is that some highly mathematically qualified and highly motivated teachers are unsuccessful, and why it is that the students of some mathematically unqualified teachers receive top results. The role of the teacher and the professional knowledge currently required is more complex and sophisticated, and has changed in response to the major societal, economic, cultural, and political changes, which have taken place (Hattie, 2003).

Concern over the mathematical knowledge of primary school teachers, has been expressed for many years (Ball, Thames & Phelps, 2008; Ma, 2010). Linking the professional knowledge of teachers, to the relationship between classroom practice, and student understandings as a result of those practices, has thus been a focus of researchers in recent times (Ball et al., 2008; Chick, Baker, Pham, & Cheng, 2006; Schoenfeld, 2011). Much of the recent research has been founded on the work of Shulman (1986, 1987), who was one of the first researchers to identify the complexities associated with different categories of knowledge teachers require for students' mathematics learning. Shulman introduced the term pedagogical content knowledge (PCK) as being of particular interest to teachers, as it contains a special type of knowledge which distinguishes teaching from other professions.

Recent years have seen more use of the term *numeracy* in education (Perso, 2006). Often the terms mathematics, and numeracy, are used interchangeably and yet some argue that there is a difference in meaning (Coben, 2000; Perso, 2006). Mathematics is about the exploration and use of patterns and relationships in quantities, space and time, about representing and symbolising these ideas, and eventually learning to abstract and generalise (Bobis, Mulligan, & Lowrie, 2013; Ministry of Education, 2007). The development and conceptualisation of the term *numeracy* has been an important influence on the teaching of mathematics, and was first attributed to the United Kingdom's Crowther report in 1959, where numeracy was described as the mirror image of literacy (Tout & Motteram, 2006). Perso (2006) argued that prior to the 1950s school mathematics focussed on computation and it was with introduction of computational tools, and the associated need for higher-order thinking skills, that the need for people to be able to transfer their mathematics understandings to everyday life became greater. Perso (2006) questioned whether in the current cultural and social context of schooling, educators are primarily teachers of mathematics, or teachers of mathematics for numeracy? She argued that there

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 429–436. Sunshine Coast: MERGA.

needs to be a shift in focus from pure mathematics, to a focus on mathematics as the fundamental prerequisite for numeracy for all children throughout their schooling, as they prepare for life skills requirements beyond the classroom.

The teaching of mathematics in schools throughout the twentieth century saw six identifiable phases, each with its unique emphasis: drill and practice, meaningful arithmetic, new maths, back to basics, problem solving, and standards and accountability (Lambdin & Walcott, 2007). Each of these phases introduced what was seen as new and innovative practices, for that particular period of time. In more recent times education reforms emphasised that learners of any age will not succeed at mathematics unless they are taught in ways which enable them to bring their intelligence, rather than rote learning, into use when learning their mathematics (Skemp, 1989).

One contributing factor often cited as part of the reason for poor mathematics proficiency, is the focus that was previously on developing procedural knowledge, at the expense of conceptual understanding (Skemp, 2006). Thus, the current standards-based education system supports a curriculum that emphasises concepts and meanings, rather than rote learning, and promotes integrated, rather than piecemeal usage of mathematical ideas (Howley, Larsen, Solange, Rhodes, & Howley, 2007). Ma (2010) asserted that in order to facilitate conceptual learning, teachers need to emphasise and promote the connections between, and among ideas that for non-teachers are implied. Ma described this as well-developed, interconnected, knowledge packages, made up of a thorough understanding of mathematics, having breadth, depth, connectedness, and thoroughness. She referred to this as *profound understanding of fundamental mathematics* (PUFM). She noted that “although the term ‘profound’ is often considered to mean intellectual depth, its three connotations, deep, vast and thorough are interconnected” (Ma, 2010, p. 120).

The emphasis on teaching concepts and meanings positions mathematical knowledge as a social process, whereby children construct mathematical ideas from their understanding and experiences, of the world in which they live (Ross 2005). The ‘drill and practice’ of basic facts and taught routines, will not prepare children for a technological world. Current teaching focuses on the structure underlying numbers and number operations (Anghileri, 2006; Mulligan & Mitchelmore, 2009). The single most influential factor on student learning is the teacher (Hattie, 2003).

Methodology

Aim of the Study

This main purpose of this study was to identify the strengths and weaknesses in the professional knowledge of primary school teachers, when teaching mathematics for numeracy in the multiplicative domain, and the impact these have on student learning. These might then be addressed in professional learning sessions, to assist in teacher development.

Research Design

A multiple-case study design was used. Multiple-case study design refers to the investigation of more than one participant, where the focus is both within and across the cases (Creswell, 2008). The ability to conduct a number of case studies may then bring with it a need to form some type of generalisability, which was required in this research.

The qualitative and quantitative data collected merged as the various data sets from the four case study teachers, were analysed and interpreted.

Research Setting and Participants

Two teachers from two schools were the four case-study teachers, who along with the children in their classes formed the basis of this study. School A was a full primary (Years 1 to 8) inner city school, while School B was an urban primary school (Years 1 to 6). The case-study research was based around the senior classes of each school: the teacher of the Year 5 and 6 class (Andy), and the teacher of the Year 7 and 8 class (Anna) from School A, along with two teachers of Year 5 and 6 classes from School B (Beth and Bob). The teachers at School A taught their own class for maths, while School B grouped their classes by ability. Bob's class was third in ranking (one being the top class out of the six), and Beth's class fourth class in ranking.

Research Approach

A mixed-methods approach was employed to collect data. Mixed-methods research is often described as research in which the investigator collects and analyses data, integrates the findings, and draws inferences using both qualitative and quantitative approaches and methods, in a single study or programme of inquiry (Cohen, Manion & Morrison, 2000;). The rationale behind mixed-methods research is that more can be learned about a research topic if the strengths of qualitative research, are combined with the strengths of quantitative research, while compensating at the same time for the weaknesses of each (Cohen et al., 2000).

Data Sources

Data collection came from multiple sources including questionnaires, assessments, observations, and interviews. Classroom observations were the key part of data collection which focused primarily on the professional knowledge of teachers in action. In order to validate the observations of the lessons, field notes were written, photos taken, and lessons both audio-taped and video-recorded. This meant that the researcher could return to the details of the lessons and cross-check details at a later date.

Questionnaires were administered to the teachers and their students at various times throughout the study. Questionnaire data were later compared to in class observations. An initial questionnaire was given to the teachers containing three sections: (1) teachers' views about mathematics; (2) multi-choice questions around aspects of subject matter knowledge and (3) scenarios about the teaching of mathematics, where judgements were required in relation to mathematical understanding.

This research related to the teaching of multiplication and division. Pre-unit and post-unit assessment tasks designed by the researcher were administered to the students. The tasks were based on key aspects of knowledge students at Years 5 and Years 6 are expected to implement according to Level three of the New Zealand Curriculum (Ministry of Education, 2007) and the National Mathematics Standards (Ministry of Education, 2009).

The two lessons from each of the case-study teachers were subsequently coded for detailed analysis. Following transcription of lessons the qualitative data was exported into the computer programme NVivo 10 which was used for the coding. Coding stripes were used to group information about particular themes together. The basis for the coding used in this research, were the categories identified on the PCK framework developed by Chick, Baker, Pham and Cheng, (2006), which became one of the key features in determining the professional knowledge of teachers of primary school mathematics.

Results

Pre-Unit Assessment

Prior to teaching the multiplication and division unit of work the students were given nine assessment tasks to ascertain current knowledge (Figure 1). The students were asked to solve each problem, explain how they worked it out, and where possible draw a diagram to show their thinking Most of the tasks were at Levels 2 and 3 in The New Zealand Curriculum, and the majority of children should have been capable of correctly solving these (Ministry of Education, 2007; 2009).

Task 1 Mult as repeated addition 4+4+4+4+4+4=24 How would you write this as a multiplication fact?	Task 2 Draw a Diagram of 3 x 5 =	Task 3 Division Partitive 20÷4	Task 4 Division Quotitive 20÷4	Tasks 5 & 6 Commutative Property 2 x 5	Task 7 Using x5 Basic Facts: I have 6 groups of 5 cubes and know to write this as 6 x 5 = 30. How could I use this to work out 6 x 4 = ?	Task 8 Using known Basic Facts: I know that 4 x 7 = 28. How can I use this to work out 4 x 14 = ?	Task 9 Division with remainders: 30 apples into 4 equal sized bags

Figure 1: Pre-Unit assessment task types

The pre-assessment results showed that there were only two tasks where greater than fifty percent of the children in any class, were able to give a correct answer. Task 7 saw 55% of Beth's class give a correct answer, while 75% of Anna's solved Task 8 correctly. Task 4 saw the poorest result with one child correct in two classes, two correct in one class, while no-one solved the problem correctly in the other class. The correct responses on the other tasks, ranged from 5% of Beth's class on Task 3 (partitive division), to 40% of Bob's class on Tasks 5 and 6 (understanding of commutativity).

Multiplication Lessons Observed

Two lessons were observed for data collection: one at the start of a six week unit on multiplication and division, and one at the end of the unit. All teachers began the first lesson by establishing the meaning of the multiplication ('×') symbol. In mainstream New Zealand classes the first number of a multiplication expression represents the multiplier and the second number the multiplicand. The lesson focused on use of the commutative property of multiplication unpacking the difference in representation between the two different equations, for example 5 × 3 and 3 × 5. Two teachers (Anna and Bob) became confused themselves when explaining the difference and this led to confusion among the students in their classes. The final lesson differed for each teacher, according to the progress the students had made throughout the unit.

Clearly PCK

One of the greatest weaknesses in relation to the teaching of multiplication of all the teachers was their curriculum knowledge. The teachers were unclear as to exactly what they should be teaching students at Level 3 (in Anna's case level 4) of the curriculum. Stages 6 and 7 of the Number Framework (Ministry of Education, 2008a), directly align to

Curriculum Levels 3 and 4, and the teachers did not immediately recognise what strategies and knowledge the students were expected to utilise. The lesson expectations were consistently below national expectation, and the teachers made little attempt to probe the students and push them along.

The teachers struggled to identify cognitive demands of the tasks and aspects that affected their complexity, from the viewpoint of their students. The main problem was the students' difficulty in understanding the multiplier as the first number in the equation, and the multiplicand as the second number, and the significance of acquisition of this knowledge for the students as they moved on to more complex problems with double digits.

All teachers identified a learning intention for their lesson, which began with 'We are learning to…' (referred to as the WALT). While the WALT provided a focus for each lesson, it also became a hindrance, as many opportune moments were missed for the students to bring their own thinking to their problem solving. Observations suggested there was a two-fold reason why the teachers maintained focus on the WALT: management of the children; and apprehension of coping with something mathematical that may arise, to which the teacher may not know the answer. So long as the WALT was at the forefront of the lesson, they were prepared to answer any questions that may be asked, during the lesson.

The nature of the lesson depended on whether the teachers recognised the misconceptions the students currently held. The initial lesson taken by Andy and Bob was very teacher directed. The children were given little opportunity to discuss ideas together and responses to questions were directed at specific students. These students generally had raised their hands because they knew the answer to the given question, and while incorrect responses were sometimes given, it was generally due to inaccurate computation rather than misunderstandings, or misconceptions. Beth's students all had manipulatives available to them, which allowed her to visually see many of the misconceptions the students had. The models the students had constructed along with the discussion as the students explained their thinking, allowed her to recognise misunderstandings the students may have had.

Content Knowledge in a Pedagogical Context

The frequency with which the teachers were required to deconstruct content also aligned to the nature of the lesson. In the first lesson the teachers were very much involved in the problem solving with the lessons being teacher directed throughout. This meant the teachers were able to clarify uncertainties immediately, as they were 'right on the spot' to do so. In the latter lesson the teachers posed problems, and the students were left to solve them on their own more. Thus the teachers were not always in a position to be aware of students' difficulties until discussions were held later in the lessons. As they deconstructed content the teachers discussed the relationship between repeated addition and multiplication, the link between repeated addition and the array model of multiplication, the importance of recognising patterns in mathematics, and the need to have some basic facts as instant recall to assist in working out other facts.

There was little evidence of what was originally referred to by Ma (2010) as *Profound Understanding of Fundamental Mathematics* by any of the teachers. Lessons appeared to be planned and procedurally implemented and as students struggled with understandings, the teachers lacked the depth and breadth of knowledge required to reframe questions and offer explanations in alternative ways. Seldom were connections made between or within ideas. While the teachers could solve problems themselves, their number sense was weak, and of concern.

Pedagogical Knowledge in a Content Context

Classroom techniques, or generic classroom practices, of raising hands to ask/answer questions, using manipulatives to explain thinking, and using a modelling book during group work, were implemented by all of the teachers. The teachers asked the students to share ideas with others, and discuss problems together, but this seldom occurred. The students 'talked' together, but rarely 'discussed' ideas or justified findings.

Knowledge of assessment was limited by all of the teachers. Prior assessment data was under-utilised. The pre-unit assessment data was not used to identify gaps and weaknesses, which could then be incorportaed into the planning of lessons. Similarly it appeared that the results of other matheamtics assessment tools had not been used.

Questioning was very much of the supportive nature and seldom did the teachers extend the thinking of the students. The teachers readily accepted answers given by students, and when a problem was answered correctly, they acknowledged the response and continued with the lesson. The teachers did not ask for justification of responses, and seldom pushed the students to the next level with questions such as, "What would happen if we changed…" or "If we changed this number (for example the multiplier), what affect would it have on this number (the multiplicand**)**?"**.**

Post-Unit Assessment

At the conclusion of teaching the multiplication and division unit of work, the students were given nine assessment tasks (note: Tasks 5 & 6 were combined and shown as one task to report data) similar to those of the pre-unit assessment (Figure 2). Of the four classes and eight tasks (32 counts in total) there was a percentage decline in the number of students who solved the problem correctly on 14 occasions, an increase of correct responses on 15, while 3 remained the same. The results showed that more than 50% of Bob and Beth's students were correct on Task 1, with more than 50% of Anna's students correct on Task 7. All other tasks saw less than 50% of the children correct with a range of zero on task 3 from Anna's class, and Beth's class on tasks 4 and 8, through to 48% correct on tasks 5 and 6 from Andy's class.

Task 1	Task 2	Task 3	Task 4	Tasks 5 & 6	Task 7	Task 8	Task 9
Mult as repeated addition 5+5+5+5=20 How would you write this as a multiplication fact?	Draw a Diagram of 3 x 6 =	Division Partitive 12÷3	Division Quotitive 12÷3	Commutative Property 3 x 5	Using x5 Basic Facts: I have 6 groups of 5 cubes and know to write this as 6 x 5 = 30. How could I use this to work out 6 x 6 = ?	Using known Basic Facts: I know that 3 x 10 = 30. How can I use this to work out ☐ x 5 = 30?	Division with remainders: 26 apples into 4 equal sized bags

Figure 2: Post-Unit assessment task types

Discussion

Overall, the results were of both considerable interest and concern. The pre-unit assessment results showed that generally the students were below, and in many instances well below, their expected levels (Ministry of Education, 2009). This should have been an indication to the teachers that there was a great deal of knowledge teaching required for the students to understand the concepts associated with multiplication and division. The post-unit assessment showed that little progress had taken place throughout the six weeks, with less than half of the tasks showing an increase in the number of students obtaining correct responses. The teachers had taught many of these ideas in class, and questions must be asked as to why the expected improvement did not occur. Some of these can be attributed to the students themselves, while close analysis of the teaching also highlighted gaps in teachers' professional knowledge.

The teachers' lack of curriculum knowledge and uncertainty of exactly what is required of them in their teaching is of concern. Teachers must understand the requirements of the Curriculum Levels (Ministry of Education, 2007) and align these to the Number Framework Stages (Ministry of Education, 2008), and the Mathematics Standards expectations (Ministry of Education, 2009). The alignment needs to be instantly recognisable if effective decision making during a lesson is to be made. What questions to ask, what problems are given, how far to extend the students in their thinking, are all dependent on having at their fingertips an understanding of the progressions of learning.

There were times when both the teachers and children displayed misconceptions. The term 'misconception' suggests wrong understanding of concepts. Rather than wrong understanding it would be more pertinent to suggest it was often a muddled, or confused, understanding. The teachers seldom exhibited a deep and thorough conceptual understanding of aspects of the mathematics they were teaching (Chick et al., 2006), referred to by Ma (2101) as Profound Understanding of Fundamental Mathematics (PUFM). This contributed to their confusion within the key mathematical concepts they were teaching, and the significance of consistently using correct mathematical language. With current teaching focusing on the structure underlying numbers and number operations (Anghileri, 2006; Mulligan & Mitchelmore, 2009), the teachers PUFM could be narrowed down to the need for a stronger understanding of number and number sense (SUN).

While it is essential that students are aware of the learning intention of each lesson, teachers must take care not to let the focus over-ride the opportunity for new learning to occur. Opportune and teachable moments must not be overlooked, as addressing an issue when it arises will often mean the student will make more sense of the solution and retain the newfound knowledge. This does not mean taking each lesson in a totally different direction from the planned purpose, but if students are to remember key ideas from the lesson, then the learning experience must be meaningful to them.

Problem solving and the associated skills of discussion and justification are now an accepted part of classroom practice. This study showed that while the students were given problems to solve together, they often worked as individuals within their groups, and struggled with the notion of challenging each other's thinking. The students seldom participated in 'friendly argumentation'. Similarly, while the teachers supported the students in their solution methods, there is a definite need for them to extend given ideas by questioning the students thinking more. This would also assist in the students progressing through the Number Framework stages and curriculum levels.

Conclusion

The mathematics classroom of today places a significant emphasis on conceptual understanding, and the importance of making mathematics meaningful beyond the classroom. This suggests that teachers are now teachers of mathematics for numeracy, challenging them to consider the mathematical concepts being taught as well as the contexts within which they are taught. The professional knowledge required by teachers is complex and multi-layered, requiring ongoing attention to the many aspects of PCK originally mooted by Shulman, if students are to achieve, and move beyond, their expected levels.

References

Anghileri, J. (2006). *Teaching number sense.* (2nd ed.). London, UK: Continuum.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 3-9), Palmerston North, NZ: MERGA.

Bobis, J., Mulligan, J., & Lowrie, T. (2013). *Mathematics for children: Teaching children to think mathematically.* Frenchs Forest, NSW: Pearson Australia.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Krátká, & N. Stehliková (Eds.), *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 297-304. Prague, The Czech Republic: PME.

Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult numeracy development: Theory, research, practice.* Cresskill, NJ: Hampton Press Inc.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education (5th Ed).* London, UK: Routledge Falmer.

Creswell, J. W. (2008). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research.* Upper Saddle River, NJ: Pearson Education.

Hattie, J. (2003). Teachers make a difference: What is the research evidence? Presented at *Australian Council for Educational Research Annual Conference on: Building teacher quality.* Retrieved from http://www.education.auckland.ac.nz/uoa/fms/default/education/staff/Prof

Howley, A., Larsen, W., Solange, A., Rhodes, A. M., & Howley, M. (2007). Standards-based reform of mathematics education in rural high schools. *Journal of Research in Rural Education, 22*(2). pp. 1-11. Retrieved from http://jrre.psu.edu/article/22-2.pdf .

Lambdin, D., & Walcott, C. (2007). Changes through the years: Connections between psychological learning and theories and the school mathematics curriculum. In W. G. Martin, M.E. Strutchens, & P. C. Elliott (Eds.). *The Learning of Mathematics: sixty-ninth Yearbook.* Reston, VA: NCTM.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Ministry of Education (2007). *The New Zealand curriculum*. Wellington: Author.

Ministry of Education (2008). *Book 1: The Number Framework: Revised edition 2007.* Wellington, NZ: Author.

Ministry of Education (2009). *The New Zealand Curriculum mathematics standards for years 1– 8.* Wellington, NZ: Author.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33-49.

Perso, T. (2006). Teachers of mathematics or numeracy? *Australian Mathematics Teacher, 62*(2), 36-40.

Ross, V. (2005). Pointing the way: Possible avenues of development in the field of mathematics education. *Curriculum Inquiry, 35*(2), 235-246.

Schoenfeld, A. H. (2011). *How we think: A theory of goal-oriented decision making and its educational applications.* New York, NY: Routledge.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Skemp, R. R. (1989). *Mathematics in the primary school.* London, Great Britain: Routledge.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School, 12*, 88-95.

Tout, D., & Motteram, G. (2006). *Foundation numeracy in context.* Camberwell, Vic: ACER Press.

Determining a Student's Optimal Learning Zone in Light of the Swiss Cheese Model

Patricia Morley
Luckmor
<trishmorley@luckmor.com>

Simone Zmood
Monash University
<simone.zmood@monash.edu>

Participation in society is increasingly dependent on educational achievement. Accordingly, society as a whole is committing more resources to education to prevent the adverse outcome of students moving through the school system only to emerge without the knowledge and skills that they might be expected to attain. In this paper, we explore the application of two models developed to prevent adverse outcomes in industrial and medical settings to the issues involved in providing an optimal mathematics education for all children.

Introduction

Teachers and parents may dream of optimal learning for their students and children, respectively, but defining what this means and putting it into practice is complex. Vygotsky's (1978) Zone of Proximal Development (ZPD) suggests that a student learns optimally in the zone requiring guided learning which is beyond what can be accomplished solely by independent learning. It can be thought of as the *stretch* zone where students are being challenged but able to learn through the guidance of a more knowledgeable teacher or peer. When there is insufficient challenge, students will *coast* and if the learning expectations are too great then the student may *crash*, having been overwhelmed by the cognitive or affective load.

This paper provides a theoretical discussion on the practical implications of determining a student's ZPD in light of the diversity of learning and understanding even within one individual. Both the Swiss Cheese Model and the enhanced Hot Cheese Model are used to explore 'holes' which impact on mathematics learning from a system, classroom, curriculum, and student perspective. We suggest that both models have the potential to clarify issues involved in the assumptions made about student knowledge, and the role and interpretation of assessments. In particular, this paper focuses on determining what the base level is that a student can move forward from in his or her learning – an important starting point for ensuring students are learning within their ZPD.

Models of Student Learning and Mathematical Errors

The implication of the ZPD is that teaching and learning are effective when instruction is tailored to current level of understanding of the child. Care, Griffin, Zhang, and Hutchinson (2014) describe a project which uses assessment to identify where children are in their learning development in order to enable differentiated instruction.

The Swiss Cheese Model (SCM) was introduced as a metaphor to explain how the combination of several factors can lead to industrial accidents in complex systems and as a framework for the investigation of those accidents systems (Reason, 1997). It allows for consideration of multiple factors that lead to adverse outcomes, rather than placing emphasis on the final straw. *Hazards* are known potential causes of problems. *Defences* are the actions taken to prevent hazards contributing to *adverse outcomes*. Figure 1 shows a

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 437–444. Sunshine Coast: MERGA.

generic Swiss cheese model. Each slice of cheese represents a *defence layer* within the system, the holes in each slice of cheese represent gaps or imperfections in the defence, and when the holes in several slices of cheese align creating a *hazard trajectory* then this accumulation of multiple failures can result in harm. The SCM is a useful focus of attention for investigating unwanted outcomes in order to put in place layers of defence against future harm. A key feature of this model is its flexibility. The number of layers of defences can be adjusted to suit the situation. The SCM, despite its simplicity, has been widely used to draw attention to the multi-faceted nature of adverse events. In particular, it encourages a more holistic view through recognition of contributing factors in addition to the most proximate cause.

The Hot Cheese Model (HCM) refines the SCM by including interactions between the defence layers of the system which are referred to as *feature interactions* (Li & Thimbleby, 2014). The HCM explicitly recognises that a system of defences – the slices of cheese – is active and not passive nor unchanging. It is not enough to put multiple layers of defence in place with non-aligning holes as any new defence layer introduced may end up causing new errors and thus harm. Li and Thimbleby (2014) provide an interesting example of feature interactions. In Detroit, a monitoring camera was installed as safety device in a nuclear reactor. However, it fell and blocked a coolant drainage hole. The blockage resulted in temperatures that destroyed sensors, leaving a nuclear meltdown to go undetected by the reactor operators.

From its origin in industrial accidents, the SCM has been transferred and adapted to other fields such as medicine, demonstrating that transfer of these ideas from one field to another field was not only possible but useful. Both the SCM and the HCM allow clearer thinking about complex situations. We will now discuss how the SCM, and also the HCM, can be used in the educational context.

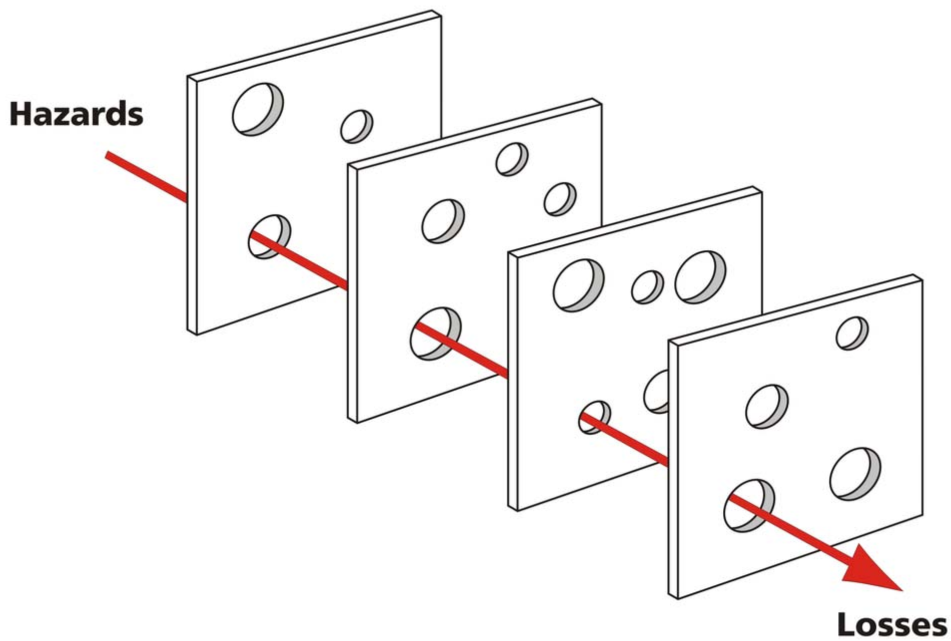


Figure 1. Generic Swiss Cheese Model (SCM) (Mack, 2014)

In Education, we might think of the process of education as one of obtaining knowledge and skills. A situation, condition, or event that might impede learning is, in this context, a hazard. The layers of Swiss cheese in this model are defensive layers that are put in place to prevent the impediments from affecting the desired outcome of learning. The arrow moving through the Swiss cheese represents a student's failure to learn despite the defences in place. Education of a population is a large endeavour in terms of resources and time required from a wide range of people. Despite this, too many children leave school without having attained some of the basic knowledge and skills that they might reasonably be expected to have. In Education, this is the adverse outcome, or losses, that we consider below.

It is acknowledged that measurement of outcomes is a necessary precondition to understanding the success and adverse effects of any process. Whether the measurement of outcomes is modelled as a defence layer in the SCM, or not, is dependent on the situation, as discussed here. Even where the measurement of outcomes is neutral, the feedback of information into the system is not necessarily neutral. To illustrate this, we draw on research on the effective use of measurement to reduce workplace injuries. The measure of work time that was lost due to injury was adopted as an Occupational Health and Safety (OHS) measure, but was found to be problematic (Blewett, 1994). Lost Time Injuries (LTI's) were used as a measure of workplace safety. However, this measure was unsatisfactory for a number of reasons, including that it was "far more sensitive to claims and injury management processes than to real changes in safety performance" (Blewett, 1994, p. 29).

In Education, much has been written about potential and actual adverse effects of assessment that interfere with the main goal of education. Many of these concerns are related to how the information is used, rather than about the measurement itself. Unlike other contexts, where measurement is neutral until it is fed back into the system, testing provides an opportunity for a learner to organise information and increase learning.

It is widely recognised that educational assessment performs a variety of functions, ranging from the use of large-scale assessments to inform policy to in-class assessments to inform teaching practices (Care et al., 2014). Educational outcomes are to some extent measured by the outcomes of assessments, and we consider them to be intrinsic to educational processes, rather than a neutral measurement. Accordingly, in this paper assessments are treated as defence layers within the model, rather than neutral measurements external to the model.

Following is a discussion on how the SCM and HCM can be used to analyse mathematics learning in school at four levels: the education system as a whole, the mathematics classroom, the mathematics curriculum, and the individual student.

Education System

At the education system level, it is recognised that good educational outcomes depend on a suitable physical environment, a well-structured curriculum, competent teachers, and student attendance. Accordingly, employing the terminology of the SCM, a poor physical environment, an inadequate curriculum, incompetent teachers, and poor student attendance each may be considered as a *hazard*, that is something which might contribute to the adverse event of students not learning at the appropriate level.

The defence layers that are in place to prevent the adverse outcomes described above are: a suitable budget for school building and maintenance; a structured curriculum;

minimum teacher qualifications and professional development; system-wide students assessments such as NAPLAN and Year 12 examinations; and student mandatory attendance. These protective layers are put in place via legislation and Education Department policy.

Mathematics Classes

At the classroom level, children need good rapport with the teacher, a teacher who has the requisite knowledge to teach mathematics, and learning resources such as manipulatives, textbooks and ICT. Associated hazards are impediments to learning such as: when the children are absent; when children are disengaged from the subject material; and when connections are not made with previous knowledge. The protective layers are accordingly: roll calls or attendance lists; lesson plans; monitoring of students' learning via quizzes, exams; review of homework books; and projects. Roediger III, Putnam, and Smith (2011) identify ten benefits of testing, ranging from identifying gaps in knowledge and providing feedback to teachers, to encouraging active learning by encouraging "students to study", producing "better organisation of knowledge", improving "transfer of knowledge to new contexts", and improving "metacognitive monitoring". An accurate understanding of the source of a *gap* is essential in order to match the teaching intervention to the type of error, and at the appropriate level of the individual student or the whole class (Holmes, Miedema, Nieuwkoop, & Haugen, 2013). The responsibility for these layers derives from the individual teacher, who operates within the larger framework of the school and wider educational policies.

Mathematics Curriculum

The mathematics curriculum is both a defence layer in the complex system of education – across the educational system and within the mathematics classroom – and a system in its own right. It provides a framework for teaching the desired mathematical knowledge in a structured way so that concepts and procedures are built up on previous knowledge. If one applies the traditional SCM at the curriculum level, hazards could be that the curriculum expects too much (or too little) of students, and presuming that students have actually mastered previous teachings and are ready to learn new material. Thus, in a somewhat recursive situation, the curriculum and its periodic revision act as defence layers.

However, transforming the SCM, one can look at each slice of cheese as a study area of the mathematics curriculum. To illustrate this, the Australian curriculum outlines the scope and sequence for three interlinked branches of mathematics: number and algebra; measurement and geometry; and statistics and probability. Each of these branches of mathematics is broken down into more detailed study areas. For example, the number and algebra branch comprises six strands: number and place value; fractions and decimals; real numbers; money and financial mathematics; patterns and algebra; and linear and non-linear relationships.

If each of these study areas represents a slice of cheese, then some holes in conceptual understanding will impact mastery of learning in another slice. Roman numerals are often introduced in primary school maths to explore alternative number systems and emphasise place value. However, provided a student has conceptual understanding of place value, a hole in knowledge of Roman numerals is unlikely to be detrimental in the long-term, other than on an assessment with test items for Roman numerals. In contrast, an understanding of fractions is critical for developing working knowledge of algebra which, in turn, is important in developing an understanding of calculus. Following this line of reasoning, some study areas of mathematics are pre-requisites for mastery of other study areas and any gaps may have escalating consequences and long term implications. For example, misguided early number concepts (Mazzocco, Murphy, Brown, Rinne, & Herold, 2013), fractions and algebra (Gray & Tall, 1992a, 1992b).

Individual Student

Another transformation of the SCM is useful to consider the implications for an individual student. Picture a vertical stack of sliced cheese, with lower slices representing previous years of mathematics education and each slice of cheese consisting of the mathematics curriculum taught at that specific year level. Holes in understanding in earlier slices may mean that the student does not have the foundation to develop robust understanding in some areas of the current intended curriculum. There are many factors which might contribute to an individual not having the knowledge and skills in place to accommodate learning the current topic in the classroom, ranging from difficult personal circumstances, previous poor teaching, or learning difficulties. It is important for a teacher to be aware of whether the difficulties experienced by students are due to learning disabilities, and to be able to act accordingly (Butterworth, Varma, & Laurillard, 2011). Defences against such hazards include factors such as having a supportive family who values mathematics learning and having a teacher who knows how to identify the sources of gaps in knowledge and what interventions are most appropriate to fill the gaps.

Implications for Teaching

Ultimately, the SCM and HCM were developed to both pro-actively mitigate the risks of gaps aligning resulting in an adverse outcome and also to analyse what the root causes of any failures were. In this section, we discuss what the potential sources of error may be, implications for assessments and interventions, and link this back to the important task of determining a individual student's ZPD.

Sources of Error

There has been considerable research on the causes of errors in mathematics. Skemp (1976) differentiates between instrumental understanding which is understanding what to do, and relational understanding which is "knowing both what to do and why" (p. 2). Others use the terms procedural and conceptual understanding to differentiate between learning a collection of procedures or algorithms and developing a deeper understanding of what is happening mathematically. Both have their place. Procedural knowledge that is not underpinned by conceptual understanding can lead to learning lots of *rules* which only apply in certain situations. Strong conceptual understanding without the fluency of procedural knowledge can drive a student to derive formulas and rules from first principles. Whilst this is a valuable skill, there is not always sufficient time during testing situations to *derive knowledge* and a measure of fluency is useful. The ideal is strong conceptual foundations which underpin procedural fluency and flexibility.

Holmes et al. (2013) identify three sources of mathematics error: (1) vocabulary errors which are gaps in knowledge or misinterpretations; (2) computational errors; and (3) erroneous beliefs or misconceptions. They found that it can be difficult to differentiate between computational errors and misconceptions, and yet this teacher knowledge is crucial for determining the appropriate teaching interventions. Teachers also need to "make judgement calls as to what gets addressed in the class setting and what becomes an individual student intervention" (Holmes et al., 2013, p. 32).

There are other possible sources of error. Some students may not lay out their work properly on the page and or have illegible handwriting. Some students could have issues with test taking or working under pressure. An underlying source of any error could be working memory overload, when students are trying to deal with more information than they can manage and do not have good working practices to reduce the demands on their working memory. Self-management skills can be taught to help students develop their metacognition and thus improve their learning and production.

From a teaching point of view, there is no such thing as a *silly mistake* because every error has an underlying cause. Identifying the gap, identifying its cause, and understanding the implications of the gap continuing, are all important factors in optimising a student's mathematics learning.

Implications for Assessments

In Education, one very important way of ascertaining whether successful mathematics learning has taken place is through a variety of assessments including tests, quizzes, exams, assignments and projects, problem solving journals, review of homework books, class discussions and conversations with individuals. These assessments can be designed and implemented at various levels such as large-scale national testing, school-based exams, class assessments, and individual conversations with students. The different levels of assessments match up with three of the interpretations of the SCM presented above: the education system; the mathematics class; and the individual student.

Large Scale Testing. NAPLAN is an example of a defence layer in the HCM, performing a monitoring role required for understanding the effectiveness of the education system at various points. This section discusses the role of NAPLAN under the SCM and the HCM.

The inclusion of NAPLAN in the SCM requires an understanding of the role that NAPLAN plays in the education system. If the NAPLAN assessments were considered as having a neutral role in measuring outcomes, it would not be necessary to include NAPLAN explicitly in the model. This view, however, is simplistic. NAPLAN assessments have a role in identifying areas of strengths and weaknesses in educational outcomes, and therefore NAPLAN would be included as a defence layer at the system level.

Under the HCM, it is recognised that the defence layers can interact with other defence layers in the system. As others have previously highlighted, the measurement role that NAPLAN plays is not neutral. On a positive note, NAPLAN is expected to illustrate curriculum expectations and consequently shape teacher practices in improving students' mathematics and numeracy performance. On a negative note, pressure to improve scores may have the undesired impact of encouraging shallow teaching practices.

Limits of written testing. It is important to recognise that a student's mathematical understanding is only assessed to the extent of the questions contained on a test. Some holes do not show up on typical assessments. Multiple choice tests are an example of where a student might get the answers correct on a test but actually have a hole in conceptual understanding. A teacher might pre-test the class on a given topic to help with future lesson planning. Although the test may be effective as a whole, there are likely to be some children whose capabilities are over or under-estimated. Holmes et al. (2013) emphasise that "identifying student misconceptions from their work is a matter of identifying and categorizing patterns. Thus, it is extremely important when developing assessments to have multiple questions targeting the same concept in order to better classify misconceptions" (p. 32).

Benefits of talking with students. Holmes et al. (2013) suggest that "a good way to discover what students may be thinking when examining their answer for a particular problem is simply to ask them" (p. 38). Gray and Tall (1992a) go further stating that "in general classroom activity it is essential for the teacher to talk to individual children and to listen to how those children are performing their arithmetic calculations" (p. 13) and warn that "simply allowing them to carry out idiosyncratic procedures may actually be leading them up a cul-de-sac of eventual failure at more advanced arithmetic" (p. 13).

Another method of getting inside the student's head is to ask students to keep a journal, where they explain their thinking when solving certain problems, in order to diagnose inappropriate strategies. This can be a good way to record and capture the development of a student's understanding. However, journals do lack the immediacy and interactivity of a conversation and thus may require some back and forth to dig deeper into the student's reasoning.

In summary, assessments provide evidence of student's capability to understand and respond to assessment items. While it is easy to misconstrue the extent of a student's understanding, the SCM suggests two things: firstly, that sources of errors might not be the immediately obvious; and secondly, that the imperfections of any type of assessments are appropriately compensated for by using a variety of styles of assessment, including conversation, thereby exposing different strengths and weaknesses. Despite their limitations, formative, summative, and large-scale testing provide information that might suggest further lines of inquiry which might be otherwise unavailable.

Implications for Interventions - Dealing with 'holes'

One of the necessary assumptions of any lesson plan is that students are able to absorb the material. Gaps, or *holes*, in students' understanding weaken this assumption. The SCM suggests that holes are to be expected, therefore teachers need to develop approaches which accept and address the existence of holes. One possibility may be to emphasise pre-requisite knowledge over revision of procedural activities.

We interpret the SCM as suggesting that students, with a perspective differing from a teacher's perspective, might be able to identify issues that a classroom teacher may not have seen. Empowering students to actively participate in identifying the prerequisite knowledge required for a new task may also have the desirable effect of promoting relational understanding of material prior to instrumental understanding.

Determining a student's ZPD

Pre-testing a student's knowledge before teaching a new unit is a well-established way to determine a student's readiness to learn new material or whether they may already have acquired the intended knowledge. Written or online quizzes are the most common format used, and teachers should keep in mind the limitations of written testing discussed above.

Once any gaps in knowledge or understanding are identified, teachers need to decide whether the gap is of a critical nature which will impact future learning and, if so, how best to fill the gap. This process can help teachers clarify the base level of the student's ZPD, the demarcation between coasting and being stretched. The upper level of the student's ZPD – the demarcation between being stretched and crashing – is a topic for another time.

Conclusion

Reason's Swiss Cheese Model (SCM), although somewhat simplistic, has been successfully used in the engineering and medical fields to illustrate and model complex systems, in particular to highlight the multifactorial aspect of adverse events. This paper has introduces the SCM model to the field of Education, based on the consideration that children failing to learn important concepts and skills despite all efforts to the contrary is an example of an adverse event. The SCM model is flexible, and examples have been given on how the SCM can be applied at the individual student level, the mathematics classroom level, the curriculum level, and the system level.

The inherent simplicity of the SCM limits the usefulness of the model for complex situations. Li and Thimbley's (2014) Hot Cheese Model, based on the SCM, includes the potential for unintended interaction of components of measures that are intended to prevent adverse outcomes. This paper has suggested that these models might be pertinent to assessments in education, whether in the classroom or system wide.

References

Blewett, V. (1994). Beyond lost time injuries: Positive performance indicators for OHS, summary paper. *Positive Performance Indicators: Beyond Lost Time Injuries: Part 1- Issues*, 1-55.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: from brain to education. *Science*, 332(6033), 1049-1053.

Care, E., Griffin, P., Zhang, Z., & Hutchinson, D. (2014). Large-scale testing and its contribution to learning. In C. Wyatt-Smith, V. Klenowski, & P. Colbert (Eds.), Colbert, P. (2013). *Designing assessment for quality learning*. Netherlands: Springer.

Gray, E., & Tall, D. (1992a). Success and failure in mathematics: Procept and procedure, 1. a primary perspective. Retrieved from http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1992k-primary-procept.pdf

Gray, E., & Tall, D. (1992b). Success and failure in mathematics: Procept and procedure 2 – secondary mathematics. Retrieved from http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1992l-secondary-procept.pdf

Holmes, V-L., Miedema, C., Nieuwkoop, L., & Haugen, N. (2013). Data-driven intervention: Correcting mathematics students' misconceptions, not mistakes. *Mathematics Educator*, *23*(1), 24-44.

Li, Y., & Thimbleby, H. (2014). Hot cheese: A processed swiss cheese model. *The Journal of the Royal College of Physicians of Edinburgh*, *44*, 116-121.

Mack, D. (2014). Swiss cheese model of accident causation. Retrieved from https://en.wikipedia .org/wiki / Swisscheese model#mediaviewer/File:Swiss cheese model of accident causation.png

Mazzocco, M. M. M., Murphy, M. M., Brown, E. C., Rinne, L., & Herold, K. H. (2013). Persistent consequences of atypical early number concepts. *Frontiers in Psychology*, *4*, 486.

Reason, J. T. (1997). *Managing the risks of organizational accidents*. Aldershot: Ashgate Publishing.

Roediger III, H. L., Putnam, A. L., & Smith, M. A. (2011). Ten benefits of testing and their applications to educational practice. *Psychology of Learning and Motivation - Advances in Research and Theory*, *55*, 1-36.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 20-26.

Vygotsky, L. S. (1978). *Mind and society: The development of higher mental processes*. Cambridge, MA: Harvard University Press.

Student and Parent Perspectives on Fipping the Mathematics Classroom

Tracey Muir
University of Tasmania
<Tracey.Muir@utas.edu.au>

Traditionally, the domain of higher education, the 'flipped classroom' is gaining in popularity in secondary school settings. In the flipped classroom, digital technologies are used to shift direct instruction from the classroom to the home, providing students with increased autonomy over their learning. While advocates of the approach believe it is more engaging and effective than traditional instruction, there is little empirical research into the benefits of this approach, particularly in relation to mathematics instruction. This paper adds to the limited research by reporting on students and parents' experiences with a flipped classroom in a senior mathematics class. The results indicated that there were five main components that influenced students' motivation to engage with the flipped classroom approach. The study has particular implications for students and secondary mathematics teachers who have limited time to make the curriculum comprehensible for students and to prepare them for external assessment tasks.

In the flipped classroom, teachers typically record and narrate screenshots of work they do on their computer screens, create videos of themselves teaching or curate video lessons from internet sites such as TED-Ed and Khan Academy (Hamdan, McKnight, McKnight, & Arfstrom, 2013). Benefits of the approach include differentiated teaching for a range of student abilities, greater student motivation and increased student-teacher interaction (Bergman & Sams, 2012). Despite its growing popularity, there is little empirical research on the flipped classroom outside of higher education settings, with Abeysekera and Dawson (2015) labelling the area as under-evaluated and under-theorised. This paper adds to the limited research in the field by investigating senior secondary students' and parents' experiences with flipping the mathematics classroom. It adds to a previous study by Muir and Chick (2014) through targeting a different cohort of students and documentation of parental perspectives. Specifically, this paper aims to answer the following research question: What are student and parent perspectives of the benefits or otherwise of adopting a flipped classroom approach in the teaching of senior secondary mathematics?

The study is important because it documents an alternative approach to traditional mathematics instruction. There is continued concern in Australia, and internationally, over the lowering levels of engagement with mathematics (Attard, 2010), and research has shown that there is a definite decline in school mathematics engagement of many young adolescents compared with their primary school counterparts (NSW Department of Education & Training, 2005). As noted by the Department of Education and Early Childhood Development (2009), there is a persistent and progressive decline in middle school students' attitudes towards, and interest, in science and mathematics. This is of concern as disengagement with mathematics can lead to exclusion from courses requiring specific levels of mathematics and generally limits one's capacity to understand life experiences through a mathematical perspective (Sullivan, Mousley, & Zevenbergen, 2005). According to Attard, the pedagogical relationship between students and teachers appeared to have a significant effect on students' engagement in mathematics, and that students were highly engaged when working on computers. The flipped classroom caters

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 445–452. Sunshine Coast: MERGA.

for students' propensity to be online, and is consistent with MCEETYA's (2003, p. 4) statement that "students will use online curriculum content to expand and deepen their understanding at a pace, in a place and with an educational purpose that suits them". The pedagogy, however, must transform learning, engage students in ways not previously possible (MCEETYA, 2005), and give them greater control over how, where and when they learn (ACARA, 2014).

The flipped classroom also reconceptualises the paradigm of traditional mathematics homework. It is common practice in Australian secondary classrooms to allocate regular homework, often involving the use of the classroom textbook, and requiring the completion of a number of exercises. In the home environment, completion of homework tasks can be problematic, particularly as students move into higher grades, and the mathematics becomes more challenging. Mathematics homework often becomes a source of tension between parents and children (Civil, 2006) and many parents feel largely uninformed about contemporary mathematics teaching methods (Muir, 2009). In the flipped classroom, traditional homework tasks are completed in class where the teacher can provide targeted assistance as students work through activities designed to help them master the material.

Theoretical Framework

Regarded as the pioneers of flipped learning, Bergman and Sams (2012) reported that flipping their classroom led to greater student interaction in class, and more targeted individual instruction. A range of benefits associated with flipping the classroom have been identified for students, including differentiation of teaching, allowing the "pausing and rewinding' of teachers in recorded presentations, informed parents, a more transparent classroom, greater student motivation and interest, and improved classroom management (Bergman & Sams, 2012).

A key feature of the flipped classroom is the shifting of direct instruction to outside of the group learning space, and maximising one-on-one interactions in the classroom that more actively involve students in the learning process (Hamdan et al., 2013). The reduction of in-class time spent on teacher presentations and explanations allows the teacher to target their teaching to specific areas which may be particularly challenging and provide for greater monitoring of individual student progress. Instructional benefits of the flipped classroom approach include active learning, increased one-to-one interaction, priming, reduction in cognitive load and catering for diverse learners (Hamdan et al., 2013).

A theoretical model proposed by Abeysekera and Dawson (2015) provides an appropriate lens for investigating the flipped classroom approach. Although developed in a higher education setting, it contains a number of elements that would be relevant in a secondary school setting. The model, which is depicted in Figure 1, shows five components of the flipped approach that have the potential to cater for motivation and cognitive load. These components are: sense of competence, sense of relatedness, sense of autonomy, tailoring to expertise and self-pacing.

Motivation, which is closely linked to engagement, can be defined as 'the willingness to attend and learn material in a development program' (Cole, Field & Harris, 2004, p. 67). According to Pintrich and De Groot (1990), motivation is linked strongly with self-regulated learning and contains three components: an expectancy component, which includes students' beliefs about their ability to perform a task, a value component, which includes students' goals and beliefs about the importance and interest of the task, and an affective component which includes students' emotional reactions to the task. In essence, students' motivation is related to their beliefs about whether or not they can perform the task and that they are responsible for their own performance (Pintrich & De Groot, 1990). This is consistent with Xu and Wu's (2013) research on self-regulated learning in relation to homework management. They suggested that the use of self-regulatory strategies are influenced by goal orientation (purpose for engaging in a task), task value (the importance and utility of a task), and task interest (the appeal of a task or an activity).

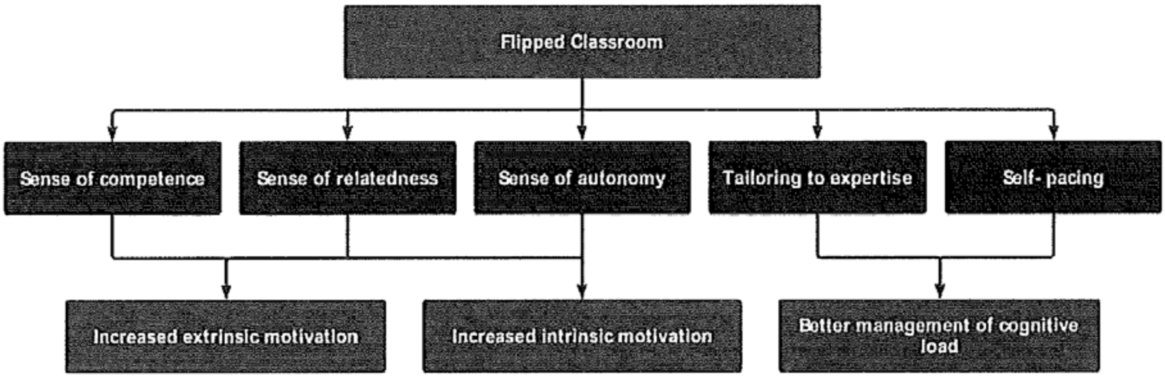


Figure 1. Theoretical model for the flipped classroom (Abeysekera & Dawson, 2015, p. 10)

As homework is primarily an individual task, undertaken outside of a scholarly environment, with the goals typically set by others, it requires students to be motivated in order to complete it. As mentioned earlier, homework is often seen as a source of tension between students and parents (Civil, 2006) and students complain about homework tasks being frequently boring, too easy or too hard, or irrelevant to their lives (Xu & Wu, 2013). As depicted in Figure 1, intrinsic and extrinsic motivation is closely linked with characteristics such as competence, relatedness and autonomy, which in turn all relate to self-regulated learning. Students develop a sense of competency through a belief that they can perform a task, are motivated to perform the task if they can relate to it as being importance and interesting, and are more likely to complete the task if they have a sense of autonomy or belief that they are responsible for their own performance. They are also more likely to manage the cognitive demands associated with a task if the instruction is tailored to their expertise, and there is provision for self-pacing, such as manipulating the pace of video tutorials. These aspects are particularly applicable to students' engagement with homework tasks, including those set within the context of a flipped classroom approach.

Methodology

The study employed a mixed-methods approach (Creswell, 2003) to investigate students' and parents' perceptions of their experiences of a flipped mathematics classroom. Within this methodology, the researcher used sequential procedures (Creswell, 2003) where data collected from the surveys were used to inform the interview schedule, allowing more detailed exploration with a few cases or individuals. The study was undertaken with a senior secondary mathematics class from a large metropolitan secondary school in Tasmania. Mathematics Methods is a senior secondary pre-tertiary course which covers topics such as functions, calculus and statistics. The teacher, Mr Smith, (pseudonym) was a fully qualified mathematics teacher, with over 20 years' teaching experience and had been teaching the course for several years. He had trialled the use of videos in 2013 (see Muir & Chick, 2014), but 2014 was the first year in which he used a fully flipped approach to teach his class. The student participants were in Grades 11 or 12 (approximately 16-17 years of age); there were 24 students in the class, all of whom completed the online survey (15 male and nine female), and 10 participated in the student interviews (seven male and three female). Six parents participated in the parent interviews.

The procedure involved the completion of an online survey using Qualtrics and for some students, participation in a follow-up interview. The survey contained 24 items, two of which required responses in a Likert format (see Table 1 for example items). There was also the provision for open-ended responses. The survey took approximately 15 minutes to complete. Semi-structured interviews were conducted with students after the completion of the survey. The interviews were audio-recorded and transcribed, and took approximately 15 minutes. Students were given the option of individual or focus group interviews, and with one exception, ('Rose') they all participated in focus group interviews. Parent interviews were conducted early in 2015 after students had finished the course and received their results. These were conducted individually, either in person or over the phone and varied from between 15-40 minutes duration. The teacher was also interviewed.

Quantitative data from the survey were analysed using Qualtrics, with responses to the Likert scale items expressed in percentages for ease of comparison. Qualitative data from the surveys and interviews were transcribed and analysed using reflexive iteration (Srivastava, 2009) whereby each sentence in the transcripts was coded, initially through emerging themes. The transcripts were then re-analysed and instances of the components contained in Figure 1 were identified. This process limited researcher bias in that the researcher was open to the possibility of other themes emerging and not restricted to narrowing the data to pre-determined themes. Initially 11 codes were ascribed to the data, and these were able to be further classified into the five components in Figure 1. For example, references to 'convenience' or 'easily accessible' were included in 'sense of autonomy' and 'targeted work' in 'tailoring to expertise'. The results section has been organised to report against the themes identified in the student data, supplemented by teacher and parent interview data.

Results and Discussion

Survey data showed that 100% of students had a computer and internet access at home and that 88% of students had accessed Mr Smith's pre-prepared online tutorials that year. Items from the student surveys that are relevant to this paper are presented in Table 1. Qualitative data from the survey were drawn primarily from three main open-ended questions which asked students to identify the advantages of the online resources as compared to the text book and the teacher, and whether or not they would recommend the practice to others.

Sense of Competence

Responses in this category included references to being helpful in terms of understanding the mathematical content and/or achieving success, thereby establishing a 'sense of competence' in the user. Table 1 shows that 96% of students agreed that online tutorials helped with their learning and helped them to learn a concept. Furthermore, 92% indicated that they found the tutorial helpful and 88% indicated that watching the tutorial contributed to success in both tests and class work. Qualitative comments from students included "Online resources are good for clarification of understanding" and "I liked Mr Smith's videos because they are easy to understand and they're more based on the questions we're answering". Jess, in a focus interview, indicated that "It helped because he was actually like teaching how to do everything and [it was] easier than looking in the book and trying to figure it out for yourself".

Mr Smith explained that:

> the flipped classroom enabled me to do the easier examples, set the scene, bit of drill and practice, you know what's differentiation of trig functions about - do these examples, so that when we got to class, they were ready to go [In class]. I was able to do a more sophisticated example so I didn't waste 20 minutes starting from scratch so that the homework, instead of doing lots of problems which I could have got them to do if I wanted to, was to just get the topic consolidated, the knowledge consolidated, the easier questions done, the rule, whatever it was, so that they were ready to go when we got in there

Parents were ambivalent in their perceptions of whether or not the flipped classroom approach impacted upon their child's success with the subject and with their overall grades. Donna, for example, believed it definitely benefited her daughter and "definitely helped her in terms of her results". Sue, however, felt that her son, Andrew, "thought that by watching these videos, I'm going to understand this maths and then when they ask questions I'm going to be able to do it … but he didn't – had no idea". Sue's comments show that, while not the intention, her son tended to passively watch the videos, which was in contrast with other students who generally indicated that they regularly paused and rewound the videos, and took notes throughout. Other parents, perhaps not surprisingly, were reluctant to attribute their child's success or otherwise in the subject to the flipped classroom approach, due to extraneous variables and no opportunity to compare with other approaches.

Sense of Relatedness

Table 1 shows that 88% of students accessed Mr Smith's online resources, compared with 25% who accessed other online resources. Reasons for this included relevance, with many students' comments showing that they particularly connected with, or related to, Mr Smith. Illustrative survey comments included, "I preferred Mr Smith's videos [over other online tutorials] because they were explained well and easy to understand". They were also impressed with Mr Smith's commitment to helping them learn:

> [In class] he'd get everyone involved and like the amount of effort he put into these videos – like he'd spend his periods where he was free, recording like he was talking to himself on his iPad and just the amount of effort he put in was really good. [Jack, focus interview]

> I like Mr Smith – he's really good and I understand him, but if it was like [Mr T, another mathematics teacher], I have no idea about half the stuff he's saying, so I probably wouldn't understand his videos, but I understand Mr Smith's. [Ella, focus interview]

The parents also communicated a sense of relatedness to Mr Smith. Sue, for example, acknowledged that it was "almost like having your teacher coming into your home environment and you don't feel so isolated". Interestingly, only 33% of students agreed that they used the tutorial to explore mathematics of their own, despite finding them engaging. This is perhaps not surprising as the emphasis was on the prescribed work that needed to be covered in the course, and exam preparation, indicating that students were extrinsically motivated to access them.

Table 1
Student Responses to Selected Likert Scale Items (n=24)

Statement	SA/A	Undecided	D/SD
I use online resources to help me with my learning	96%	4%	0%
I have used online resources (not prepared by my teacher) to help me with my mathematics this year	25%	25%	50%
I have used online resources prepared by my teacher to help me with my mathematics this year	88%	0%	12%
The tutorial helped me to understand a concept	96%	0%	4%
The tutorial was about the right length	71%	16%	13%
I watched all of the tutorial from beginning to end	71%	8%	21%
I found the tutorial helpful	92%	4%	4%
I found the tutorial boring	25%	42%	33%
I think I did better in the test because I watched the tutorial	88%	8%	4%
I think I understood the work better in class because I watched the tutorial	88%	8%	4%
I used the tutorial to explore mathematics of my own	33%	42%	25%
I used the tutorial to explore ideas about mathematics begun in class	58%	38%	4%

Sense of Autonomy

In order for students to be motivated through integrated regulation, the need for autonomy needs to be satisfied (Abeysekera & Dawson, 2015). Students' survey and interview data showed several references to this aspect of the framework. Rose, for example, in her interview, recommended the use of videos and stated:

> If you're doing your homework at home on a Saturday night, and you don't understand something, then rather than waiting for next maths lesson, you could just go online and access the video straight away

Elsa's comment, "I always watch them because I don't want to get behind", demonstrates that she sees herself as in control of her own learning, and also indicates that she is extrinsically motivated by wanting to maintain her grades.

In terms of identifying advantages over asking the teacher or using a text-book, six student responses included references to the capacity to view the clips multiple times and pause and rewind them. Five student responses also mentioned accessibility (e.g., "You can work on the topic at home").

Parents also appreciated their children taking control over their own learning, particularly as they felt unable to assist with this level of mathematics homework. Trudy, for example, stated that:

> I think it's really good that she's had the videos to watch because before, and in other years, she might have been working from a textbook and get stuck, and I couldn't help her, whereas with this

> way, if she's stuck, she can watch the video and she can write down questions and she can either email her teacher or she can ask him the next day, and it just seems to take all the stress out of the homework that used to happen before when she couldn't do things.

Tailoring to Expertise and Self-pacing

In the framework, these components lead to better management of cognitive load. Students typically referred to these aspects when discussing the affordances of the medium, such as "You can go back and revise it whenever you need it", "It [Mr Smith's tutorial] was more specific to what we were studying", "You can pause and rewind the video" and "You can choose what part to watch and what you need help with".

The ability to differentiate the learning and allow students to monitor their own progress was what led Mr Smith to trial the flipped classroom approach, particularly as it gave him added capacity to cover all the material required. He also stated:

> It had spin offs that I didn't expect, so I had [names student] was [overseas] for a couple of weeks, and he was basically learning, he had his text book with him, he was watching my videos, he was doing the problems I set, and his mother was quite appreciative of the fact that he could do that.

Conclusions and Implications

The results indicate that Abeysekera and Dawson's (2015) framework was useful for interpreting perceptions of the flipped classroom approach. Although originally developed for use in higher education settings, the senior secondary mathematics students in this study referred consistently to the five components in the framework when talking about their experiences with the flipped classroom. Reference was also made to these components by the classroom teacher and parents of the students in the class. The results showed that students had a purpose for engaging in the task (Xu & Wu, 2013) in that they were motivated to succeed in the subject and felt that watching the video tutorials helped them understand the work better and perform successfully in assessment tasks. The tutorials helped them with developing a *sense of competency* and a *sense of autonomy* in that they could use the video tutorials to consolidate and extend their learning when and where it suited them. Creating *a sense of autonomy* particularly resonated with parents, who felt unable to assist with mathematics homework at this level.

It appeared that Mr Smith was particularly influential in developing *a sense of relatedness* in his students. The results showed that students and parents were appreciative of the time and effort involved in producing the videos, and they recognised that he could select examples and provide directed teaching when necessary to capitalise on the flipped classroom approach. Class time previously spent working through examples on the board became more targeted towards specific instruction. In this way Mr Smith was *tailoring to students' expertise* and providing students with opportunities to *self-pace* their learning.

Overall, the study shows that the flipped classroom approach has merits in terms of creating an environment where students can be intrinsically and extrinsically motivated to achieve learning goals. While it could be argued that the students in this study were already motivated to succeed as they chose to study the course, it is evident through the data that they could identify factors that influenced their engagement with the course. With the exception of Sue, who raised concerns about the passivity of the approach, students and parents favoured it over traditional mathematics homework practices. The study, therefore, has implications for teachers, parents and policy-makers who may need to re-consider traditional approaches to senior secondary teaching and homework practices in light of these findings.

References

Abeyseker, L., & Dawson, P. (2015). Motivation and cognitive load in the flipped classroom: definition, rationale and a call for research. *Higher Education Research & Development, 34*(1), 1-14.

ACARA (2014). *Information and Communication Technology (ICT) capability*. Available from: http://www.australiancurriculum.edu.au/GeneralCapabilities/Pdf/ICT

Attard, C. (2010). Students' experiences of mathematics during the transition from primary to secondary school. In L. Sparrow, B. Kissane & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 53-60). Fremantle, WA: MERGA.

Bergman, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class everyday.* Washington, DC: International Society for Technology in Education.

Civil, M. (2006). Working towards equity in mathematics education: A focus on learners, teachers, and parents. In S.Alatorre, J.L. Cortina, M. Sáiz, & A. Méndez (Eds.), *Proceedings of the Twenty Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, 30-50). Mérida, Mexico: Universidad Pedagógica Nacional.

Cole, M., Field, H., & Harris, S. (2004). Student learning motivation and psychological hardiness: Interactive effects on students' reactions to a management class. *Academy of Management Learning and Education, 3*(1), 64-85.

Creswell, J.W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Department of Education and Early Childhood Development (2009). *Energising science and mathematics education in Victoria*. Melbourne: Education Policy & Research Division Office.

Hamdan, N., McKnight, P., McKnight, K., & Arfstrom, K. (2013). *A review of flipped learning.* Available: http://flippedlearning.org/cms/lib07/VA01923112/Centricity/Domain/41/LitReview_FlippedLearning.pdf

MCEETYA. (2005). Pedagogy strategy: Learning in an online world. Available: http://www.curriculum.edu.au/verve/_resources/ICT_LearningOnlineWorld-PedagogyStrategy.pdf

MCEETYA. (2003). Contemporary learning: Learning in an online world. Available: http://www.curriculum.edu.au/verve/_resources/Contemp_Learning_Final.pdf

Muir, T. (2009). At home with numeracy At home with numeracy: Empowering parents to be active participants in their child's numeracy development. In R. Hunter, B. Bicknell, & T. Burgess (Eds.) *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, pp. 395-402). Wellington, NZ: MERGA.

Muir, T., & Chick, H. (2014). Flipping the classroom: A case study of a mathematics methods class. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 485-492). Sydney: MERGA.

NSW Department of Education and Training. (2005). Students - middle years. Available from http://www.dec.nsw.gov.au/about-us/statistics-and-research/public-reviews-and-enquiries/

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology 82*(1), 33-40.

Sullivan, P.A., Mousley, J., & Zevenbergen, R.L. (2005). Increasing access to mathematical thinking. *Gazette of the Australian Mathematical Society*, *32*(2), 105-109.

Srivastava, P. (2009). A practical iterative framework for qualitative data analysis. *International Journal of Qualitative Methods, 8*(1), 76-84.

Xu, J., & Wu, H. (2013). Self-regulation of homework behaviour: Homework management at the secondary school level. *The Journal of Educational Research*, *106*, 1-13.

Authority and Agency in Young Children's Early Number Work: A Functional Linguistic Perspective.

Carol Murphy
University of Waikato
<carolmm@waikato.ac.nz>

This paper presents a preliminary study of three six year-old children's use of functional language when engaging collaboratively on a mathematics task. The analysis is presented as an illustration of young children's authority and agency in mathematics as evidenced in their discourse. Modality, as a function of language, was seen to indicate reasoning as a semantic process that expressed a state of knowledge as the children explored number comparison relationships. It is proposed that the children's use of modality indicated an element of internal authority in arbiting mathematical correctness and that related to the nature of the task.

Introduction

It would seem a worthwhile aim to encourage learners to participate as creative agents, who think and reason for themselves in mathematics, rather than being passive recipients of knowledge (Boaler & Greeno, 2000). Bandura (1997) considered how self-efficacy related human agency to the capacity to coordinate learning and motivations. Hence self-efficacy and agency would seem important in supporting learners to be creative agents.

This paper presents a preliminary study of a task that intended to support collaboration and self-efficacy in young learners. Three six-year old children worked together on a puzzle designed to encourage the children to think and reason for themselves. The episode was analysed in relation to the children's use of language as evidence of authority. Whilst just one episode, the analysis provides a rich interpretation of the children's engagement with the task, from which more can be learnt about children's agency and self-efficacy in mathematics, and the nature of supporting tasks.

Exploring Agency and the Use of Language in Mathematics

Lange (2009) defined human agency as the "faculty to act deliberately according to one's own will and thus to make free choices" (p. 2588). This interpretation of agency is extended further by Pickering's (1995) and Cobb, Gresalfi, and Hodge's (2009) distinctions between human agency (choice and discretion of a learner), conceptual agency (developing meanings and relations between concepts and principles), and disciplinary agency (the established procedures of a discipline). Pickering referred to the free and forced moves between these distinctions as a 'dance of agency.' As such, the choices made in learning mathematics are forced or tempered by the intrinsic authority of the discipline of mathematics, and agency in mathematics is further defined as the opportunity to exercise discretion in making choices by drawing on mathematics ideas to solve problems (Grootenboer & Jorgensen, 2009).

Traditionally the teacher has authority in influencing, or controlling, the flow of ideas in a mathematics classroom (Amit & Fried, 2005). That is, the teacher controls the 'dance of agency' and shapes the authority of knowledge for the students. However, if students control the 'dance of agency', they learn to rely on the disciplinary agency of mathematics, and not the authority of the teacher. Such students would have the confidence to become "arbiters of mathematical correctness" (Schoenfeld, 1992, p. 62). This ability would seem intrinsic to the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 453–460. Sunshine Coast: MERGA.

relationship between self-efficacy and agency, and so lead to students as creative agents who think and reason for themselves.

Studies regarding the distribution of authority in teaching mathematics have shown how secondary students rely on their teacher's authority (Wagner & Herbel-Eisenmann, 2009). So it would seem desirable to establish agency with students from an early age. Recent studies have considered play and agency with kindergarten children (e.g. Erfjord, Carlsen, & Hundeland, 2015), but little research has been carried out with primary school children. Whereas there is the potential for free choice in the play activities of pre-school children, primary school children are introduced, more formally, to key mathematical ideas that are often modelled by the teacher. Hence primary school children are required to engage with conceptual and disciplinary agencies of mathematics in a more structured way. That is, they are being led by the teacher to engage in the dance of agency. So, how might we develop tasks where the children are managing this dance of agency, rather than the teacher?

Furthermore, consideration is needed on how to investigate agency and authority in the mathematics classroom. One way is to analyse discourse as patterns of interaction between teachers and students (Wagner, 2007). Other methods of discourse analysis focus on the functional use of language. In relation to Halliday and Matthiessen's (2004) theory of systemic functional linguistics (SFL), functional linguistics provides a way of examining meaning making in a given environment. In examining agency in relation to language use, a key distinction is made between the primary tense, that expresses what is present at the time of speaking, for example 'it is' or 'it isn't,' and modality that expresses certainty or possibility, for example 'it has to be' or 'it can be.' Modality is further divided into deontic and epistemic. Deontic modality indicates the necessity or possibility of acts, that is, socially regulated behaviour, and these are more commonly known to young children (for example, 'you have to sit still' or 'you can't go out to play'). Epistemic modality indicates the speaker's beliefs based on the available evidence (for example, 'that has to be …').

Much mathematical language relies on the use of modality, both deontic and epistemic. The use of deontic modality suggests authority through the control of behaviour in how to carry out a procedure, and epistemic modality suggests the certainties or possibilities regarding mathematics, and hence is part of the dance of agency in relating to the discipline of mathematics. De Freitas and Zolkower (2010) have focused on modality in studying authority and agency of the teacher in mathematics classrooms, but modality has not been used as a focus to study young children's interactions.

Developing the Task

I had been working with a class teacher over a school year to develop tasks to encourage collaboration with six year-old children. The intention was to move away from direct instruction, and to shift authority away from the teacher. As such, the nature of the task was important in providing access to conceptual and disciplinary agency, where the students were put in charge of making decisions. The task presented here was developed as a puzzle with intrinsic logic (see figure 1). In solving a puzzle, as in playing a game (van Oers, 2014), there are rules: rules of the puzzle (how to act based on the rules of the puzzle) and conceptual rules (how to act based on specific concepts). As such the task resembled a play activity, where students make choices, but where the correctness of a choice is based on the rules of the puzzle.

A further aim in developing the task was to support the children's learning in number. In particular we focused on the comparison relationship 'more or less than'. Finding a number with say two more or two less, relies on the comparison of two cardinal units and involves more than counting. The relational nature of numbers is abstract. Relations between numbers do not refer directly to concrete objects; they can only be represented by concrete or symbolic objects (Steinbring, 2005). This abstract notion was seen as important in supporting the children's learning in number, as they often relied on counting processes.

Structural dot patterns, based on ten frames (see Figure 1), were used to represent cardinal units for comparison, as a way to encourage part-whole thinking rather than counting. Developed from earlier research on young children's counting models and subitising abilities (Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983), the importance of pattern and structure in early mathematics learning has now become fully recognised. Studies on children's use of representations and structure, such as egg boxes as six and ten frames, have been shown to support part-whole thinking (Young-Loveridge, 2002). Further studies have examined young children's awareness, recognition, and visualisation of pattern and structure. (Mulligan & Mitchelmore, 2009), and the examination presented here provided an opportunity to explore the use of pattern in comparing numbers.

The task required the students to complete a rectangle by placing ten frame cards, which were more or less than the previous one, according to a given condition recorded on the arrows around the rectangle (Figure 1). For example, the ten frame following the start 10 ten frame could be either two more or two less than ten. Eight ten frame cards with values from three to ten were provided to complete the task. The rectangle was to be closed, meaning that the last ten frame had to meet both the previous and the final conditions. As can be seen in Figure 2, the 9 ten frame has been placed, so that it is three more than the 6 ten frame and one less than the 10 ten frame.

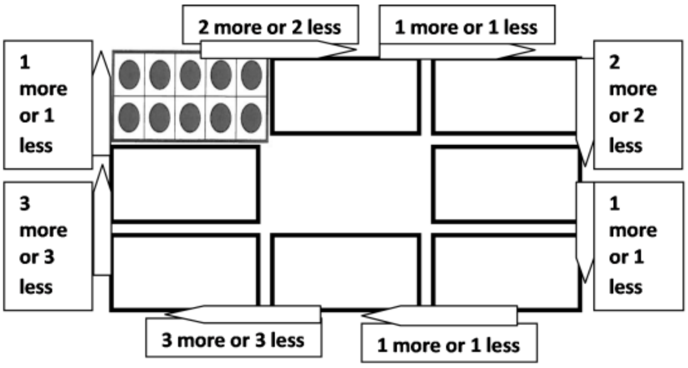


Figure 1. The More or Less task

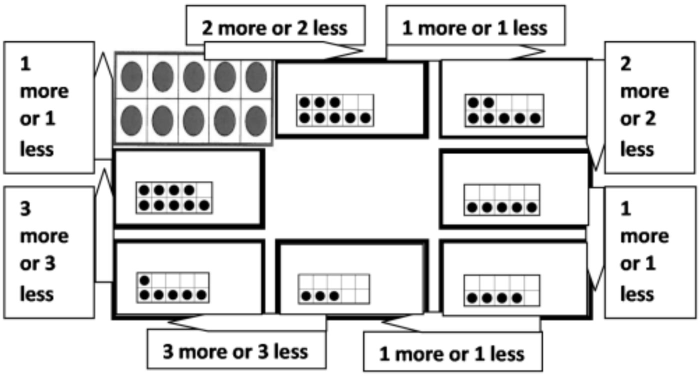


Figure 2. A completed version of the More or Less task

Research Methods

In the episode examined in this paper, the focus is on how the pattern structures, within the nature of the task, might mediate the students' arbitration in determining the correctness of their choices. The task was presented by me, as the researcher, to three six year-old children, Kim, Emma, and Helen. The children had worked together, and with me, on previous collaborative tasks, but the More or Less task was a new introduction. The episode was video recorded and transcribed. The research method followed the principles of the clinical interview (Ginsburg, 1997). As the researcher, I observed, probed, and prompted the children as they worked on a task. The intention was to enter the children's minds, but through discourse analysis that focused on functional linguistics, and in particular the children's spontaneous uses of primary tense and modality. The use of the primary tense indicated what was present and known to the children, and modality indicated the children's reflections on possibilities or certainties. In introducing the task to the three children, Kim, Emma, and Helen, I emphasised that they needed to close the rectangle, so that, for the last space, the ten frame would have to be one more or one less that the first card (the 10 ten frame).

Examples of Critical Incidents in Use of Language

As this was just one episode, analysis was carried out through viewing the video material alongside the transcript. The students' use of present tense and modal terms in critical incidents were identified. Transcripts of the critical incidents are presented below. Actions are presented in italics in parenthesis for clarification. References are made to figures 3 to 6, showing images taken from the video recording.

1. Helen: What shall we do, two more or two less? Two less, two less. *(Helen placed the 8 ten frame next to the 10 ten frame.)*
2. Kim: One less. (*Kim read from the next arrow.)*
3. Helen: It's eight. *(Helen counted the dots on the 8 ten frame and Kim placed the 7 ten frame next.)*
4. Kim: Two more or two less. *(Kim read from the next arrow.)*
5. Helen: Two more. *(Emma handed Helen the 9 ten frame. Helen pointed to two dots on the 9 ten-frame.)*
6. Helen: See there's two more. *(Helen placed the 9 ten-frame.)* (See Figure 3.)
7. Kim: We need to decide which one goes where. Do you want to do one more or one less?
8. Helen: One less, I mean one more *(The children looked at the ten frames they had left (3, 4, 5, and 6) and Helen pointed to the 9 ten-frame and the next space on the rectangle.)*

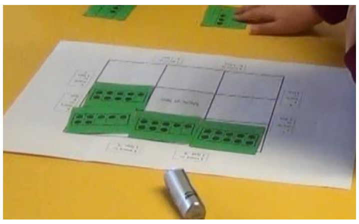


Figure 3. The children's first attempt at completing the task

9. Helen: So that will be one less, it'll have to be one less.

10. Kim: It will have to be eight. No. We have to do one more, because we don't have…. *(Kim looked to the 10 ten frame)* Oh no we have to start again. (*The children removed all the ten frames, apart from the 10 ten-frame, and started again.)*

11. Helen: What about we do this – more, less, more, less, more, less *(Helen pointed to the spaces around the rectangle.)*

12. Kim: But there's no bigger number, that's the biggest number. (*Kim pointed to the 10 ten-frame.*)

13. Kim: Why don't we do this one less, this one less, this one more, this one more, this one less, this one less? What do you want that to be? *(Kim pointed to the spaces around the rectangle and then stopped at the last closing space.)* (See Figure 4)

14. Emma: More.

15. Helen: No less, less.

16. Kim: That has to be nine. *(Kim pointed to the space before the 10 ten-frame and Helen placed the 9 ten frame in the space.)* (See Figure 4)

17. Kim: Two more? *(Kim indicated the space next to the 10 ten-frame.)*

18. Helen: Two less, seven. *(Helen pointed to the space next to the 10 ten frame, and Kim placed the 8 ten frame.)* (See Figure 4.)

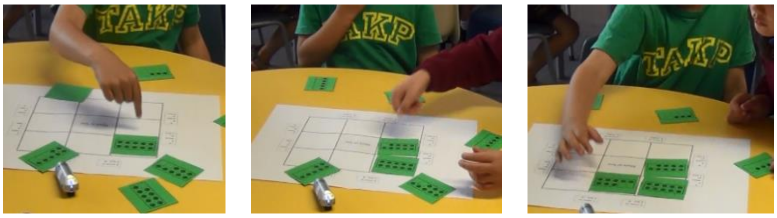


Figure 4. The children start again on the task

At this point the children realised the positioning of the 9 ten frame before the 10 ten frame and, even though Helen had stated seven as two less than ten, Kim placed the 8 ten frame after the 10 ten frame (Figure 4). They then became confused over the positions of the 8 and 7 ten frames.

19. Kim: One more or one less. Eight where's eight? *(Kim suggested eight as one more than seven.)*

20. Helen: On no, this is an eight. *(Helen picked up the 8 ten frame and counted the dots.)* It's eight. That's seven. *(Helen moved the 8 ten frame and placed the 7 ten frame between the 8 ten frame and the 10 ten frame.)*

The children then chose which ten frames to place next, with Kim asking "Which do you want, one more or one less?" but they did not place the ten frames according to the conditions given in the arrows (Figure 5).

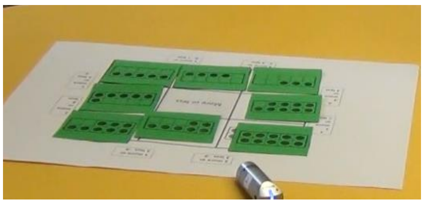


Figure 5. An incorrect solution to the task

All three children turned to the researcher to say "Done!" The researcher returned to the group and asked the children to explain their decisions for placing the ten frames.

21. Researcher: So you decided that one had to be a nine. *(Researcher pointed to the 9 ten-frame.)* Why did you decide that?
22. Kim: Because ten's the biggest number. We can't do one more, that would be eleven and we don't have eleven.
23. Researcher: So why did you decide to put this one there? *(Researcher pointed to the 7 ten frame next to the 10 ten frame.)*
24. Kim: Cos it's, what's this? Because it's…
25. Helen: No it's three less.
26. Kim: It's supposed to be two less. *(Kim swapped the 7 with the 8 ten frame.)*
27. Researcher: *(Researcher pointed to the new position of the 7 ten frame.)* Why wouldn't you put the 9 ten frame there? You could have made that one more?
28. Kim: Cos maybe you couldn't put anything here. *(Kim pointed to the 9 ten frame before the 10 ten frame.)*
29. Researcher: So where do you go after that? You need two more or two less. Can you use nine? *(Kim shook her head.)* So what you are going to have to use?
30. Kim: Five? *(Kim moved the five next to the 10 ten frame but then moved it away and replaced it with the 4 ten frame.)*
31. Helen: No you need the five, you need the five, you need the five there. *(Helen moved the 4 ten frame away and replaces it with the 5 ten frame.)*

The students needed reassurance from the researcher in placing the last three ten frames but they did complete the task with the correct solution (Figure 6).

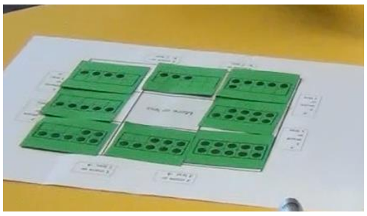


Figure 6. The children complete the task with a correct solution

Analysis and Discussion

In transcript lines 1 to 7 the children used the primary tense. Helen asked, "What shall we do?" suggesting a free choice. The children also used the primary tense in stating "It's eight" or "There's two more." The numbers and the number relationships were present and known to the children. Even though Kim used a deontic modal term "We need to decide," she then asked what they wanted to do, not what they had to do, again suggesting free choice. In transcript lines 8 to 9, it seemed the children realised they did not have a ten frame more than nine, so they decided they needed a ten frame less than nine, and, for the first time, they used epistemic modality in the phrases, "It'll have to be one less" (transcript line 9) and "It will have to be eight" (transcript line 10). As Kim noticed the 8 ten frame had been used, she then used deontic modality in the phrase, "We have to do one more" (transcript line 10). The children were beginning to reason what they had to do, and what numbers were needed to meet the rules of the puzzle, rather than referring to free choice. Hence, the children were beginning to relate to conceptual rules and the rules of the puzzle.

As the children started a second attempt (transcript lines 11 and 13), and plotted possible positions around the rectangle, they seemed to experiment with systems. This systemic approach, whilst still tempered by free choice in choosing a system, suggested the children were attempting to relate to authority within the rules of the task. Their experimentation with the systems also led them to look at the final closing position. Kim used the primary tense in noting that there was no bigger number than ten (transcript line 12), and then used epistemic modality in the phrase, "That has to be nine," in determining the value of the ten frame in the closing position (transcript line 16 and Figure 4). Ten, as the biggest number, was present and known to the students, but Kim then used epistemic modality to realise a necessary number for the closing ten frame.

The children made an error in stating seven as two less than ten, but the 8 ten-frame was placed next. The confusion with the 8 ten-frame and the 7 ten-frame (transcript lines 19 and 20) resulted in the children checking by counting the dots. The children referred to the numbers on the ten frames and used the primary tense "…this is an eight….It's eight. That's seven," in confirming knowledge that was presented to them. The children then further confused the completion of the task by swapping the 8 and 7 ten frames, but later, after questioning from me, Kim and Helen changed the placing of the 8 ten frame as two less than ten (transcript lines 24 and 25). Kim's phrase, "It's supposed to be…" (transcript line 26) showed use of epistemic modality that linked knowledge of the number on the ten frame with the number relationship and the rules in the task.

Children's use of both the primary tense and modality were further evidenced as I questioned the positioning of the ten frames. For example, Kim's statements "We can't do one more... we don't have eleven" (transcript line 22) and "you couldn't put anything here" (transcript line 28) further suggested Kim was linking the knowledge of the number on the ten frame with the number relationship and the rules of the task. Later, Helen was clear in her use of necessity as deontic modality in the repeated phrase "you need the five" (transcript line 31). Whilst deontic, this use of modality must also have related to epistemic modality in noting that the next ten frame had to be 5.

Analysis of the children's dialogue indicated that, as they realised the limiting conditions in completing the task ("But there's no bigger number...[than ten]"), they began to use modal terms both deontic ("So we have to..." and "You need…") and epistemic ("It'll have to be one less" and "That one has to be nine..."). This suggested they were linking knowledge that was present to them in the numbers as quantities, the number comparison relationships, and the intrinsic logic or rules of the task. As they realised the limitations of their choices, the children moved from free choice (human agency) to conceptual agency. Whilst the children received some prompting from me, as the researcher, the rules of the puzzle in the task had the potential to mediate the children's arbitration in determining the correctness of their choices. Hence authority was determined within the task.

Concluding Remarks

From the analysis of the use of modality in this episode, it is proposed that young children are capable of conceiving of possibilities and certainties and reflecting on these. Modality was seen to indicate reasoning as a semantic process, where it depended on understanding the meaning of the premises and expressed a state of knowledge. It is further proposed that tasks presented as puzzles, with an intrinsic logic, have the potential to support students in determining the correctness of their choices, and in realising the dance between human, conceptual and disciplinary agency. The task was new to these children, and it remains to be seen if further use of such tasks would enable young children to work independently in completing the task. Whilst this task was based on the comparison relationships, more than and less than, the closed rectangle could be further investigated with other number relationships, including multiplicative thinking, and other mathematical functions.

Further studies are needed to investigate how extended use of such tasks might effect a shift of authority in learning from the teacher, and so support learner agency with young children. However, this preliminary study of one episode has shown the potential of investigating agency through discourse analysis focusing on modality as a function of language. More extensive studies, and finer analysis of the use of language using software such as NVivo, would be important in further understanding how tasks can be developed to support young children's self-efficacy and agency in relation to learning mathematics.

References

Amit, M., & Fried, M. (2005). Authority and authority relations in mathematics education: A view from an 8th grade classroom. Educational Studies in Mathematics, 58(2), 145-168.

Bandura, A. (1997). Self-efficacy: The exercise of control. New York, NY.: W. H. Freeman and Company.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), Multiple perspectives on mathematics teaching and learning (pp. 171-200). Westport, CT: Greenwood Press.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classroom. Journal for Research in Mathematics Education, 40(1), 40-68.

de Freitas, E., & Zolkower, B. (2010). Discursive authority in the mathematics classroom: Developing teacher capacity to analyze interactions in terms of modality and modulation. Paper presented at the Mathematics Education and Society 6 Proceedings, Berlin, Germany.

Erfjord, I., Carlsen, M., & Hundeland, P. S. (2015, 4-8 Feb 2015). Distributed authority and opportunities for children's agency in mathematical activities in kindergarten. Paper presented at the CERME 9.: 9th Congress of European Research in Mathematics Education, Prague, Czech Republic.

Ginsburg, H. P. (1997). Entering the child's mind: The clinical interview in psychological research and practice. NY: Cambridge Press.

Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. Eurasia Journal of Mathematics, Science and Technology Education, 5(3), 255-266.

Halliday, M. A. K., & Matthiessen, C. (2004). An introduction to functional grammar. Third edition. London: Arnold.

Lange, T. (2009). "Tell them that we like to decide for ourselves" - children's agency in mathematics education. Paper presented at the Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education. January 28th - February 1st 2009, Lyon

Mulligan, J., & Mitchelmore, M. (2009). Awareness of Pattern and Structure in Early Mathematical Development. Mathematics Education Research Journal, 21(2), 33-49.

Pickering, A. (1995). The Mangle of Practice: Time, Agency, and Science. Chicago: University of Chicago Press.

Schoenfeld, A. (1992). Reflections on doing and teaching mathematics. In A. Schoenfeld (Ed.), Mathematical thinking and problem solving (pp. 53-70). Hillsdale, NJ: Erlbaum.

Steffe, L. P., & Cobb, P. (1988). Construction of arithmetic meanings and strategies. New York, NY.: Springer-Verla.

Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). Children's counting types: Philosophy, theory, and application. New York, NY.: Praege.

Steinbring, H. (2005). The construction of new mathematical knowledge in classroom interaction: an epistemological perspective (Vol. 38). Berlin, NY.: Springer.

van Oers, B. (2014). The roots of mathematising in young children's play. New York, NY.: Springer.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. Mathematical Thinking and Learning, 9(1), 31-50.

Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. Educational Studies in Mathematics, 72(1), 1-15.

Young-Loveridge, J. (2002). Early childhood and numeracy: Building an understanding of part-whole relationships. Australian Journal of Early Childhood, 27(4), 36-42.

Examples in the Teaching of Mathematics: Teachers' Perceptions

Lay Keow Ng
National Institute of Education, Singapore
< ng_lay_keow@moe.edu.sg>

Jaguthsing Dindyal
National Institute of Education, Singapore
<jaguthsing.dindyal@nie.edu.sg>

As part of a study examining how teachers in Singapore select and use examples for teaching mathematics, 121 teachers from 24 secondary schools responded to three open-ended questions about the use of examples in teaching. The results show that students' abilities and the difficulty level of the examples were among the topmost considerations teachers have when introducing mathematical ideas or when selecting homework tasks. This paper also reports on teachers' perceptions of a *good* example.

The use of examples by teachers in the mathematics classroom is a well-established practice. While researchers have attended to the roles of sub-categories of examples, research into how teachers integrate examples into their teaching remains scarce (Zodik & Zaslavsky, 2008). Research has also shown that the use of examples, or exemplification in short, is neither arbitrary nor straightforward, where prospective teachers (Huntley, 2013) and experienced teachers (Zodik & Zaslavsky, 2008) both face problems, hence summoning the need for research in this area.

Literature has also revealed a strong connection between teachers' knowledge and their use of examples in teaching. Rowland, Huckstep, and Thwaites (2005) found that teachers' ability in selecting suitable mathematical examples was strongly related to their mathematics content knowledge for teaching. Also, Chick (2010) stressed that the capacity of teachers in crafting effective examples relies heavily on their pedagogical content knowledge too.

Teachers use examples in various ways, often to introduce an idea or illustrate a concept. Also, examples are used by teachers in the assignment of specific tasks, such as homework, which in Singapore is a common practice. Several factors may affect the choice of specific examples by teachers. This paper focuses on the following three questions.

1. What factors do secondary mathematics teachers consider when choosing examples for introducing new mathematical ideas?
2. What factors do secondary mathematics teachers consider when selecting examples for homework tasks?
3. What are the characteristics of a *good* example used for teaching mathematics in the eyes of secondary teachers?

Examples in the Teaching of Mathematics

The significance of examples is summarised by Watson and Mason (2002): "learning mathematics can be seen as a process of generalizing from specific examples" (p. 39). Examples are therefore paramount in mathematical teaching and learning.

The definition of examples used by researchers generally refers to an example as an illustration of a larger class. This broad definition can include geometrical figures, demonstrations of solving problems, tasks, and worked examples, as long as the mathematical object is offered or perceived as an example of something. In this study, a task can be an exercise, problem, or assessment assigned to students for completion during or beyond curriculum time. The same task may differ in operation and learning outcomes,

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 461–468. Sunshine Coast: MERGA.

depending on the intentions of the author, the aims and knowledge of the teacher, the goals, knowledge, and experiences of the students, and on the learning environment. The role of teachers therefore lies in setting up appropriate tasks.

Example selection is, however, not merely choosing or implementing good examples, but entails leveraging on coherent example sets to build students' understanding in order to attain instructional goals. Watson and Mason (2006) claimed, "the starting point of making sense of any data is the discernment of variations within it' (p. 92). They proposed to systematically change certain aspects of a task while keeping others invariant, to help learners better perceive the mathematical structure. In addition, Skemp (1971) advised educators to reduce the *noise* in examples during concept formation so as to draw learners' attention to the key characteristics of the concept.

Empirical findings from work with teachers have also revealed principles that guide teachers in making their example choices. One common approach was the use of simple first examples (Bills & Bills, 2005) that include keeping the numbers small and ordering examples in increasing complexity. To scaffold students' learning, teachers have also proposed using examples that build on students' prior knowledge (Bills & Bills, 2005) and keeping unnecessary work to a minimum (Zodik & Zaslavsky, 2008). Sometimes, teachers tend to craft and use examples that allow them to attend to common errors and misconceptions to forewarn their students (Zodik & Zaslavsky, 2008) or to include uncommon cases to increase students' exposure.

Teacher Knowledge and the Use of Examples in Teaching Mathematics

A closer scan of the literature on mathematical examples highlights the close connection between teachers' examples and their knowledge. In particular, content knowledge and pedagogical content knowledge (PCK) have been identified to directly influence teachers' exemplification abilities. Content knowledge is the knowledge of the subject matter content. PCK is the "blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8). Ball, Thames, and Phelps (2008) sub-divided PCK into knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). KCS includes an awareness of topics that students will find easy or difficult and their common conceptions and misconceptions. KCT comprises of knowledge on the sequencing of examples and the use of appropriate representations. Finally, KCC encompasses knowledge of educational goals, assessments, and the sequencing of topics across grade levels.

Rowland et al. (2005) observed how content knowledge and PCK contributed to the decisions and actions of their participants. Of the four units of their Knowledge Quartet framework, transformation or knowledge-in-action was strongly tied to teachers' example choice. Variables, sequencing, representations, and learning objectives were also identified as related to teachers' awareness in exemplification.

Noticing the lack of research between teachers' PCK and their exemplification practices, Chick and her colleagues (see Chick, 2007) studied the instructional practices of Australian elementary teachers and were successful in surfacing moments where aspects of PCK were enacted through the teachers' examples. Chick (2007) also noted that most of the examples that the teachers used were planned and selected based on the examples' structures and qualities. The selection process was much guided by the teachers' PCK, especially on what affordances they perceived the examples could offer. Even when teachers have to come up with an example on the spot, their ability to do so is greatly influenced by their PCK (Chick & Pierce, 2008). Similarly, Zodik and Zaslavsky (2008) who carried out an in-depth study with five secondary teachers concluded that content knowledge, PCK, and knowledge of students' learning, a sub-category of PCK, shape teachers' examples.

Methodology

This study surveyed the exemplification practices of secondary mathematics teachers in Singapore for which a purposeful sample of experienced teachers was used to provide richer information. Participants were chosen from teachers who had taught mathematics for at least five consecutive years and had some experience in teaching at the upper secondary level. A questionnaire was then constructed and distributed to teachers who fit the criteria.

The questionnaire was pilot-tested with 16 teachers from two schools and thereafter refined. Of the 128 questionnaire returns from 24 secondary schools, seven were invalid as three had only lower secondary (grade 7 and grade 8) teaching experience and four had taught for less than five years. The remaining 121 teachers had a mean of 12 years of teaching and 89 of them had experience in teaching Additional Mathematics: an advanced level of mathematics that is offered to more mathematically able students in upper secondary and includes topics like plane geometry proofs and introductory Calculus. Of these 121 teachers, 44 teachers taught one other subject and the rest taught mathematics only. All respondents had a first degree and a teaching qualification. 25 of the teachers had a masters degree of which 19 were masters in mathematics or mathematics education. The gender composition was almost 50:50 (57 females). 119 indicated their age group and the age distribution is shown in Table 1.

Table 1
Age Group of 119 Teacher Respondents

Age	Under 30	30-39	40-49	50-59	60+
Number of teachers	7	58	32	17	5

The purpose of the questionnaire was to explore teachers' opinions on mathematical examples, their mathematical knowledge of teaching, and their mathematical beliefs. For this paper, the focus is on the three questions that surveyed the teachers' exemplification considerations. The first question read "list down two factors you consider when selecting examples to *introduce* a new concept/procedure/rule/principle". Research has shown that teachers like to begin with a simple or familiar first example and order examples in increasing degree of difficulty (Rowland et al., 2005). Teachers also reported to be conscious of the importance to reduce the *noise* in examples so as to focus learners' attention on the critical aspects (Skemp, 1971). Hence, the objective of this question was to elicit teachers' decisions in selecting their first few examples in order to focus on those teachers who can better justify their choice of mathematical examples.

The second question asked teachers to list down two factors they considered when selecting homework tasks. Hiebert et al. (1996) proposed that teachers look for tasks that can offer situations that students will perceive as problematic and that provide platforms for students to think about important mathematics. Tasks should also connect to some part of the students' knowledge so that they are attainable by students. Hence, it is worthwhile to investigate how teachers decide on homework tasks.

Finally, teachers were asked to write down three characteristics of what they think a *good* example would have. Zaslavsky and Lavie (2005) defined a good example as one “that conveys to the target audience the essence of what it is meant to exemplify or explain” (p. 2). They described good examples as transparent, can foster generalisation, and aid in explaining and resolving mathematical subtleties. Thus, the third question was to elicit what teachers believed that a good example would entail.

Results and Discussion

The data collected for this study involved teachers’ responses to the three questions. Teachers’ responses for each question were categorised and 13 category codes were created to facilitate the analysis and discussion both within and between the questions. In all 13 categories, some were common. Table 2 presents the percentage category frequencies for each question, ordered in decreasing frequencies for question one.

Table 2
Categories of 121 Teachers’ Exemplification Considerations

Category Code	Category Description	Teach Mathematics Idea (%)	Select Homework (%)	Good Example (%)
SA	Students’ Abilities	25.5	17.4	13.1
DL	Difficulty Level	21.3	23.0	16.1
FC	Familiar Context	18.3	-	8.36
LO	Learning Objectives	8.09	8.12	5.97
EC	Exemplify Content	8.09	-	10.7
VE	Variety of Examples	6.81	19.2	10.1
CE	Clarity of Examples	5.11	-	15.8
TI	Thinking and Interesting	3.83	-	9.25
CM	Common Misconceptions	2.13	0.855	4.18
CH	Classwork and Homework	0.851	5.98	-
NE	Number of Examples	-	9.83	-
RL	Reinforce Learning	-	8.94	4.78
AU	Assess Understanding	-	6.41	1.49

RQ1. What Factors do Secondary Mathematics Teachers Consider when Choosing Examples for Introducing New Mathematical Ideas?

A total of 235 teachers’ considerations, when they teach new mathematical ideas, were gathered in which the first three categories surfaced more often. From Table 2, *Student Abilities* (SA) was reported as the major concern teachers have when introducing new content (60 counts). SA consisted of responses on students’ abilities, prior knowledge, and the need to scaffold students’ learning. The comments included “must suit students’ ability” and examples should be able to “link to prior knowledge”. Some teachers, like the mentors in Bills and Bills’ (2005) study, also advocated instructional scaffolding via examples like “easy ones first, then progressively more challenging ones”.

The second most common category was *Difficulty Level* (DL) which pertains to whether the examples were easy or hard (50 counts). Many teachers echoed that they would take note of the difficulty level of examples. Others proposed to use an example that is "easy to understand" and this resembles the key theme in another study, which was keeping things simple (Bills & Bills, 2005). A related category was to use *Familiar Context* (FC) that students can easily relate to by linking to the "personal experience of students" or "real-world situations", of which there were 43 counts. In a way, SA, DL, and FC encompassed one of the guiding principles teachers in Zodik and Zaslavsky's (2008) study demonstrated which was to start with a simple or familiar case.

Of the participating teachers, 19 were concerned if examples used could "address the instructional objectives" and prepare students for examinations (LO). This factor was also identified by Rowland (2008) in his study. Teachers were also mindful when selecting the first few examples that could exemplify a new content (EC), so as reduce the noise (Skemp, 1971) by selecting only those that were able to "highlight the key points".

There were 16 comments on using different examples, *Variety of Examples* (VE), when presenting a new mathematical idea whereas some included examples that "show the application of the new concept". 12 wrote about the *Clarity of Examples* (CE) that examples should be clear, "should not be overly tedious to solve", and should involve "small numbers, positive integers if possible". This partially reflected the approach by teachers in another research to draw attention to relevant features (Zodik & Zaslavsky, 2008). Arousing interest and stimulating thought processes, *Thinking and Interesting* (TI) was also raised (9 counts). Fewer (5 counts) attended to the need to address *Common Misconceptions* (CM) and only two teachers selected examples that "can help them [students] to solve questions given for homework later" (*Classwork and Homework*-CH). Since the teaching of a new mathematical idea was the focus of this question, it was logical that the following categories: *Number of Examples* (NE), *Reinforce Learning* (RL), and *Assess Understanding* (AU), were not part of the teachers' considerations.

RQ2. What Factors do Secondary Mathematics Teachers Consider when Selecting Examples for Homework Tasks?

There were 234 written factors where the top three categories, DL, VE, and SA were more frequently cited. Similar to teachers' choice of the first few examples, when they plan homework, DL (54 counts) and SA (41 counts) were important too. What differs in DL was teachers were more prone to choose challenging over simple homework tasks. "Tasks should be reasonable within ability of students" so that "students can manage the homework". Hiebert et al. (1996,) considered SA as vital too as teachers should select tasks that "students can see the relevance of the ideas and skills they already possess" (p. 16).

A key approach by many (45 counts) was to expose students to varied examples (VE), as a limited range of examples might lead to an incomplete or erroneous understanding. "Direct application of concepts, challenging questions, and integrated mathematics and real-life situations" should be tasked for a "comprehensive coverage of exercise".

The next three codes, NE (23 counts), RL (21 counts), and LO (19 counts) had comparable ratings. Some teachers carefully considered the "time taken to complete homework questions" by reminding themselves to give "manageable number of questions" (NE). However, this category was absent in the teachers' exemplification considerations when they introduced new concepts or when they identified good examples.

Some teachers were concerned whether homework could "reinforce classroom teaching" (RL). The "purpose of the homework task" (LO) to cover the school's scheme of work or to "prepare students for examinations" was also raised. 15 teachers suggested that the role of the homework is "to assess students' understanding" (AU) and that "tasks should give feedback on students' learning". Slightly fewer (14 counts) shared that their homework selection was based on the classwork and that for the homework they "will give questions similar to the work done in class". Only two stated that they would include "questions that can surface common mistakes or misconceptions".

It was noticeable that the teachers did not consider FC, EC, CE, and TI when they set homework tasks. Since homework served mainly for students to develop their skills, teachers reported that they tended to expose students to different types of problems rather than focus on context familiar to them (FC). The same can be said for EC and CE, which were more relevant to mathematical understanding. What was more conspicuous was the absence of thinking and interesting aspect in homework tasks, as this is fundamental in Singapore mathematics framework (Ministry of Education, 2012).

RQ3. What are the Characteristics of a "Good" Example used for Teaching Mathematics in the Eyes of Secondary Teachers?

The respondents gave 335 written descriptions of their concept of good examples. Likewise, when teachers look for critical attributes in examples, DL (54 counts) and SA (44 counts) were pivotal. Interestingly, over 75% were more likely to pick an "easy to understand" example over one that "can stretch their thinking". A good example should also be "pitched at the right level for the class" and be able to "link with prior knowledge". Unlike the previous two questions, there were five teachers who favoured the use of "illustrations and diagrams" to "assist in the conceptualisation", which Rowland et al. (2005) found to be tied to teachers' exemplification practices.

A substantial number of teachers (53 counts) felt that good examples are "clear" (CE) and "well-crafted", where they "test students on the concept but not on the English". "Ease in calculation" and having "no complicated equations" reflected the *keep unnecessary work to a minimum strategy,* discussed earlier in Zodik and Zaslavsky (2008).

Teachers (36 counts) also characterised those that "highlight the salient points" (EC) and enable one "to generalise ideas or rules" as good examples. Hence, good examples are transparent and promote generalisation (Zaslavsky & Lavie, 2005). Others (34 counts) see examples as a set of "varied examples" (VE) to provide "sufficient coverage", to "link concepts together", and to allow the "application of concepts across topics".

Another desirable attribute of an example is if it is "able to provoke thinking" and "arouse students' interest" (TI). Of this type, 31 counts were identified and we can draw a parallel between TI and what Hiebert et al. (1996) meant by tasks that problematised the subject, so that they will "pique the interests of students and engage them in mathematics" (p. 18). Following next, is teachers' preference (28 counts) for examples "related to everyday experiences of students" (FC) or "has real-life application".

Twenty teachers indicated that a good example "delivers the lesson objectives" (LO) and some felt that it should be "similar to the examination syllabus type of questions". Fewer comments (16 counts) highlighted examples that "reinforce concepts or skills taught in class" (RL). 14 felt that good examples offer "opportunities to sieve out misconceptions in students" (CM) so as to attend to students' errors (Zodik & Zaslavsky, 2008). There were only five comments on choosing examples that can "assess students' understanding" (AU).

Connections to Teacher Knowledge

The three questions discussed in this paper were not based on any specific mathematical content. However, another section of the questionnaire examined teachers' mathematical knowledge. The data suggested that there were obvious connections between teachers' PCK and their use of examples. When teachers present new content, KCS is exhibited in how they considered students' prior knowledge (SA) and the difficulty level (DL) of the topic. As such the teachers try to choose examples that students can relate to (FC) and find interesting (TI) to make learning more manageable and meaningful for the students. Furthermore, knowledge of students' conceptions and misconceptions (CM) means that teachers prefer examples "that should not be clouded by other concepts or difficult algebra manipulation" (CE) so as not to confuse their students (Ball et al., 2008). Each of the above-mentioned categories requires teachers' knowledge of how students learn the mathematical content or KCS in short.

Teachers' example choice is influenced by their KCT too. They select examples that are able to exemplify the mathematical idea (EC) and also provide students with sufficient contact with the mathematical content through varied examples (VE). Teachers' KCT guide them in the sequencing of homework tasks in "ascending difficulty" (SA) in order to scaffold students' learning. In addition, teachers tend to pick those tasks that are able to reinforce what has been taught (RL) or by relating homework tasks to what have been covered in class (CM), in order to help students retain knowledge and gain fluency in their mathematical competency (Rowland, 2008). Furthermore, challenging tasks (DL) are also utilised to bring students deeper into the topic.

Finally, teachers' knowledge of the curriculum (KCC) sensitises them to those examples that are able to address and deliver learning objectives stipulated in the mathematics syllabus, as well as prepare students for assessment (LO) by making available to them examples that are similar to those tested in examinations. At the same time, teachers leverage on examples that "provide good feedback about students' understanding" (AU) in order to improve students' learning.

Conclusion

Teachers will continue to use examples in teaching their students, for whom examples may be a primary means for learning mathematical concepts. The use of certain examples for teaching a particular topic may not be universal, which implies that the survey of the teachers from Singapore who participated in this study may be very context-specific. It is important to be aware of the limitations in using questionnaire findings to study teachers' pedagogical practices since what is written may not be used in actual lessons. Nevertheless, this study brings us some insights into the exemplification perceptions of experienced mathematics teachers in Singapore. Teachers are most concerned over students' abilities and the difficulty level of examples when choosing examples. However, when selecting examples for different purposes, the considerations differ to some extent. For instance, when introducing new content, teachers favoured examples that connect with students' experiences whereas for homework, they are more concerned with providing students with varied exposure.

Finally this research reveals the potential direction for further research into the reasons teachers considered as critical factors in their choice of examples and points to a connection between teacher knowledge and beliefs about what constitutes effective teaching and learning of mathematics through the use of mathematical examples.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389–407.

Bills, C., & Bills, L. (2005). Experienced and novice teachers' choice of examples. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), Building connections: research, theory, and practice (Proceedings of the 28th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 146–153). Melbourne, Australia: MERGA.

Chick, H. L. (2007). Teaching and learning by example. In J. Watson & K. Beswick (Eds.), Mathematics: Essential research, essential practice. (Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 3–21). Sydney, Australia: MERGA.

Chick, H. L. (2010). Aspects of teachers' knowledge for helping students learn about ratio. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education*. (Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, pp. 145–152). Fremantle, Australia: MERGA.

Chick, H. L., & Pierce, R. (2008). Issues associated with using examples in teaching statistics. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the Joint Meeting of PME 32 and PME-NA XXX* (Vol. 2, pp. 49–55). Mexico: Cinvestav–UMSNH.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, *25*(4), 12–21.

Huntley, R. (2013). Pre-service primary teachers' choice of mathematical examples: Formative analysis of lesson plan data. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow*. (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, pp. 394–401). Melbourne, Australia: MERGA.

Ministry of Education (2012). *O– & N(A)–Level mathematics teaching and learning syllabus*. Singapore: Curriculum Planning and Development Division.

Rowland, T. (2008). The purpose, design and use of examples in the teaching of elementary mathematics. *Educational Studies in Mathematics*, *69*(2), 149–163.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255-281.

Shulman, L. S. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–22.

Skemp, R. R. (1971). *The psychology of learning mathematics.* Harmondsworth, UK: Penguin.

Watson, A., & Mason, J. (2002). Extending example spaces as a learning/teaching strategy in mathematics. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Conference of the international Group for the Psychology of Mathematics Education* (Vol. 4, pp. 378–385). Norwich: PME.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning, 8*(2), 91–111.

Zaslavsky, O., & Lavie, O. (2005). *Teachers' use of instructional examples*. Paper presented at the 15th ICMI Study Conference, Águas de Lindóia, Brazil.

Zodik, I., & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics, 69*(2)*,* 165–182.

How Inquiry Pedagogy Enables Teachers to Facilitate Growth Mindsets in Mathematics Classrooms

Mia O'Brien
Griffith University
<mia.obrien@griffith.edu.au>

Katie Makar
The University of Queensland
<k.makar@uq.edu.au>

Jill Fielding-Wells
University of Tasmania
<jill.wells@utas.edu.au>

Jude Hillman
Jindalee State School
<jhill98@eq.edu.au>

Growth mindsets are vital for effective lifelong learning. Students with growth mindsets are more willing to learn new things, take risks, and embrace challenges. Students with fixed mindsets have limiting beliefs about their abilities, and will attribute success in learning to factors beyond their control. Inquiry in mathematics classrooms may have the potential to facilitate growth mindsets. This paper provides an analysis of inquiry mathematics in a primary classroom and reflects upon its potential to foster growth mindsets in classrooms.

What is the Problem?

The Australian Academy of Science (AAS) has expressed concerns that "Australia with be unable to produce the next generation of students with an understanding of fundamental mathematical concepts, problem-solving abilities and training in modern developments to meet projected needs and remain globally competitive" (2006, p. 9). The AAS is not alone, the research-based Australian National Numeracy Review Report (National Numeracy Review Panel, 2008), which came about in response to a need for improving numeracy and mathematics learning within Australia, recommended that:

> From the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking (2008, p. xii).

There is emerging evidence that innovative teaching approaches can significantly improve students' attitudes and engagement in learning (O'Brien, under review). Within mathematics classrooms, inquiry pedagogies are linked to observable improvements in students' levels of engagement, performance and interest in mathematics (e.g., Allmond & Huntly, 2013; Fielding-Wells & Makar, 2008). Building a classroom culture of thinking results in significant gains in improving student thinking and reasoning abilities (Ritchhart & Perkins, 2005); but doing so relies on the effective development of certain types of student dispositions–the propensity for open-mindedness, curiosity, attention to detail and evidence, imaginativeness, scepticism, and a high tolerance for ambiguity (Ritchhart, 2002; Ritchhart & Perkins, 2005).

Dweck's (2006) research on dispositions, or mindsets shows that students can hold beliefs about their personal qualities that reflect either a positive, flexible disposition towards learning and knowing (a growth mindset) or a limited, inflexible disposition towards learning and knowing (a fixed mindset). In this paper we present an analysis of a primary mathematics inquiry classroom to illustrate how the distinctive pedagogical practices of mathematical inquiry can foster growth mindsets in students.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 469–476. Sunshine Coast: MERGA.

Literature and Theoretical Framework

Growth Mindsets

Beliefs play an important role in learning (Hofer & Pintrich, 2012). Dweck's (2006) recent research synthesises the complex but interrelated sets of beliefs about one's personal qualities and abilities as *mindsets*–noting the distinction between having a *fixed mindset* and having a *growth mindset.* This work can be illustrated by the diagram in Figure 1.

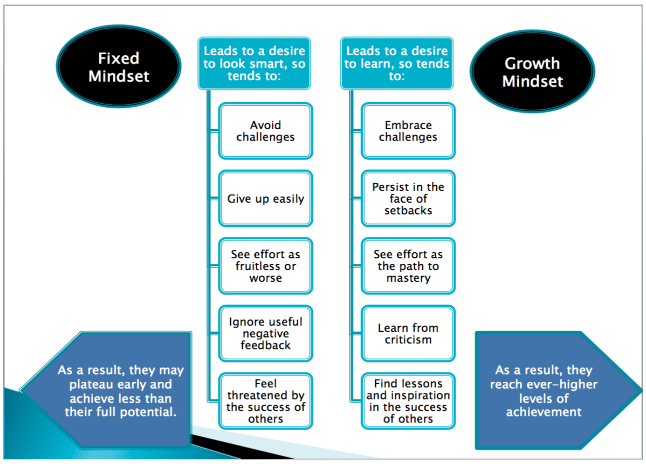


Figure 1. Fixed vs. growth mindsets (Press, 2014).

Having a fixed view of one's self means that you see personal qualities as stable and predetermined. In order to feel secure in a social context, students with fixed mindsets work hard to project a *positive* impression. They want to look smart, and they avoid challenges that can potentially reveal uncertainty or ignorance as they believe this to be unacceptable. They see any kind of feedback or guidance as a negative affirmation of their inabilities, and in turn, feel threatened by the success of others (*they know, I don't*).

In contrast a student with a growth mindset fundamentally believes that his or her personal qualities–intellectual ability, personality, character, preferences, and beliefs–are changeable and in a constant state of growth. As such, growth minded students are oriented to learning and feel less defensive in social settings. They embrace challenges and persist in the face of setbacks (a common occurrence in learning!). They value effort and see its contribution to mastery learning, easily learn from feedback or criticism, and are encouraged and inspired by the learning successes of others (*if they can do it, so can I*). In this way core beliefs about intellectual abilities and personal dispositions influence and shape the way students approach learning.

Inquiry Pedagogy

Cobb, Wood, and Yackel (1993) describe mathematical inquiry as an apprenticeship where ways of thinking are developed within classrooms. Students are supported to work with an ambiguous, ill-structured problem (Makar, 2012); ill-structured problems being those that are ambiguous or have a number of open constraints such that they require negotiation (Reitman, 1965).

In guided forms of inquiry, "The teacher provides the students with the problems or questions and the necessary materials. The students have to find the appropriate problem-solving strategies and methods." (Bruder & Prescott, 2013, p. 812). Throughout the process of solving an inquiry problem, students are scaffolded and challenged by the teacher to plan for, identify, and provide mathematical evidence. The need for negotiation, decision-making, reasoning, and collaboration is somewhat different from usual practices in school mathematics that centre on clarity, structure, and lack of ambiguity (Baber, 2011). Working with ambiguity and open-endedness requires flexibility and a willingness to *not know yet*; to see learning as an opportunity to build new knowledge and new ways of thinking, and to be prepared to take risks and work collaboratively on the creation and testing of ideas and solutions.

For those students who already have a growth mindset, inquiry potentially provides an engaging learning experience that offers a degree of openness, challenge, and autonomy. Provided there is appropriate scaffolding and support for skill development and the reinforcement of related dispositions (such as evidence-based reasoning, mastery learning, and resilience in the face of challenge), inquiry learning may further enhance and promote a growth mindset. It is an opportunity to learn to work confidently with the unknown, to learn how to learn from and with others; to take risks, explore ideas, to reflect on one's own learning process, and to question taken-for-granted assumptions and ideas. In this paper we identify pedagogical practices that, while inherent within inquiry pedagogies, can promote and scaffold a *growth mindset* within inquiry mathematics classrooms.

Methodology

This paper draws on classroom video data from the first year of a larger collaborative project between the authors that is investigating potential links between positive learner identity and mathematical inquiry (O'Brien, Makar, & Fielding-Wells, 2013; 2014). In the first stage of the project, the aim was for the researchers to learn to recognise characteristics of positive learning identity that occurred in mathematical inquiry lessons. Positive mindset is part of the first pillar of positive learning identity and among the first aspects of positive learning identity we are analysing in depth. This paper presents a case study from one of the classrooms in the project.

Study Context, Participants and Data Collection

For this paper, we focused on video data from a lesson in a Year 5 classroom in a suburban, middle class primary school in Queensland. The lesson took place 9 months into the first year of the project from a classroom with an experienced inquiry teacher and 27 students (aged 9-10 years old). The class was working on the inquiry question, "What is the best one litre container I can build with one face that is 125cm^2?" This was the third inquiry unit the students had completed in the year so they had by this time developed a classroom environment that supported mathematical inquiry. We selected this lesson because it was one in which the class was at a point in their inquiry in which they were stuck and the teacher had taken the opportunity to stop the class and discuss the issues they were having. We recognised that *being stuck* was a productive context in which to observe characteristics of positive mindset (Dweck, 2006).

Data Analysis

The video data went through a process of analysis adapted from Powell, Francisco, and Maher (2003) who describe seven stages: intent viewing, describing the video data, identifying critical events, transcribing, coding, constructing a storyline, and composing narrative. A log was created from the video to provide time stamps, screen shots, and brief excerpts to provide a running summary of the lesson (*intent viewing; describing the video* *data*). Using the video log, the researchers re-watched segments to identify those that were potentially useful to characterise positive mindset (*identifying critical events*) and discussed the potential narratives within these critical events as explained by Powell and his colleagues, recognising how narrative and critical events "co-emerge" (p. 417). The critical events were transcribed and annotated (*transcribing; coding*) with characteristics of positive mindset (Figure 1). The researchers discussed the insights provided by the coded critical events and chose a small number of excerpts which would coherently and succinctly illustrate links between mathematical inquiry and the key characteristics of positive mindset (*constructing a storyline*). Finally, these insights were composed as the narrative written in the paper (*composing narrative*).

Results

This learning episode has been selected from a sequence of lessons in which the teacher had implemented a unit of inquiry in her mathematics classroom. In our analysis, we identify the opportunities in the lesson that reinforced the need for a *growth mindset* in learning, in particular with reference to the need to:

- embrace challenges
- persist in the face of setbacks
- see effort as the path to mastery
- learn from criticism
- find lessons and inspiration in the success/learning of others.

Embrace Challenges, Persist in the Face of Setbacks, See Effort as the Path to Mastery

Facing ambiguity and doubt can be a challenging experience, and one that students might initially be inclined to shy away from. However in inquiry pedagogy, the challenge that ambiguity presents is actively embraced and reinforced as highly valuable in order to deepen mathematical understanding and decision-making. In this particular episode, the students faced the challenge of devising a one litre container and realised they were yet to learn specific mathematical concepts that might help them when one group's container required finding the volume of a cylinder.

In the excerpt below, one of the students had researched the formula for calculating the volume of a cylinder; however Ms Thomson, the teacher, wanted the students to develop conceptual understanding of volume rather than move to a formula so quickly. The teacher reassured them that while that may be a challenge, they have much they already know that they can draw on to respond to that challenge, and in doing so, reinforced that challenge is an expected feature of learning.

Noah:	Actually I have an answer for Isabelle and whoever said you can't measure a circle. We in my group we found it easy to make this circle. What we did is we umm we got the diameter and then we halved the diameter which is the radius … We put the scratchy bit (referring to the point of a compass) at the end and we twirled it all the way around and then we cut this scratch and it made this thing here. And, to measure a circle you actually need to halve the diameter which is from this side to this side
Student:	Radius times the pi?
Noah:	Radius times radius times pi.
Ms Thomson:	OK, now that is a formula that you will learn later on for measuring the area of a circle … If you don't know any formulas, how do you measure the area of the base of that cylinder? … How could you do it? Think of all your years

	of schooling and what you've done in the past how could you measure it? What could you do? Benjamin?
Benjamin:	I'm confused.
Ms Thomson	... [to the class] Think outside the box, [if you] don't use the mathematical formulas. …
Student:	Circles aren't square centimetres….
Students:	(several students in unison) You can— (long pause)
Ms Thomson:	At least you are thinking. How can you use square centimetres to measure, or how could you measure area of the base when it's a circle? What would you do, Alkina?
Alkina:	I don't know how to measure [a circle].
Ms Thomson:	[to the class] No idea? Oh, come on, think, think … If I gave it to a five year old and said 'How many square centimetres do you think are on the bottom of this?' What would they do? ...
Arnav:	They'd get blocks like this so you could put it on the bottom and then trace it around and whatever is left you could estimate how much of the block is, how many blocks are left that it hasn't covered and then you'd probably get a close answer.
Ms Thomson:	You would! You'd get a pretty close answer. Do you understand what he's talking about? What else could you do?

You can see in this exchange that the teacher did not give the students an answer, or a way forward, but rather continued to probe and question until the students suggested a solution. The students therefore experience that they can find the answers within themselves if they persist: that challenge is an opportunity to think about something more deeply rather than a stopping point, which links closely to Dweck's (2006) third point: effort as the pathway to mastery.

Learn from Criticism

As this same episode continued, the teacher asked students to explore various options for responding to the task, and where necessary she acknowledged how difficult it was, but affirmed that such difficulty is to be expected. By doing so, she modelled to the entire class that their approach or preliminary solutions may be incorrect or not working, but that is to be expected; and that feedback and critique on that approach is a valuable part of the learning experience, on the path to mastery. As the teacher continued to question each group and specific students about where they were up to and how they were approaching the problem, she provided feedback and facilitated critique from herself and other students on their progress thus far:

Chloe:	… if you keep that one the same then make that the height we need it be to equal a litre, um, then we need to keep this one the same as well.
Ms Thomson:	Why?
Chloe:	Because if you change that [side], it would kind of be like the sides would curve instead of being straight and that's not really what we want, so [pause]. You could kind of, I think you could do that. [pause] Well, it's going to be open because the base like, the other side of it needs to be open.
Ms Thomson:	OK. Isla?
Isla:	Well, if we make it higher, then the base will have to change and that might not equal a litre.
Ms Thomson:	Why? Sorry what did you say?

Isla:	If they make like them higher to 8cm, they’ve got to change the size of their base so it fits.
Alkina:	No, ‘cause that’s going to be the base so if you just make it higher it won’t change the base. We’re just making these higher. It won’t change the base.
Isla:	OK.
Zhang:	You just have to change these sides.
Ms Thomson:	What did you say Zhang?
Zhang:	I reckon that’d work, I think if you just make it like 3 cm higher, all the other sides, um, then that would be 8 [cm] and then it would work.
	[The group of girls are talking quietly about what they need to do.]
Ms Thomson:	So do you guys know where you are going with that?
Chloe:	Now we do because we didn’t, we thought we knew before Josh’s group did their’s because they were doing the same thing that we are doing. But then we realised it wasn’t going to work, so yeah.
Ms Thomson:	… Have you got anything to add to that, Alkina?
Alkina:	No
Ms Thomson:	Harrison. Harrison, your turn to share what you’ve found so far.

In this episode, the students illustrate their willingness to rely on one another to challenge and develop each other’s thinking. The critique from peers became a resource for learning rather than an indication that they were performing poorly. This suggests that the students in this class were building positive mindsets through their collaborative wrestling and critique of each other’s thinking.

Find Lessons and Inspiration in the Success or Learning of Others

In this last extract from the final stages of the episode, the teacher asked students who had worked out a satisfactory solution to explain what they found and how, providing evidence for their claims. In doing so, she purposefully brought in the experiences of other students into the lesson as a shared experience of learning–an opportunity for students to learn from (and appreciate) the success of others as an inspiration for their own learning process and experience:

Ms Thomson:	So when you said your container, you don’t know how high it is and you didn’t know how high to make it … But do you understand why you have to make it that high?
Alexander:	Because that equals a litre.
Ms Thomson:	Why? …
Alexander:	125 x 8 = 1000
Ms Thomson:	1000 what?
Alexander:	Cubic centimetres.
Ms Thomson:	Harrison?
Harrison:	What we could do is we could make it 8cm high with a base of 125 square centimetres and that will be 1 litre . …
Ms Thomson:	Why? [pause] Benjamin?
Benjamin:	Because 125 centimetres x 8 = 1000
Alexander:	1000 cubic centimetres and that’s what we need to make it equal a litre.
Students:	Why? (several students, anticipating the teacher’s next question)
Ms Thomson:	… You made the container and then what did you do?

Zhang:	And then we—
Max:	Tested it.
Zhang:	Then we tested it.
Ms Thomson:	How? …
Max:	Well we had a cup equalled 500 ml or approximately it said apparently according to someone and we put some sand in it to fill the 500ml mark and poured that in twice.
Ms Thomson:	Anybody going to add, anyone going to say something? They tested it; does anyone want to say anything? … No one? …
Lucy:	Well we did the same way of testing except we did it like 5 times, to make sure it was perfect.

Hearing about the solution pathways that other students in the class had developed was an important learning opportunity for all students. They were exposed to a variety of approaches to the problem and a set of strategies for finding a solution that they might not have considered themselves. By making these explicit, the teacher actively encouraged all students to consider adopting these strategies in the next stage of the learning task. She also reinforced the value of learning from what others had done–whether ineffectively or successfully–as a valid part of the learning experience. The teacher was quite deliberate about paraphrasing the puzzling out that has occurred with some groups (e.g., the opening sentence in this extract). In doing so, she invited *all* of the other students to participate in the unpacking and development of the possible solution pathways. Even students who had reached an impasse could reconnect with, and contribute to the dialogic exploration of the solution. Lastly, exploring the various approaches taken by different groups explicitly valued both the successes *and* failures (however temporary) of the class. While the inquiry offered the teacher an opportunity to scaffold learning and direct thinking and reasoning, her role was primarily to bring all learning experiences into view and to highlight and value each for their contribution to the learning of the class as a whole.

Discussion

Engaging effectively in learning and in life requires flexibility, determination, resilience, and a host of high-level intellectual capabilities (Dweck, 2006). While mathematics classrooms can provide comprehensive opportunities to develop mathematical knowledge and concepts, a more deliberate pedagogical approach is required if we are to foster the kinds of *dispositions* that accompany significant gains in student thinking and reasoning (Ritchhart & Perkins, 2006), and mathematical capabilities (NNRP, 2008).

In this analysis we identified specific pedagogical practices that foster and reinforce a *growth mindset* amongst students. In general, mathematical inquiry pedagogies elicit (and require) the kinds of open-mindedness and willing flexibility that is the hallmark of the growth mindset. However a key feature of inquiry is the time, encouragement, and scaffolding of students' exploration of solutions in a shared, collegial way. Where there is an open-ended solutions pathway, there is the possibility of running into dead-ends, of becoming lost, and of encountering difficulties and disagreements. Such setbacks, and the effort we must exert to navigate them, are all part of engaging positively in a field that we eventually master. The teachers' role in inquiry is to monitor the momentum of the inquiry carefully, to look for opportunities to redirect, to gently guide and to offer hints, clues or support (both intellectual and psychological) to students, and to generally inject needed momentum along the way. By doing so, the teacher reinforces a range of dispositions and personal qualities that are characteristic of growth mindsets. This paper has outlined how the pedagogy of inquiry within mathematics classrooms can enable the teacher to model and scaffold–and the students to experience first-hand–what it means to embrace challenges, persist, mobilise effort in the pursuit of mastery, learn from criticism, and find lessons in the learning and successes of others. In doing so, the features of a growth mindset are integrated into the students' experiences of mathematical thinking and reasoning.

Acknowledgements

The authors would like to acknowledge funding from the Australian Research Council (DP140101511). We thank Jude for her expertise and insights in teaching this class.

References

Allmond, S. & Huntly, K. (2013). Achievements and challenges encountered by classroom teachers involved in a research project: A reflection. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne: MERGA.

Australian Academy of Science [AAS]. (2006). *Mathematics and statistics: Critical skills for Australia's future*. Retrieved from *http://www.review.ms.unimelb.edu.au*

Baber, R.L. (2011). *The language of mathematics: Utilizing math in practice.* New York; Wiley.

Bruder, R., & Prescott, A. (2013). Research evidence on the benefits of IBL. *ZDM – The International Journal on Mathematics Education, 45*(6), 811-822.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking and classroom practice. In E. A. Forman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development*. New York: Oxford University Press.

Dweck, C. (2006). *Mindsets: The new psychology of success.* New York: Random House

Hofer, B., & Pintrich, P. (2002). (Eds). *Personal epistemology: The psychology of beliefs about knowledge and knowing.* New York: Routledge.

Fielding-Wells, J., & Makar, K. (2008). *Student (dis)engagement with mathematics*. Paper presented at the Australian Association of Research in Education (AARE), Brisbane, Australia. Retrieved from http://www.aare.edu.au/08pap/mak08723.pdf

Makar, K. (2012). The pedagogy of mathematical inquiry, In R. Gillies (Ed.), *Pedagogy: New developments in the learning sciences* (pp. 371-397). Hauppauge, NY: Nova Science.

National Numeracy Review Panel (NNRP). (2008). *National numeracy review report*. Canberra: Author.

O'Brien, M. (under review). Developing pedagogical expertise to support the development of *positive* learning resources in Primary School students. *Educational Action Research.*

O'Brien, M., Makar, K., & Fielding-Wells, J. (2014). *Characterising positive learner identity within mathematics inquiry classrooms.* Paper presented at the 2014 AARE/NZRE conference, Brisbane.

O'Brien, O., Makar, K., & Fielding-Wells, J. (2013). Investigating connections between mathematics pedagogies and student efficacy and engagement. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne: MERGA.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, *22*(4), 405-435.

Press, L. J. (2014, October 29). The mindset of growth and success [Web log post]. Retrieved from https://visionhelp.wordpress.com/2014/10/29/the-mindset-of-growth-and-success

Reitman, W. R. (1965). *Cognition and thought: An information processing approach*. New York: Wiley.

Ritchhart, R. (2002). *Intellectual character: What it is, why it matters, and how to get it*. San Francisco: Jossey-Bass.

Ritchhart, R., & Perkins, D. N. (2005). Learning to think: The challenges of teaching for thinking. In K. J. Holyoak & R. G. Morrison (Eds.), *The Cambridge handbook of thinking and reasoning* (pp. 775-802). New York: Cambridge University Press.

Challenging the Mindset of Sammy: A Case Study of a Grade 3 Mathematically Highly Capable Student

Linda Parish
Australian Catholic University
<linda.parish@acu.edu.au>

This case study narrative reports on the journey of *Sammy as her mindset as a learner of mathematics is challenged. Often students who are mathematically highly capable are viewed as being privileged, they are rarely placed with the cohort of struggling students. Children like Sammy who are mathematically highly capable or gifted, however, are simply students who learn differently and therefore require a different type of teacher support. [*Sammy is a pseudonym].

> I think the problem is that she is really good at everything, and she's always been good at everything, and she doesn't know how to fail. It freaks her out completely, and she won't even get close to it because at the first little thought that something's going to go wrong she'll just shut down. (Sammy's teacher)

Generally students who are considered mathematically gifted or highly capable are not perceived as being classroom strugglers. Students categorised as 'vulnerable' or 'at risk' are most likely to be those who are not achieving minimum standards, those who require intervention or specialist assistance to help them 'catch up' to their peers in order to be able to participate successfully in the regular classroom (Gervasoni, et al. 2013). This study, however, considers the reality that there are students who, even though they may be highly capable mathematically, may in fact be vulnerable in terms of realising their true capabilities within the classroom. Indeed, it may be *because of* their unusually high natural aptitudes for understanding mathematical concepts that they develop a skewed view of what the process of learning entails, which in turn may stifle further learning opportunities.

Sammy is a case in point. Sammy, an eight-year-old, Grade 3 student, was a participant in a research project, 'Supporting the learning of students who are mathematically highly capable or gifted'. She proved to be a girl with issues in terms of perception of herself as a learner, which was curtailing her ongoing learning potential. This paper is her story: a story of how Sammy began to transform, through targeted teacher intervention, her self-limiting mindset to a much more confident and positive mindset.

Context

Classroom teachers have the responsibility to understand and cater for the learning needs of all students in their classes. This is essential for students who are mathematically gifted even though, or maybe because, these students are already successful in mathematics.

During the past decade there has been educational research focusing on identification of mathematical giftedness (e.g., Bicknell, 2009), on understanding mathematically highly capable students and how they learn (e.g., Leder, 2008), on providing suitable programs for them (e.g., Chesserman, 2010) and best approaches for teaching them, for example, differentiation (Kronburg & Plunkett, 2008), and acceleration (Hannah, James, Montelle & Noakes, 2011). However, the discourse about *why* it is necessary to consider the specific needs of mathematically highly capable or gifted learners has historically been based around benefits to society and our "globally competitive economy" (Office of the Chief

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 477–484. Sunshine Coast: MERGA.

Scientist, 2014), with provision for the gifted even being described as "human capital development" (Ibata-Arens, 2012, p. 3). Thankfully this is beginning to change. In the latest Victorian Government Department of Education's *Strategy for gifted and talented children and young people*, the benefits to the individual are highlighted first "The chance to realise their potential, pursue a passion and develop a love of learning…" precedes the more common general benefits to society, "…gifted and talented children and young people are the potential leaders of tomorrow" (DEECD, 2014, p. 5).

Developing a 'love of learning' is a necessary element of 21st Century education. In an ever-changing technological era where types of jobs for future school leavers is highly unpredictable (Robinson, 2006), students need to learn how to continue to learn beyond the classroom. Unfortunately classroom environments can actually paint a skewed picture of what successful learning is, and what it feels like. Without providing students with work that requires perseverance and sustained personal effort we run the risk of turning naturally successful learners into students who are intimidated and fearful of effort and initial difficulty (Williams, 2014). Children who are mathematically gifted or naturally highly capable may have attracted many positive comments from a very early age from parents, friends of parents, pre-school teachers, even complete strangers. It is normal for many adults in western cultures to recognise and want to praise children's mathematical abilities (Bishop, 2002). The problem that may develop, however, is that without praise directed at effort and perseverance exhibited in performing a task, and often children who are highly capable mathematically do not need to apply much effort or perseverance in early mathematics tasks, we may be inadvertently nurturing a self-limiting mindset: a mindset that leads to children avoiding failure at all costs, which may mean avoiding suitably challenging tasks, or the opportunity of learning something new (Dweck, 2006). The development of a self-limiting mindset in students who are mathematically highly capable or gifted could have severe consequences.

In this study, I was testing three conjectures about mathematically gifted students: (1) Mathematically gifted students who are not challenged sufficiently develop a limited view of the process of successful mathematics learning, which results in a self-limiting mindset, but this mindset can be changed; (2) Mathematically gifted students who possess a self-limiting mindset will require teacher support when approaching challenging tasks so as not to feel overwhelmed and/or distressed; and (3) With a positive mindset about themselves as learners, mathematically gifted students can be taught, or may only need to be given permission, to challenge themselves, by being creative, delving deeper, and exploring further their own curiosities. Sammy's story describes an initial analysis of my findings.

Sammy's Story

Sammy was identified as mathematically highly capable through a multi-faceted process: teacher nomination; independent analysis of previous mathematics assessments and work samples; a parent questionnaire asking about her prior-to-school mathematical aptitude; individual semi-structured interviews carried out with both Sammy and her teacher; and a one-to-one task-based mathematics interview, assessing her ability to perform novel and creative tasks (as opposed to assessing previously learned mathematics content). It is important to note that my research was not aiming to identify *all* mathematically gifted or highly capable students, only to identify a small number of students to follow in a case study.

I observed Sammy in four mathematics lessons in a one week period as a participant observer – asking her questions, suggesting further challenges, and providing support if necessary. This was followed by collaborative discussion with her classroom teacher, exploring suggestions for supporting Sammy's ongoing mathematics learning (see below). These suggestions were then implemented by the teacher within regular mathematics lessons over the following twelve weeks. Further classroom observations followed – three mathematics lessons in a one week period – with me as a participant observer. I then re-interviewed both Sammy and her teacher, and conducted a second one-to-one task-based mathematics interview with Sammy.

All interviews and in-class conversations were audio-taped and transcribed. Classroom observations were journalled, and student work samples were collected and/or photographed. This formed the data for a priori analysis.

Data analysis was based on the observation of classroom involvement and participants' perspectives, establishing themes for description, reflection and interpretation (Creswell, 2013). It consisted of making a detailed description of the case – supporting mathematical giftedness – and its context – within the mathematics classroom (Hébert & Beardsley, 2001). The study assumes a social constructivist framework which places emphasis on the role of others in the learning process, including an actively involved teacher and the shared experience of other children.

Sammy Before

In the initial one-to-one assessment interview Sammy presented as a bright, friendly, very self-assured girl. She answered questions quickly and confidently, even when she was incorrect she was very quick and confident with her answer. I was initially unsure just how outstanding her natural ability was, but in hindsight, it's possible she was 'playing it safe' with many of her responses, and answering quickly rather than being seen to be having to put in any effort.

When I asked her "How do you know someone is good at maths?" her reply was, "[They] always finish their work in time. They're always going 'done', and always get the right answer…" (Sammy, May 2014)

My first observation of Sammy in a maths lesson in her regular classroom was in mid-June. The lesson was about arrays (visualising multiplication), and the focus of the lesson was 'writing number sentences to describe arrays'. The introductory session covered what an array is [arranging dots/counters in equal rows and columns], and the class was then sent off to explore various quantities (12, 15, 18, etc.), using counters to make arrays, and writing number sentences to describe the different arrays they could make with each of these quantities. Sammy set to work quickly and quietly by writing down a list of number sentences – 12x1, 1x12, 3x4, 4x3, 2x6, 6x2 – and then dutifully drawing the arrays next to each equation. She was going through the motions, reproducing work she could already do quite confidently. And she was happy.

I decided to intervene to show her some other possibilities she could explore. I explained that I could see another number sentence that could be written for an array of 12 she had drawn (Figure 1): I could see three rows of three on the top and another row of three along the bottom – 3x3+3 (Figure 2). She immediately saw two rows of three plus another two rows of three – 2x3+2x3 (Figure 3), so I left her to see what else she could discover within some of the other arrays she had drawn. As I walked away I heard her exclaim to others at her table, "This is so cool!", and she eagerly set to work.

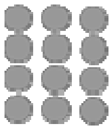


Figure 1.
12=4x3

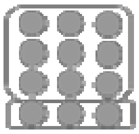


Figure 2.
12=3x3+3

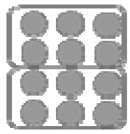


Figure 3.
12= 2x3+2x3

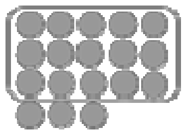


Figure 4.
18=3x5+3

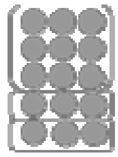


Figure 5.
15=3x3+3+3

When the teacher called the class back to the mat, Sammy was very eager to share what she had discovered about ‘array busting’, but what she chose to share was that she was able to write 18 as a number sentence: 3x5+3. Unfortunately, although the number sentence was correct, and what she drew certainly represented the equation (Figure 4), she had lost sight of the array focus. The dilemma was that the rest of the class was looking puzzled at her non-array representation, and some began to question it. To overcome the awkwardness of the situation I got Sammy to compare her drawing with a 15 array that was already drawn on the board and asked her to ‘bust’ the 15 array in the way she was trying to describe. She was a little flustered initially, but ended up with 15=3x3+3+3 (Figure 5).

It wasn’t until the teacher directed the class to the follow-up activity, and the students dispersed from the mat, that I realized that Sammy was sobbing! Initially she couldn’t explain what was wrong because she was sobbing so hard, but eventually pointed to the 18 ‘array’ on the board and choked out, “I can’t do it!” After some reassurance she eventually settled back to work, but reverted to writing and drawing basic number sentences for her arrays (16=4x4, 2x8, 8x2 etc.), and remained miserable for the rest of the lesson.

Sammy had been fascinated and excited about the possibilities of more complex array partitioning, but when the class discussion led her to believe she was wrong, she became highly distressed, and subsequent learning opportunities in that lesson were completely stifled.

From the interview with Sammy’s teacher it seems that this kind of reaction, while not an everyday occurrence, was not uncommon (I witnessed it twice in the seven lessons I observed), and it was the sort of thing that Sammy dwelled on…

> I said, "Why don't you try something else with that and doing the ‘explore more challenging things’", and she's like, "No, I think I'm ok with this." And she brought up that lesson [the arrays lesson], like from however long ago it was, and I was actually really surprised. But she had brought it up a few times since then, like I've heard about it a few times ... (Sammy’s teacher, July 2014)

Sammy also tended to be very self-critical. ‘I’m no good at maths’ was a regular utterance when she was asked a question she didn’t know the answer to immediately. Within the first 30 minutes of a lesson on ‘How many designs can you make that are 1/4 yellow and 3/4 red?’ she had uttered, "Let's do the easy ones first", “Oh my gosh, you guys are fast! How do you do it [come up with design ideas] so fast?" “I’m not very good at this”, “I’m not good at maths”, “But it’s too hard”, and “I can’t do this”, all the while successfully working on not only coming up with creative designs but also exploring and understanding non-contiguous three-quarters designs, and non-uniform parts (see Figure 6). This, however, was new territory for her that required some thinking and she constantly wanted to go back to ‘the easy ones’.

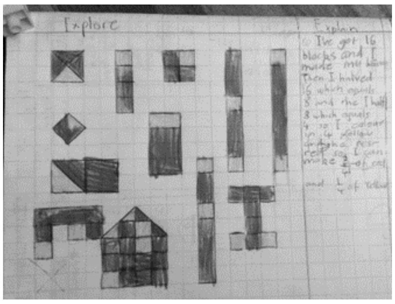


Figure 6. Sammy's ¼ yellow ¾ red designs

Sammy was undoubtedly highly capable mathematically. She learnt new concepts quickly, she was readily able to transfer new knowledge to novel situations, she was able to reason abstractly, and she was able to explain her reasoning to others (cf. Krutetskii, 1976). However, she seemed to have issues with having to put in effort, with not knowing the answer straight away, with having to think hard about a problem. These things, to her, seemed to be an indication that she was 'no good at maths'. The challenge with students like Sammy is not just in providing them sufficiently challenging work. Students like Sammy also need to be sensitively supported through these often foreign feelings of floundering, of cognitive conflict, that are, in fact, a necessary part of higher level learning (Roche, Clarke, Sullivan & Cheeseman, 2013). To us Sammy's responses of devastation and uncontrollable sobbing may seem extreme, but it is not uncommon for highly capable or gifted students to exhibit intense emotional sensitivity (Dabrowski & Piechowski, 1977), where every little setback is felt as earthshattering. These feelings are very real and these children need to be given strategies to help cope with their intense emotions.

The Intervening Period

The next stage in the study was to meet with Sammy's teacher and discuss together ways she could provide sufficiently challenging tasks for Sammy, and how to support her in this. Most of the suggestions discussed were useful and effective strategies for all students in the class. The following list was agreed upon:

- Establish a classroom understanding that learning requires effort and hard thinking, and that is what is expected in a mathematics class. Hard thinking is a good thing, not a sign that you are not good at maths.
- Establish that when I (the teacher) ask a question, I am posing a problem I want you to think about. I don't want a quick answer (I am not testing you). What I require is a well thought out explanation, the answer is the by-product of this.
- Model that there is always more you can explore. Teach them how to think deeper (if necessary); there is a skill in learning how to learn. Generate a classroom environment that values creativity. Encourage students (especially the highly capable students) to run with their own ideas. Constantly ask questions like "How are you challenging yourself?", "What's next?", "How can you be creative with this?" The aim is for these questions to become part of a student's own self-talk.
- Give them permission and time to explore and investigate further – to follow their own curiosities.

- Be aware of, and challenge self-limiting mindset statements such as “I’m no good at maths”, or “This is too hard”, as well as statements like, “This is easy!”, or “I’m bored”, or “I’m finished!”

Sammy’s teacher was already putting into practice task differentiation for varying abilities within her class. However, putting the onus of challenge, in part, onto the student who is mathematically highly capable allows for even further meaningful differentiation. Sometimes what we, as teachers, think will be challenging turns out to be either too easy, or just way too hard. Sometimes, something we think will be too easy turns out to be quite challenging due to misconceptions or gaps in their learning. It also avoids the tragedy of statements like, “At University they get you to actually learn things yourself, instead of school where they tell you everything and get you to do it a certain way…" by Jacob Bradd, on acceleration to university at age 14 (McNeilage, 2014, para 18). Or the advice to parents to allow their children ‘mental health days’, “…days on which gifted kids are given an opportunity to [stay home] to learn more. They don’t have to sit in a room waiting for the other kids to catch up. They can unfurl their wings and fly” (Bainbridge, n.d., para 8).

Sammy After

In the follow-up classroom observations with Sammy three months later, I witnessed a child who was more willing to take risks. She still became excited and animated when faced with new ideas to explore, but was now much more willing to stop and think through things that didn’t initially make sense. This allowed her to learn even more. Her teacher had also noticed this:

> … there was another time when she could have absolutely lost it – they were talking about recording the area of a certain object, so imagine they measured this bench and they recorded that it took 50 large playing cards. James, the pre-service teacher, was writing '50' and then 'large cards' next to it, and she [Sammy] is like, "and you should put like a little square on the top of it, it's a number 2 and it means squared," because she was trying to tell him about squared centimetres. And he was like, "but is this playing card square?" And she's like, “Well no ..." and then she made the connection that "Oh, it's actually centimetres squared because they are squares!", and that was the whole reason behind it. But usually she would just freak out because he said, "but that's not a square". (Sammy’s teacher in post interview, October 2014)

In this lesson, when the teacher questioned Sammy’s suggestion of putting ‘a little square on the top of it’, she stopped and reflected and was able to identify a critical concept about area measurement, that the unit of measure for measuring area is an actual square. This was in complete contrast to the previous lesson I had observed with arrays where she had an emotional meltdown because she thought she was wrong in front of the whole class.

The other very notable change in Sammy was her lack of negative comments such as ‘I’m no good at maths’. Not once in the three follow-up lessons observed did she mention this. When asked about this in the post interview Sammy said that her teacher had been helping her learn how to not say things like that, that her teacher had drawn up a chart to help her change her mindset, and she drew an example of the chart for me:

> …like, 'I can't do it', and she has all negative stuff here [indicating the left side of the chart], and then she reversed them here into positives [indicating the right side of the chart] to something like, 'I'll work hard to get the answer, but I might not be able to get it right just now'. (Sammy, November 2014)

When I asked if it was she was consciously choosing not to say ‘I’m not good at maths’, she stopped and thought and seemed quite surprised before exclaiming, “I *don’t* think it anymore…It’s just kind of worked like magic!”

When I asked her this time how she knew she was good at maths her response was, “I know I'm good at maths because I did that [pointing to a task she’d just persevered with for over 30 minutes] and I thought it was too hard but I did it!” (Sammy, November 2014)

Discussion and Conclusion

Sammy certainly appeared to have self-limiting beliefs about what it means to be ‘good at maths’. To be good at maths she believed she needed to be able to work quickly and get the answers right. When we began to challenge her, even though she initially enjoyed the challenge, if she couldn’t get it right straight away, or if she perceived she may have done something wrong, she immediately came to the conclusion that she wasn’t good at maths, and this was quite distressing for her. Her ‘safe’ responses in the initial one-to-one assessment interview may also have been an example of underachievement due to this misconception.

The good news is that Sammy’s self-limiting mindset was turned into a more positive mindset in a reasonably short period of time. However, this was a scary and sometimes overwhelming venture for her that required intensive and sensitive support from her teacher who had to learn to understand where Sammy was coming from, and how to deal with her hyper-sensitivities.

As far as encouraging Sammy to challenge herself by being creative, delving deeper and exploring further, there seems to have been quite a deal of ‘unlearning’ of what she believed was expected in the classroom. She has certainly come a long way in that she is now prepared to take risks and explore unknowns if suggested to her, which is a necessary first step, but she is yet to show any initiative in this.

The data reported on in this paper suggest that challenging Sammy’s mindset about her beliefs about herself as a mathematics learner proved to be a challenging journey for both Sammy and her teacher.

> Probably one of the biggest things I've learnt this year is because she is really great at everything you wouldn't necessarily look at her and think "This kid's struggling" but she is probably struggling more than anyone in the class, but in a different way. She's been my biggest struggler this year. (Sammy’s teacher, October, 2014)

There is much to learn and understand about students who are mathematically highly capable or gifted. Contrary to popular belief that they are a ‘privileged’ group who work easily without the need for help from the teacher, they are actually just children who learn differently and therefore may require a different type of teacher support. The classroom observations and follow-up interview demonstrated that Sammy’s mindset had changed to a much more positive one, but this only came about due to the diligence and perseverance of her teacher who was prepared to learn as much as she could about things like hyper-sensitivities. As with so many things, prevention is better than a ‘cure’. If more teachers were aware of the impact of not catering for students who are mathematically highly capable or gifted, in terms of making sure they understand that hard work, effort and perseverance are a normal and expected part of learning, it is possible we may see less underachievement in this cohort of students, and may even pave the way for happier and more successful adults in the future.

> It wasn’t because Newton and Einstein were geniuses that they were successful, it’s because they made the transition from learning, to thinking, to creating. (Jacob Barnett, 2012, 14 year old astrophysicist)

References

Bainbridge, C. (n.d.). *Giving kids an emotional and mental break.* Retrieved from http://giftedkids.about.com/od/socialemotionalissues/qt/mental_day.htm

Barnett, J. (2012, April 9). *Forget what you know.* [Video file]. Retrieved from http://tedxteen.com/talks/tedxteen-2012/111-jacob-barnett-forget-what-you-know

Bicknell, B. (2009). *Multiple perspectives on the education of mathematically gifted and talented students* (Unpublished doctoral dissertation). Massey University, Auckland: New Zealand.

Bishop, A. J. (2002). Critical challenges in researching cultural issues in mathematics education. *Journal of Intercultural Studies, 23* (2), 119-131.

Chesserman, A. M. (2010). *Teacher attitudes and effective teaching practices for gifted students at Stage 6* (Unpublished doctoral dissertation). The University of New South Wales, Sydney: Australia.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches.* Thousand Oaks: SAGE Publications Inc.

Dabrowski, K., & Piechowski, M. M. (1977). *Theory of levels of emotional development: Vol. II – From primary integration to self-actualization.* Oceanside, NY: Dabor Science.

Department of Education and Early Childhood Development (DEECD) (2014). *Aiming high: A strategy for gifted and talented children and young people 2014-2019*. DEECD: Melbourne.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. New York: Random House.

Gervasoni, A., Parish, L., Livesey, C., Croswell, M., Bevan, K., Hadden, T., & Turkenburg, K. (2013). Longitudinal progress of 6-year-old students who participated in an Extending Mathematical Understanding mathematics intervention program. In V. Steile (Ed.), *Mathematics education: Yesterday, today and tomorrow,* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, pp. 346-353). Melbourne: MERGA.

Hannah, J., James, A., Montelle, C. & Noakes, J. (2011). Meeting the needs of our best and brightest: Curriculum acceleration in tertiary mathematics. *International Journal of Mathematical Education in Science and Technology, 42*(3), 299-312.

Hébert, T. P., & Beardsley, T. M. (2002). Jermaine: A critical case study of a gifted black child living in rural poverty. In S. B. Merriam and Associates (Eds.), *Qualitative research in practice: Examples for discussion and analysis.* (pp.). San Francisco: Jossey-Bass.

Ibata-Arens, K. C. (2012). Race to the future: Innovations in gifted and enrichment education in Asia, and implications for the United States. *Administrative Sciences, 2*, 1-25.

Kronburg, L., & Plunkett, M. (2008). Curriculum differentiation: An innovative Australian secondary school program to extend academic talent. *The Australasian Journal of Gifted Education, 17*(1), 19-29.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children*. Chicago: University of Chicago Press.

Leder, G. (2008). High achievers in mathematics: What can we learn from and about them? In M. Goos, R. Brown & K. Makar (Eds.), *Navigating currents and charting directions.* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, pp. 195-202). Brisbane: MERGA.

McNeilage, A. (2014, December 27). Study: Gifted children benefit from bypassing school for university. *The Sydney Morning Herald*. Retrieved from http://www.smh.com.au/national/education/study-gifted-children-benefit-from-bypassing-school-for-university-20141227-12cnf0.html

Office of the Chief Scientist (2014). *Science, Technology, Engineering and Mathematics: Australia's Future.* Australian Government: Canberra.

Robinson, K. (2006, February). *How schools kill creativity*. [Video file]. Retrieved from http://www.ted.com/talks/ken_robinson_says_schools_kill_creativity?language=en

Roche, A., Clarke, D., Sullivan, P., & Cheeseman, J. (2013). Strategies for encouraging students to persist on challenging tasks: Some insights from work in classrooms. *Australian Primary Mathematics Classroom, 18*(4), 27-32.

Williams, G. (2014). Optimistic problem-solving activity: Enacting confidence, persistence, and perseverance. *ZDM - The International Journal in Mathematics Education*, *46*, 407–422.

Facebook as a Learning Space: An Analysis from a Community of Practice Perspective

Sitti Maesuri Patahuddin
University of Canberra
<Sitti.Patahuddin@canberra.edu.au>

Tracy Logan
University of Canberra
<Tracy.Logan@canberra.edu.au>

This study investigates the potential of Facebook as a medium and process for teachers' learning about mathematical and pedagogical knowledge. Participants' (N=117) responses towards four inter-related posts regarding division-of-fractions were captured and systematically analysed to gain insight about the participants' engagement. The results suggested the potential of Facebook to support informal teachers' learning. This was evidenced by the existence of the three main elements of community of practice (CoP): mutual engagement; negotiated joint enterprise; and development of a shared repertoire.

> Until the education community comes up with a formal means of professional development that is free, user friendly, and timely, Facebook teacher groups and similar forms of social media should be seen as an effective supplement to traditional teacher professional development (Rutherford, 2010, p. 69).

The citation above reflects the awareness of the educational potential of Facebook (FB). In fact, FB has become one of the most prominent social network sites, having 1.35 billion monthly active users worldwide as of September 2014 (Facebook, 2015). Furthermore, FB has been an object of research since 2005, one year after FB was created. Four review papers by Hew (2011), Aydin (2012), Nadkarni and Hofmann (2012), and Manca and Ranieri (2013) together informed us of; the effects of FB usage, students' attitude towards FB, educational aspects of FB, reasons for people using FB (e.g., FB as a part of a formal learning environment), and "the extent to which its pedagogical potential is actually translated into practice" (Manca & Ranieri, 2013, p. 487). Manca and Ranieri concluded "pedagogical affordances of FB have only been partially implemented and that there are still many obstacles that may prevent a full adoption of FB as a learning environment such as implicit institutional, teacher and student pedagogies, and cultural issues" (p. 487). These reviews suggest that studies on FB within the domain of mathematics education are sparse. Despite FB's popularity, most FB studies (through surveys) have not explored its potential for teachers' learning in mathematics or mathematics pedagogy.

In addition, four main studies were found that specifically highlighted FB and teacher professional development (Bissessar, 2014; Goodyear, Casey, & Kirk, 2014; Ranieri, Manca, & Fini, 2012; Rutherford, 2010). All suggested that further exploration in this area was warranted. We found no studies that specifically focused on the use of FB and teachers' mathematics learning. Therefore, there is a need to gain insight on teachers' learning through FB by analysing data from their authentic conversations about mathematics or mathematics pedagogy. This study will provide additional understanding of the potential of FB as a space and process for teacher professional development on an informal basis. This is neither a survey-based research nor an experimental design. This was not designed as a formal professional learning site. The researcher was not a teacher trainer or part of the Government body. Rather the results of this study are from simply, a part of life activities where the researcher routinely shared ideas, opinions, photographs, and web links. Therefore, we argue that FB used in this study is considered an informal site for learning. This paper considered how the FB environment contributed to a sociocultural

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 485–492. Sunshine Coast: MERGA.

approach to Indonesian teachers' professional learning through the emergence of a community of practice (CoP) (Wenger, 1998). This study was guided by our research question: How does engagement within the FB environment build towards a community of practice for teachers' learning?

Theoretical Underpinnings

This paper is underpinned by a sociocultural approach to learning through investigating the emergence of a community of practice (CoP) (Wenger, 1998) within the FB environment. Wenger, McDermott and Snyder (2002, p. 4) defined CoPs as "groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis." Within a CoP, members jointly develop and share practices, learn from their collaborations with group members, and have opportunities to develop personally, professionally, and/or intellectually (Wenger, 1998). Table 1 briefly describes the three defining characteristics of CoP.

Table 1
Description of the three defining elements of Wenger's community of practice

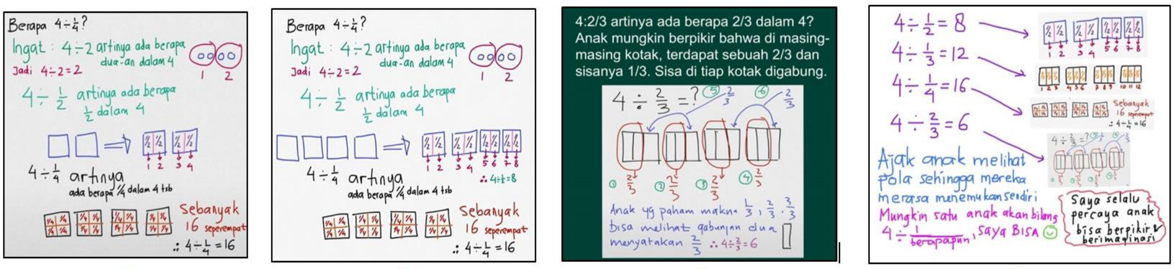
Element	Description
Mutual engagement	**How does it function**: people are engaged in actions whose meanings they negotiate with one another, through diversity, relationships and opportunity?
Joint enterprise	**What is it about**: negotiated common interests and collective goals with mutual accountability?
Shared repertoire	**What capability has it produced**: communal resources (routines, tool, artifacts, discourse, styles, etc.) that members have developed?

Note: Adapted from Wenger, 1998.

The Context of the Study and Method

Facebook has been heavily used in various communities in Indonesia. It falls within the top five countries for FB users, with over 64 million users who actively access their accounts monthly (The Jakarta Post, 2013). Although the exact number of teachers joining this network is unknown, the identification of over 100 FB groups, created for Indonesian educators with over 50,000 members, reflects teachers' interest on FB. In addition, Indonesia is characterised by word-of-mouth communication or oral culture, and is one of the top users of mobile phones, through which FB can be accessed easily even for teachers in remote areas. It appears that teachers may find that FB is a quick solution for them to find the information they need, to report or to solve their problems, or to seek support within uncertain political situations and limited educational resources.

The data presented in this paper are drawn from a larger virtual ethnographic study (Hine, 2000) on the use of FB for Indonesian mathematics teachers' informal professional learning. This paper investigated 117 (F1-F117) Indonesian FB users' engagement with four inter-related FB posts concerning division of a whole number by a fraction, a concept often taught using the rule "invert and multiply". Of the participants, 70 were identified as teachers, 30 were not mentioned as teachers but their posts or profile reflected having educational background or interest, and 17 with unknown educational background. Data were collected within a one-week period. Figure 1 illustrates the posts uploaded by the first author and shows a visual model for dividing a whole number by a fraction. The model has been drawn incorrectly in Post 1, with Post 2 showing the correct representation. Post 3 shows 4÷2/3 and Post 4 links the model to the rule, with the author suggesting in the red cloud in the corner that students, if given the opportunity, will often find these rules and patterns on their own.



1 2 3 4

Figure 1. The four images of the inter-related FB posts

These four images were initially only posted on the first author's own FB wall. However, she shared and others in turn shared these posts to not only their own walls, but various Indonesian education FB group walls also. This resulted in comments and likes on the author's wall, as well as on other "outside" walls.

Data Analysis

A content analysis was utilised in order to identify themes of responses from the comments and shares and hence, the approach to coding the data was naturally grounded. All the responses (like, share, comments) were downloaded and imported into Microsoft Excel for descriptive statistics and NVivo where emerging themes were noted and the data coded. The coding system was continuously refined after recognising the similarities and differences since qualitative data analysis requires flexibility and open mindedness. Coding reliability measures (e.g., two coders independently coded the data) showed an initial agreement on 87% of the codes, with the remaining 13% agreed upon after discussion. Coding was simplified from 13 major codes to be only 6 main categories (see Table 2).

Table 2
Coding of the six main categories of responses to the four inter-related posts

Types	Meaning/Indicators	No. of comments
Appreciation	Comments were appreciative of the information.	56
Opening opportunities for others	Permission to share the posts; Used the facilities on FB to share to their own wall or to a group wall with or without description/message/comments; Inviting others to discuss.	85
Enriching the conversation	Attempting to correct; Asking questions related to mathematical ideas, pedagogical, and mathematical pedagogical ideas; Answering questions; Clarifying ideas/additional explanations.	70
Direct impacts	Changing opinion about mathematics; Increasing confidence; Applying the ideas in their teaching; Expecting more of these types of posts.	15
Misalignment with practice	Not being critical of the posts per se, but being critical of the existing practices of teaching and of systems.	9
Other	Comments/questions not related to the specific posts or private conversations.	18
Total		264

Mutual Engagement

Wenger (1998) suggested that certain contexts enable mutual engagement. Two main enabling factors were identified in this study: the tool (FB) and the participants. The affordances of FB itself enabled engagement, such as: the features of liking, sharing, commenting; through to the ability to upload images and video easily; and the user-friendly navigation of the features. However, the participants themselves also enabled engagement: through joining FB; developing a profile; having the time to read, respond to posts and interact with other users; and through their own network of FB friends. These subtle and underlying affordances highlight the ease with which CoP can begin to develop.

There was evidence of engagement from geographically diverse participants (89 people from 20 regions in Indonesia; 8 Indonesians in 6 other countries; 20 others with unknown locations). Table 2 shows that the four posts together gained traction in engagement in a one-week period, suggesting that this type of resource is useful to the Indonesian education community. The participants' engagement was shared among one another, where even the smallest involvement contributed to the overall conversation. People's involvement consisted of showing appreciation, or inviting others to join the conversation or to apply the ideas; while others contributed to enriching the mathematical and pedagogical discussion. Other people were directly impacted and attempted to implement the idea with students or children. Hence, a variety of different types of engagement were contributing to the discussion.

Within the data, there was evidence of negotiated actions to develop meaning within the community. For example, the themes of enriching the conversation revolves around attempting to correct mistakes; asking questions or prompts of both the posts and of the conversation; answering questions posed by others; and clarifying understanding and thinking. Interestingly, the first post accidentally contained an incorrect representation, where the first author intended to model 4÷1/2 using 4 boxes, but only drew 2 boxes. As a result, the comments suggested: “It seems two boxes missing because the result should be 8.” Another suggested: “Maybe, [author’s name] meant to write 2÷1/2 because there are two boxes.” Some people informed other people that this was actually the incorrect representation and gave links to the second post, which had the correct representation.

The following examples illustrate the mutual engagement and highlights that responses were thoughtful and enriching.

> Thank you very much for your explanation. It is very important to teach children about concepts. If they already understand, what is next? For example for 4÷2/3, how many 2/3 are there. And they found 6 2/3 [there were 6 two thirds]. Now, Can we then direct them to 4×3/2? [Post 2-IN; F102]

> Can the method be applied to large numbers such as 5÷12/13? [Post 4-OUT; F107]

The teachers asked questions and contributed ideas such as in relation to the representation and explanation of 4÷1/2 = 8 and 4÷8 = 1/2 as presented below.

> Can you please show this with pictures, the difference between 4÷1/2 = 8 and 4÷8 = 1/2. [Post 4-OUT; F89]

Some of the questions asked in the discussion were explicitly answered. The transcript below highlights the interactive nature of the discussion:

> **Question**: May I share this Mam? The explanation is very clear. But how should I explain 4÷1/4 = 4×4/1? [Post 2-IN; F102]

> **Answer**: F102, it would be better if the children can find the patterns themselves. Because we give them many rules without meaning, any child can be wrong in remembering the rules. Through many examples and having mental images as the models above, the child can understand why an integer divided by a fraction gives a greater result. In the picture the child can see the number of ¼ box is 4×4 = 4 + 4 + 4 + 4. [Post 2-IN; F93]

The notion of mutual engagement as defined by Wenger is shaped here by the people themselves and the affordances of the FB environment. There is a dynamic interaction in this community, and it involved people from very different contexts and with different expertise. This virtual community evolved naturally over a short period of time and with further stimulation, has potential for greater impact.

Joint Enterprise

The joint enterprise is the common focus of the community. The initial enterprise proposed by the author was an approach to help children understand fractions without forcing them to use a rule that may not make sense. It shows the connection between the idea of dividing a whole number by a whole number, “how many [divisor] within [dividend]?” and dividing a whole number by a fraction. For example 4÷2 means “how many twos in the four?” Similarly 4÷1/2 means “how many halves are there in a four?” This differs from the general accepted notion of “dividing means equal distribution”, namely “share the four equally to two people, and how many does each person get?”

The four inter-related posts became a joint object of conversations about mathematics and teachers’ own pedagogical practice and their own mathematics understanding in an informal and less confronting space. 70 comments were coded under the enriching the conversation category with many of these comments related to mathematical ideas, pedagogical, and mathematical pedagogical ideas. These became the joint enterprise through which the participants engaged with these ideas.

For example, with regard to mathematical knowledge, participant F105, wrote the following thought-provoking post. The question posed by this participant was outside of the examples given in the four posts, which indicated that this person had thought about the model and tried to apply it to their own example:

> This is very inspiring…….. By the way, I tried to solve 4÷3/4. [using the model method] and I found 5+1/4. But when I solved it using the instant method 4÷3/4 = 4×4/3, the result was 5+1/3. Please give advice on this. [Post 4-IN; F105]

To demonstrate that the enterprise was jointly constructed, F2 responded:

> F105, if you use the method of dividing the 4 bars with 3/4, the results are the same. If you do, you will be able to find 5 with the rest of the log would be a quarter. But the quarter itself is a third of three quarters. So the answer is still 5+1/3. [Post 4-IN; F2]

This illustrates how the mathematical idea of dividing by fractions can be challenging and not always as simple as the examples in the four posts. This conversation above shows how a deeper understanding of mathematical knowledge is often needed to solve similar problems. As mentioned in F2's response, there is a need to understand that the remaining part is 1/4 of the original whole, yet only 1/3 of the new unit (3/4).

Some comments were quite general and therefore they were categorised as general pedagogy, that is, general ideas about teaching and learning. For example:

> Concepts should be learned by learners before introducing them the procedure or technique to make the learning process as meaningful as possible. [Post 4-IN; F43]

> Come on lovely teachers, cultivate the process in the teaching and learning and do not always give students smart solutions. Let them find their own smart solutions. [Post 4-OUT; F2]

The most critical ideas were categorised as pedagogical mathematical knowledge. These comments were related to how to teach mathematics and addressed particular student mathematical difficulties.

> Fractions in the early elementary school stage should start with a concrete => picture => symbol. Most teaching directly goes to the symbol, they cannot wait… even though the children do not understand yet. As a result, many still think that 1/3+1/2 = 2/5 [Post 4-OUT; F90]

One teacher shared his teaching experience following on from a comment from another teacher's findings about students' difficulties in adding fractions.

> I also adopted this method for my vocational students as you wrote above. [many students did 1/3+1/2 = 2/5]. As a result, I kindly taught them using "biting" [manipulatives], I prefer this than having complaints from my school principal. [Post 4-OUT; F76]

One teacher prompted others to explain a method of teaching 4÷1/2 and 4÷8. Two teachers replied:

> You can draw half a kilogram of sugar for 4÷1/2 … For 4÷8, prepare 4 "biting" and share them to eight children ….Welcome for any suggestions. [Post 4-OUT; F76]

> 4÷8 can be described as 4 apples for eight children, so it can be 1/2. But if it is 4÷1/2, this is difficult to teach to students (perhaps: half those 4 apples, so there are 8), welcome for any corrections. [Post 4-OUT; F89]

The evidence from the comments and ideas supports the emergence of joint enterprise within the discussion. The participants themselves developed these ideas through their explicit engagement and own expertise, practices and experiences. The production of the joint enterprise has led to a shared repertoire among the participants.

Shared Repertoire

The affordances of FB allow the notion of a shared repertoire to evolve naturally. The design of FB encourages the sharing of ideas and resources. As a result, there is the potential to reach many people with one post even though the original poster may not know it. Facebook affords to document all the conversations that can be found/revisited by FB friends anytime. Furthermore, the content of the posts becomes a shared resource and becomes part of the community's shared repertoire. Since the joint enterprise was related to the mathematics knowledge, general pedagogy and mathematical pedagogical content knowledge, the model and the associated processes of teaching and learning also become part of the shared repertoire associated with that joint enterprise as reflected by the comments: "*I am very happy to have this knowledge. I have kept all the images that you uploaded. Thanks Mam*."

Further evidence was a message received from a primary teacher six weeks after the fourth post was uploaded, who exhibited no engagement during the one-week data collection. She expressed her strong interest in the posts and reported challenges from her further explorations on the modelling idea of division of fractions. She even stated that she had tried to solve 1/6÷1/3 and had thought about it for three days:

> Mam, I always take a chance to read your FB wall because there many lessons that I could have, many of your pictures I have downloaded and kept for my learning and to give to my students at school. One point, it is interesting to explain mathematical concepts to kids using pictures. The lesson of fraction division you gave is very interesting. But now I find it difficult … when I have to explain a fraction division, where the divisor is greater than the number divided, for example a sixth divided by a third. I have been working on this for 3 days, trying to find the solution but I haven't been able to solve it.

Although some people may not have been identified as participants in terms of the initial data collection, we cannot assume that their lack of response meant that there was no interest or no wider learning occurring from the four-post conversations. This is further evidenced by a teacher posting their own videos of their method and explanation of solving the problem visually and symbolically on a teacher group wall. This attracted many comments, likes and shares. Hence, the response of the teacher was "public" in the FB world and became a shared resource for others to use, learn from and implement in their own classrooms.

We argue that the use of the model to illustrate the division of fractions became a shared repertoire since this seemed logical and easy to follow for many people compared to a rule-based teaching "invert then multiply"; it stimulated discussions and was widely shared and encouraged to be utilised by the community or participants. In this case, FB as a social networking site mediates the knowledge building within the community.

Unlike other studies (Bissessar, 2014; Rutherford, 2010), the professional learning identified in this study was not created for a specific purpose, the participants in this study engaged actively and willingly in shaping this informal CoP. However, it is difficult to pinpoint the most determinant factor for why they engaged with the posts. It could be related to: (a) the type of posts, that is, the mathematical content of the posts; (b) how the content is misaligned to the current context of mathematics teaching; (c) the types of comments on these posts which stimulated further conversations; or (d) the reputation of the person who initially posted them. Alternatively, it could be that this sort of environment is seen as a safe space for people to ask questions, contribute ideas and learn from each other without being attacked or ridiculed. Understanding about such factors is an area for further research.

Conclusions and Implications

This study identifies that the beginnings of a CoP emerged through the online interactions in Facebook via the four inter-related posts. The mutual engagement and interactions of the participants are apparent. The shared enterprise was developed through insightful discussions on mathematical knowledge and mathematical pedagogical knowledge, which were captured in a relatively short timeframe and a variety of resources, both concrete and intellectual, were jointly developed. The four inter-related posts were not designed to be a CoP. It was a part of the first author's routine to share experiences and ideas through FB. However, the systematic analysis of these posts provided opportunities to gain insight about the potential of FB for teachers' learning or even to communicate mathematics to wider audience. This adds another perspective on research about FB where the previous studies were dominated by their finding of FB as mainly for social networking (Nadkarni & Hofmann, 2012).

An implication for further research could be the use of CoP as a theoretical framework to analyse the potential of FB, as not only a medium for engagement but also to understand the process of teachers' professional learning. That is, developing a better understanding about FB as a virtual space for professional learning and how the CoP framework, along with the space itself, can facilitate such teacher learning processes. CoP is also a promising framework to help us to optimise FB as a tool. This potentially gives us opportunities to address issues regarding the structure and sustainability of teachers' learning in this new connected digital world.

References

Aydin, S. (2012). A review of research on Facebook as an educational environment. *Educational Technology Research and Development, 60*(6), 1093-1106.

Bissessar, C. S. (2014). Facebook as an informal teacher professional development tool. *Australian Journal of Teacher Education, 39*(2), 9.

Facebook. (2015). Company Info | Facebook Newsroom. Retrieved 19 January, from http://newsroom.fb.com/company-info/

Goodyear, V. A., Casey, A., & Kirk, D. (2014). Tweet me, message me, like me: using social media to facilitate pedagogical change within an emerging community of practice. *Sport, Education and Society, 19*(7), 927-943.

Hew, K. F. (2011). Students' and teachers' use of Facebook. *Computers in Human Behavior, 27*(2), 662-676.

Hine, C. (2000). *Virtual ethnography*. London: Sage Publocations.

Manca, S., & Ranieri, M. (2013). Is it a tool suitable for learning? A critical review of the literature on Facebook as a technology-enhanced learning environment. *Journal of Computer Assisted Learning, 29*(6), 487-504.

Nadkarni, A., & Hofmann, S. G. (2012). Why do people use Facebook? *Personality and Individual Differences, 52*(3), 243-249.

Ranieri, M., Manca, S., & Fini, A. (2012). Why (and how) do teachers engage in social networks? An exploratory study of professional use of Facebook and its implications for lifelong learning. *British Journal of Educational Technology, 43*(5), 754-769.

Rutherford, C. (2010). Facebook as a source of informal teacher professional development. *In education.* Retrieved October 3, 2014, from http://ined.uregina.ca/ineducation/article/view/76/513

The Jakarta Post. (2013). Facebook has 64m active Indonesian users. Retrieved March 18, 2015, from http://www.thejakartapost.com/news/2013/06/18/facebook-has-64m-active-indonesian-users.html

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*: Cambridge university press.

Wenger, E., McDermott, R. A., & Snyder, W. (2002). *Cultivating communities of practice: A guide to managing knowledge*. Harvard Business Press.

Strategies for Solving Fraction Tasks and Their Link to Algebraic Thinking

Catherine Pearn
The University of Melbourne
<cpearn@unimelb.edu.au>

Max Stephens
The University of Melbourne
<m.stephens@unimelb.edu.au>

Many researchers argue that a deep understanding of fractions is important for a successful transition to algebra. Teaching, especially in the middle years, needs to focus specifically on those areas of fraction knowledge and operations that support subsequent solution processes for algebraic equations. This paper focuses on the results of Year 6 students from three tasks from a Fraction Screening Test that demonstrate clear links between algebraic thinking and students' solutions to fractional tasks involving reverse processes.

The National Mathematics Advisory Panel (NMAP, 2008) stated that the conceptual understanding of fractions and fluency in using procedures to solve fractions problems are central goals of students' mathematical development and are the critical foundations for algebra learning. Teaching, especially in the primary and middle years, needs to be informed by a clear awareness of what these links are before introducing students to formal algebraic notation.

Sixty-seven Year 6 students from an eastern suburban metropolitan school in Melbourne were tested using our Fraction Screening Test (Pearn & Stephens, 2014). This paper aims to identify and examine students' responses to three tasks from the test that demonstrate clear links between algebraic thinking and students' solutions to fractional tasks involving reverse processes.

Previous Research

According to Wu (2001) the ability to efficiently manipulate fractions is "vital to a dynamic understanding of algebra" (p. 17). Many researchers believe that much of the basis for algebraic thought rests on a clear understanding of rational number concepts (Kieren, 1980; Lamon, 1999; Wu, 2001) and the ability to manipulate common fractions. There is also research documenting the link between multiplicative thinking and rational number ideas (Harel & Confrey, 1994).

Siegler and colleagues (2012) used longitudinal data from both the United States and United Kingdom, to show that, when other factors were controlled, competence with fractions and division in fifth or sixth grade is a uniquely accurate predictor of students' attainment in algebra and overall mathematics performance five or six years later. They controlled for factors such as whole number arithmetic, intelligence, working memory, and family background. We need to extend these important findings to highlight for teachers those specific areas of fractional knowledge that impact directly on algebraic thinking.

Lee and Hackenburg (Lee, 2012; Lee & Hackenburg, 2013) conducted research with 18 middle school and high school students. Their research showed that fractional knowledge appeared to be closely related to establishing algebra knowledge in the domains of writing and solving linear equations and concluded: "Teaching fraction and equation writing together can create synergy in developing students' fractional knowledge and algebra ideas" (p. 9). Their research used both a Fraction based interview and an Algebra based interview. The two interview protocols were designed so that the reasoning involved in the Fraction based interview provided a foundation for solving problems in the Algebra

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 493–500. Sunshine Coast: MERGA.

Interview. In both Interviews students were asked to draw a picture as part of the solution. For the Fraction tasks they were also asked to find the answer whereas in the Algebra tasks they were asked to write an appropriate equation but not solve it. Examples of one of each of the Fraction and Algebra Tasks are shown in Table 1 below.

Table 1
Examples of tasks used by Lee and Hackenburg

Fraction Task	*Algebra Task*
Tanya has \$84, which is $\frac{4}{7}$ of David's money. Could you draw a picture of this situation? How much does David have?	Theo has a stack of CDs some number of cm tall. Sam's stack is $\frac{2}{5}$ of that height. Can you draw a picture of this situation? Can you write an equation?

After analysing the data, Lee (2012) constructed models to determine the fraction schemes used by students and their reasoning about unknowns and writing equations. However, the important point that these authors make is that the thinking required to solve this type of fraction task is very similar to the kind of thinking required to "solve for *x*" in a corresponding algebra equation. Both the Fraction Task and the Algebra Task from the Lee and Hackenburg study (2013) shown in Table 1 require multiplicative thinking to move from a given fraction to the whole, and relating these actions to the corresponding quantities. They cannot be solved additively, for example, by saying "I have to add another three-sevenths". We notice that in the Fraction Task above students are not asked to explain their thinking or what the picture represents. Moreover Lee and Hackenburg do not discuss the range of possible methods that students might use to solve the fraction task, presenting instead an example of a picture and associated comments by one student. Students are not required to solve the algebra equation ($S = \frac{2}{5}T$ where S and T represent the number of CDs that Sam and Theo have).

Stephens and Pearn (2003) identified Year 8 proficient fractional thinkers as students who demonstrated a capacity to represent fractions in various ways, and to use reverse thinking with fractions to solve problems. This research also showed that effective reverse thinking depends on a capacity to apply multiplicative operations to transform a known fraction to the whole. This capacity will later be fundamental to the solution of algebraic equations. In this study we identify algebraic thinking in terms of students' capacity to identify an equivalence relationship between a given collection of objects and the fraction this collection represents of an unknown whole, and then to operate multiplicatively on both in order to find the whole. Jacobs, Franke, Carpenter, Levi, and Battey (2007) also emphasise the need to "facilitate students' transition to the formal study of algebra in the later grades (of the elementary school) so that no distinct boundary exists between arithmetic and algebra" (p. 261). Three distinct aspects of algebraic thinking identified by Jacobs et al. (2007) and by Stephens and Ribeiro (2012) are important for this study. They are students' understanding of equivalence, transformation using equivalence, and the use of generalisable methods.

This Study

Unlike the Lee and Hackenburg study (2013) which used both a Fraction Interview and a separate Algebra Interview, our study is based on analyses of students' performances in a single paper and pencil test of fractional thinking. Previously Pearn and Stephens (2007) used a Fraction Screening Test and Fraction Interview using number lines to probe students' understanding of fractions as numbers. Results from these showed that successful students demonstrated easily accessible and correct whole number knowledge and knew relationships between whole and parts.

The current version of the Fraction Screening Test (Pearn & Stephens, 2014) includes items that require students to use reverse or reciprocal thinking. The Fraction Screening Test was divided into three parts. Part A included 12 tasks, 11 tasks had been trialled in previous work (Pearn &Stephens, 2007). Part A tasks included routine fraction tasks such as equivalent fractions, ordering fractions and recognising simple representations. Part A also included a simple reverse thinking task showing a collection of four lollies and saying: "This is one-half of the lollies I started with. How many lollies did I start with?" This task was correctly answered by the majority of students and was one of the easiest questions in Section A. Part B included five number line tasks with four tasks trialled in previous work. One number line task involving reverse thinking gave a number line showing "where the number $\frac{1}{3}$ is. Put a cross (x) where you think the number 1 would be on the number line." Part C included three questions which required students to use reverse thinking using less familiar fractions (see Figure 1).

Our Sample

Sixty-seven Year 6 students from an eastern suburban metropolitan school in Melbourne were tested using our Fraction Screening Test (Pearn & Stephens, 2014). Students completed the tests in approximately 30 minutes. After analysis of the 67 sets of responses, 19 students were chosen for closer analysis. These 19 students had correctly solved each of the three questions shown in Figure 1 and provided adequate explanations of their thinking. They were asked to provide a more detailed written explanation of their solution to one question only in order to confirm their thinking.

Our Three Key Questions

The analysis for this paper is based on these three items from Part C which specifically required students to use reverse or reciprocal thinking in which their task is to find a whole collection when given a part of a collection and its fractional relationship to the whole.

We devised these three items to offer students opportunities to use more explicit algebraic thinking which was not needed in the earlier task relating to one-half. Each of the three questions was marked out of three. Only one mark was given if there was some evidence of correct diagram or of an initial representation which the student did not take further (starting point). Two marks were given for a correct answer but without explanation and three marks were given for an adequate explanation.

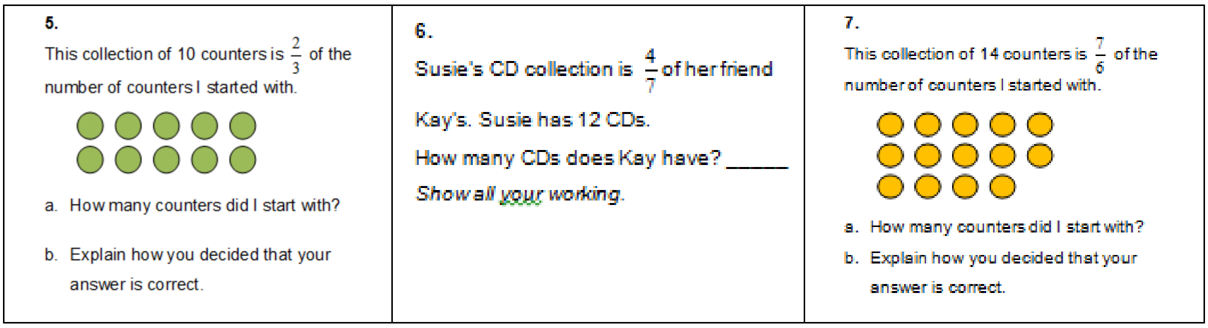


Figure 1. Questions 5 – 7

Like Kieran (1981) and Jacobs et al. (2007) we do not restrict correct algebraic thinking to students' ability to use pro-numerals or unknowns or necessarily to set up formal algebraic equations. We expect that these Year 6 students who have not necessarily been exposed to formal algebra will employ a variety of successful representations to solve these problems. We also expect that some students may use a routine algorithm to solve the problem. Simply using a routine without an appropriate explanation may not be convincing evidence of algebraic thinking. However, we also expect that some students may choose to solve the same problems in non-algebraic ways.

Results

Among the 67 students, five groups were identified: Group A (19 students) who correctly answered and adequately explained each of the three questions (scoring 9 marks out of a possible 9). Group B (9 students) answered the three questions correctly but gave an incomplete explanation or no explanation for one of their correct answers (scoring 7 or 8). All Group B students scored a 3 for Question 5. Group C (14 students) all had correct answers to Question 5, with 12 providing adequate explanations (scoring between 4 and 6). Group D (11 students) scored between one and three marks on the same three items. All 11 students omitted to answer at least one of the questions. Four students had correct answers to Question 5, with three providing adequate explanations. No student in this group correctly answered Question 7. Group E, (14 students) scored 0 on all three questions, providing insufficient evidence of performance.

Forty-six of the 67 students (69%) gave correct answers and 43 gave adequate explanations to Question 5. The diagram accompanying this problem may have assisted students to solve the problem. Some students' explanations used reverse thinking to show that one-third was equivalent to 5 dots and therefore the whole needed to be 15. Other students' explanations involved additive one-step thinking saying that one more row was needed to make the whole. Either explanation is suitable for this question.

Question 6 was correctly answered by 38 students (57%). Not being supported by a diagram, it appeared more difficult. Using one-step additive thinking is not helpful in solving this problem. It was necessary for students to calculate the number of CDs represented by $\frac{1}{7}$ and to scale up that quantity to make a whole. More difficult was Question 7 involving an improper fraction $\frac{7}{6}$ even though it was supported by a quantitative representation. Question 7 was correctly answered by 32 of the 67 students (48%). Some successful explanations applied a fractional lens to decode the 14 dots shown, arguing that each pair of dots represents $\frac{1}{6}$ and that the whole can be found by subtracting two dots. This solution, as is Emily's solution to Question 6 (see Figure 2), involves similar two-step thinking as those students who first divide the 14 counters by 7 to find how many counters are represented by $\frac{1}{6}$ and then to multiply (scale up) by 6 to find a whole. These two questions, even with a diagram provided for Question 7, were more difficult than Question 5.

Analysis

In this section, we focus on the 19 students (Group A) who gave completely correct responses and adequate explanations to all three questions. Some explanations were briefly written leaving some thinking unstated and raising a question of whether these students may have been using a routine. Each of the 19 students was asked to provide a short written elaboration of their initial explanation to one question selected by the researchers. In looking at their initial responses and their subsequent elaborations our goal was to identify those features that could be confidently taken to indicate evidence of algebraic thinking. Our focus was to identify instances of student thinking that could be clearly classified as algebraic; namely, understanding of equivalence, transformation using equivalence, and use of generalisable methods. Students in Group A offered the best chance to show this.

Confident Reverse Thinkers

Responses of Group A students show that confident reverse thinkers are able to step back from a visual representation, and to relate the fraction to the numerical quantity it represents. These students know how to scale down and scale up fractions and the quantities they represent to obtain a measure for the whole. Scaling down and scaling up is a reliable two-step procedure for finding the whole. It may even be compacted into one-step. These students are not dependent on using additive strategies which may be appropriate for simple fraction problems like the one-half task in Part A.

From the 19 fully correct responses four different types of responses were evident: *Response Type 1*. Eleven students employed equivalent operations using fractions and whole number quantities in parallel. See for example Emily's response to Question 6 in Figure 2 where she wrote $\frac{4}{7} \div 4 = \frac{1}{7} \times 7 = \frac{7}{7} = 1$ on one line and $12 \div 4 = 3$, $3 \times 7 = 21$ on the one underneath tracking both fractional and whole number computation in parallel.

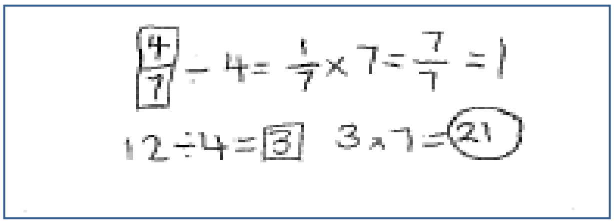


Figure 2. Emily's response to Question 6

Emily's response can be directly compared to a two-step solution for $\frac{4}{7}\,x = 12$. Like some other students, Emily uses an equal sign idiosyncratically to connect her steps as in the first line of her response. However, Emily clearly understands the need for equivalent operations to relate the two lines of her solution. Other students write "equivalence relationships" involving fractions and whole numbers together. For example, in Question 6 some students wrote: $\frac{4}{7} = 12$, $\frac{1}{7} = 3$, $3 \times 7 = 21$

Sometimes a two-step reverse operation is compacted into one step as Kenneth's response to Question 5 as shown in Figure 3.

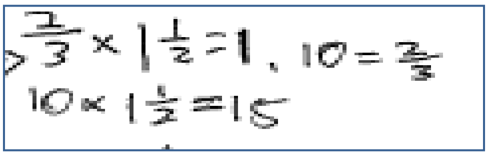


Figure 3. Kenneth's response to Question 5

Kenneth's response mirrors very closely the kind of transformational thinking needed to solve the algebraic equation $\frac{2}{3}\,x = 10 \;\rightarrow x = 10 \times 1\frac{1}{2}$.

Response Type 2. Six students left the fraction unstated and operated directly on the whole number quantity. While scaling up the fraction is left invisible, this transformation clearly guides the operations on the associated whole numbers using equivalence: For example, one two-step response to Question 6 was $12 \div 4 = 3, 3\ \times 7 = 21$; or by another student on the same question: $12 \times 7 = 84$, $84 \div 4 = 21$ or in one compacted step by another student for Question 7 was $14 \div \frac{7}{6}$. These strategies explicitly show the kind of generalisable algebraic thinking needed to solve the equation $\frac{7}{6}\ x = 14$

Response Type 3. Symbolic representation using an unknown was used by one student only. Figure 4 shows Julie's response to Question 6: 12 is $\frac{4}{7}$ of *x*, meaning that $x = 12 \div \frac{4}{7}$ which is now $\frac{12}{1} \times \frac{7}{4}$

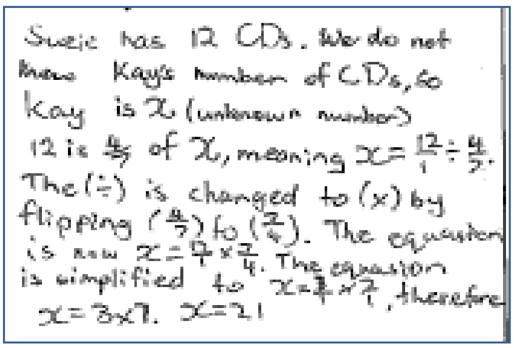


Figure 4. Julie's response to Question 7

Julie's response shows a clear understanding of equivalence and transformation. It is also generalizable unlike Response Type 4 which relies on written descriptions involving continued adding. This was used by one student for Question 6 who stated: " $\frac{4}{7}$ of Kay's CD collection is 12. That means that $\frac{1}{7}$ is 3. I started adding 3 onto 12 until it reached $\frac{7}{7}$. That number is 21". Multi-step responses like this correctly establishing that one-seventh is equivalent to 3 then rely on additive strategies to achieve the whole. This is a more limiting strategy than shown in the preceding Response Types which demonstrate reciprocal thinking.

Mixed Methods

Julie, who used a pro-numeral expression for Question 6, used the second and also generalisable method to solve other questions (e.g. $10 \div \frac{2}{3}$ to solve Question 5). While some Group A students tended to use either the first or second method consistently, most used a mix of methods. We wondered, for example, if the student who wrote $14 \div 7 = 2 \times 6 = 12$ might be using a routine, but this student later explained that "14 was split into seven numerator groups". Adding, "I could have taken one group away".

A similar range of methods, excluding symbolic representation, was evident among students in Groups B and C. However, among students in Groups C and D additive processes became more evident, like this Type 4 explanation from a student in Group C for Question 5: "I had to halve 10 because $\frac{2}{3}$ is 10, halve 2 to get 1, and so I did this to get 5. I just added it (5) on after (to get 15)."

Among students in Group D explanations begin to show less evidence of multiplicative (reverse) thinking: "Started with 10 to get 15"; or "Every 5 is $\frac{1}{3}$"; or "Because there are 5 in each row and 10 is $\frac{2}{3}$ of 15"; or "$\frac{1}{3}$ = 5, $\frac{2}{3}$ = 10, 1 = 15". There is clear evidence of equivalence but these additive strategies have less algebraic potential compared to the efficient multiplicative (reverse) strategies shown by those using Response Types 1, 2, and 3. Algebraic thinking, as we have defined it, requires more than use of equivalence. It needs to be reflected in confident and appropriate transformations of the fractional entities involved.

Conclusion and Implications

Confident reverse thinkers are able to scale down and scale up (or scale up and then scale down) based on the meaning of the particular fractional relationship. This is exactly what is required in "solving for *x*" in corresponding algebraic representations. Their working shows that scaling down and scaling up of fractional quantities must be accompanied by equivalent changes in the quantities represented by a particular fraction. These methods and their resulting mathematical relationships are indicative of algebraic thinking, by which students demonstrate that they can manipulate the fractional and numerical quantities independently of any diagram or visual representation.

The algebraic significance of these findings is that they draw attention to three quite specific aspects of fractional operations that are not sufficiently emphasised in earlier studies. The first is being able to transform (operate on) a given fraction in order to return it to a whole, regardless of whether the fraction is expressed in proper or improper form. The second is students' understanding of equivalence, meaning that the operations that are required to restore a fraction to a whole need to be applied to the corresponding numerical quantities represented by the fraction. The third is to utilise efficient and generalisable multiplicative methods to achieve this goal; in contrast to other methods, usually additive, which may work only with simple fractions. All three aspects are essential for the subsequent solution of algebraic equations. Teachers especially need to help students identify and use these efficient and generalisable strategies.

References

Harel, G. & Confrey, J. (1994). *The development of multiplicative reasoning in the learning of mathematics*. New York: SUNY Press.

Jacobs, V., Franke, M., Carpenter, T., Levi, L. & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary School. *Journal for Research in Mathematics Education 38*(3), 258–288.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics 12(*3), 317-326.

Kieren, T. E. (1980). The rational number construct: Its elements and mechanisms. In T. E. Kieren (Ed.), *Recent Research on Number Learning* (pp. 125-149). Columbus: Ohio State University. (ERIC Document Reproduction Service No. ED 212 463).

Lamon, S. J. (1999). *Teaching Fractions and Ratios for Understanding: Essential Knowledge and Instructional Strategies for Teachers*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lee, M.Y. (2012). *Fractional Knowledge and equation writing: The cases of Peter and Willa*. Paper presented at the 12th International Congress on Mathematical Education 8 – 15th July, 2012, Seoul, Korea. Last accessed 19th March 2014 from http://www.icme12.org/upload/UpFile2/TSG/0766.pdf

Lee, M.Y. & Hackenburg, A. (2013). Relationships between fractional knowledge and algebraic reasoning: The case of Willa. *International Journal of Science and Mathematics Education 12*(4), 975-1000

National Mathematics Advisory Panel [NMAP] (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel.* U.S. Department of Education. http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Pearn, C. & Stephens, M. (2014). Fractional knowledge as a signpost to algebraic readiness. Presentation at the annual meeting of the Mathematical Association of Victoria, *Maths Rocks* at La Trobe University, December 4-5.

Pearn, C. & Stephens, M. (2007). Whole number knowledge and number lines help develop fraction concepts. In J. Watson & K. Beswick (Eds), *Mathematics: Essential research, essential practice*. Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Vol. 2, pp. 601–610. Adelaide: MERGA.

Siegler, R., Duncan, G., Davis-Kean, P., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M., & Chen, M. (2012). *Early Predictors of High School Mathematics Achievement*. Retrieved from http://pss.sagepub.com/content/early/2012/06/13/0956797612440101

Stephens, M. & Pearn, C. (2003). Probing whole number dominance with fractions. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics Education Research: Innovation, Networking, Opportunity.* Proceedings of the Twenty-sixth Annual Conference of the Mathematics Education Research Group of Australasia, pp. 650-657. Sydney: MERGA.

Stephens, M. & Ribeiro, A. (2012). Working towards Algebra: The importance of relational thinking. *Revista Latinoamericano de Investigacion en Matematica Educativa, 15*(3), 373–402.

Wu, H. (2001). How to prepare students for algebra. *American Educator, 25*(2), 10-17.

Mentoring to Alleviate Anxiety in Pre-Service primary mathematics Teachers: an orientation towards improvement rather than evaluation.

Timothy Perkins
University of Notre Dame Australia
<tim.perkins@nd.edu.au>

Increasing numbers of primary pre-service teachers (PSTs) enrolled in Education degrees in Australia enter university with insufficient mathematical content knowledge (Livy & Vale, 2011) and low confidence levels about their ability to teach and do the mathematics required for their intended role as classroom teachers (Wilson, 2009). Mentoring of PSTs by highly capable and experienced classroom teachers, within the framework of a structured and well-planned mentoring programme (Hudson & Peard, 2006), has the potential for developing the confidence, and thus alleviating the mathematics anxiety exhibited by PSTs.

Introduction

Pre-service teachers often complete their professional experience without having improved their skills, confidence, content knowledge, or repertoire of pedagogical skills in mathematics. In fact, it could be postulated, that due to a lack of exposure to excellent teaching of mathematics, their skills do not improve and the anxiety around having to eventually become responsible for the teaching of mathematics to a class on their own, actually increases. This is clearly not best practice and the consequence is a perpetuation of the limited capacities of the pre-service teachers leading to poor teaching and impeded learning opportunities for their students. What is even more disturbing is the clear anecdotal evidence that despite lack of confidence with mathematics content knowledge and lack of interest in mathematics as a subject, surprisingly few pre-service primary teachers are concerned about their ability to teach mathematics.

One way of addressing and potentially arresting and reversing this trajectory of negativity is a planned and focused mentoring programme (Hudson & Peard, 2006). A highly skilled and qualified mentor has the potential to ameliorate the mathematics anxiety often experienced by pre-service teachers and the scope to set them on the path to quality mathematics teaching and learning for themselves and their future students.

Literature review: Mathematics Anxiety

Mathematics anxiety has been defined in various ways, each of which shares some fundamental and common characteristics. According to Wilson and Gurney (2001) mathematics anxiety is “a learned emotional response, characterised by a feeling that mathematics cannot make sense, of helplessness, tension, and lack of control over one’s learning.” (p.805). Chewning (2002) defines mathematics anxiety as: “an intense emotional feeling that people have about their inability to understand and do mathematics. People who suffer from math anxiety feel that they are not capable of doing any course or activity requiring mathematics” (p.1). This, clearly, would make the teaching of mathematics by the sufferer extremely problematic.

Harding, cited in Harding and Terrell (2006), defines mathematics anxiety as a “learned emotional response which usually comes from negative experiences in working

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 501–507. Sunshine Coast: MERGA.

with teachers, tutors, classmates, parents or siblings" (p.2). Greenwood (1984) supports such causality stating that mathematics anxiety often emanates from "A bad experience in a math class or with a math teacher" and elaborates by stating that mathematics anxiety, "results more from the way subject matter is presented than from the subject matter itself " (p.3).

What these definitions share is a predication on the belief that mathematics anxiety indeed exists and that it is both debilitating for the sufferer in terms of being able to perform mathematical tasks and also that the mathematics anxiety regularly stems from a negative experience in mathematics class or more specifically a negative inter-personal experience with a mathematics teacher.

Mentoring

Mentors can play a significant role in shaping pre-service teachers' practices (Hudson & Hudson, 2010). Previous research has shown that mentors choose to be involved in mentoring programmes for pre-service teachers because of a desire to "influence the quality of pre-service teacher education" (Hudson & Hudson, 2010, p.1). As stated by Edwards (1998), "the role of mentor has considerable pedagogic potential for the development of pre-service teachers" (p.48) while according to Hudson and McRobbie (2004), mentoring can lead to improved classroom practices. However, the potentially positive relationship between mentoring and mathematics anxiety has not been adequately explored. Mentoring needs to be a planned and intentional process (Christensen, 1991) where the "the job of a mentor is to put the mentees' interest in the foreground of the relationship" (Lennox, Skinner, & Foureur, 2008, p.9).

Sullivan (2011), in analysing the Japanese *Lesson Study* approach, as described by (Inoue, 2010), explored the value of teachers watching, critically reflecting, and supporting each other as a way of developing profoundly valuable mathematics lessons. He suggests that: "by building trust between teachers and emphasising an orientation to improvement as distinct from evaluation, this approach will result in powerful mathematics teacher learning. (p. 59)" Although Sullivan is referring to fully qualified teachers working together within the framework of a very specific methodology, this could be extrapolated to the context of mentor and pre-service teacher working together in the same way as planned for this study. Sullivan goes on to state that: "the principles of collaborative planning, with observation and review of the lesson rather than the teacher, can be effectively incorporated into the practicum experiences of prospective teachers. (p. 60)" It could be suggested that Sullivan's focus on improvement rather than evaluation is more likely to occur outside the professional experience blockwhich is based around evaluation.

Mentoring of pre-service teachers by experienced teachers occurs most often during their professional experience block. Many teachers who supervise and mentor pre-service teachers on their placements have received no "professional development in mentoring to support pre-service teachers in the school context" (Hudson & Hudson, 2010, p. 5). Consequently, the scope for mentoring and supervising to be of a high quality, and to significantly add to the pre-service teacher's developing expertise, may be compromised.

Exposure to best practice in the teaching of mathematics by supervising teachers as well as support and opportunities to lead mathematics lessons in a structured, planned and non-threatening environment, is essential for pre-service teachers. PSTs would benefit from observing creative, engaging, challenging, differentiated lessons modeled by teachers displaying confidence, sound pedagogy, and having sound content knowledge. However, this is not always the case. Mathematically anxious pre-service teachers are not always partnered with a competent teacher during their professional experience blocks.

Professional Experience Block

The logical forum for PSTs to benefit from exposure to informed mathematics teaching is the professional experience block. Professional experience or practicum blocks form part of the assessable component of pre-service teachers' degree courses. Supervising teachers observe and assess the teaching of their pre-service teachers and grade the students on their efforts. University supervisors also monitor student professional experience units and work in conjunction with the supervising teachers to make decisions about whether or not the pre-service teachers meet the required standards in order to *pass* the professional experience and, therefore, progress further in their degree. If the students are deemed to have performed poorly, they can *fail* the professional experience. The *on the job* pressure of being monitored in such a way is a valuable way of ensuring that future teachers meet the necessary standards for entry into the profession. An unintended and perhaps unavoidable side effect of this experience is increased anxiety and stress in pre-service teachers who are uncertain about aspects of their practice and unfamiliar with being monitored in such a potentially high stakes forum. Such a forum is conducive to pre-service teachers limiting risks, *playing it safe* and masking any inadequacies they may have. They will swim in the shallows for fear of being found out for not being proficient once their feet can no longer touch the bottom.

For this reason, a fundamental aspect of this study, and its key point of difference from other such mentoring approaches is that it separates the mentoring process from the professional experience block. The study involves pre-service teachers being mentored for a school term with experienced and highly capable mentor teachers in the mathematics classroom, outside of their professional experience. The pre-service teachers who nominate to take part in the study are encouraged to take risks with their teaching in an exclusively supportive environment with a deliberate and planned focus on diminishing the fear of judgement or failure.

Bly (1988) suggested one difference between the traditional classroom and the playful classroom:

> In a traditional mathematics classroom there are a set of rules and if you get something wrong, it leads to shame. In a playful mathematics classroom, there are a set of guidelines and if you do something different, it leads to conversation" (cited in Breen, 2001, p.46).

It is possible that the source of fear and anxiety felt by pre-service teachers in relation to their mathematics has: "more to do with the personality and style of the teacher than with the content of the mathematics and their ability to cope with it" (Breen, 2001, p. 45). It would be fair to suggest that the pre service teachers who self-nominated to be a part of this study may have experienced the shame and humiliation of the traditional classroom and more specifically the traditional teacher. It is for this reason that each experience that the pre-service teachers has during the process is exclusively instructive and reflected on in such a way that future improvement leading to best practice was the goal.

Breen (2001) refers to the work of Davis (1996) identifying three forms of listening which he believes limit or enhance the thinking and self concept of the pre-service mathematics teacher. These three forms of listening are: *Evaluative listening* based on judgement; *Interpretive listening* based on subjective nuance; and *Hermeneutic listening* which is based in respect for the teller, where the teller and the listener are engaged in a shared purpose and the views of each are valued as worthy of consideration and the desire to come to a shared and improved understanding is mutual. Such an approach requires the deliberate addressing and breaking down of the traditional power dynamic between teacher and student or mentor and mentee.

The aim of this study is to develop a strategy that will allow university mathematics educators and school staff development teams to address issues of mathematics anxiety through a practical and manageable mentoring programme as a means to improve the confidence of pre-service teachers and ultimately, the quality of mathematics outcomes for their students. It is aimed at utilising the capacities and skills of expert teachers as mentors for those entering the profession in a collegial, supportive, and non-threatening environment. The research question being addressed in this aspect of the study is: In what ways can a mentoring relationship outside of the professional experience block increase the confidence of pre-service teachers who identify as suffering from mathematics anxiety?

The Mentoring Model Proposed for this Study

Several aspects of both Hudson and Skamp's (2005) and Rogoff's (1995) models have been incorporated in a proposed model, which focuses on the following requirements for the selection of the mentors:

- Experience–a minimum of 5 years' teaching experience
- Professional Responsibility–a desire to improve the profession by working with pre-service teachers to improve their ability to teach mathematics. Along with a desire to read and learn from articles about mentoring best practice.
- Mathematical Confidence–a very confident disposition towards both doing and teaching mathematics based on a subject matter knowledge
- Teaching Expertise–a proven track record of successful maths teaching evidenced through positive dispositions of their own students towards mathematics, creative approaches, positive classroom environments, engaging lessons, differentiated opportunities for students
- Appropriate personal attributes–a demeanour appropriate for mentoring pre-service teachers who lack confidence in this subject. Characteristics such as: good listener and communicator, empathy, sense of humour, supportive, encouraging, ability to deliver feedback positively and constructively
- Time–A preparedness to plan and review lessons with the PST's and to respond to their reflections of the experience.

Methodology

This investigation was carried out as a case study of eight 3rd and 4th year students from a metropolitan university in Sydney. The students were selected as a result of self-nominating to be part of a mentoring programme to address mathematics anxiety from which they had indicated they were suffering. 227 3rd and 4th year students completed an adapted Mathematics Anxiety Rating Scale (Richardson & Suinn, 1972)) survey with 38.29% (n=85) indicating that they suffer from mathematics anxiety according to Wilson and Gurney's (2011) definition of mathematics anxiety as a "learned emotional response, characterised by a feeling that mathematics cannot make sense, of helplessness, tension, and lack of control over one's learning". From these, 8 pre-service teachers (PSTs) were selected based on the extent to which they were negatively impacted by their mathematics anxiety. The PSTs were paired together to form 4 groups.

Mentors were from Catholic primary schools in the Sydney archdiocese, nominated by their school principals according to their meeting the six criteria listed above. The mentoring process took place in Term 4, 2014 with 2 PST's working with 1 mentor teacher in their primary classroom for a 1-hour mathematics session once per week for 8 weeks. Where possible, the team of 3 met for approximately 10 minutes prior to and/or 10 minutes after each session to explore what would be happening in class that day.

These mentoring sessions were underpinned and supported by the reflective practice of journaling via the use of a blog specifically established for this project. Pre-service teacher mentees in the programme were expected to reflect on their experience and practice, celebrate their successes, and support their fellow pre-service teacher mentees via the blog on a weekly basis. The mentors were also expected to blog their own thoughts and responses to the mentees postings. At the completion of the project, mentors and mentees completed a post survey and a semi-structured interview with the researcher. Due to work and travel commitments, only 6 of the 8 mentees and only 2 of the 3 mentors have been interviewed thus far.

Results and Discussion

One of the most prevalent themes to emanate from the interviews conducted with the students after the mentoring programme was the relationship between professional experience and the mentoring programme. Professional experience, also known as prac, is the time that students spend in classrooms with cooperating teachers, to improve their professional practice. In NSW it is mandated that all teacher education students spend a minimum of 80 days, which equates to 16 school weeks, working in classrooms during their four-year degree.

There has been much criticism of teacher training in NSW in the media over the past 12 months and one of the areas being challenged is the professional experience blocks. The pre-service teachers in this study complete extensive professional experience with their course requiring them to complete 31 weeks of professional experience during their 4-year programme. One of the great difficulties universities face is the sourcing of professional experience opportunities for their students. With increasing numbers of students enrolled in education courses, there is increasing pressure on the Professional Experience Office at universities to place their students. The professional experience block should be an opportunity for students to work with experienced and highly capable teachers in a mentoring relationship, which adds a practical component to the theoretical input at university. A concern outlined by the government reports into teacher training (Piccoli, 2012) is that the cooperating teachers in the professional experience are not given sufficient training in mentoring and can sometimes provide a less than perfect learning opportunity for their prac students.

> AMac: It was good to be guaranteed a good teacher and someone you knew wanted to help you however they could. And she was always willing to answer our questions … which sometimes on prac is a little bit challenging.

Of the 8 students involved in this study, all 8 suggested that the mentoring approach was superior to their professional experience. When questioned as to why this was the case, two prominent themes emerged. Firstly, and most importantly, was the relationship between the student and the mentor as opposed to the relationships they have experienced with their cooperating teachers during professional experience placements. All 8 students suggested that the relationship with the mentors was more collegial and relaxed with a distinct focus on improving their skills.

TW: This project was all about bringing our weaknesses out into the open, exploring them, and actively working towards strengthening them.

Secondly, the mentees suggested that the *high stakes* aspect of professional experience meant that they were always under pressure, that the environment was stressful, they felt that they were being judged, that they were expected to be highly proficient, and that as a result, they were disinclined to take risks and that they kept their deficiencies to themselves for fear of being judged and potentially failed by their cooperating teachers.

AM: On prac, you are watched like a hawk. You're paranoid the whole time just to perform well. Prac is smoke and mirrors.

SA: [T]hat pressure just not being there. It was so much more human, the relationship with the teacher, which is not something you have on prac."

This consequently led to low level activities during professional experience, based around keeping all students busy and trying to make themselves look capable rather than rich, challenging tasks that may not be immediately successful. One of the mentors stated:

MM: [A] key difference, they don't have that fear of failing. They don't have the fear of being assessed and they could just get on with it.

A second major theme to come out of the study is the reduction in fear felt by the mentees that they had to be experts in Mathematics and know all the content from the syllabus in order to be able to teach it well. Many of them commented on the fact that they knew a lot more than they thought they did.

AM: What I really picked up on - I have just gone, 'Maths as a subject, I don't know it: I'm hopeless at it.' because that's just the attitude I've always had. But being in the classroom over the eight weeks I went 'There is a lot I actually know.'

TW: I also learnt more about myself, my skills and abilities. I walked into the project thinking that I wouldn't be able to complete any Stage 3 mathematical content. I now know that there are only a few gaps and I can work on filling those gaps with knowledge.

The mentors supported these thoughts. For example:

MM: I said to them, teaching's not about knowing everything. I said, look, we teach Maths, I said look at me, look what I'm teaching in history and science, I said I can't possibly know all of this at any one point in time.

There are many more themes to explore and draw out from this research project. What has been interesting is the unanimous consensus among the mentees that there has been some shift from anxiety towards greater confidence as a result of completing the mentoring trial. The precise reasons for this movement towards greater confidence need to be teased out more specifically but at this stage, the results look promising. What is also noteworthy is that the mentors found the process valuable and productive which augers well for future trials.

References

Breen, C. (2001). Coping with fear of mathematics in a group of preservice primary school teachers. *Pythagoras 54*, 42-50. Retrieved from http://web.uct.ac.za/depts/educate/download/pythagfear.pdf

Chewning, S. (2002). Overcoming math anxiety. Retrieved from http://www.odu.edu/~jritz/oted885/CurriculumForOvercomingMathAnxiety.pdf

Christensen, L. (1991). *Empowerment of preservice teachers through effective mentoring: Course requirements.* Alabama, US: University of Alabama.

Davis, B. (1996). *Teaching mathematics: Towards a sound alternative*, New York: Garland Publishing

Edwards, A. (1998). Mentoring student teachers in primary schools: assisting student teachers to become learners, *European Journal of Teacher Education*, *21*, 47–62.

Greenwood, J. (1984). Soundoff: my anxieties about math anxiety. *The Mathematics Teacher, 77,* 662-63.

Harding, G., & S. L. Terrell (2006) Strategies for alleviating math anxiety in the visual learner. Math Anxiety Workshop. University of Maryland University College (UMUC).

Hudson, P., & Hudson, S. (2010). Mentor educators' understandings of mentoring preservice primary teachers. *The International Journal of Learning, 17* (2), 157-170.

Hudson, P., & McRobbie, C. (2004). Evaluating a specific mentoring intervention for preservice teachers of primary science. *Action in Teacher Education, 17* (2), 7-35.

Hudson, P., & Peard, R. (2005) Identifying mentoring practices for developing effective primary mathematics teaching. In *Eighth International Conference: Reform, Revolution and Paradigm Shifts in Mathematics Education*, 25 Nov - 1 Dec 2005, Universiti Teknologi Malaysia (UTM), Johor Bharu, Malaysia.

Hudson, P., & Skamp, K. (2005). Development of an instrument: Mentoring for Effective Primary Teaching. *Science Teacher Education*, 658-674.

Inoue, N. (2010). Zen and the art of neriage: Facilitating consensus building in mathematics inquiry lessons through lesson study. *Journal of Mathematics Teacher Education*, *14*(1), 5-23. doi:10.1007/s10857-010-9150-z

Kamath, A. (2006) *Think positive and things will go right.* New Delhi, India: Lotus Press.

Lennox, S., Skinner, J., & Fourer, M. (2008). Mentorship, preceptorship and clinical supervision: three key processes for supporting midwives. *New Zealand College of Midwives, 39* (7-12).

Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Teacher Education and Development, 13* (2) 22–43.

Monk, D. H., & King, J. A. (1994). Multilevel teacher resource effects in pupil performance in secondary mathematics and science: The case of teacher subject matter preparation. In R. G. Ehrenberg (Ed.), *Choices and consequences: Contemporary policy issues in education* (pp. 29-58). Ithaca, NY: ILR Press.

NSW Department of Education and Training (DET). (2003). *Quality teaching in NSW public schools. Discussion paper*. Sydney, NSW: Professional Support and Curriculum Directorate.

Piccoli, A. (2012). Ministers introduction in great teaching inspired learning: Discussion paper. Department of Education and Communities: Sydney. Retrieved from http://www.schools.nsw.edu.au/media/downloads/news/greatteaching/gtil.pdf

Richardson, F.C., & Suinn, R.M. (1972). The Mathematics Anxiety Rating Scale: Psychometric data. *Journal of Counseling Psychology, 19*(6), 551-554.

Rogoff, B. (1995) Observing sociocultural activity on three planes: participatory appropriation, guided participation and apprenticeship, in: J. Wertsch, P. del Rio & A. Alvarez (Eds.), *Sociocultural Studies of Mind.* Cambridge: Cambridge University Press.

Sullivan, P. (2001). *Teaching Mathematics: Using research-informed strategies*, Camberwell: ACER Press.

Wilson, S. (2009). "Better You Than Me": Mathematics anxiety and bibliotherapy in primary teacher professional learning. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia, Vol 2), Palmerston North, NZ: MERGA.

Wilson, S., & Gurney, S. (2011). "My self-esteem has risen dramatically": A case study of pre-service teacher action research using bibliotherapy to address mathematics anxiety. In J. Clark, B. Kissane,, J. Mousley, T. Spencer, & S. Thornton (Eds.) *Mathematics: Traditions and (New) Practices* (Proceedings of the AAMT-MERGA conference 34th annual conference of the Mathematics Education Research Group of Australasia, pp. 804-812), Alice Springs: MERGA.

Spatial Visualisation and Cognitive Style: How Do Gender Differences Play Out?

Ajay Ramful
University of Canberra
<Ajay.Ramful@canberra.edu.au>

Tom Lowrie
University of Canberra
<Thomas.Lowrie@canberra.edu.au>

This study investigated potential gender differences in a sample of 807 Year 6 Singaporean students in relation to two variables: spatial visualisation ability and cognitive style. In contrast to the general trend, overall there were no significant gender differences on spatial visualisation ability. However, gender differences were prevalent among students who possessed high spatial visualisation ability, in favour of boys. In terms of cognitive style, there were significant gender differences in the spatial imagery and verbal information processing dimensions. Boys gave higher ratings to their spatial-imagery encoding and processing preferences than their verbal information processing preferences. Some of these findings are in contrast to studies undertaken in the educational-psychology literature. Implications are drawn regarding pedagogical practices in Singaporean schools.

There has been sustained interest from both mathematics educators and psychologists to understand how spatial ability operates and develops. Due to its strong correlation to performance in mathematics and science (Sinclair & Bruce, 2014), spatial ability continues to attract research attention. Spatial ability is generally regarded in terms of mental rotation, spatial relation, spatial orientation and spatial visualisation, although these concepts are not always used with the same consistency, to some extent because of the complex relationships among them (Carroll, 1993; Clements & Battista, 1992; Höffler, 2010). Relatedly, the inconsistencies in the definition of spatial constructs and their measurement by different standardised spatial tests make the comparison between studies problematic (Voyer, Voyer, & Bryden, 1995). This study focuses particularly on spatial visualisation that involves "the ability to 'see', inspect, and reflect on spatial objects, images, relationships and transformations" (Battista, 2007, p.843). It may involve elements such as holding a visual pattern in memory, comparing visual patterns, or doing a mental transformation and requires the manipulation of internal (mental) representations. Although there has been much interest in understanding how boys and girls operate on spatial visualisation tasks (Voyer et al., 1995), what has not been fully explained, is the significance of cognitive style in the relationship between spatial visualisation and gender, especially at the primary level in mathematics education. This research gap is the rationale for the current study.

Spatial Visualisation and Gender Differences

There is considerable evidence pointing to the fact that boys and girls differ in their spatial abilities (Battista, 1990; Ben-Chaim, Lappan, & Houang, 1988; McGuinness, 1993; Voyer et al., 1995). This tendency is equally observed in terms of spatial visualisation (Mayer & Massa, 2003). Different explanatory factors have been put forward to explain why boys and girls differ in spatial ability, recognising the contribution of both learner-related factors (such as cognitive variables) and environmental factors (such as activities in which boys and girls are engaged in their daily life). In terms of learner-related factors, substantial attention has focused on the ways in which boys and girls encode and process

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 508–515. Sunshine Coast: MERGA.

information, what is commonly referred to as cognitive style (Arnup, Murrihy, Roodenburg, & McLean, 2013; Kozhevnikov, 2007; Mayer & Massa, 2003).

Cognitive Style and Gender Differences in Mathematics Learning

Blazhenkova & Kozhevnikov (2009) distinguished among three categories of learners, namely object imagers, spatial imagers and verbalisers. Object imagers prefer to use colourful, concrete, high-resolution and pictorial images of objects to interpret information. Spatial imagers prefer to represent schematic images and spatial relations. The third category of people, verbalisers, has a preference to process information verbally. This study is framed within this three-tier categorisation of cognitive style. There is research to suggest that boys and girls differ on cognitive dimensions. For instance, Arnup et al. (2013) observed that boys with an analytic imagery cognitive style had higher mathematics performance compared to corresponding girls. Anderson, Casey, Thompson, Burrage, Pezaris, and Kosslyn (2008) reported that girls with high spatial-imagery cognitive style performed better on geometry tasks, compared to those who had lower spatial-imagery scores. Blazhenkova, Becker, and Kozhevnikov (2011) found that males scored higher on the spatial-imagery dimension while females had higher object-imagery scores, with no gender differences on the verbal information processing dimension.

Condensing the findings from the cognitive style and spatial visualisation literature, the following two patterns emerge: (1) boys tend to fare better than girls in spatial visualisation tasks and (2) boys tend to use more spatial imagery information processing than girls. Building on these findings, we hypothesised that the extent to which students use spatial imagery would be a significant determinant in their spatial visualisation ability. In particular, girls who have high spatial imagery cognitive style would have high spatial visualisation ability.

Examining the relation between spatial imagery as a cognitive style and its relation to spatial visualisation as an ability is premised on the assumption that the latter involves processing requirements shared by the former. A corresponding question is then to what extent do boys and girls process spatial information differently and how are these related to spatial visualisation ability? Thus, we posed and revisited the following two questions:

1. How do boys and girls vary in terms of spatial visualisation ability and cognitive styles?
2. Does gender interact with cognitive style on spatial visualisation ability?

Method

This paper emanates from a larger cross-cultural study (Lowrie, 2013) designed to investigate the ways in which students process mathematical information from two different cultures, Singapore and Australia. The participants (age range 11-12 years) for this paper were the Singapore cohort and included 807 Grade 6 students (392 boys and 415 girls) from 8 Singaporean schools (6 government and 2 government-aided). The schools were chosen from different regions of Singapore on the basis of their willingness to participate in the study. Two instruments were used to collect data in April 2013. Both instruments were administered on the same day by the research team according to the guidelines of the tests (Ekstrom, French, & Harman, 1976; Blazhenkova et al., 2011). Correlations, t-test, and factorial ANOVA were used to analyse the data in line with the two research questions.

Instrument 1: Measurement of Cognitive Style

The Children's Object-Spatial Imagery and Verbal Questionnaire (C-OSIVQ) (Blazhenkova et al., 2011) is premised on three dimensions of cognitive styles: (i) object-imagery, (ii) spatial-imagery, and (iii) verbal information processing. The instrument consists of 15 items from each dimension. Participants rated the 45 items on a 5-point Likert scale (1 = total disagreement; 5 = total agreement). The scores in each of the three sets are averaged to produce an object-score, a spatial-imagery score and a verbal information processing score. A sample item from each of the three dimensions are presented for descriptive purposes: (i) My visual images are like colorful photographs, or pictures of real objects and scenes (object-imagery), (ii) I can easily imagine and rotate three-dimensional figures in my mind (spatial-imagery) and (iii) My verbal abilities would make me a good writer (verbal information processing).

Instrument 2: Measurement of Spatial Visualisation Ability

The Paper Folding Test (PFT) (Ekstrom et al., 1976) is a commonly used instrument for measuring spatial visualisation ability both in Educational Psychology and Mathematics Education. In this timed test, students are required to visualise the folding and unfolding of a square sheet of paper with a punched hole (see Figure 1). The PFT consists of 20 items. A correct item is given a score of 1 mark. Incorrect items are negatively marked. The total score is calculated as follows: Number of items marked correctly minus one-fifth the number marked incorrectly (Mayer & Massa, 2003).

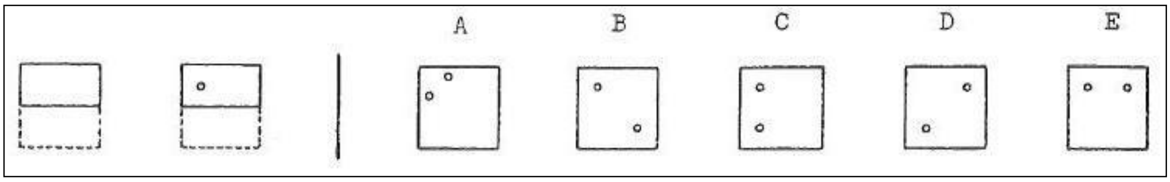


Figure 1. Paper Folding Test[1]

Results and Discussion

Descriptive Statistics

Table 1 presents the mean performance of boys and girls on the two instruments.

Table 1
Distribution Characteristics of the Instruments

Test	Mean		Standard Deviation	
	Boys	Girls	Boys	Girls
PFT	10.14	9.82	4.40	3.95
Object	3.73	3.81	0.68	0.67
Spatial	3.57	2.95	0.71	0.74
Verbal	3.16	3.29	0.67	0.65

[1] *The Paper Folding* Test is reproduced with license and permission of Educational Testing Service, New Jersey, USA.

Boys' scores were higher than that of girls' on the Paper Folding Test. Similarly boys had higher spatial-imagery scores. We comment on the statistical significance of these differences at a later point.

Table 2 shows the correlation between the three dimensions of cognitive style and spatial visualisation, interpreted from a gender perspective. Spatial visualisation, as measured by PFT, was correlated to object imagery for boys and for both boys and girls for the spatial imagery dimension, although the value of the correlation coefficient was larger for boys. There were no significant correlations between verbal information processing and spatial visualisation. It is to be noted that there were correlations among the three dimensions of the C-OSIVQ questionnaire.

Table 2
Correlation Among Variables with Focus on Gender

Measure	Object		Spatial		Verbal		PFT	
	B	G	B	G	B	G	B	G
Object	1	1	.51**	.43**	.54**	.57**	.14**	.02
Spatial			1	1	.34**	.34**	.29**	.16**
Verbal					1	1	.04	.05
PFT							1	1

Note: ** $p < 0.01$

Research Question 1: How Do Boys and Girls Vary in Terms of Spatial Visualization Ability and Cognitive Styles?

Gender differences on the Paper Folding Test. Overall, there were no gender differences on the Paper Folding Test (Boys: M = 10.14, SD = 4.40; female: M = 9.82, SD = 3.95), $t(802) = 1.059$, $p = 0.290$. The students' scores on the PFT were split into three categories to determine whether there were gender differences among students with different levels of spatial visualisation ability. The participants were classified into Low-SV (bottom 25% of the distribution, PFT score <6.8), High-SV (top 25% of the distribution, PFT score >13) and Medium-SV (middle 50%, PFT score between 6.8 and 13). Table 3 shows that gender differences were only significant among those students who had high spatial visualisation ability, with boys faring better than girls.

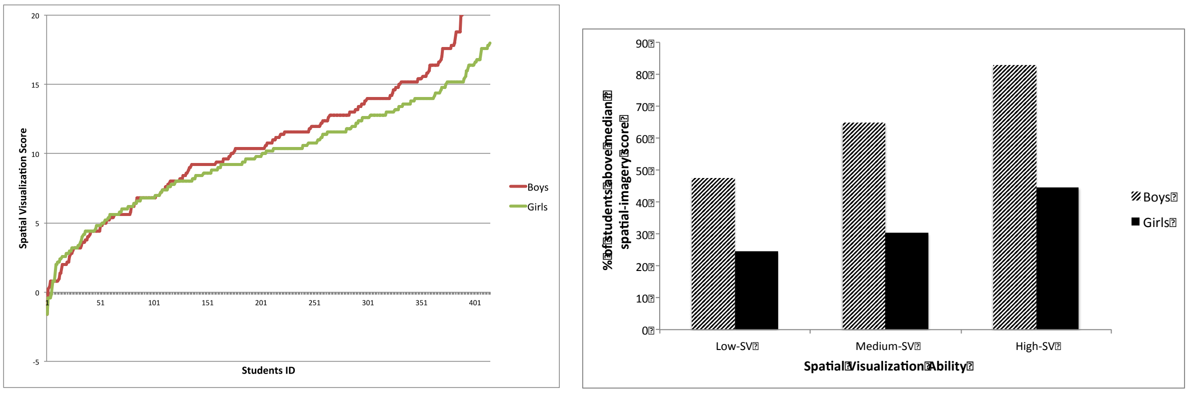
Table 3
Comparison of Boys and Girls Spatial Visualisation Ability from PFT

Level of SV	Number		Mean		t-value	Sig.
	B	G	B	G		
Low-SV	102	101	4.33	4.51	$t(201) = -0.666$	$p = 0.506$
Medium-SV	186	224	10.44	10.16	$t(408) = 1.708$	$p = 0.088$
High-SV	101	90	15.46	14.98	$t(189) = 2.216$	$p = 0.028$

The spatial visualisation scores for boys and girls were sorted separately in ascending order and plotted in Figure 2(a). Noteworthy, the gap between genders in spatial visualisation ability begins to appear as the score on the PFT crosses 10 points.

Gender differences on the C-OSIVQ questionnaire. In terms of cognitive styles, there were significant gender differences between the spatial imagery ($t(741) = 11.555$, $p = 0.000$) and verbal information processing dimensions ($t(758) = -2.500$, $p = 0.013$). For spatial imagery, the male scores were higher; for verbal processing, female scores were higher (see Table 1). Further, boys gave higher ratings to their spatial-imagery encoding and processing preferences than their verbal information processing preferences. The opposite tendency was observed for girls. No significant gender differences were observed for the object imagery dimension ($t(754) = -1.543$, $p = 0.123$). In their study, Blazhenkova et al. (2011) noted a similar pattern for the spatial imagery dimension, however they did not find differences in verbal information processing but rather on the object dimension, in favour of girls.

In Table 2, we observed that there were significant correlations between spatial-imagery and PFT for both boys and girls. Figure 2(b) shows in further detail how the level of spatial visualisation ability is related to spatial-imagery differentially for boys and girls. In Figure 2(b), the vertical axis represents the percentage of boys and girls whose scores on the spatial-imagery scale were higher than the group median. Thus, this category of students would be regarded as having a preference for spatial imagery. Across all three levels of spatial visualisation ability, there were almost twice as many boys as girls who had spatial imagery scores above the median. Further, the percentage of boys and girls who preferred to use spatial imagery were higher in the high spatial visualisation (High-SV) group than in the low spatial visualisation (Low-SV) group. This gives further evidence that there is a relationship between spatial visualisation ability and spatial imagery as a cognitive style.



(a) (b)

Figure 2. Differences between boys' and girls' scores on (a) PFT, and (b) spatial-imagery scores

Research Question 2: Does Gender Interact with Cognitive Style on Spatial Visualisation Ability?

Students were grouped in 8 categories, depending on whether they were below or above the medians in each of the three dimensions of the cognitive style (2 object x 2 spatial x 2 verbal). We coded the scores below the median as 1 and above the median as 2. For example, a student who scored low on the object-imagery, spatial-imagery and verbal information processing respectively, was coded as 111 while a student whose scores were above the median in all the three categories was coded as 222. This categorisation partitioned the sample into 8 classifications of students as shown in Table 4. The majority of boys and girls were either in the category 111 or 222. Boys who had high spatial visualization ability were primarily from group 222 (high object imagery, high spatial imagery and high verbal information processing) and similar girls were from 111 or 222.

Table 4
Distribution of Students by Cognitive Style

Cognitive Style	111	112	121	122	211	212	221	222
Boys (%)	20.1	6.6	17.4	7.5	4.2	3.3	12.9	27.9
Girls (%)	30.2	9.8	3.3	5.2	9.3	19.1	5.7	17.4
Total (students)	178	58	70	44	48	81	64	156
High-SV Boys (%)	13.8	2.3	21.8	8.0	1.1	1.1	16.1	35.6
High-SV Girls (%)	25.9	11.1	7.4	2.5	4.9	14.8	9.9	23.5

A factorial ANOVA was carried out with spatial visualisation as dependent variable and cognitive style and gender as independent variables. There was a significant main effect of cognitive style $F(7, 683) = 6.110$, $p < 0.000$, $\omega^2 = 0.05$), indicating that it influenced the participants' score on the spatial visualisation test. The non-significant effect for gender ($F(1, 683) = 2.564$, $p < 0.110$) showed that it did not influence the spatial visualisation scores, other things being equal. However, the significant interaction effect between gender and cognitive style ($F(7, 683) = 2.142$, $p < 0.037$, $\omega^2 = 0.01$) demonstrated that the influence of cognitive style on spatial visualisation was different for male participants than it was for females. Figure 3 shows the variations in cognitive style against performance in PFT.

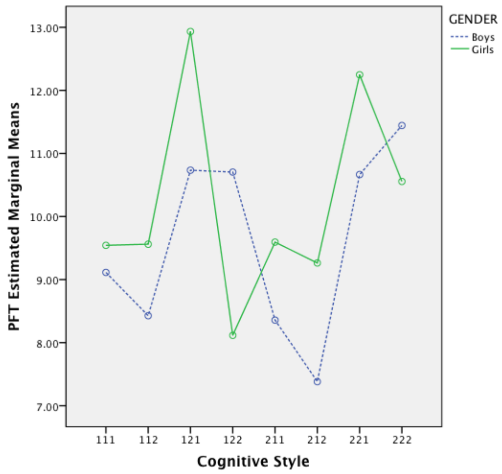


Figure 3. Variation in cognitive style and spatial visualization score

Conclusion

We make two conclusions in terms of gender differences in spatial visualisation ability, cognitive style and their interactions. Firstly, in contrast to research findings (e.g., Battista, 1990), there were no gender differences overall in spatial visualisation ability for this cohort of Singaporean students. The only significant differences were among students with high spatial visualisation ability, in favour of boys. One possible explanation for the absence of gender differences overall for the Singaporean students (as compared to the general trend), is that visualisation is explicitly emphasised in the Singaporean curriculum (Ministry of Education Singapore, 2012). Thus, the nature of the mathematics curriculum may be an influential factor in explaining gender differences related to spatial ability.

Secondly, in terms of cognitive style, boys gave higher ratings to their spatial imagery information processing mode in contrast to girls. Further, as the level of spatial visualisation increased from low, medium to high, the percentage of boys and girls who used spatial imagery increased. This consolidates the finding that spatial imagery information processing is related to spatial visualisation. Correlations between spatial visualisation and spatial imagery were higher for boys than for girls. The significant interaction between gender and cognitive style suggests that spatial-imagery may be operating differently for boys and girls. Although the present findings do not provide strong evidence for a direct relationship between spatial imagery and spatial visualisation ability, the results do suggests that cognitive style is an influential factor in manipulating mental images as is characteristic of spatial visualisation. It is acknowledged that there are other factors besides cognitive style that explains why boys and girls performed differently on spatial visualisation tasks.

The results of this study are dependent on the operational definition of constructs and instruments that were used to measure spatial visualisation and cognitive style. We focused on cognitive style from the object-spatial-verbal dimension while we measured spatial visualisation from only one instrument, i.e., the PFT. As we make further attempts to understand the ways in which cognitive style plays out in spatial visualisation, it is important to use different instruments and consider diverse conceptual underpinnings to unfold the link between unobservable constructs as in processing mathematical information and spatial skills. For instance, it may be insightful to qualitatively follow boys' and girls' responses to the spatial visualisation tasks in the PFT in future interview-based investigations.

The current study contributes in expanding the knowledge base on gender differences on spatial reasoning based on a relatively large sample of students. It revisits an issue that requires the attention of educators. Methodologically, it highlights the necessity to perform analysis by level of students to understand the underlying structure or to reveal patterns that may not be visible otherwise.

As we design curricular experiences to develop spatial skills in school mathematics, it is important to understand general trends in which boys and girls may differ in processing spatial information. Due to its methodological approach the current study may not provide direct instructional guidelines but the disparities in spatial visualisation ability and spatial imagery cognitive style between boys and girls do suggest that there is a necessity to support girls more explicitly so that they develop a spatial habit of mind, an aspect that may not be explicitly fostered socially and educationally.

References

Anderson, K.L., Casey, M.B., Thompson, W.L., Burrage, M.S., Pezaris, E., & Kosslyn. S.M. (2008). Performance on middle school geometry problems with geometry clues matched to three different cognitive styles. *Mind, Brain, and Education*, *2*(4), 188-197.

Arnup, J. L., Murrihy, C., Roodenburg, J., & McLean, L. A. (2013). Cognitive style and gender differences in children's mathematics achievement. *Educational Studies*, 39(3), 355-368.

Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. *Journal for Research in Mathematics Education, 21*(1), 47-60.

Battista, M.T (2007). The development of geometric and spatial thinking. In F. K. Lester Jr (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843-908). Charlotte, NC: Information Age Publishing.

Ben-Chaim, D., Lappan, G., & Houang, R.T. (1988). The effect of instruction on spatial visualization skills of middle school boys and girls. *American Educational Research Journal*, 25(1), 51-71.

Blazhenkova, O., & Kozhevnikov, M. (2009). The new object-spatial-verbal cognitive style model: Theory and measurement. *Applied Cognitive Psychology, 23*, 638-663.

Blazhenkova, O., Becker, M., & Kozhevnikov, M. (2011). Object-spatial imagery and verbal cognitive styles in children and adolescents: Developmental trajectories in relation to ability. *Learning and Individual Differences, 21*(3), 281-287.

Carroll, J.B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. New York: Cambridge University Press.

Clements, D.H., & Battista, M.T (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-464). New York: Macmillan.

Ekstrom, R. B., French, J. W., & Harman, H. H. (1976). *Manual for kit of factor-referenced cognitive tests*. Princeton, NJ: Educational Testing Service.

Höffler, T.N. (2010). Spatial ability: Its influence on learning with visualizations – A meta-analytic review. *Educational Psychology Review*, 22(3), 245-269.

Kozhevnikov, M. (2007). Cognitive styles in the context of modern psychology: Toward an integrated framework of cogntive style. *Psychological Bulletin, 133*(3), 464-481.

Lowrie, T. (2013). Cross Cultural Comparison of Student Sense-making: The development of a mathematics processing framework. In V. Steinle, L. Ball, & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, eBook, pp. 751-754). Melbourne, Australia, MERGA, Inc.

Mayer, R. E., & Massa, L. J. (2003). Three facets of visual and verbal learners: Cognitive ability, cognitive style and learning preference. *Journal of Educational Psychology*, 95(4), 833-846.

McGuinness, D. (1993). Gender differences in cognitive style: Implications for mathematics performance and achievement. In L. A. Penner, G. M. Batsche, H. M. Knoff & D. L. Nelson (Eds.), *The challenge in mathematics and science education: Psychology's response* (pp. 251-274). Washington, DC: American Psychological Association.

Ministry of Education Singapore (2012). Primary Mathematics Teaching and Learning Syllabus, Singapore

Retrieved January 20, from http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2013.pdf

Sinclair, N., & Bruce, C.D. (2014). Spatial reasoning for young learners. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), Proceedings of the Joint Meeting of PME38 and PME-NA 36 (Vol. 1, pp. 173 – 174). Vancouver, Canada: PME.

Voyer, D., Voyer, S., & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: A meta-analysis and consideration of critical variables. *Psychological Bulletin, 117*(2), 250-270.

The Practice of Teacher Aides in Tasmanian Primary Mathematics Classrooms

Robyn Reaburn
University of Tasmania
<Robyn.Reaburn@utas.edu.au>

This paper describes a pilot study investigating Teacher Aides (TAs) in primary mathematics classrooms. Three teacher/TA teams were investigated. The teachers were asked about the role of TAs in their classrooms, and the TAs were asked about their confidence in mathematics and in the desirability of professional learning. The TAs indicated a need to update their knowledge of mathematics content and terminology. It is also suggested that more research to investigate methods of assisting students with learning difficulties is required.

In Australian Schools Teacher Aides (TAs), also known as Teacher Assistants, are employed to assist teachers in classrooms; this assistance may be in the form of support for students with disabilities, support for students with learning difficulties, or support in preparing materials (Forlin, 2010). These TAs may have completed TA training, may be qualified teachers, or may have not received any training. They may be employed to assist a specific student, a group of students, or as general classroom aides. They may work within the classroom or work with students in a separate environment. It has been demonstrated, however, that even if they are employed to assist one particular student, they influence the whole class environment, as they usually interact with students around them (Blatchford, Russell, Bassett, Brown, & Martin, 2007).

Research has shown that whilst the presence of TAs in a classroom is seen as beneficial by teachers, this presence does not necessarily lead to improved academic outcomes for the students in mathematics (Farrell, Alborz, Howes, & Pearson, 2010). There is also research to suggest that their presence may be detrimental to students' mathematical outcomes (Webster et al., 2010). This paper describes a pilot study of TAs and teachers to investigate the role of TAs in Tasmanian primary schools.

Literature Review

In 2011 there were approximately 6900 teachers and 1900 Teacher Aides (TAs) in Tasmania (Garsend, 2011). Although this means that there are approximately two TAs for every seven teachers, previous research in other countries shows that many teachers have no training in working with TAs during their pre-service courses (Webster, et al., 2010). In addition, in Tasmania it is not essential that a TA should have any training in instruction (Department of Education [DoE], 2008). Despite this lack of training for TAs, it is part of their duties to "Prepare teaching aides and other material to support teaching and learning programs including supporting the implementation of individual student education and behaviour programs" (DoE). As evidence from overseas shows that TAs may be employed to help students with learning difficulties in mathematics, the most vulnerable of students may be receiving the least qualified help (Gerber, Finn, Achilles, & Boyd-Zaharias, 2001).

TAs have been found to have a positive effect on the classroom environment. Teachers assisted by a TA report higher job satisfaction, as the support from the TA lowers the teacher's level of stress and workload by relieving them of their administrative duties. The

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 516–523. Sunshine Coast: MERGA.

presence of TAs also reduces off-task behaviour and disruption and allows the teachers more time to teach. TAs can also have a positive impact on the personal and social development of pupils, and can encourage parental involvement in their children's learning (Woolfson & Truswell, 2007).

When TAs are prepared and trained, and have support and guidance from the teacher, TAs can also have a positive effect on the academic progress of the students (Webster et al., 2010). Blatchford, Russell, Bassett, Brown, and Martin (2007) show that well trained TAs can improve the learning outcomes in literacy for students when they are running targeted programs. The results in relation to numeracy, however, are less positive (Farrell, Alborz, Howes, & Pearson, 2010). In fact, there is evidence that shows that TAs may have a negative effect on the academic progress of the students they assist. This result persists even when the results are controlled for level of disability and socio-economic status. Even more disturbing is that the negative effect is more pronounced for the students with more serious problems. Students with TAs may have worse academic outcomes when compared to similar students without a TA (Webster et al, 2010).

It has been posited that one of the reasons for these reduced academic outcomes is that TA-supported pupils become separated from their teachers and the curriculum as a result of spending more time with the TAs (Radford, Blatchford, & Webster, 2011). Another reason for these reduced academic outcomes may be the type of interaction that takes place between the student and TA. It has been demonstrated that whereas "teachers spent more time explaining concepts, provided more feedback, linked the current lesson to students' prior knowledge, and attempted to promote students' thinking and cognitive thinking in a task" (p. 328), TAs are reactive – responding to the needs of the student and lesson at the moment. As a result, they may give confusing and inaccurate explanations. In addition, they found that whereas teachers tend to ask questions in a lesson that encourage students to open up and to offer their opinions, TAs tend to close down discussion. This may be because the TAs believe that teachers place a greater value on written work completion rather than discussion. In addition, because TAs may wish to help their students avoid failure, they often supply the answers without any scaffolding questions. This problem is exacerbated by the TAs' lack of mathematical knowledge.

It is clear from the statement of duties from the DoE in Tasmania that TAs should be working under a teacher's supervision. It is of concern that this may not always be the case. This research was partly prompted by the researcher's experience as a mathematics teacher educator. At the institution where the researcher teaches there are many students, from all over Australia, who are TAs working towards a full teaching qualification. It is clear from posts on the discussion boards that at least some of these students are finding the material they use for mathematics interventions themselves, without the assistance of a teacher.

The literature on the use of TAs in Australia is sparse outside of the area of giving assistance to students with physical disabilities. One aim of this study was to examine the practice of TAs giving assistance in mathematics classrooms in primary schools to find out where more research is needed. The other aims of the study were to examine TAs' confidence in mathematics instruction, to examine TAs' use of questioning, and to determine if there is a need for professional development for TAs in mathematics.

Methodology

This investigation was carried out via a case study. Case studies involve the collection of detailed data on an entity to enable understanding of that entity. Case studies are often carried out via observation and interviews, and allow "an investigation to retain the holistic and meaningful characteristics of real life events" (Burns, 2000, p. 460). They can be useful as pilot studies, as they may "bring to light variables, phenomena, processes and relationships that deserve more intensive investigations" (p.460). It was intended that the close examination of a small number of participants would allow the investigator to collect data that would enable a well-targeted broader study in the future. The investigation was also carried out using a grounded theory approach (Strauss & Corbin, 1998). With this approach, the "researcher begins with an area of study and allows theory to emerge from the data" (Strauss & Corbin, 1998, p. 12).

The participants were three volunteer Teacher/TA teams from three government primary schools. Each teacher took part in a semi-structured interview. Each TA took part in a semi-structured interview, an observation of a lesson, and then a follow up semi-structured interview. All the interviews were carried out on an individual basis. All of these interviews and observations were audio-recorded.

The teachers were asked about the level of TA support they received, the guidance they gave the TAs, the selection of materials and resources, and whether or not they found the presence of TAs beneficial. In addition, they were asked about any training they had received in working with TAs. The TAs were asked about their training and qualifications, the length of time they had been working as a TA, the nature of their work, the level of guidance they received from the teachers, their confidence in working in mathematics, and if they would like training in mathematics instruction.

The transcripts of the interviews and observation were analysed using the process of open coding and grouping (Strauss & Corbin, 1998). As a consequence the results were categorised as follows: *Confidence*, *Resources*, *Questioning of Students*, *Awareness of Student Needs*, *Relationships with Teachers*, and *The Desire for Training*. In addition another category, *Other Issues*, was added to describe themes of interest that did not fit into the other categories.

Results

The levels of experience, the time taken by each TA in mathematics instruction, and their qualifications are summarised in Table 1. All of the TAs and teachers in the study were females.

Confidence

The TAs were asked about their confidence in mathematics instruction. TA1 and TA2 both stated that they were confident in their mathematics instructions, although TA2 stated that she had been less confident when she had been an Aide in Grades 5 and 6 in previous years. TA3, who was new to mathematics instruction, was less confident, stating that she didn't know if she was "doing it right". She had more experience in literacy support and felt more confident in this area. For both literacy and numeracy, however, she stated that she would be "very keen to get some feedback". TA1 was confident in working with the books in the mathematics program used at her school (Go Maths, Origo Education) and was careful to follow the resources provided in this program when required. She felt that her mathematics knowledge had increased by following the books in the program. All of the TAs stated that they did not teach first concepts but helped to reinforce learning of work previously taught.

Table 1
Details of Teachers Aides in the study

	Years of Experience	Qualifications	Time spent on mathematics instruction	Grade	Place of work	Number of students
TA1	16 years	Certificate IV in education	One one-hour session five days per week.	Grade 6	Outside of normal classroom	2
TA2	19 years	Certificate III in business, First Aid	One session per day five days per week.	Preparatory Grade	Within classroom	4-5
TA3	5 years	*	40 minutes to one hour once per week	Grade 1	Outside of normal classroom	8

* Information not collected

Resources

The TAs were asked to identify the resources used and how they accessed them. They identified computer and hands-on learning tools such as dominoes, blocks, unifix blocks, paddle-pop sticks, card games and computer games. TA2 also used workbooks that were part of the *Go Maths* program. All three TAs stated that these resources were chosen by the teacher, although they would occasionally make their own suggestions. For example, TA1 made the suggestion that her students use magazines to cut out pictures and make a shop in the classroom. The level of independence given to the teachers in the use of these resources varied. TA2 was the most highly supervised, and her teacher stated that she modelled all the work for the TA before any resources were used.

Questioning of Students

Each TA worked with a small group of children during a lesson that was observed by one of the investigators. TA2 played a card game with the students that involved the terminology of “tall, short and medium”. In this lesson, cards were used which had pictures of pictures with different colours and different sizes. She gently reminded the children to use these words when they had a tendency to use the words “big” and “little” and then asked them to choose which word they would use.

Student: I got two pencils, the small one is green.

TA2: Small or short?

Student: Short.

TA2: Good!

TA1 conducted one-on-one lessons with two boys that involved fractions and graphing. It was noticeable that this TAs used prompting questions to assist the students to come to the answer.

Student 1 (reading from text book): If it takes ten hours to sail around an island, how long does it take to do it twice?

Pause

TA1: What's twice – how many times do you do something if you do it twice?

Student 1: Two times

TA1: So what's the answer?

Student 1: 20 hours!

TA1: Good, how did you know?

Student 1: 2 times 10 is 20.

Student 2 (working on multiplication tables): 8 times 6 – I can't work it out.

TA1: What is 8 times 3?

Student 2: I can't work it out.

TA1: What is two times 8?

Student 2: 2 times 8 is sixteen.

TA1: Can you add 8?

Student 2: 24.

TA1: Good! What is double 24?

Student 2: 48.

TA1: Good, so 8 times 6 is 48.

Awareness of Student Needs

All the TAs were keenly aware that they were dealing with students who had varying levels of learning ability and to be aware of the students' needs. For example, TA2 stated she kept constant watch for tiredness and made sure that she had a variety of activities planned. One of her students very much disliked changes in routine so she generally kept the structure of each day alike. She felt that it was very important to know the students well as individuals. In addition, she had become aware to allow the students "thinking time" after questions were asked. TA1 was aware that some of the children took longer to understand a concept but she needed to balance that with "slogging over the same thing" each day. TA3 was strongly aware that one of her students was "lower than the rest" and had trouble grasping the concepts.

Relationships with Teachers

All three TAs stated that they were very happy with the relationships they had with the teachers they worked with. All felt they could ask for help, clarification and advice when needed, that they could make suggestions, could talk about lessons they felt did not work well, and had regular meetings with their teachers. These meetings were usually held at the end of the lesson or at the end of the day. The teacher of TA2 stated that she encouraged the TA to "use her initiative" but always wanted to know what had happened. Occasionally she felt the TA had missed the point of the lesson. All TAs felt that they were treated as equals by the teachers and all the teachers spoke very appreciatively of their current TAs. However one teacher indicated that while she had a good working relationship with her present TA she had had problems in the past with TAs who overstepped their role. She was concerned that "some TAs do not realise they are not the teacher, and it is not their job to interfere with a child's behaviour and work".

Desire for Training

All three TAs were keen to have further training in mathematics support. It was interesting to note that TA1 used computers extensively in her instruction, but TA2 felt that she needed training in Interactive White Boards, computers and Ipads. She noticed that the students were confidently using IWBs and felt inadequate because she could not. All TAs stated that mathematics methods had changed since they were at school. TA2 spoke about her time as an Aide in Grade 5 and her problem with subtraction:

> TA2: [My problem was that] subtraction had changed, completely altered. We would take ten from down the bottom, and now it's the other way around. When we first did it I said to the children: That's not right! And they looked at me like I was quite alien.

All were keen to have more training in the language of mathematics because they did not feel confident that they were using the terminology correctly. TA3 stated this forcefully:

> TA3: Any training, any feedback, even if it's little. Any guide as to what I am doing, someone who speaks the same language as the teachers do. I want to give the same message to the children that they get from [their] teachers.

Interestingly, the teacher of TA3 stated that the TAs in the school had all worked in the school "enough to understand the models and language."

Other Issues

The teachers of TA2 and TA3 expressed their concern, quite strongly, that recent cuts in education funding had resulted in the employment of fewer TAs in Tasmania. The teacher of TA3 expressed her concern that the TA did not have time to sit in on mathematics lessons where she was not involved, so she could observe lessons with the whole class. She also stated that there was a high demand for her assistance so that the TAs in her school were "spread very thin." TA2 also spoke of the variety of work she was required to do that might include changing a nappy to helping with instruction.

Discussion

An area of particular interest with this study was the use of questioning and prompts used by the TAs. Radford, Blatchford and Webster (2011) suggest that the questioning methods used, and the tendency for TAs to "do" their work for the students, was actually detrimental to the students' learning. For these three TAs, however, this was not observed. All three avoided giving answers directly to their students and were careful to give prompting questions only. However, it was not clear during the observation that when TA2 asked the student to "double 24" he was aware of how this strategy worked.

The TAs were concerned about their lack of current mathematical knowledge. They were usually, although not always, confident that they knew the content that they were required to teach, but not confident that they were using current terminology and methods. TA1, who had access to the materials used by the whole school, was much more confident in this area as she had learnt from the *Go Maths* books that were being used by the school as a whole. It is apparent that while the teacher of TA2 always modelled the work the students were doing in front of the TA the other teachers expected their TAs to carry out work they were given without such modelling. This research suggests that teachers may need to take more care to give more explicit instruction. This, however, will take more time and it is apparent that, like teachers, TAs may already be working under a tight schedule.

This research has brought up some issues that need consideration by teachers. Farrell et al. (2010) found that teachers believed they benefited from having TAs in their classes, and the three teachers in this study all spoke enthusiastically about the help they received. It was evident, however, that there was a gap between the teachers' confidence in their TAs and the TAs confidence in their own knowledge. Whereas one of the TAs always worked within the classroom, two of the TAs were working outside of the main classroom for extended periods. For these TAs the teachers could not be sure that the TAs were using the appropriate mathematical language or were always accurate in their explanations, and these TAs were acutely aware of this and concerned that they might be making errors.

All of the TAs were aware of their limitations and were keen to have professional development in the area of mathematics instruction. Hurst and Sparrow (2012) demonstrated that TAs whose content knowledge increased after professional development then experienced increased confidence in their teaching. All of the students observed in this research had learning difficulties of some kind, some severe. It is of concern that some of the most vulnerable students are given mathematics instruction by people with little to no training in the area.

It could be argued that these students with learning difficulties should be instructed by teachers with specialised knowledge of teaching such students. This needs to be balanced, however, by the importance of involvement of TAs who are from the same community as the school students. For example, in Hurst and Sparrow's study it was important that there was involvement by the local Aboriginal and Islander Education Officers in the schools. In cases such as these the involvement of community members may well outweigh the benefits of specialist teachers, and the running of professional development to increase the skills and knowledge of community members could be the option.

Conclusion

It was apparent from the interviews with the TAs that they were all keen to give their students the best learning experiences possible. They were very aware that most of the students they were instructing had difficulties in some way. They were also aware that they might be doing something different to the teachers and were concerned that they might be doing something detrimental to their students' learning. In particular, they were concerned about their use of mathematical language.

From this small study it appears that TAs involved in mathematics instruction are keen to have professional development in this area. More research into the content and type of this professional development is required. In addition, more research into the best ways of giving instruction to students with mathematical difficulties is also needed. Should we continue as now, where TAs give instruction with varying levels of supervision, should we put resources into the professional development of TAs, or should all such instruction be given by specialist teachers?

Acknowledgements

I would like to thank Dr Tracey Muir for her assistance in the setting up and running of this project and for her helpful suggestions in the writing of this paper.

References

Blatchford, P., Russell, A., Bassett, P., Brown, P., & Martin, C. (2007). The role and effects of teaching Aides in English primary schools (Years 4 to 6) 2000-2003. Results from the class size and pupil-adult ratios (CSPAR) KS2 project. *British Educational Research Journal, 33*(1), 5-26.

Burns, R. (2000). *Introduction to research methods*. Australia: Longman.

Department of Education [DoE] (2008), *Statement of duties: Teacher aide*, Tasmania, DoE.

Farrell, P., Alborz, A., Howes, A., & Pearson, D. (2010). The impact of teaching Aides on improving pupils' academic achievement in mainstream schools: A review of the literature. *Educational Review, 62*(4), 435-448.

Forlin, C. (2010). The role of the support teacher in Australia. *European Journal of Special Needs Education, 16*(2), 121-131.

Garsend, J. (2011). *ACTU Independent Inquiry into Insecure Work in Australia: AEU Tasmanian Branch Submission*. Tasmania: AEU.

Gerber, D., Finn, J., Achilles, C., & Boyd-Zaharias, J. (2001). Teacher aides and students' academic achievement. *Educational Evaluation and Policy Analysis, 23*(2), 123-143.

Hurst, C., & Sparrow, L. (2012). Professional learning for teaching Aides and its effect on classroom roles. In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), *Mathematics education: Expanding horizons* (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, Singapore, pp. 332-369). Adelaide: MERGA.

Origo Education (nd). *Go Maths*. Details available from http://www.origoeducation.com/gomaths?mageloc=AU

Radford, J., Blatchford, P., & Webster, R. (2011). Opening up and closing down: How teachers and TAs manage turn-taking, topic and repair in mathematics lessons. *Learning and Instruction, 21*(5), 625-635.

Strauss, A., & Corbin, J. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. London: Sage Publications.

Webster, R., Blatchford, P., Bassett, P., Brown, P., Martin, C., & Russell, A. (2010). Double standards and first principles: Framing teaching Aide support for pupils with special educational needs. *European Journal of Special Needs Education, 25*(4), 319-336.

Woolfson, R., & Truswell, E. (2007). Do classroom Aides work? *Educational Research, 47*(1), 63-75.

Qualitative Facets of Prospective Elementary Teachers' Diagnostic Proceeding: Collecting and Interpreting in One-on-one Interviews

Simone Reinhold
Leipzig University
<simone.reinhold@uni-leipzig.de>

The research presented in this paper focuses on the cognitive diagnostic strategies that prospective elementary mathematics teachers (PTs) use in their reflections of one-on-one diagnostic interviews with children in grade one. Thereby, it responds to the detected lack of knowledge regarding qualitative facets of diagnostic proceeding in interview assessments. Results include facets of collecting data and facets of interpreting within a diagnostic micro-process. The discussion takes up the relevance of these findings for teacher education.

The challenges of every-day classroom situations include the design of appropriate learning opportunities, which refers to adaptive teaching competence and includes diagnostic competence (cf. Wang, 1992). To meet these demands, beginners and experienced teachers benefit from a constructivist view of their students' individual progress in developing mathematical concepts. A powerful method to gain particular information on children's mathematical conceptions is provided with diagnostic one-on-one interviews which stem back to the clinical method of interviewing developed by Jean Piaget (cf. Ginsburg, 2009). Standardised task-based interviews enable access to the range and depth of children's thinking as (in-service) teachers actively explore qualitative facets of children's approaches to mathematical tasks. Prepared interview tools and empirically based growth points for the analysis may guide through these one-on-one interviews and thereby foster teachers' professional development (e.g., ENRP task-based assessment interview/CMIT/EMBI; cf. Clarke, 2013; Bobis et al., 2005; Peter-Koop et al., 2007).

Additionally, there is a need to sensitise *prospective* elementary mathematics teachers (PTs) for the variety, range, and depth of young children's mathematical thinking and to qualify them for informal formative assessment. In this sense, preparing, conducting, and analysing students' mathematical conceptions in one-on-one interviews offers substantial learning opportunities and supports the development of PTs' diagnostic attitude (cf. Peter-Koop & Wollring, 2001; Prediger, 2010; Sleep & Boerst, 2012). Yet, qualitative facets of the diagnostic proceeding during a one-on-one interview have only been scarcely studied so far. This includes facets of interpretation and facets of data collection; that is, the question how actions or utterances are taken up before being used for interpretation.

Theoretical Framework

The Concept of Diagnostic Competence and Domains of Teacher Knowledge

Recent studies on diagnostic competence mainly focus on measuring the accuracy of teachers' judgments (cf. Südkamp et al., 2012). With an emphasis on those numerical indicators, diagnostic competence is most often "operationalized as the correlation between a teacher's predicted scores for his or her students and those students' actual scores" (Helmke & Schrader, 1987, p. 94). Here, questions of *qualitative* aspects of diagnostic competence and its acquisition remain unanswered, and *processes of diagnosing* which lead to the evaluation of an individual student's development are not taken into account.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 524–531. Sunshine Coast: MERGA.

Ball et al. (2008) suggest that *pedagogical content knowledge* (PCK) includes knowledge about *common* mathematical conceptions or misconceptions that are frequently encountered in the classroom. Options to achieve this kind of knowledge may arise from analysing *individual* cases, which refers to *knowledge of content and students* (KCS) defined as subdomain of PCK (Ball et al., 2008, p. 403). Thus, the capability of "eliciting and interpreting individual students' thinking" can be found among the set of "high-leverage practices" novices should be familiarised with (cf. Ball et al., 2009; Cummings Hlas & Hlas, 2012). Sleep & Boerst (2012) conceptualise this particular "high-level practice" as subcomponent of the domain "assessing student thinking" (p. 1039). In this sense, analysing an individual's mathematical concept may contribute to a deeper understanding of widespread (mis)conceptions. It may develop KCS, improve a teacher's practices in terms of diagnostic attention, and thereby enrich his or her diagnostic expertise.

Modelling Phases of the Diagnostic Process

In the field of elementary mathematics education research (which intensely deals with *qualitative* aspects of children's wide-ranging learning developments), expertise in this area reaches beyond teachers' accuracy in measuring children's achievements. It additionally includes rather vague aspects like diagnostic sensitivity, curiosity, an interest in children's emerging understanding, and learning or the aptitude to gather and interpret relevant data in non-standardised settings (e.g., Prediger, 2010). Following this process-oriented attitude towards diagnostic competence, activities of formative assessment in a one-on-one interview can be seen as a multidimensional cyclic process (Klug, 2011; Klug et al., 2013). According to this model, a *pre-actional phase* (e.g., considerations of preparing diagnostic activities; choice of tasks or methods) prepares an *actional phase* (including data collection and data interpretation), which is followed by a *post-actional phase*. The latter implies taking the necessary action from data collection/interpretation, which leads to the design or the evaluation of a concept for an individual support in a repeated run through phases of this diagnostic *macro-process*.

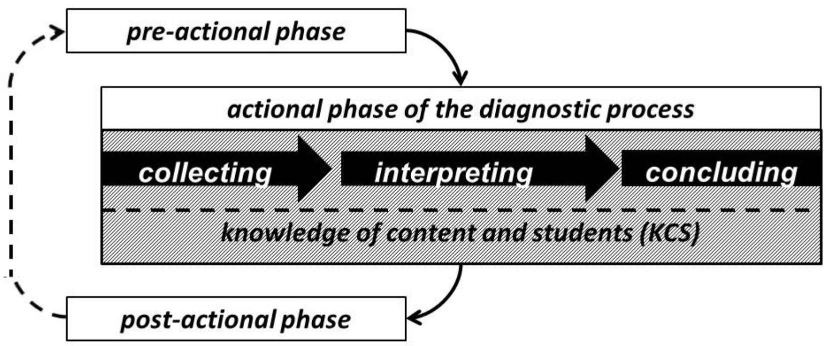


Figure 1. The macro-process of diagnosing and differentiation of the micro-process in the actional phase

Researchers in mathematics education have partially specified the challenges that teachers face within such diagnostic macro-processes. Focusing on *micro-processes within the actional phase of diagnosing*, collecting data, interpreting, and drawing further conclusions have deep impact on the diagnose via an interview and are based on different kinds of knowledge (e.g., KCS, see Figure 1). In this sense, proceedings in a one-on-one diagnostic interview are vitally influenced by cognitive processes and a person's (verbal) articulation (e.g., ways of questioning, confirming) and intentional decisions (e.g., switching between tasks) may reveal facets of these ongoing internal considerations: When conducting a one-on-one interview, there is no direct access to students' conceptions. Instead and in terms of cognitive activity, those conceptions "must be reconstructed by interpreting their utterances" (Prediger, 2010, p. 76). Yet, we have little knowledge on how this *interpretation* takes place or what is taken into account when an interviewer is "gathering information" (Klug et al., 2013, p. 39). This refers to *collecting and interpreting within the actional phase of the diagnostic process*.

Collecting as a Source for Interpretation and Conclusion

Collecting valuable information is obviously of high importance as this information is the source for interpretation and conclusion. Sleep & Boerst (2012) point out that the available information initially relies on the (previous) choice of tasks for the diagnostic situation as tasks "yield sound and useful information about student learning of particular content" (p. 1038). For one-on-one interviews, these tasks are usually chosen in the pre-actional phase, but they obviously influence opportunities for data collection in the actional phase, too. Moyer & Milewicz (2002) identified general questioning categories (check-listing/instructing/ probing and follow-up questions) used by PTs while collecting data in one-on-one interviews. Furthermore, interpreting within any diagnostic situation is also based on a substantial perception of the diagnostic situation. This "includes the ability to structure the situation cognitively, the ability to change the focus of attention and the willingness and ability to adopt other perspectives" (Barth & Henninger, 2012, p. 51). Thus, attention and the capability to focus this attention tend to be crucial prerequisites for collecting within the actional phase. Attending as integral element of "professional noticing of children's mathematical thinking" refers to the skill of "being able to recall the details of children's strategies" (Jacobs et al., 2010, p. 172).

In the actional phase of diagnosing in a one-on-one interview, noticing and collecting includes the motivation to listen and watch, the ability to observe with keen eyes, the capability to detect important details, or the attitude to value particular aspects in children's utterances or actions. Yet, little is known about the *facets of collecting* PTs use in one-on-one interviews they prepare and conduct with children: *How* is all this information *gathered*, *what kind of information* is it and what characterises PTs' interpretation as they *act systematically*?

Research Questions

Aiming at an empirically grounded theoretical framework for a qualitative view on PTs' cognitive activities in one-on-one interviews with children, the main purpose of the project *diagnose:pro* is to detect traits of diagnostic strategies: We intend to find out what cognitive elements characterise the PTs' diagnostic strategies when they diagnose individual arithmetic approaches in one-on-one mathematics interviews with first-graders and try to reconstruct how these strategic elements interact. This paper directs the attention to facets of collecting and interpreting PTs use in their diagnostic proceeding:

- What *kind of information* is collected to supply an interpretation and conclusion during the actional phase of the diagnostic process?
- What *differences in the way this information is collected* can be detected?
- What *facets of interpreting* occur?

- (How) do differences concerning the choice of collected information, concerning the way of collecting or facets of interpreting *influence the type of diagnostic strategies* that can be reconstructed from retrospective interviews?

Methods

In the sense of theoretical sampling (Corbin & Strauss, 2008), data collection was intended to capture the range of PTs' practices and proceedings and focused on re-interviews of one-on-one diagnostic interviews. All PTs attended mathematics methods courses in the last year of their university studies (Master of Education). In cooperation with an elementary school, these courses provided the opportunity to prepare, conduct, and analyse individual diagnostic interviews with up to 6 first-graders per PT. Drafts for these interviews were prepared at the beginning of the course where the PTs could make use of theoretical work on concepts of arithmetic learning trajectories and the method of task-based mathematics interviews (e.g. EMBI; Peter-Koop et al., 2007). Until Autumn 2013, 7 PTs from these courses agreed to take part in retrospective interviews that focused on the video-recording of an interview they had conducted shortly before.

With a deliberately general advice at the beginning of the retrospective interviews, the PTs were asked to *analyse the interview* while watching the video-recording. The interviewee was requested to stop the video at any scene in order to comment on the diagnosis he or she would derive from this specific situation. If comments were rather short or pure in detail, the interviewee was asked to explain what knowledge, information, or evidence warranted his or her uttered hypothesis. In addition to this concrete task (diagnosis of the child's conception or knowledge), the PT reflected on his or her proceeding in a more general way. Referring to the preliminary design of the interview, the PTs were asked to comment on the choice of some selected tasks, on the wording of questions, on their own gestures, or on deviations from the sketch: What prompted them to react to a child's response? What was taken into account to confirm a diagnosis? These retrospective analyses of diagnostic interviews offered the chance to narrow the focus and to pay attention to details. In this sense, PTs' data collection and interpretation obviously differed from real-time practice in an interview that requires being concurrently aware of many more details.

The analysis of all interviews was based on Grounded Theory methodology; therefore, codes were derived from data via open, axial, and selective coding or contrasting comparison of the data. Use of the software ATLAS.ti enabled video-data to be coded directly. To approach the aim of capturing identified characteristics of diagnostic proceeding in whole range ("saturated", Corbin & Strauss, 2008, p. 143), we also include data which consists of written comments of 31 PTs (collected in 2011) and video/audiotaped peer-talks among 28 PTs about video-scenes of diagnostic interviews (collected in 2012).

Findings

Analyses of the study's data supported the notion that cognitive elements of PTs' ways of diagnostic proceeding in one-on-one interviews often resemble processes in qualitative data analysis. This includes acts like *collecting*, *interpreting,* and *concluding* within diagnostic micro-processes (see Figure 1). The findings also contribute to the identification of sub-categories of collecting, interpreting, or concluding and to interrelations among these sub-categories that hint at distinct types of diagnostic strategies.

Facets of Interpreting: Comparing, Contrasting, Coding

Excerpts from re-interviews with Ann and Sue, Masters students in their last year of studies, display exemplary *facets of interpreting within the diagnostic micro-process of the actional phase.*

In her interview with six-year old Tom, Ann offers empty boxes for ten eggs and some chestnuts. The boxes of ten are partitioned in four fields (see Figure. 2, cf. Besuden, 2003) since Ann intends to find out how children use these structures for counting or for abbreviated enumeration (i.e., counting strategies including subitising parts of an amount).

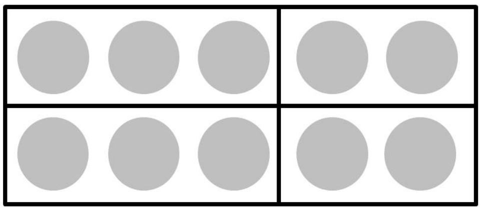


Figure 1. Structured box used in one-on-one interviews Ann and Sue conducted with first-graders

During the re-interview, Ann stops the video and comments on a scene where she has just put five chestnuts into the box (forming a row). Tom is asked to add further chestnuts in order to get a result of eight and fills two, then one more into the box. Answering Ann, he remarks, "Because I left two free, one more'd be nine, then ten."

> Ann (07:08): And there I noticed that he, eh, always took ten as a starting point for the higher numbers, well, for eight and a moment ago for nine. He remembers, okay there are ten in the package, and then he always counts backwards.

In her comment, Ann compares and refers to Tom's previous work ("a moment ago"). *Comparing* details to a child's previous utterances or actions, to that of others or to the PTs own concept may also occur in terms of *contrasting* different scenarios:

> Ann (08:30): Here, he saw, okay, there are four in one box and there are another four in the second box, well, four plus four equals eight, but he didn't do it that way in the next task. There he'd count single ones, it was done quite *differently*.

Sue uses the same kind of tasks in her interview with six-year old Ben. She wants him to find out how many chestnuts have to be added to four chestnuts (which are presented in the "square" on the right side of the box) to get a result of seven. Ben replies by first adding two (forming a "rectangle"), then one more to reach seven (Ben: "These are six, then seven."). Sue codes these actions by creating the new term "auxiliary calculation":

> Sue (05:40): "Responding to my enquiry, how he'd done this, now, how many he'd add, actually, I only wanted to hear 'three', well, he would seize on his, let's say *auxiliary calculation*, six plus one equals seven."

PTs are similarly *coding* observed phenomena as they try to grasp unfamiliar, but obviously central aspects of a child's conception. Codes are often referred to later in the interviews (e.g., Sue's reference to the code "auxiliary calculation", 22:30) and may also substitute established terms (e.g., "shortcut" instead of subitising).

Facets of Collecting: From Observing to Tracking, Recognizing or Sorting

Findings of the study also reveal that collecting information within the actional phase of a diagnostic micro-process may vary concerning the type of collecting and concerning the choice of information, as the following excerpts display. In *our* analyses of the PTs' process-oriented analyses we took into account that facets of data collection may include observations which are not mentioned by the PTs. Subconsciously grasped information (e.g., on a child's hidden insecurity, fear to fail when working on the given task, or motivation while working on a task) could also have an influence on a conclusion which is drawn later on. In this sense, we are restricted to focus on the mentioned items. Besides, there is no way to tell data collection in the interview from data collection that can *definitely* be assigned to the re-interview.

PTs' data collection was coded as *observing* when we considered the PTs to watch closely what was happening in the diagnostic situation. All PTs did listen attentively to the child's utterances. They paid attention to significant details, but they most often (also) noticed the (singular) occurrence of micro-incidents that were only loosely connected. In this sense, data collection included various details (see list in table 1) and often ended up in collections that resembled a "colorful bunch of flowers".

On a higher level, facets of collecting coded as *tracking* refer to the skill of following a series of activities or utterances. This includes to follow a child's action over a longer sequence and to maintain attention during the diagnostic situation. This can be seen in the following protocol of Lisa's re-interview on an interview with 6-year old Sam. Sam is asked to take five chips (one side blue, the other side red) and comment on possible ways of displaying an addition with these manipulatives. Sam starts with spreading the chips over the table and starts to sort them, "Three red ones and two blue ones", as Lisa stops the video:

Lisa (01:51):	To comment on this, I'd say he separated red and blue from the beginning and named what was lying on the table.

Later on, Lisa tracks this idea and collects further information from subsequent situations that refer to this issue (sorting and considering position of colors).

Lisa (02:16):	Here, it is clear that he separated the colours from the beginning."
Lisa (10:20):	We wanted them to find that *sorting* the possible additions helps to find all of them, yes and he is arranging them in *any* kind of structure, but … not the one we had intended them to find … But in a way he *does* sort the possible arrangements because in this corner here, the blue ones are closer together. In the next row, the blue ones stick closely together, too, and there the red ones."

PTs' data collection was coded as *recognising* when they repeatedly identified details they had already noticed in previous situations. In contrast to *tracking*, this was restricted to single incidents. *Sorting* in PTs' data collection was identified when they found or intentionally searched for groups or patterns in children's utterances or actions. A further analysis of PTs' comments also reveals a wide range of mentioned details (see examples in Table 1).

Table 1
Various sources for interpretation: What is collected?

Collected	Example
Verbal utterance	"This boy, he was able to identify the summands and he said, 'This number and this number equals this number.'" (Anne)
Activity	"He's drawing a circle around *this* piece of the pattern." (Pam)
(In)correctness of solution	"He was supposed to draw a circle around repeating parts of the pattern, but he failed." (Pam)
(Elements of) strategy	"He used counting strategies, saw 4 and continued counting from that first summand." (Sue)
Eye movement	"He hesitated and looked the other way." (Anne)
(Subtle) movements of lips, head or hands	"I see he is nodding and I guess he's counting up to five here." (Lisa)
Emotional state	"I got the impression he'd start crying." (Anne)
Interviewer's behaviour	"Okay, I liked what I did in this situation as we decided to accept 'wrong' answers, too." (Sue)

Discussion

The study responds to the detected lack of knowledge regarding qualitative facets of diagnostic proceeding in one-on-one interviews and thereby contributes to strengthen the "power of task-based one-on-one interviews" (Clarke, 2013) in daily practice. Even if the reported findings are restricted to a certain type of tasks (arithmetic issues) and that they refer to a rather small number of participants (n=28 in peer-talks; n=7 re-interviews), the study takes *a look behind the s*cenes"of PTs' diagnosing in one-on-one interviews.

PTs' attention was most often attracted by children's obvious or prominent activities or utterances. Items were also collected if the PTs found surprising deviations from what they had expected before. Furthermore, other incidents obviously exactly matched what they had expected. This emphasises the importance of KCS (e.g., knowledge of common (mis)conceptions) as both deviation and alignment can only be stated if there is knowledge which can be used for this comparison. Additionally, this underlines the close relationship between collecting data and reasoning about the collected details (*interpreting* and *concluding*). Yet, this relationship does not necessarily appear as a linear process in PTs' diagnostic proceeding. Instead, PTs may run through these intertwined micro-processes in circles: a type of diagnostic strategy we call a *branched interpretation.* At the same time, we detect other diagnostic strategies, namely the strategy *descriptive collector*, when the PTs focus on collecting and describing the child's actions and neglect both interpreting and concluding.

This reveals *hidden* diagnostic practices that have to be uncovered in order to make them explicit. They are assumed to be of great importance for teacher education. Hence, further investigations in the project *diagnose:pro* will explore, for example, how elements of diagnostic strategies and types of strategies can be taken up in discussions of university courses. This includes making explicit what problems may occur when the strategy *descriptive collector* is predominant. Prospective research in this field will have to examine if awareness of (elements of) diagnostic strategies and types of diagnostic strategies (including awareness of *strategic diagnostic tools*) may contribute to appropriate interpretations of children's utterances in interviews. This might help to identify a theoretical and practically relevant framework for *high-leverage diagnostic practices* (including various facets of collecting and interpreting) to cope with diagnostic challenges in the classroom.

References

Ball, D. L., Thames, L., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Ball, D. L., Sleep, L., Boerst, T., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal*, *109*(5), 458-474.

Barth, Ch., & Henninger, M. (2012). Fostering the diagnostic competence of teachers with multimedia training–A promising approach? In I. Deliyannis (Ed.), *Interactive Multimedia* (pp. 49-66). Rijeka, Croatia: InTech.

Besuden, H. (2003). Rechnen mit Eierkartons. *Grundschule*, *35*(5), 51-54.

Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R. B., Young-Loveridge, J., & Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking. *Mathematics Education Research Journal*, *16*(3), 27-57.

Clarke, D. (2013). Understanding, assessing and developing children's mathematical thinking: task-based interviews as powerful tools for teacher professional learning. In A. M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-30). Kiel: PME.

Corbin, J., & Strauss, A. L. (2008). *Basics of qualitative research: Grounded theory procedures and techniques* (3rd Ed.). Thousand Oaks, CA: Sage Publications.

Cummings Hlas, A., & Hlas, Ch. S. (2012). A review of high-leverage teaching practices: Making connections between mathematics and foreign languages. *Foreign Languages Annals*, *45*(s1), s76-s97.

Ginsburg, H. P. (2009). The challenge of formative assessment in mathematics education: children's minds, teachers' minds. *Human Development*, *52*, 109-128.

Helmke, A., & Schrader, F.-W. (1987). Interactional effects of instructional ability and teacher judgement accuracy on achievement. *Teaching and Teacher Education*, *3*(2), 91-98.

Jacobs, V. R, Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169-202.

Klug, J. (2011). *Modelling and training a new concept of teachers' diagnostic competence*. Darmstadt: TU Darmstadt.

Klug, J., Bruder, S., Kelava, A., Spiel, Ch., & Schmitz, B. (2013). Diagnostic competence of teachers: A process model that accounts for diagnosing learning behavior tested by means of a case scenario. *Teaching and Teacher Education*, *30*, 38-46.

Moyer, P. S., & Milewicz, E. (2002). Learning to question: categories of questioning used by preservice teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education*, *5*, 293-315.

Peter-Koop, A., & Wollring, B. (2001). Student teacher participation in interpretative classroom research. *Mathematics Teacher Education and Development*, *3*, 4-15.

Peter-Koop, A., Wollring, B., Spindeler, B., & Grüßing, M. (2007). *Elementar Mathematisches BasisInterview*. Offenburg: Mildenberger.

Prediger, S. (2010). How to develop mathematics for teaching and for understanding. *Journal of Mathematics Teacher Education*, *13*(1), 73-92.

Sleep, L., & Boerst, T. A. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teacher and Teacher Education*, *28*, 1038-1048.

Südkamp, A., Kaiser, J., & Möller, J. (2012). Accuracy of teachers' judgements of students' academic achievement: A meta-analysis. *Journal of Educational Psychology*, *104*, 743-752.

Wang, M. C. (1992). *Adaptive teaching strategies: Building on diversity*. Baltimore: Brookes.

Describing the nature and effect of teacher interactions with students during seat work on challenging tasks

Anne Roche
Australian Catholic University
<anne.roche@acu.edu.au>

Doug Clarke
Australian Catholic University
<doug.clarke@acu.edu.au>

As part of a project that is examining how to support teachers in the use of challenging tasks and those teacher actions that encourage students to persist, we focused on the activities of students and teachers during seatwork. We describe the nature of teacher interactions with students, student behaviours when working on challenging tasks, and the relationships between the two. Interactions that seemed most beneficial were brief, and usually preceded by the teacher watching and listening to the students at work.

Background

The Encouraging Persistence Maintaining Challenge (EPMC) project has been investigating the ways teachers can be supported to use challenging tasks in mathematics and what teacher behaviours might encourage students to persist (Sullivan et al., 2011). We use the term *persistence* to describe student actions that include students concentrating, applying themselves, believing that success is possible, and making an effort. We describe tasks as *challenging* in that they allow for the possibility of sustained thinking, decision making, and risk taking.

Three elements considered key to helping students engage with, persist at, and learn from challenging tasks are "the ways in which the tasks are posed, the interactive support for students when engaged in the tasks and collaborative reviews of the class explorations" (Sullivan et al., 2013, p. 1). The research team has previously reported on a proposed structure of the lesson (Sullivan et al., 2014), ways of introducing challenging tasks (Cheeseman, Clarke, Roche, & Walker, under review), and the effective use of the summary phase (Walker, 2014).

The three key elements mentioned tend to occur in one of the three phases of the lesson: Launch-Explore-Summarise (Lampert, 2001). In this paper, we examine one aspect of these key elements or lesson stages: the explore phase. Japanese teachers use the term *kikan-shido* to mean *between desk instruction*, describing that phase in the lesson when students participate in seatwork, sometimes individually or in groups, while the teacher roams around the classroom, providing support and interacting with students as necessary. The activities and function of these interactions have been documented across several countries in the secondary context (O'Keefe, Xu, Li Hua, & Clarke, 2006). O'Keefe et al. (2006) developed a list of teacher activities during kikan-shido that were common across 12 countries in 8th grade classrooms. The four principal functions for these activities were: (1) monitoring student activity; (2) guiding student activity; (3) organisation of on-task activity; and (4) social talk. For a detailed description of each function and the related activities, refer to O'Keefe, et al. (2006).

Stein, Grover, and Henningsen (1996) examined the extent to which the implementation of a task remained consistent with how it was set up and the factors that appeared to be associated with the decline of task demand, particularly when the task had high cognitive demand. Some of the reasons for the demand declining were teachers over-explaining the task, students failing to engage with the task, and teachers providing too

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 532–539. Sunshine Coast: MERGA.

little time for students to explore and think about the task.

As students work in small groups to solve problems, Yackel, Cobb, Wood, Wheatley, and Merkel (1990) asserted the importance of social interactions (between teacher and student and among students) that provide opportunities for students to explain their thinking and to understand one another's thinking.

The research questions that guided this aspect of the project were:

1. What is the nature of teacher interactions with students as they are working in pairs on challenging tasks?
2. Which teacher interactions seem to be the most productive for student work?

The Project Context and Data Collection and Analysis

In 2014, 47 teachers from Years 3 and 4 at 13 Victorian primary schools began their involvement in the project. The data reported in this paper were collected from two Year 3 classrooms in an independent girls' school. Each class consisted of 16 students. The teachers in these classrooms were each videotaped teaching three of the ten lessons provided by the EPMC project during a professional learning day. The content for the ten lessons was addition and subtraction, with an emphasis on mental strategies. As well as a single camera on a large tripod set up to capture the teacher's movement and words, four small cameras were placed on tables to capture pairs of students as they attempted to solve tasks. The five cameras enabled us to film the teacher interactions with eight students in each classroom. The students completed a pre- and post- online test on similar content to the lessons. The teachers were interviewed after each lesson about their perceptions of the students' engagement and learning, and these interviews were transcribed. Work samples were collected from all students in every lesson.

Each lesson consisted of a main task, possible prompts, and a consolidating task. An important feature of the lesson documentation was the inclusion of *enabling prompts* for students who have difficulty making a start on the main task and *extending prompts* for students who finish quickly. The intention was that the student who succeeds on the enabling prompt(s) could then proceed with the original task (see Sullivan, 2011). During the professional learning day, the teachers were introduced to the idea that a lesson may have three phases: Launch, Explore, and Summary phases. The Explore phase was suggested as the time when the teacher would roam around, observe students, and ask them to explain their strategies. During this time, the teachers were encouraged not to tell students how to solve the problem, but rather to provide enabling or extending prompts as required, to select students for sharing at the summary phase, and to allow students time to struggle with the task and not to intervene too quickly. One helpful idea we have used throughout the many iterations of this project is the *zone of confusion*. Teachers were encouraged to discuss with their students the notion that for genuine learning to occur, it is likely that at some stage they will be in this zone of confusion. Teachers reported that students responded very well to this notion.

For brevity, only one of the three lessons (for each teacher that was observed) will be discussed. This lesson was called *Finding ways to add in your head* and the main task was: Work out how to add 298 + 35 in your head. What advice would you give someone on how to work out answers to questions like this in your head? The enabling prompts were:

- Work out the answer to 28 + 7 in your head.
- Work out the answer to 98 + 7 in your head.
- Work out the answer to 198 + 7 in your head.

The extending prompts were:

- Work out how to add 98 + 97 + 67 in your head.
- Work out how to add 295 + 96 + 79 in your head.

In relation to the main task (298 + 35), Fuson et al. (1997) provided a very detailed analysis of students' methods in multi-digit addition and subtraction calculations, grouping them into two primary classes (*decompose tens and ones*, and *begin with one number* methods), as well as a third category of mixed strategies. In discussions of the task at the professional learning day, we anticipated the following strategies, and used the names given in parentheses:

- (Change both numbers) (298 + 2) + (35-2) = 300 +33 = 333
- (Overshoot) 300 + 35 – 2 = 333
- (Jump) 298 + 10 + 20 + 3 = 333
- (Split) 200 + (90 + 30) + (8 + 5) = 333
- (Other partitioning) e.g., 290 + 35 + 8 = 333

Interestingly, using Fuson et al.'s categories, the fourth strategy involves decomposing, the third involves beginning with one number, and the other three are mixed strategies.

The consolidating task consisted of a worksheet of four additions (each a 3-digit plus a 2-digit number), with the request to show in writing how they worked it out. No student was given the consolidating task in the lessons we observed.

All conversations in which the teacher participated during kikan-shido were transcribed, and two coders independently classified the teachers' actions, using the 16 categories of O'Keefe et al. (2006). Where the coders disagreed, discussion eventually yielded agreement. The videos of the pairs of students were observed and three that demonstrated a range of success on the task were transcribed for further analysis.

Results

We now describe three aspects of the data: the teacher activities and their frequency during kikan-shido; descriptions of some students' strategies and behaviours during seat work; and pre- and post-test results on an item of similar content to that of the lesson.

Teacher Activities During Kikan-shido

Drawing upon O'Keefe's four principal functions during kikan-shido and their related teacher activities, Table 1 shows the frequency of these activities in each of the two teachers' lessons (lessons A and B) and the time spent on kikan-shido and the proportion of the lesson spent on kikan-shido.

Not surprisingly, there are similarities and differences in the number of occurrences of each activity between the two lessons. Given the lessons were being recorded for the purposes of the project and were at Year 3 level (not Year 8), we were not surprised that there was no time spent monitoring homework completion or arranging the room. In neither lesson did the teacher need to *re-direct a student* who was perceived to be not paying attention. In both lessons, all students appeared to maintain engagement with the task. It was interesting to note that in these lessons neither teacher chose to *Give advice at the board* during kikan-shido. This was the case in all of the six lessons we observed.

Table 1
The Frequency of Kikan-shido Activities Across Two Lessons

		Lesson A 20 mins (40%)	Lesson B 17 mins (30%)
Monitoring	Selecting work for sharing	2	0
	Monitoring progress	15	7
	Questioning student	10	2
	Monitoring homework completion	0	0
Guiding	Encouraging student	12	7
	Giving instruction/advice at desk	15	9
	Guiding through questioning	2	4
	Re-directing student	0	0
	Answering a question	12	5
	Giving advice at board	0	0
	Guiding whole class	0	4
Organisatio-nal	Handout materials	0	0
	Collect materials	2	0
	Arranging room	0	0
	School related	1	0
Social talk	Non-school related	3	1

In lessons A and B, the teachers made 42 and 35 visits, respectively, to pairs of students at tables and on 10 and 5 occasions, respectively, the teachers looked and listened to students working on the task and left without speaking to them (one type of *monitoring progress*).

For our analysis, we chose to code the teachers' action of providing an enabling or extending prompt to a student or pair of students as *Giving instruction/advice at desk*. In both lessons, all students received one or other of the prompts during kikan-shido.

It is clear that while the teachers spent similar amounts of total time on kikan-shido, Lesson A had a much greater frequency of activities generally, and of *monitoring progress*, *questioning students*, and *answering questions*, in particular.

Student Behaviours During Seatwork

The students were sent to their seats to write their solution strategies for 298+35. Prior to this, they had had quiet, individual time on the floor to come to a solution without pencil and paper. At the request of the researchers, each pair of students was given one A3 page with the main task, so that they might share their strategies aloud. The eight students in each class filmed during seatwork were spread across the room with the intention of varying which students were observed over the three lessons that were videotaped.

We now provide some examples of student strategies and teacher interventions. Due to space constraints, only three pairs of students will be discussed. In each case, the student behaviours prior to a teacher's interaction (and its code), including the teacher action of providing a prompt (coded as *giving instruction*), and the subsequent student actions as a result, are described. We now describe the three events, and then reflect on them.

Event 1. Molly and Gene wrote two methods for solving 298+35 after first checking the correctness of their mental solution by using the conventional vertical written algorithm (described as *algorithm* from now on). Molly used a *jump* strategy and wrote: You could do 298+30 which equals 328, then you add 5 which equals 333. Gene wrote: You could work systematically so 298+10=308; 308+20=328; 328+2=330; 330+3=333.

At 9 minutes into seatwork, the teacher asked them to describe their solutions and then left them to think about whether there was a more efficient way. This interaction was coded as *questioning student* and lasted 70 seconds. At 13 minutes, Gene (using the strategy of *changing both numbers*) said, "You could go plus 2 is 300. Let's do it an easy way. Take the five apart into 2 and 3". She wrote: Take the five apart into two and three and then go 298+2 = 300, then add 30 equals 330, then 330 + 3 = 333.

At 16 minutes, the teacher approached and asked them to explain their most efficient strategy and then gave them the extending prompts. This interaction was coded as *questioning student* and *giving instruction*. Molly read the first one aloud ("Work out how to add 98+97+67 in your head"). They thought silently for 89 seconds. Gene said, "I haven't got the answer but if you have, what is it?" Molly answered, "257". Gene used the algorithm to check and got 262. Seatwork ended at this point.

Event 2. Sue and Nell began by discussing possible strategies for adding 298 and 35. Nell explained her strategy (*other partitioning*) and wrote: First I had 298 and then I took away the 8 and added the 35 from the number. I added 8 and got 333. Sue was unable to come up with any solution strategy. She suggested, "Counting on the ones in your head and then adding the tens." She also indicated the possibility of using an empty number line, but Nell wondered how this would be possible in your head. Sue suggested that the algorithm would also be hard in your head and that counting-on by ones "would take ages."

At 9 minutes, the teacher approached and asked them if they were ready for something "tricky". The girls enthusiastically said, "Yes." The teacher gave each girl a copy of the extending prompts, without first reading their solutions or asking them to share what they had written on the main task. Both girls were visibly perplexed by the new tasks. Sue said, "Okay, this is a bit harder than I thought it would be. … I'm in the zone of confusion." Nell said:

> How do you work this out? … I can imagine inside my head there's a big box and there's no doors and I'm trying to find my way out … I can just see it. Me in a box and I'm trapped … it's like, help.

At around 13 minutes, Sue showed her paper to the camera, which demonstrated she had written two incorrect answers. At 16 minutes, the teacher asked them to "tell me what you did for 98+97+67." Nell responded by describing a strategy that adds the numbers left to right in this way: "98+2=100; that leaves 95; 95+5=100; that leaves 62." At this point (*before* Nell added the 100+100+62 that she had created), the teacher asked Nell to describe the steps again. During Nell's responses, the teacher asked eight clarifying questions, gave one piece of advice and made four affirming statements. This was coded as *guiding through questioning*. In the second iteration of Nell's explanation, she came to the conclusion that 64 remained on the last step, hence leading to an incorrect solution. The teacher noticed this wasn't the same as Nell's first response and suggested she try again. This interaction lasted 2 minutes. Seatwork ended about one minute later.

Event 3. Zita and Sandy demonstrated solving the main task using the written algorithm and seemed unable to move beyond this solution strategy. Zita wrote the algorithm and explained the steps as: "8+5=13; so you put the 3 there, put the 1 there;

10+3 is 13; 2+1 is 3.” However they both agreed this wasn’t an easy method “to explain how you did it in your head.” At 9 minutes, the teachers asked them to explain their strategy (*questioning student*). The teacher commented that they were using an algorithm and left them to think about how they might do it in their head. After 16 minutes, without advancing any further in their thinking, the teacher intervened and provided the enabling prompts (28+7; 98+7; 198+7) (*giving instruction*). Both girls discussed their thinking and agreed on an appropriate mental strategy (*overshoot*; 28+10=28; 28-3=35). For 3 minutes, Zita wrote (while Sandy waited): I first turn the seven into a ten. Now you know that 28+10 =38, Now you know that 7 is 3 away than 10 so now it’s 38 take away 3 and that is 35. She repeated this method for the next two prompts. Seatwork ended here.

Reflection 1. Gene and Molly seemed to benefit from the teacher not providing any additional explanation on how to solve the task other than “there may be a more efficient way.” This appeared to inspire the students to consider more options, leading to their discovery of one of the most efficient strategies for this addition. From the lengthy silence, it was clear that the extending prompt was challenging for the students. Neither student had trouble putting their solution strategies in writing as well as engaging with the task with minimal teacher support. However, seat work ended before they completed the extending prompts.

Reflection 2. The two students were originally clear about what it meant to solve something *in their head*, as opposed to a written method, though only Nell described a mental method to actually solve 298+35. Providing the extending prompts for both students caused a challenge as noted by their comments, and they clearly were *in the zone*. However, Sue’s lack of progress on the main task meant the extending prompt was likely to be too great a challenge and this proved correct. It may be that the enabling prompt would have been more helpful for Sue to make progress. We also noticed that the teacher’s desire for clarification by asking many questions interrupted Nell’s thinking and made it hard for her to keep the steps of her solution in her head. It may be the teacher was struggling with making sense of Nell’s strategy on the run.

Reflection 3. Zita and Sandy struggled to move beyond the written algorithm on the main task, but the provision of the enabling prompt seemed to provide just the right challenge so that Zita could access a successful mental strategy. She then proceeded to use it for all three prompts. Sandy did not solve the main task, and was left waiting as Zita solved and recorded the enabling prompt. Seatwork ended before they could go back to the main task. It may be that sharing the worksheet as a pair (as per the authors’ request to the teacher) may have contributed to some students waiting for their turn to write a solution.

Student Pre and Post Test Results

The students were pre- and post-tested using an online assessment that included ten mathematics items and some survey items. The item most closely connected to the lesson *Finding Ways to Add in Your Head*, was: What is 5 + 5 + 5 + 295 + 295 +295? All six students discussed earlier were incorrect in the pre-test on this item. All but two (Sandy and Sue) were correct in the post-test. In the two classes, overall, four of the 32 students were correct on the pre-test (12.5%), increasing to 16 on the post-test (50%). This compared to an increase across all Year 3s in the project from 22.2% (n=752) to 47.9% (n=624).

Discussion

We have only reported three events of teacher and student activity during seatwork, but they support some general points and challenges that have emerged from these and other lessons we and our colleagues observed and analysed in the broader study.

In both lessons, the teachers did a number of things well. They held back from telling students how to solve the problems, selected students for sharing, and allowed students to struggle. The introductions (though not discussed here) were engaging and provided motivation for students to engage with the task, and for most students the task demand was maintained. In both classrooms, the students were familiar with the term *zone of confusion* and they understood when they were in it. In most cases, the teacher interventions were brief, and sometimes the intervention involved only watching and listening. Both teachers commented in a pre-interview that their students had had not much experience writing down their thinking and that they anticipated it might be "a challenge." However, we noticed that most students did this well. It was clear in both classrooms that the students were encouraged to share their thinking and listen respectfully. As one teacher said to the whole class, "When people were talking to each other they were looking at each other in their eyes, and they were really explaining to each other. Who thinks they learnt something from their partner?"

During the project's professional learning day the project team had encouraged teachers to allow students time to work on the task, first by themselves, then in pairs or groups. We noticed that even in pair work, during genuine struggle the students chose to think quietly by themselves. That is, without being prompted by the teacher, the students naturally took that silent time to think through the task by themselves first.

We also noticed some challenges for the teachers and students. One challenge was how to help students move beyond the written algorithm in attempting to solve tasks like these. While most students seemed to have no trouble differentiating between a solution strategy obtained mentally and a written method, some students initially struggled with deriving a mental strategy. It seemed in one instance at least, that providing a task with smaller numbers (the enabling prompt) was enough to help students make this transition. While the use of enabling prompts assisted student thinking in the lessons described, seatwork ended before there was time to revisit the main task. On some occasions we noticed that the decision to give an extending prompt without first checking students' success or understanding of the main task seemed unjustified and unhelpful for the students' progress.

Sometimes we noticed that making sense of a student's strategy and attending to the mathematics in what they were saying was difficult. Successful improvisation (Borko & Livingston, 1989) is more likely when the teacher has taught the content before and can anticipate students' responses more easily. This lesson and its structure were new for these teachers. We noticed sometimes that extended teacher questioning (coded as *guiding through questioning*) interrupted the student's flow of thinking, and added unnecessarily to their working memory. It seemed that the most productive teacher interactions were short, well-timed interventions and preceded by respectful watching and listening (Fennema, Carpenter & Peterson, 1989).

We were very encouraged by the two classes' post-test results on an item of similar content. We noted that some students who seemed to struggle but had some success (even if the success was not on the main task, but on the easier enabling prompt) were also successful on the post-test item. We cannot be sure of course that the learning that contributed to such improvement on this item only occurred as a result of this lesson.

In summary, we have added to the body of knowledge on kikan-shido, with our focus on Australian *primary* mathematics classrooms. However, a number of questions still remain. The desirable amount of time allocated to seatwork, the recommended proportion of students who receive prompts, and the appropriate balance between individual and pair work are all areas worthy of further research.

References

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematics instruction by expert and novice teachers. *American Education Research Journal, 26*(4), 473-498.

Cheeseman, J., Clarke, D., Roche, A., & Walker, N. (under review). Introducing challenging tasks: Inviting and clarifying without explaining and demonstrating. *Australian Primary Mathematics Classroom.*

Fennema, E., Carpenter, T. P., & Peterson, P. L. (1989). Learning mathematics with understanding: Cognitively guided instruction. In Brophy (Ed.), *Advances in research on teaching* (pp. 147-164). New York: Macmillan.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., & Olivier, A. I., et al. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28,* 130-162.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven, CT: Yale University Press.

O'Keefe, C., Xu, Li Hua, & Clarke, D .J. (2006). Kikan-shido: Between desks instruction. In D. J. Clarke, J. Emanuelsson, E. Jablonka, and I. A. H. Mok (Eds.). *Making connections: Comparing mathematics classrooms around the world* (pp. 73-106). Rotterdam: Sense Publishers.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 32*(2), 455-488.

Sullivan, P. (2011). Teaching mathematics: Using research-informed strategies. *Australian Education Review 59*. Camberwell, Victoria: Australian Council for Educational Research.

Sullivan, P., Cheeseman, J., Michels, D., Mornane, A., Clarke, D., Roche, A., & Middleton, J. (2011). Challenging mathematics tasks: What they are and how to use them. In L. Bragg (Ed.), *Maths is multi-dimensional* (Proceedings of the 48th Annual Conference of the Mathematical Association of Victoria, pp. 33-46). Melbourne: MAV.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education, April,* DOI 10.1007/s10857-014-9279-2.

Sullivan, P, Aulert, A., Lehmann, A., Hislop, B., Shepherd, O. & Stubbs, A. (2013) Classroom culture, challenging mathematical tasks and student persistence. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Research Group of Australasia, pp. 618-625). Melbourne: MERGA.

Walker, N. (2014). Improving the effectiveness of the whole class discussion in the summary phase of mathematics lessons. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 629-636). Sydney: MERGA.

Yackel, E., Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s.* Reston, VA: National Council of Teachers of Mathematics.

Teachers' talk about Robotics: Where is the Mathematics?

Annie Savard
McGill University
<annie.savard@mcgill.ca>

Kate Highfield
Macquarie University
< kate.highfield@mq.edu.au >

Programming and the use of robotics present affordances for mathematics learning with application across a broad range of ages. However, realising these affordances in the classroom requires educators to recognise and build apron these potential opportunities for learning. This paper reports one component of a larger study, examining teacher discourse in semi-structured focus group as they review engagement with robotics. Data highlights limited engagement in mathematisation and the key role of mathematical pedagogic content knowledge (PCK).

Background

Robotics in Mathematics Learning

The use of robotics and programming has a long-standing history in mathematics education with tools such as 'turtle' geometry or Logo explored in classrooms for over three decades. Here, research suggests that children engaging with programming robots to move have opportunity to explore spatial concepts, problem solving, measurement, geometry, and engage with meta-cognitive processes (Clements & Meredith, 1993; Yelland, 1994). Papert's seminal work in this area suggested that Logo programming, and the visual nature of this tool, was a way to "externalize" learner's ideas and make concepts "more accessible to reflection" (Papert, 1980, p. 145). The visual nature of these tools, and the use of dynamic representation enables engagement in mathematics learning and opportunities for exploration of both content within mathematics and processes of mathematics learning.

A growing number of studies promote the use of robotics in engaging children in problem solving and learning (Bers, 2010; Bers & Ettinger, 2012; Bers, Seddighin & Sullivan, 2013; Horn & Jacob, 2007; Horn, Solovey, & Jacob, 2008; Horn, Solovey, Crouser, & Jacob, 2009; Sullivan & Bers, 2012). These studies suggest that robotics can be engaging learning opportunities (Kazakoff, Sullivan, & Bers, 2013; Stoecklemayer, Tesar, & Hoffman, 2011) and promote collaboration and problem solving, with tangible interfaces and hybrid graphical-tangible tools enabling participation both younger and older learners. Highfield's research, using simple robotics with young children, suggests a range of mathematical content that can be explored and highlights the key role of the task in promoting mathematics learning (Highfield, 2010; Highfield & Mulligan, 2009). Goodwin and Highfield (2013) suggest that the manipulable nature of these tools affords opportunity for problem solving and reasoning; with the task at hand, combined with the tool, enabling mathematical thinking. However, robotics alone do not enable mathematical engagement, with the key role of the educator, the task, and the context of learning also playing integral roles in extending mathematics learning.

Pedagogical Content Knowledge for Teaching

The role of the teacher in mathematics learning is essential, with research suggesting the intersecting domains of pedagogical knowledge, and content knowledge as particularly key in mathematics learning (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). While a teacher of mathematics must know how to solve the problems they provide to their students,

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 540–546. Sunshine Coast: MERGA.

such knowledge of content alone is insufficient. A teacher of mathematics must also know how to represent a solution to such a problem with a picture, explain why the solution works, and identify common mistakes made by students as they solve such problems (Hill, Rowan, & Ball, 2005; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). Thus, pedagogical content knowledge is comprised of both knowledge of content and pedagogy, and would be displayed by one knowledgeable of the best ways of representing some concept for students, as well as the ability to explain such concepts in order to address students' conceptions (Schulman, 1986).

Ethnomathematics as a Tool to Examine Mathematical Engagement

Savard's (2008) ethnomathematics model presents different context in the mathematics classroom: mathematical; sociocultural; and citizenship. This framework presents the starting point of a lesson as situated in the sociocultural context, where an object or a phenomenon was studied within a situation. The mathematical modelization of the situation brings students into the mathematical context. The implications of the mathematical results are studied within the sociocultural and the citizenship context. Formulation of results during the classroom discussions can help students develop citizenship competencies such as critical thinking reflection and decision-making (Savard, 2008). Thus, within this robotics project, we studied different contexts in the teachers' discourses to situate their epistemological point of view, as well as opportunities for students to develop their mathematical competencies. The robotics project was considered as the sociocultural context in which the sociocultural objects were studied in order to develop different kind of knowledge.

Given this, the robot itself might be studied using movies, stories or visual arts. The tasks to be performed by the robot, that is, the missions, are also parts of the sociocultural contexts. Coding the robot using mathematics is part of the mathematical context. The citizenship context is interpreted as what is involved living in society, including political, economic, and societal rules. The mathematical context is rich and offers huge potential when it is time to code a robot. However, this could only be realised if teachers were able to recognise and engage with this mathematical context and learning afforded. The study drew on this framework and examined the following research questions:

1. What was the focus of teacher attention when planning and implementing a robotics project in the classroom?
2. To what extent were teachers able to identify and articulate the mathematical context within this robotics project? and
3. How did teachers identify and extend on mathematics learning?

Based on that, we could define the nature of the teachers' sensitivity to the milieu (DeBlois, 2006; Savard, Freiman, Larose, & Theis, 2013) when they used inquiry-based learning to integrate mathematics in the robotics project. The teachers' sensitivity to the milieu might be defined by what teachers are paying attention to when planning, teaching, or evaluating students.

Methodology

The robotics project took place in September 2010 and ended in June 2011. Six French Canadian elementary school teachers from Grades 1 to 6 volunteered and registered for this project offered by their School Board. The School Board provided all the robotics material. In addition, two mathematics consultants and two computer technology consultants provided training and support for the teachers. The training and the support were provided over six days of meetings through the school year with computer technology consultants and mathematics education consultants alternating presentation and attendance at meetings.

Within this project the researcher acted in the role of mathematics consultant and conducted the semi-structured focus group.

The project focused on two main points of data collection including: (1) data collected from the classroom context, including teacher plans and robotic tasks, referred to as "missions"; and (2) a semi-structured focus group with the teachers was also conducted to explore teachers' implementation of the project in their classroom. This paper refers only to this second data component. Within the focus group, teachers began by discussing how the robotic project was conducted in their classrooms, more specifically outlining what they did with their students. The discussion was held in French. This discussion was video-recorded and transcribed by a research assistant and translated into English. The teachers' discourse was analysed using the afore mentioned framework (Savard, 2008) to explore teacher's sensitivity to the mathematical context and to mathematisation of learning with robotics.

Results and Analysis

Through the discussion among elementary schools teachers, three School Board consultants, and the researcher, two milieus emerged from our corpus of data.

The First Milieu: Learning Opportunities for Students

The first milieu that emerged from our data is related to the learning opportunities for students. The robotics project enabled students to learn about and use different kinds of robots, to explore and their use as well as constructing and programming robots using *Lego NXT* or *Lego WeDo*. The learning opportunities are in fact activities that are related to the content to be learnt within the robotics activities. Along with technologies, those teachers identified mathematics, language arts, and visual arts as content to be learnt by students.

For technologies, teachers mention robots as one item of content. Here, they wanted students to learn about robotics, especially what makes a robot a robot, such as sensors. They also paid attention on how to program or code the robot, using a computer-program. As one teacher stated:

> Then, I went to the computer lab to look at the program SCRATCH with the students, looking at the different colours, controls and movements. (Grade 1 teacher Sophie).

Mathematics was an articulated goal for some teachers when using the robotics with students. First, the tasks involved mathematical knowledge such as geometry and measurement. For example, in Grade 6, the robot had to do a path made of square of one-meter squared or a rectangle where the lengths needed to be double the width.

Then, there were some mathematical concepts needed to code the robots:

> Just before the holidays, I showed them the program on the board and the little presentation. I created four small missions, for example one of them was to make the robot move forward in a straight line for a meter. For the second mission, the robot needed to turn by a quarter. We worked on that in Math, the rotations by a quarter to the left or right. The second mission was only on rotation, then I had planned to make them do a square, but we did not get to that. (Grade 4 teacher Priscilla).

In the above example, the task outlined facilitated engagement with measurement content, with the teacher demonstrating an understanding of pedagogy and content in mathematics learning, harnessing the robotic tool to facilitate mathematical engagement.

Language arts were also outlined, with some teachers identifying the need to have students know the vocabulary associated with the robot. Thus, students learnt the names of the pieces used for building the robot, because they need this information to build it. In one of the Grade 1 classroom, those words were studied along with the regular vocabulary words:

> I focused really on the vocabulary and the right terms. It is not a thing, but actually a bolt for example. The importance of using the right term, where it is appropriate. (Grade 1 teacher Nancy).

In one Grade 4 classroom, students had to write a story about robots doing mission on Mars. The robotics gave a nice theme to explore for students:

> They will imagine it as if it occurred for real that the robot landed in Mars. Then, many imaginary things would be able to occur. Their robot can even have emotions; we bring the project to the next level. Here we continue by focusing on French and expression. (Grade 4 teacher Priscilla).

In addition to the Language Arts focus, one Grade 1 teacher mentioned visual arts: she asked her students to build a robot in team of two using recycling material as the starting point of the project:

> I started with a Visual Arts activity. I asked them first what a robot was in their opinion, and I also asked them to bring recycled materials that they would use to make their artwork in groups. (Grade 1 teacher Sophie).

The Second Milieu: Learning Conditions

The second milieu that emerged from our data is related to the learning conditions for implementing robotics. Teachers referred to time, material, classroom management, and motivation for students as main learning conditions.

Time was discussed as the length students used to complete some tasks with the robots. It is also related to plan the use of the computer lab, as well as the material. Because students were required to build the robots using Lego bricks, they have to carefully plan the time allowed to it:

> When we are at the point of programming, it is not necessary to do it all at once. Like classifying the pieces, we have no choice, but to do it all at once. The construction part too, I found it hard to cut that part in two. When we do a bloc, we get settled and everything is there, ready to build it all, but for the missions one period and "one flapping time" is enough. (Grade 4 teacher Priscilla).

The material brings also one constraint: as there was not enough material for every student; they had to share the material. This led teachers to talk about teamwork and classroom management:

> It is possible that we do robotics all together, but for the mathematics aspect of it I prefer that they are only two to work on the robot. After that, it was the construction of the robot itself. It was not easy for them to be on the same page and to each respect their own role. You give out the pieces, you build, etc. Half of the students were able, but the other half was not. There was always one that wanted to hold on to the pieces. Teamwork is hard and they do not have the maturity. (Grade 4 teacher Priscilla).

The Grade 6 teacher talked about how students divided the work of building the robot, coding and testing with the robot:

> They assigned each other the tasks, but they rotate. It is not always the same person doing programing; therefore, they each get to try different tasks. (Grade 6 teacher Phil).

Finally, they spoke about how the robotics project motivated students: they were thrilled to work with the robots. As a grade 1 teacher said:

> Yes, boys just like girls were really motivated. They had their eyes wide open. They were eating the information. Afterwards, I presented the robots with a PowerPoint presentation once again. (Grade 1 Teacher Nancy).

The Grade 4 teacher Priscilla expressed how those learning conditions were tied together:

> The first two missions everyone had the chance to complete them. The third one only one team almost completed it. They did not want to stop. It was December 22nd in the afternoon and we were working on robotics. Usually we do other things, but I said that we would work and have fun while working on robotics. They were very happy. Even though they had something hard to do and that they were tired, it went well. But at the end, they could not take it anymore. (Grade 4 teacher Priscilla).

Discussion

Overall, an analysis of dialogue in this focus group indicates that the teachers spent more time discussing the learning conditions than the learning opportunities for their students. Outlined above as the second milieu data from this focus group suggests that teachers were paying more attention to the implementation of the robotics project than the learning process of their students. Thus, those learning conditions seemed very important for them to share among their colleagues. We can look at those learning conditions as important aspects to consider facilitating the learning opportunities. It seems that the pedagogical knowledge for teaching involved was important for facilitate students learning, but it was not directly aimed toward some specific concepts to be learnt, such as addressing students alternative conceptions (Savard, 2014). In this case, the milieu they were paying attention belongs to the citizenship context, where all learning conditions refers to how to live in society: planning time, dividing work, rules and norms as a group and motivation to do something.

When they discussed the learning opportunities for their students, they talked more about the tasks completed than the mathematics concepts to be learnt. It is also surprising that they did not mention learning science and technology at all. While it was evident that teachers could address some arts (languages and visual) around the robots, there were no scientific or technical concepts involved in the projects described with this focus.

Again, the pedagogical knowledge for teaching mathematics present in the discussion was quite superficial. Discussion of mathematical context and mathematical opportunities was limited. The teachers did mention mathematics as a task to be performed by the robot and the role of problem solving as students planned and represented code for the robot to perform the task. Here, the mathematics involved to perform the task, i.e. the robots' mission, can be considered part of the sociocultural context because it is the mission to be performed by the robot. From an epistemological point of view, it does not involve any use of mathematics other than mathematics as cultural symbol or artefact. It could be any symbols on drawn on the floor for the robot roll into. The mathematical meaning given to these representations has to be connected to coding the robot to do that. On the other hand, the mathematics involved in coding the robot is part of the mathematical context because is all about using mathematics to code the robot to perform the task. There is mathematization or modelization of the situation. There are different processes involved and mathematical reasoning is absolutely necessary to code the robot in relation to the task to be performed. In our data, this is missing in teachers' discussion. They knew that the robotics project was about mathematics because they were taught and trained in this direction. But this is what they were less sensitive too. For instance, they did not talk about this knowledge on how to assess it. But it might be because they were not ready yet to think about it in their implementation process. In this case, they were not paying attention at that time. Another reason might be because they are still learning about the robots, how to code and the mathematics involved. Thus, knowing how long the robot needs to rotate in order to follow a path into a maze is not a mathematical knowledge written into the provincial curriculum and thus, they might not be familiar with.

Concluding Remarks

While this study is limited due to its small size and focus on one data set its findings are relevant, highlighting the challenges teachers face in implementing technology in classrooms. Within this study teacher's focus on the use of the tool, rather than on the mathematics learning afforded by the tool suggests. In that case, how can we support teachers to do both?

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special?. *Journal of Teacher Education*, *59*(5), 389-407.

Bers, M., & Ettinger, A. (2012). Programming robots in kindergarten to express identity: An ethnographic analysis. In B. Barker, G. Nugent, N. Grandgenett, & V. Adamchuk (Eds.), *Robots in K-12 education: A new technology for learning* (pp. 168-184). Hershey: IGI Global.

Bers, M.U., Seddighin, S., & Sullivan, A. (2013). Ready for robotics: Bringing together the T and E of STEM in early childhood teacher education. *Journal of Technology and Teacher Education, 21*(3), 355-377.

Bers, M.U. (2010). The TangibleK robotics program: Applied computational thinking for young children. *Early Childhood Research and Practice, 12* (2).

Clements, D.H., & Meredith, J.S. (1993). Research on Logo: Effects and efficacy. *Journal of Computing in Childhood Education, 4,* 263-290.

DeBlois, L. (2006). Influence des interprétations des productions des élèves sur les stratégies d'intervention en classe de mathématique. *Educational Studies in Mathematics, 62*(3), 307-309.

Goodwin, K., & Highfield (2013). A framework for examining technologies and early mathematics learning. In L. D. English & J. T. Mulligan (Eds.), *Reconceptualising early mathematics learning* (pp 205-226). New York: Springer.

Highfield, K. (2010). Robotic toys as a catalyst for mathematical problem solving. *Australian Primary Mathematics Classroom, 15,* 22-27.

Highfield, K., & Mulligan, J. T. (2009). Young children's embodied action in problem-solving tasks using robotic toys. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Paper presented at the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 273–280). Thessaloniki, Greece: PME.

Highfield, K., Mulligan, J. T., & Hedberg, J. (2008). Early mathematics learning through exploration with programmable toys. In O. Figueras, J.L., Cortina, S., Alatorre, T. Rojano, & A. Sepulveda (Eds.), *‚Proceedings of the Joint Meeting of Psychology of Mathematics Education 32 and Psychology of Mathematics Education-North American Chapter* (Vol. 3. pp. 169-176). México: Cinvestav-UMSNH.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 372-400.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*(4), 430-511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406.

Horn, M., & Jacob, R.J.K. (2007*).* Designing tangible programming languages for classroom use. Paper presented at the *First international conference on tangible and embedded interaction,* Baton Rouge, Louisiana. Retrieved from http://hci.cs.tufts.edu/tern/horn-jacob-tei07.pdf

Horn , M., Solovey, E., Crouser, R., & Jacob, R. (2009*).* Comparing the use of tangible and graphical programming languages for informal science education. Paper presented at the *27th international conference on Human factors in computing systems, Boston,* MA, USA. Retrieved from web.mit.edu/erinsol/www/papers/chi09.horn.pdf

Horn, M. Solovey, E., & Jacob, R. . (2008). Tangible Programming and informal science learning: Making TUIs work for museums. Proc. of interaction design and children, (Chicago, IL, USA, June 11-13, 2008). *Proceedings of the 8th International conference on design.* ACM, New York, NY. Retrieved from http://www.eecs.tufts.edu/~etreac01/papers/idc08.pdf

Kazakoff, E., Sullivan, A., & Bers, M. (2013). The effect of a classroom-based intensive robotics and programming workshop on sequencing ability in early childhood. *Early Childhood Education Journal, 41*(4), 245-255. DOI:10.1007/s10643-012-0554-5.

Mishra, P., & Koehler, M. (2006) Technological pedagogical content knowledge: A framework for teacher knowledge. *The Teachers College Record 108(*6), 1017-1054.

Papert, S. (1980). *Mindstorms: Children, computers and powerful ideas*. Brighton: Harvester Press.

Savard, A. (2008). *Le développement d'une pensée critique envers les jeux de hasard et d'argent par l'enseignement des probabilités à l'école primaire: Vers une prise de décision.* Thèse inédite. Université Laval, Québec.

Savard, A. (2014). Developing probabilistic thinking: What about people's conceptions? . In E. Chernoff & B. Sriraman (Eds.), *Probabilistic Thinking: Presenting Plural Perspectives.* (Vol. 2, pp. 283-298). Berlin/Heidelberg: Springer.

Savard, A., Freiman, V., Larose, F., & Theis, L. (2013). Discussing virtual tools that simulate probabilities: What are the middle school teachers' concerns? *McGill Journal of Education 48*(2), 403-424.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Stoeckelmayr, K., Tesar, M., & Hofmann, A. (2011). Kindergarten children programming robots: A first attempt. In *Proceedings of 2nd International Conference on Robotics in Education (RiE 2011)*. Vienna, Austria, September, 2011, pp. 185-192. INNOC - Austrian Society for Innovative Computer Sciences. Retrieved from http://www.rie2011.org/conference/proceedings

Sullivan, A., & Bers, M. U. (2012). Gender differences in kindergarteners' robotics and programming achievement. *International Journal of Technology and Design Education*. doi: 10.1007/s10798-012-9210-z.

Yelland , N. J. (1994). The strategies and interactions of young children in LOGO tasks. *Journal of Computer Assisted Learning, 10*, 33-49.

Teaching Statistics in Middle School Mathematics classrooms: Making Links with Mathematics but Avoiding Statistical Reasoning

Annie Savard
McGill University
<annie.savard@mcgill.ca>

Dominic Manuel
McGill University
<dominic.manuel@mcgill.ca>

Statistics is a domain that is taught in Mathematics in all school levels. We suggest a potential in using an interdisciplinary approach with this concept. Thus the development of the understanding of a situation might mean to use both mathematical and statistical reasoning. In this paper, we present two case studies where two middle school Mathematics teacher taught a lesson in Statistics where the students had the task create a pie graph representing the data. Results show us that their procedural vision of Statistics lead them to focus more on a graphical representation and thus led to avoid all statistical reasoning development (Garfield, 2002).

Introduction

In the 21st century, citizens must be able to solve complex problems that cannot always be done by applying one strategy or a particular algorithm. More creative approaches that require using high cognitive level thinking are needed in order to bring solutions to these problems. Do students have the necessary opportunities to develop the abilities needed to adapt themselves and be productive citizens in their society? Do they have the opportunities to develop their mathematical competencies along with citizenship competencies (Savard, Manuel & Lin, 2014)? We argue that students should be exposed to rich tasks in Mathematics and Statistics classrooms for them to be better prepared to face the realities and problems in society. Such tasks include problems that are: open-ended (can have multiple answers and can be solved using various strategies); complex (require many steps to find answers; require to investigate a particular situation or to pose a question to investigate; ask to make choices and justify them; or require to find patterns, generalise and prove results.); ill defined (missing necessary data that prompt students to search or define them to find answers); have different interpretations; and are contextualised (Manuel, 2010; Manuel, Freiman & Bourque, 2012). However, Mathematics and Statistics are still often taught in a way where students develop procedural understandings of concepts. Using these traditional teaching methods tends to have students see mathematics as a school subject that consists of rules, formulas, equations and algorithms to apply, thus enabling them to make links between mathematics and the real world or even between different mathematical concepts (Boaler, 2009). Yet, studies suggest that students who experience any form of rich inquiry-based learning tasks seem to enjoy learning mathematics, develop more conceptual understandings of mathematical ideas, achieve better in standardised testing, and develop the necessary abilities to solve unfamiliar and more complex problems (Boaler, 1998, Boaler & Humphreys, 2005).

In this paper, we examined how two middle school teachers support students in making interdisciplinary links between statistics and mathematical concepts, and between statistics and the context of the tasks they were solving. We suggest that a very large number of rich tasks could be used in order to study ideas in statistics, and interdisciplinary and intra disciplinary links could be developed while solving those tasks. The teachers selected come from another larger nationwide study on pedagogies used by middle school

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 547–554. Sunshine Coast: MERGA.

mathematics teachers in regions of Canada. Both teachers taught a lesson on pie charts. One of the teachers who teach Grade 7 mathematics in Quebec proposed a task to compare the frequency of the colours of candies found in Halloween bags (the lesson was conducted a week prior to October 31). The other teacher who teaches Grade 8 mathematics in New-Brunswick used technology to support her teaching and proposed problems for students to solve in her lesson. Kader and Perry (1994) argue that students can also develop statistical concepts and problem solving strategies using technology. The goals of our study were to examine the interdisciplinary links the teachers made between statistical ideas and mathematical concepts, and between statistics and the context of the tasks presented in their lessons. We also examined the teachers' representations of statistics to investigate if their representations guided their actions during the lesson. We defined representations as an intersection between a situation and the knowledge mobilised by a person according to this situation (Brun & Conne, 1990). The following questions guided our study:

- What links did the teachers created between statistical and mathematical ideas in their lessons?
- What links did the teachers create between statistics and the context of the tasks and/or other disciplines?
- What are the teachers' representations of statistics?

Theoretical Framework

The idea of integration curriculum orientation is not new in education (Lowe, 2002). In the context of major worldwide changes during in the last decades and new complex phenomena, it is essential to take into consideration the new social realities. Thus, instead of integrating disciplines as stand-alone school subjects, like it was suggested in the 70's (Lenoir & Sauvé, 1998), the new realities bring forth the need for interaction between them (Legendre, 1993). These interactions might be called interdisciplinarity and could be considered as a negotiation between disciplines, where the development of one discipline contributes to the development of others (Fourez & Larochelle, 2003). It can offer an extended perspective and allows disciplines to support each other. For example, collecting and interpreting statistical data might contribute to understand social or scientific phenomena.

Instructional curricula in Statistics, also known as data analysis (National Council of Teachers of Mathematics (NCTM), 2000), focus on exposing students to a statistical approach, where students from K-12 should develop the abilities to: 1) formulate questions that can be addressed with data and collect, organise and display relevant data to answer them; 2) select and use appropriate methods to collect data; and 3) evaluate inferences and predictions based on data (NCTM, 2000). Throughout this progression, it is aimed that students develop statistical reasoning where they will be able to use descriptive Statistics in order to clearly interpret data with fidelity and with rigor (Ministère de l'Éducation du Québec, 2010; Ministère de l'Éducation et du Développement de la Petite Enfance du Nouveau-Brunswick, 2012). Both the Quebec and New-Brunswick curricula align with the NCTM standards for Statistics. In this paper, we consider that Statistics and Mathematics are two different disciplines, because they have different epistemologies. The reasoning behind those disciplines is not the same. Statistics focuses on an interpretative reasoning that depends on variability, while Mathematics focuses on a deterministic reasoning (Savard, 2014). Statistical reasoning involves a conceptual understanding of important statistical ideas (delMas (2004; Garfield (2002). However, Mathematics might be used to solve statistical problems (delMas, 2004). We understand that in school systems, Statistics are considered as a branch of Mathematics, but in opposition with Carvalho and Solomon (2012), we do not consider that developing Mathematics and Statistics together as a form of intradisciplinarity, where topics or concepts in a same discipline are related to each other.

Method

Context and Participants

This study is a part of a larger nationwide study conducted with middle school (Grades 7 or 8) Mathematics teachers in four different regions of Canada. Four Anglophone teachers from Alberta, four Anglophone teachers from Ontario, four Anglophone and four Francophone teachers from Quebec, and four Francophone teachers from New Brunswick took part in this nationwide study in which the main objective is to describe regional differences in mathematics teaching and underlying pedagogies in Canada, and to relate these to differences in student achievement in mathematics.

In the nationwide study, each teacher had to video-record three lessons: one he/she considered as typical in his/her classroom; one that he/she considered as an exemplary lesson in his/her classroom; and an introductory lesson on an idea related to fractions. Members from the research team would then edit each video keeping the best 15-20 minutes of each lesson. The four teachers from each region and linguistic group would then meet with the research team for focus group meetings. During the meetings, they would watch the edited videos of each of their lessons and discuss the practices and pedagogies they observed in the videos. These discussions would follow a similar protocol. At first, the teacher would explain the lesson he/she did and answer preliminary questions (mostly clarifications about the lesson) other members of the group may have had. Second, they would watch the edited video of her lesson. Third, the teachers and the members of the research team would discuss the lesson. During that time, all the members of the group could ask questions, ask for clarifications, point out practices and strategies that they thought where great and give suggestions. We were members of the research team for three groups: the Quebec Anglophone teachers, the Quebec Francophone teachers, and the New Brunswick teachers.

One Francophone Grade 7 teacher from Quebec and two Grade 8 teachers from New Brunswick made a lesson on pie charts as their typical lesson for this project. We analysed the lessons from the Quebec teacher and one of the New Brunswick teachers. The teacher from New Brunswick is technology oriented. During the group discussion on their lessons (after watching the video), we took the opportunity to ask them about their representation of Statistics. The video of those lessons and the focus group discussions (for those lessons) were transcribed. We used pseudonyms for naming them.

The Lessons and Data Analysis

The two cases we selected permitted us to compare between a teacher who didn’t use technology in her class and one that did.

The teacher from Quebec, Ida, designed the lesson in three parts. In the first part, she presented the task by giving each group of thee students a bag of coloured candy and constructed the question they had to investigate with them (how much candy of each colour their bag contained). She then went over the steps for completing the table that helps construct the pie chart with the students. In the second part, the students solved the task. In the last part, the teacher discussed the task with the students by doing an example using the data from one group. We will mostly focus on the last part because this is where the links were made.

The teacher from New Brunswick, Danya, gave three problems on pie charts for her students to solve. The teacher focuses on three practices with her students: observe, practice and teach. On the previous day, they discussed how to create a pie chart (observing part). This part was not video-recorded. The two problems were for them to practice. They would then teach the concept to a partner. Unfortunately time ran out so the teacher had to do this part on the next day. The teacher and all the students have access to a laptop. The teacher thus used various technologies. She showed her students her blog and informed that they she placed a video that shows how to construct a pie chart in it. She informed her students that it was the best video she found on the internet and those who needed a guide should go on her blog and watch it. She also encouraged her students to get out their diagnostic test that they did at the beginning of the trimester (she gives a diagnostic test at the beginning of each trimester on the content she will cover in order to see where her students are at and guide her teaching throughout the semester) and use it as a guide. The students had to solve the first problem individually and then compare with their neighbour. The task is shown in Figure 1 (translated from French):

The artistic activities of Canadians are the following: photos, 46%, videos, 21%, drawing, 13%, and dance and piano, 10%. Draw a pie graph that represents the artistic activities of Canadians.

Figure 1. First Problem the teacher gave.

When the students finished solving the first task, the teacher would give them a second problem. She explained to her students that this one would serve as a formative evaluation so she could see who understands how to create a pie chart. The problem consisted of a table representing the number of students who used different means of transport to get to school. The students had to create the pie chart using the data. The teacher also challenged the students to create 2 questions about the data from the pie chart. However, the students did not have to answer the questions they invented. When the students finished solving the second task, they would have a longer problem to solve on their laptop. The teacher posted a problem on Google Doc. The students had to access it from their laptops and solve it. She also mentioned that it was an example of problems they could find on the final exam. The task is shown in Figure 2 (translated from French):

Julie will start university next September. She has a monthly budget of $ 1,000. Her expenses include: $ 90 recreation, 20% rent, transportation 2/10 and twenty-five hundredths for food budget. The remainder will be spent on other personal expenses. Construct a pie chart from the data above.

Figure 2. Second Problem the teacher gave.

The teacher constantly walked around the classroom during those tasks and guided the students if needed. She insisted a lot on clearly communicating the process while solving the tasks.

We used the corpuses to analyse and interpret the specific interdisciplinary and intradisciplinary actions related to knowledge building (Savoie-Zajc, 2000). In the results, we present the actions and the links made by the teachers.

Results

Intradisciplinary Links between Statistical Ideas and Mathematical Concepts

In the last part of the lesson, Ida discussed about the effectif (the number of candy in a certain colour in a bag), the frequency (a/b) for each coloured candy (data expressed as a fraction) and the relative frequency (data in percentage). She made links with numbers, fractions and percentages. She reminded the students that the total should be found using addition of the numbers representing the frequency. For the frequency (a/b), she came back to the definition of a fraction, which is a part of a whole. Ida explicitly asked the students why we write 3/9 and made it clear that 3 is the number of red candies and the 9 is the total number of candies in the bag. At the very end, she asked the students about how to fill the relative frequency column of the table but she did not give much importance to it. Ida focused her lesson on the representation and the organisation of data using mathematical notations. When it came to the idea of constructing a pie graph, Ida made links with fractions, the circle and with angles. She started off by asking the students what they knew about a circle. The students answered that it had a 360-degree angle. Following that, she asked the students how to determine the angle of the sectors for each coloured candy in the pie graph. A student answered that, “for the red candy, you can do 3/9 of 360”. Ida made it explicitly clear that we can use fractions in other contexts. She claimed, “It is not for nothing that we discussed about fractions, fractions of a number and so on before this. Now you see that we can apply fractions in other contexts other than in arithmetic”. She then went over different strategies of doing this calculation by saying that you could use proportional reasoning. Then she did an example with the class, focusing on the strategy of dividing 360 by 9 in order to get the measure of 1/9 and then multiplying the quotient by 3. She proceeded that way because a student suggested that strategy. She seemed very responsive to students’ strategies.

In conclusion, Ida created interdisciplinary links between the representation and the organisation of data (Statistics) and Mathematics: mathematical notation, proportional reasoning, geometrical representation and measurement. But, as delMas (2004) pointed out, the lesson did not go beyond the learning of procedures and thus did not develop explicitly a statistical reasoning.

Danya supported students who struggled with changing angles into degrees, finding fractions of a number, such as 2/10 of $1,000 and transforming quantities into degrees (angles). She would question students to support them in understanding why they use particular algorithms, but she didn’t make any links between the concepts involved. The entire lesson focused on making calculations to construct pie charts. It is possible that some links were made in the previous lesson, but we cannot make this conclusion since that lesson was not recorded. It is possible that Danya did put an emphasis on the interpretation of data when she challenged her students to make questions about the data on the graph. However, we did not have the students’ work to make any conclusion. During the discussion on her video, we questioned her about this. However, she didn’t remember what she did with the questions. She recorded the lesson 7 months prior to the meeting and did not remember what she did because she used different approaches with her students.

In conclusion, although Danya had moments where she could have made links between statistical ideas and mathematical concepts; she didn’t take this opportunity to make them. The entire lesson focused on finding data by calculation and representing them on a pie chart.

Interdisciplinary Links between Statistical Ideas and the Context of the Problems or Other Disciplines

Ida did not make explicit links between statistical ideas and the context of the problem or other disciplines. She simply focused on the procedure of representing the data on a pie graph. After she finished constructing the pie graph on the board, she stopped her lesson by asking the students if they had any questions. When there were none, she ended the lesson by giving the student problems to solve in the textbook. It seemed that Ida's focus was on creating a pie chart correctly because during the discussion, she put a lot of emphasis on how she corrected problems on assessments and what she needed to see to be able to give marks. It is possible that she chose this action because there was less than 10 minutes remaining. During the discussion group, the other members noticed the fact that she didn't spend time on the interpretation of data (NCTM, 2000) and they made that suggestion to her. The other members mentioned that for example, they could have compared the data between groups, they could have made a set of data of the whole class and them see how the data of each group is similar and different from the whole class data, and to make links with probability, business, economy and other fields that would be interesting to discuss. In conclusion, Ida focused her lesson on the idea of representing data (NCTM, 2000). No inference or predictions were made based on the data.

Danya did not make explicit links between statistical ideas and the context of the problems or other disciplines. When the students worked on the problem on Google Docs, she mentioned that this is where she can see if you can do French along with Mathematics at the same time. However, that comment was focused on students being able to understand the problem since its text was longer than the others and it was more complex. During the discussion group, Danya did realise that she did not put enough emphasis on interpreting data. In conclusion, Danya, just like Ida, focused her lesson on the idea of representing data (NCTM, 2000). No inference or predictions were made based on the data.

Teachers' Representation of Statistics

Ida had a clear representation in mind: "My representation of Statistics is to be able to represent and compare the data. In this activity, I wanted them to be able to use the data and create a graph with it". However, when related to the three ideas of Statistics grounded by the NCTM (2000), we noticed that the ideas of evaluating inferences and making predictions were not in her representations. She saw Statistics as a way to collect, organise, represent and analyse data although the question was built with the class at the beginning of the lesson and that she didn't spend time on analysing the data at the end of her lesson. She didn't mention the ideas about formulating questions in her representation of Statistics. However, in the first part of her lesson, we observed that she spent a good quantity of time stressing the importance of creating clear and rigorous questions. The classroom had a small debate on this aspect. They struggled a bit with creating the question. Ida would often ask if the idea proposed was a good question or not. For example, one mentioned that just saying the representation of the coloured candy is not enough so that it is important to say where the candy came from. The students added the ideas together to create the question. In conclusion, Ida showed a procedural vision of Statistics and her lesson focused on a graphical representation and thus led her to avoid all statistical reasoning development on the stochastics process (delMas, 2004; Garfield, 2002).

Danya also had a clear representation in mind. To her, Statistics was a branch of Mathematics. “It is collect and an organisation of data, interpretation and display them. In this activity, I wanted them to be able to interpret, analyse and display data”. However, in her lesson, she only focused on displaying data. In all the tasks, the data was given to the students. We could argue that she focused on interpreting data when she challenged her students to invent two questions about the data. We did not have access to the necessary data to make this specific claim. In conclusion, although Danya’s goals for her lesson were aligned with the NCTM (2000) standards, the ideas of making predictions and inference were missing from her representations. She saw Statistics as data given to students in order to display them and compare some of them. She also showed a procedural vision of Statistics.

Conclusion

This study compared two lessons on pie charts by two Francophone teachers from two different provinces and using two different types of lessons. Ida used an experimental approach where students collected data (the colour of the candies in their bag) and represented the results, while Danya used problem solving tasks for her students to practice creating pie graphs. The results revealed that both teachers had a similar representation of Statistics. However, their representations did not seem to influence their attempt to make interdisciplinary links with Statistics. Ida made intradisciplinary links between statistical ideas and mathematical concepts, but Danya seemed to approach Statistics as a stand-alone concept. Both lessons were oriented on procedural understandings instead of the interpretation of a phenomenon. Some factors may have influenced our results. We only had access to one recorded lesson. We had no knowledge of what happened prior and after both lessons were video-recorded. It is possible that intra- and interdisciplinary links were made during those times. Also, it is possible that the milieu influenced the choices the teachers made. For instance, in Danya’s case, all Grade 8 students from New Brunswick have to write a provincial exam at the end of the school year. Questions related to statistics on this exam are only on representing data and in some cases interpreting data. It is thus possible that Danya focused her attention on the procedural aspects of creating pie charts to prepare her students for that assessment. These results highlighted some important questions to consider. First, how do teachers’ actions and representations affect students’ learning of Statistics? Second, how does technology support students in making intra- and interdisciplinary links with Statistics? More research is needed in order to bring insight to these questions.

References

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Boaler, J. (2009). *What's math got to do with it?: How parents and teachers can help children learn to love their least favorite subject*. New-York, NY: The Penguin Group.

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning*. Portsmouth, NH: Heinemann.

Brun, J. & F. Conne (1990). "Analyses didactiques de protocoles d'observation du déroulement de situations." *Education et Recherches* (3): 261-285.

Carvalho, C., & Solomon, Y. (2012). Supporting statistical literacy: What do culturally relevant/realistic tasls show us about the nature of pupil engagement with Statistics? *International Journal of Educational Research*(55), 57-65.

delMas, R. C. (2004). A comparison of mathematical and statistical reasoning. In D. Ben.-Zvi &. J. Garfield (Ed.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 79-95). Netherlands: Kluwer Academic Publishers.

Fourez, G., & Larochelle, M. (2003). *Apprivoiser l'épistémologie*. Bruxelles: De Boeck.

Garfield, J. (2002). The Challenge of Developing Statistical Reasoning. *Journal of Statistics Education, 10*(3).

Kader, G., & Perry, M. (1994). Power on! Learning statistics with technology. *Mathematics Teaching in the Middle School, 1*(2), 130-136.

Legendre, R. (1993). *Dictionnaire actuel de l'éducation*. Montréal: Guérin.

Lenoir, Y., & Sauvé, L. (1998). L'interdisciplinarité et la formation à l'enseignement primaire et secondaire: quelle interdisciplinarité pour quelle formation? *Revue des Sciences de l'Éducation, XXX*(2), 220-240.

Lowe, A. (2002). La pédagogie actualisante ouvre ses portes à l'interdisciplinarité scolaire. *Éducation et Francophonie, XXX*(2), 220-240.

Manuel, D. (2010). *Étude de la créativité mathématique dans les solutions aux problèmes proposés dans la communauté virtuelle CASMI.* (Maitrise-ès Arts en éducation (mention enseignement)), Université de Moncton, Moncton, NB, Canada.

Manuel, D., Freiman, V., & Bourque, J. (2012). Richesse des problèmes posés et créativité des solutions soumises dans la Communauté d'apprentissages scientifiques et mathématiques interactifs (CASMI). *Éducation francophone en milieu minoritaire, 7*(1), 1-18. http://www.reefmm.org/Notrerevue/v7n1manuelfreimanbourque_000.pdf

Ministère de l'Éducation et du Développement de la Petite Enfance du Nouveau-Brunswick. (2012). *Programme d'études: Mathématiques au primaire 8e année*. Fredericton, NB, Canada: Gouvernement du Nouveau-Brunswick.

Ministère de l'Éducation du Québec. (2010). *Progression of Learning in Secondary School- Mathematics*. Québec, QC, Canada: Gouvernement du Québec.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Savoie-Zajc, L. (2000). La recherche qualitative/interprétative en éducation. In T. Karsenti & L. Savoie-Zajc (Eds.), *Introduction à la recherche en éducation* (pp. 171-198). Sherbrooke: Éditions du CRP

Savard, A. (2014). Developing probabilistic thinking: What about people's conceptions? . In E. Chernoff & B. Sriraman (Eds.), *Probabilistic Thinking: Presenting Plural Perspectives.* (Vol. 2, pp. 283-298). Berlin/Heidelberg: Springer.

Savard, A., Manuel, D. & Lin, T.W.J. (2014). Incorporating culture in the curriculum: the concept of probability in the Inuit culture. *In Education Special Issue, Part 2: [Indigenous Education]* 19 (3): 152-171. (available online at http://ineducation.ca/ineducation/article/view/125/640).

Context counts: The potential of realistic problems to expose and extend social and mathematical understandings

Carly Sawatzki
Monash University
<Carly.Sawatzki@monash.edu >

This article reports the findings of research involving more than 30 teachers and their Year 5 and 6 students in 16 Victorian primary schools. The participants experienced an educational intervention where the "Money and Financial Mathematics" substrand of the "Number and Algebra" content strand was taught and learned through challenging worded mathematical problems involving realistic financial contexts. Data related to one such example, a task involving three friends sharing the cost of movie tickets and food, are discussed. Insights into the nature of student

This article explores the outcomes of financial literacy education in Australia, and the potential of challenging worded mathematical problems involving realistic financial contexts to connect students' social and mathematical understandings. In 2012, the Organisation for Economic Cooperation and Development (OECD) Programme for International Student Assessment (PISA) included a Financial Literacy Assessment for 15-year-old students. While media reports have been congratulatory, emphasising that Australia ranked in the top five of 18 participating countries, a strong relationship between student socioeconomic background and performance was evident (Thomson, 2014). Students in metropolitan schools achieved more highly than students in provincial and remote schools; and non-Indigenous students significantly out-performed their Indigenous counterparts (Thomson, 2014). Essentially, financial literacy is no different from numeracy or literacy where disparities in educational achievement are associated with socioeconomic marginalisation (Snyder & Nieuwenhuysen, 2010). While the results point to effective financial literacy education taking place in some contexts more so than others, there is limited Australian research what might be done to improve outcomes for students in marginalised communities. Furthermore, consumer, economic, and financial socialisation research together with behavioural economics research build a compelling case that human financial behaviour may depend as much on intrinsic psychological attributes and social understandings learned at home as knowledge and skills acquired at school (de Meza, Irlenbusch, & Reyniers, July 2008).

Ajzen's (1991) theory of planned behaviour provided a theoretical framework to explore this proposition. The theory of planned behaviour argues that *attitudes, subjective norms* (expectations or perceived social pressure from socialising agents including parents and teachers), and *perceived behavioural control* (resources, opportunities, and confidence) have a direct effect on intentions and an indirect effect on behaviour through intentions. Since *values* also seem to be important to the formation and development of attitudinal and behavioural tendencies (Homer & Kahle, 1988), the possibility that they too might contribute to students' financial problem-solving and decision-making was considered. *Values* are understood to mean "...the principles and fundamental convictions which act as general guides to behaviour, the standards by which particular actions are judged as good or desirable" (Halstead & Taylor, 2000, p.169). Later, I describe how these definitions guided the data collection and analysis.

There were three assumptions. The first was that financial problem-solving and decision-making would be indicative of financial behaviour. The second was that

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 555–562. Sunshine Coast: MERGA.

classroom research examining the impact of *attitudes*, *subjective norms*, *perceived behavioural control*, and *values* on students' responses to mathematical problems situated in realistic financial contexts might give insights into the social and mathematical dimensions of student financial literacy. The third was that these insights might inform how financial literacy is conceptualised, taught and learned at school.

The Encouraging Persistence Maintaining Challenge (EPMC) project involved an educational intervention featuring five challenging mathematical problems situated in realistic financial contexts – termed "financial dilemmas" - as the basis of money and financial mathematics lessons. Financial dilemmas are open-ended, require students to draw on both social and mathematical understandings simultaneously and in synergy, involve multiple solutions, and invite students to share and explain their reasoning. Importantly, the tasks involve situations that 10-12 year old children might be familiar with and/or interested in and/or able to imagine. They are "realistic" in the sense that they feature practical, applied and contextual mathematics. The financial dilemmas were intended to be used together with the following researched pedagogies and practices that have been argued to enhance mathematics learning:

- Establishing the relevance of the task to everyday life beyond school (Mandell & Klein, 2007) and explaining the importance of both social and mathematical thinking to informed financial problem-solving and decision-making.
- Building a strong lesson introduction through literacy and other strategies that give students confidence to begin problem-solving (Draper, 2002). Strategies that seem to be particularly helpful to students include the use of role play and concrete materials (i.e., notes and coins).
- Emphasising problem-solving tools and strategies that might help students, including creating tables to organise information and/or drawing pictures (Goos, Dole, & Geiger, 2011).
- Providing time for individual thinking and problem-solving, followed by small group collaboration where students can share and discuss their problem solving approaches and solution/s (Smith & Stein, 2011).
- Facilitating critical whole-class discussions, including: all the while ensuring that a range of options (mathematical workings and explanations) are recorded, and open, sometimes provocative questions are asked to stimulate different ways of thinking (Walker, 2014).

The research question is: What insights into the social and mathematical dimensions of student financial literacy can be gained from using financial dilemmas for mathematics teaching and learning?

Some Relevant Prior Research

The following literature provided insights that shaped the snapshot of the EPMC project reported in this article. Various researchers have explored the use of worded mathematical problems involving realistic contexts and have argued the potential for these to: enhance student motivation (Middleton, 1995); provide opportunities to apply mathematical knowledge and skills (Verschaffel, deCorte, & Lasure, 1994); engage students in productive exploration of mathematics (Christiansen & Walther, 1986); provide students with opportunities to develop deeper and stronger mathematical understandings (Zbiek & Conner, 2006); and help students to see the relevance and importance of mathematics beyond school (Sullivan, 2011). These outcomes are desirable in that they contribute to educating functionally numerate citizens. Students need to be able to apply mathematics to a variety of contexts, whether in the classroom, on standardised assessments, or in the ultimate "high stakes" test – everyday life beyond school.

However, finding meaningful contexts in which to situate mathematics teaching and learning can be difficult. Borasi (1986) emphasised that students are the ultimate judge whether a problem is appealing enough to attempt to solve it, and they make this judgement based on the level of difficulty they perceive in the problem, their interest in it, and the importance they ascribe to it. Meyer, Dekker, and Querelle (2001) outlined a number of characteristics of high quality contexts, which included that a context should: support the mathematics and not overwhelm it; be real or at least imaginable; be varied; relate to real problems to solve; be sensitive to cultural, gender and racial norms; not exclude any group of students; and allow the making of models. However, they agreed with Borasi (1986) that a context that interests and motivates one student might hold no interest for another.

Stillman (2000) investigated the impact of prior knowledge of context on senior secondary students' approaches to application tasks. She classified three sources of prior knowledge: academic knowledge; general knowledge of the world; and episodic knowledge derived from personal experiences outside school or in practical school subjects. Stillman (2000) found that episodic or experiential knowledge is particularly influential in shaping the extent to which students may engage with a task context. Jorgensen and Sullivan (2010) have also written about this phenomenon, drawing on their experiences in remote Aboriginal settings. They highlighted ways by which social heritage converts to academic success, giving specific examples of items about money that were included on the 2008 Australian numeracy assessment. They argued that while particular contexts may be realistic for some students, they are well outside the everyday experiences of others, and so create opportunities for 'scholastic mortality' among those who are already disadvantaged (Jorgensen & Sullivan, 2010).

Realistic Mathematics Education (RME) in The Netherlands provides an example of how mathematics teaching and learning can be conceptualised. RME is based on the view that mathematics "must be connected to reality, stay close to children and should be relevant to society" in order to be of human value (Freudenthal, 1977 in van den Heuvel-Panhuizen, 2003, p.9). RME proposes that the imagination can serve to enhance task authenticity. This suggests that students who do not have what Stillman (2010) describes as episodic or experiential knowledge related to a context can still access unfamiliar or novel task contexts provided pedagogies that help them visualise the context are used.

On one hand, the above perspectives underline the importance of taking into consideration different understandings about money students bring to school from home based on their financial realities, and situating teaching and learning in realistic contexts that connect with students' experiences. On the other hand, if schooling is to redress the apparent disparity in financial literacy levels associated with socioeconomic background, there is merit in posing contexts that are at least imaginable and perhaps might expand students' experiences. The educational intervention intended to use realistic financial contexts as the key to strengthening students' disposition to connect social and mathematical thinking as part of their financial problem-solving, the assumption being that doing so would likely contribute to informed financial decision-making.

Methodology and Methods

The EPMC project is an example of a design-based research (DBR) project. Anderson and Shattuck (2012) draw on a range of definitions of DBR to explain it as:

- being situated in a real educational context;
- focusing on the design and testing of a significant intervention;
- using mixed-methods;
- involving multiple iterations;
- involving a collaborative partnership between researchers and practitioners; and
- promoting design principles that have an impact on practice.

DBR has become recognised as a valuable methodological approach to study, transform and evaluate the practice of mathematics teaching and learning. It is a practical research methodology that seeks to increase the impact, transfer, and translation of educational research into improved teacher practice (Anderson & Shattuck, 2012).

A series of classroom investigations took place to study the implementation of the five financial dilemmas that were included in the EPMC project (the educational intervention). Each financial dilemma included enabling, consolidating, and extending versions (Sullivan, Mousley, & Jorgensen, 2009). This article reports on data collected about one financial dilemma, "Anna and her friends", which involves three friends sharing the cost of movie tickets and food. The classroom investigation explored the use of this task by two experienced educators (pseudonyms Cara and Cate) team-teaching 55 Year 6 students in an open learning environment in a government school in provincial Victoria. The teachers described their students as being from diverse socioeconomic backgrounds. Data collected included audio and video recordings of the instructional and summary phases of the lesson, hand-written observational notes made by two researchers, and students' completed worksheets.

Post-intervention surveys were also completed online by more than 30 Year 5 and 6 teachers in 16 Victorian primary schools. The sample included teachers from Government and Catholic, metropolitan and regional primary schools. The teacher participants were asked to respond to a series of brief statements by indicating the extent to which they agreed on a 5-point Likert scale (strongly disagree, disagree, unsure, agree, strongly agree). The statements related to financial literacy education (in general), lesson planning, lesson structure, and pedagogies. For each of the five financial dilemmas, there were a further 10 brief statements that required the teacher participants to reflect upon the effectiveness of the tasks as the basis of money and financial mathematics lessons. The teachers were also invited to give feedback about the tasks and pedagogies through five open-ended questions. Responses to two particular questions with reference to "Anna and her friends" - "What is your reaction to the lesson overall?" and, "Is there a particular story you would like to share with us?" – are reported in this article.

Drawing on the theoretical model described earlier, the classroom investigation and post-intervention survey data were analysed and categorised as indicating *attitudes*, *subjective norms*, *perceived behavioural control*, and *values* (as per the definitions outlined earlier). This process was undertaken with a view to understanding the nature of the social understandings that became evident through the use of "Anna and her friends", and describing how these seemed to impact the way students connected with the mathematical dimensions of the task. For example, where data reflected a particular ideal, these data were interpreted to indicate a *value*. Where patterns of behaviour became apparent, it was inferred that *subjective norms* were influential. Scrutinising the data sources with EPMC project colleagues helped to ensure validation. In the section that follows, synergies between the data sets are examined to seek insights into the social and mathematical dimensions of financial literacy.

"Anna and her friends" was presented as follows:

Task 1:

Anna, Bernadette and Carol are going to the movies together. Tickets cost $12 each, but there is a special offer for everyone who books and pays online - buy two tickets, get the third ticket free. Anna booked and paid for the tickets online.

When they arrived at the theatre, they noticed the pricelist at the shop. The price list reads as follows:

Bottled Water	$4
Icecream	$4
Medium Popcorn	$8
Bottled Water, icecream & popcorn combo	$12

Anna wants to buy a bottle of water, Bernadette wants the ice-cream and Carol wants the popcorn. Anna pays for the combo.

What might Anna say to Bernadette and Carol about how much they owe her?

Task 2:

This version of the task requires students work in dollars and cents, and account for an online processing fee of 30c per ticket purchased.

This task was considered relatable to Year 5 and 6 students since children this age are likely to have visited the cinemas before, and may have been responsible for paying for their transactions upon doing so. In Task 1 (the learning task), the calculations required were intended to be straightforward for Year 5 and 6 students, compared with Task 2 (the consolidating task) where students work in dollars and cents, and account for an online processing fee of 30c per ticket purchased. In both scenarios, notions of friendship and "fairness" – described as social understandings - are important considerations. There are multiple ways to approach this financial dilemma, which is critical for creating an awareness of alternative possibilities, and fostering critical whole-class discussion and debate about financial problem-solving and decision-making. Readers are invited to tackle these tasks before proceeding.

Findings

In the classroom investigation, two important issues emerged that were reinforced by the post-intervention survey responses by the broader group of teacher participants. First, the task involving a realistic financial context that 10-12 year old children might be familiar with and/or interested in and/or able to imagine seemed to contribute to students being actively engaged in the lesson. Second, "Anna and her friends" revealed that Year 5 and 6 students have sophisticated social understandings about money that are at the forefront of their thinking during financial problem-solving and decision-making. Each of these findings is elaborated below.

The Importance of the Choice of Context and the Lesson Introduction

"Anna and her friends" involving a realistic financial context that 10-12 year old children might be familiar with and/or interested in and/or able to imagine seemed to contribute to the success of the lesson. The following comment reflects the view of many of the teacher participants:

> The kids had all been to the movies, so were able to relate to this task. They were also getting used to discussing deals, value for money, and "fair share".

Interestingly, context familiarity was not assumed or taken for granted in the lesson by Cara and Cate. This is important, since attending the movies as entertainment is beyond the realms of affordability for some students. In launching the lesson, they set up an impromptu role play involving three classmates in the roles of Anna, Bernadette, and

Carol. The students, in role as the characters, casually conversed about how much each person should pay towards the movie tickets and combo. This worked well to showcase two different approaches to sharing the cost. The student playing the role of Anna said, “I got my ticket for free. So if you pay me for your tickets, I’ll pay for the combo.” The student playing the role of Bernadette had another idea, “I think we should split the cost of the movie tickets - $8 each – and then pay for our own share of the combo. Carol will have to pay $2 more because the popcorn is worth a little more.” Inherent in this brief exchange were different *attitudes* and *values* about sharing costs. Cara and Cate’s pedagogical choice to use role play seemed to activate students’ imaginations and make the task more accessible to the class.

Social Understandings are at the Forefront of Students’ Financial Problem-Solving and Decision-Making

When it came to distributing the costs of the movie tickets and food, students preferred to equate “fairness” with sharing equally, at least in the first instance. On the post-intervention survey, one teacher described this approach as “taking the easy option”. Related to this, another commented that some students “were keen to get a maths answer without justifying their thinking” in relation to the context. This phenomenon suggests that students were initially motivated by a perceived need to adhere to a particular *subjective norm* – a social convention that “fair” means sharing equally. Furthermore, this motivation influenced their choice of mathematics.

As the following comments by two different teachers reveal, the teachers reported provoking or extending their students to revisit and discuss the complexities of the context:

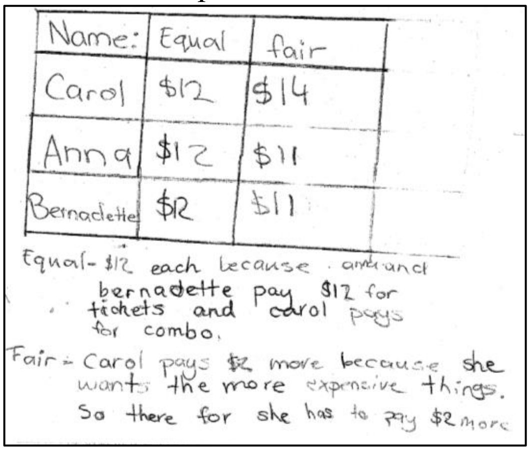
> The students were engaged, but most were happy to split the [cost] evenly. As the teacher, I needed to get the conversation moving by throwing in a few controversial ideas.

> They didn’t realise the popcorn was worth more than the water and ice-cream. They wanted to split the cost three ways - $12 each. I questioned them, “Is that fair?”

In these ways, pedagogy was pivotal to stimulating new learning. In the classroom investigation, Cara and Cate encouraged students to debate the idea that costs can be shared equally (or “evenly” – the term interchangeably used by teacher participants) or proportional to the value of items to be received. They asked students to consider, “Is sharing evenly always the fairest thing to do? Is it fair to split the cost of the combo evenly given that Carol will receive the most valuable item (popcorn)?” These questions were addressed nicely by one student, who tabulated two different solutions how much Bernadette and Carol owed Anna, as shown in Figure 1. One solution is described as “Equal,” the other as “Fair”.

On the post-intervention survey, one teacher outlined students’ diverse *values* and responses to this problem saying, “Some strongly believed that Anna should have benefited from the discount/saving, while others believed she should have [paid for] her friends”. These examples highlight how *values* can influence students to take different mathematical approaches to financial problem-solving and decision-making: if Anna retains the free movie ticket and the cost of the food is shared equally, she pays as little as $4 for the outing compared with Bernadette and Carol who pay $16 each. By contrast, if Anna pays for herself and her friends, she spends as much as $36.

In the classroom investigation, the ideas students contributed to whole class discussion and recorded on their worksheets demonstrated sophisticated social understandings about money. During Cara and Cate’s lesson, one student explained to the class that Anna should receive the free ticket but pay for the online processing fee and combo. He justified his thinking that Anna should pay slightly less than her friends ($10.90 compared with $13.50) by saying, "Paying this makes it fair in the sense that Anna did all the work." Other students also made reference to perceived social conventions (*subjective norms*). For example, one wrote, "If they were really good friends and went to the movies often, then they could just take it in turns paying the total price." Another seemed to see the opportunity for Anna to conceal the free ticket from her friends, noting on her worksheet, "I think that it really depends on how good [a] friends they are because if Anna booked the tickets online and paid for them she could have the free ticket and the two others would pay $12 each." In each of the above-mentioned options, students *valued* and were motivated by different notions of friendship and "fairness".



Name:	Equal	fair
Carol	$12	$14
Anna	$12	$11
Bernadette	$12	$11

Equal - $12 each because ~~ann~~ anna and bernadette pay $12 for tickets and carol pays for combo.

Fair - Carol pays $2 more because she wants the more expensive things. So there for she has to pay $2 more

Figure 1. Student worksheet: The difference between equal and fair.

Since financial dilemmas involve multiple solutions, the onus is on students to produce and defend an argument that is socially acceptable / "fair" and mathematically precise. Requiring students to draw on their social and mathematical understandings simultaneously and in synergy when explaining an argument revealed what *attitudes*, *subjective norms* (expectations), and *values* about money were motivating students. These factors could then be considered as part of critical whole-class discussion aimed at promoting more informed financial problem-solving and decision-making.

Conclusion

The findings reveal insights that might inform the way financial literacy is conceptualised, taught, and learned at school. "Anna and her friends" exposed that Year 5 and 6 students have sophisticated social understandings about money that, when set against realistic financial contexts, can be productively leveraged to facilitate engaging money and financial mathematics lessons. While social understandings such as *attitudes*, *values* and *subjective norms* seem to be at the forefront of students' thinking during financial problem-solving and decision-making, particular pedagogies and practices including role play and open, if not provocative questions posed by the teachers during critical whole-class discussion can help students identify and evaluate alternative ways that costs might be shared. In this way, the educational intervention served to strengthen students' disposition to connect social and mathematical thinking and, by extension, make more informed financial decisions.

The critical implication is that while financial dilemmas appeal to what Stillman (2000) describes as students' episodic knowledge, posing contexts that are at least imaginable can expand students' experiences as well as their toolkit of social and mathematical understandings. Financial dilemmas do not "stand alone" – their power lies in the associated pedagogies and practices that bring them to life. If teachers are to create and/or select realistic but perhaps unfamiliar or novel financial contexts as the basis of money and financial mathematics lessons, they need to know their students' family backgrounds, characteristics, and interests. Broader and successful implementation of the educational intervention will rely on further research of this nature, whereby new financial dilemmas are developed, trialled, studied and refined in collaboration with teachers. Professional learning opportunities designed to build teachers' capacity to use the associated pedagogies and practices with confidence will be critical. Such research is currently underway in Indigenous, rural and remote communities.

References

Ajzen, I. (1991). The theory of planned behaviour. *Organisational Behaviour and Human Decision Processes, 50,* 179-211.

Anderson, T., & Shattuck, J. (2012). Design-based research: A decade of progress in education research? *Educational Researcher, 41*(1), 16-25.

Christiansen, B., & Walther, G. (1986). Task and activity. In B. Christiansen, A.G. Howson, & M. Otte (Eds.), *Perspectives on mathematics education:* Papers submitted by members of the Bacomet group (pp.243-307). Dordrecht, The Netherlands: Reidel.

de Meza, D., Irlenbusch, B., & Reyniers, D. (2008, July). *Financial capability: A behavioural economics perspective* (FSA Consumer Research Report 69). Retrieved from http://www.fsa.gov.uk/pubs/consumer-research/crpr69.pdf

Draper, R.J. (2002). School mathematics reform, constructivism, and literacy: A case for literacy instruction in the reform-oriented math classroom. *Journal of Adolescent and Adult Literacy, 45*(6), 520-529.

Homer, P.M., & Kahle, L.R. (1988). A structural equation test of the value-attitude-behaviour hierarchy. *Journal of Personality and Social Psychology, 54*(4), 638-646.

Jorgensen, R., & Sullivan, P. (2010). Scholastic heritage and success in school mathematics: Implications for remote Aboriginal learners. In I. Snyder & J. Nieuwenhuysen (Eds.), *Closing the gap in education? Improving outcomes in southern world societies* (pp.23-36). Melbourne, Australia: Monash University Publishing.

Mandell, L., & Klein, L. (2007). Motivation and financial literacy. *Financial Services Review, 16,* 105-116.

Middleton, J.A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal constructs approach. *Journal for Research in Mathematics Education, 26*(3), 254-279.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematical discussions*. Reston VA: National Council of Teacher of Mathematics.

Snyder, I. & Nieuwenhuysen, J. (2010). *Closing the gap in education? Improving outcomes in southern world societies*. Clayton: Monash University Publishing.

Stillman, G. (2000). Impact of prior knowledge of task context on approaches to application tasks. *Journal of Mathematical Behaviour, 19,* 333-361.

Sullivan, P. (2011). *Teaching mathematics: Using research-informed strategies* (Australian Education Review, No.59). Melbourne, Australia: ACER Press.

Sullivan, P., Mousley, J., & Jorgensen, R. (2009). Tasks and pedagogies that facilitate mathematical problem solving. In B. Kaur (Ed.), *Mathematical problem solving* (pp.17-42). Association of Mathematics Educators: Singapore / USA / UK World Scientific Publishing.

Thomson, S. (2014). *Financing the future: Australian students' results in the PISA 2012 Financial Literacy assessment.* Australian Council for Educational Research [ACER], Retrieved from http://www.acer.edu.au/files/PISA_2012_Financial_Literacy.pdf

Verschaffel, L., deCorte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction, 4*, 273-294.

Walker, N. (2014). Improving the Effectiveness of the Whole Class Discussion in the Summary Phase of Mathematics Lessons. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia [MERGA]), Inc., Adelaide, pp. 597–604. Sydney: MERGA.

Zbiek, R.M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in Mathematics, 63*, 89-112.

Theorising about Mathematics Teachers' Professional Knowledge: The Content, Form, Nature, and Course of Teachers' Knowledge

Thorsten Scheiner
University of Hamburg, Germany
<thorsten.scheiner@uni-hamburg.de>

The guiding philosophy of this theoretical work lays in the argument that mathematics teachers' professional knowledge is the integration of various knowledge facets derived from different sources including teaching experience and research. This paper goes beyond past trends identifying what the teachers' knowledge is about (content) by providing new perspectives, in particular, on how the knowledge is structured and organised (form), on what teachers' draw on their knowledge (source), and whether the knowledge is stable and coherent or contextually-sensitive and fluid (nature).

Introduction

The guiding philosophy of this work is the assumption that teachers draw on a wide range of sources as they do their work, using and transforming these in various ways for the purposes of their teaching and for the needs of their students. Thus, one of the key theoretical concerns arising in the realm of teachers' professional learning and development is the question on which sources teachers draw on their work. Researchers have reflected on resources (including knowledge), identifying them, among orientations (including beliefs) and goals, as critically important determinants in what teachers do, and why they do it (Schoenfeld, 2010). The sources of particular significance for the teaching enterprise are, from the author's perspective: (a) knowledge; (b) teaching; and (c) research (see Figure 1). These three sources are viewed as playing a complementary role in relation to each other; for instance, research can inform and enhance teachers' knowledge about particular instructional strategies, as well as equipping the teacher for the rich reflection required in professional judgement. At the same time, research itself can be enriched through greater insights into the challenges and complexities of educational practice.

The last few decades have produced a considerable body of literature that describes, theorises, and conceptualises knowledge as a source for teachers doing their work. Shulman (1987), for instance, identified three dimensions of knowledge needed for teaching; namely content knowledge (knowledge of the subject matter per se to be taught), pedagogical knowledge (knowledge of how to teach in general terms), and pedagogical content knowledge (knowledge of how to teach that is specific to what is to be taught). In this and further work, Shulman (1987) makes clear that the knowledge base necessary for teaching comprises teachers' knowledge of content in the domain being taught, knowledge of learners' common conceptions, and difficulties that learners may have when learning particular content, and knowledge of pedagogical strategies that can be used to address learners' needs in particular classroom circumstances. However, less emphasis has been put on teachers' knowledge of students' learning. To put it in other words, what is missing in Shulman's (1987) contribution on various dimensions of teachers' knowledge, as argued in this work, is teachers' knowledge of learning, in particular, teachers' knowledge of theoretical frameworks of knowing and learning. However, knowledge of approaches to, and research on, learning mathematics is taken as a crucial component of mathematics teachers' resources, and a particular focus of the theoretical work reported here.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 563–570. Sunshine Coast: MERGA.

Another source of teachers' professionalisation is *the personal experience of being taught, or of teaching*. In analogy to *phenomenological primitive*s arising from experience with the physical world as described in detail by diSessa situated in his *knowledge in pieces* framework (e.g., diSessa, 1993), *pedagogical primitives* arise from experiences of being taught or of teaching. They provide powerful, mental resources useful for sense-making in the education instructional context, formed through a process in which individual teacher's ways of teaching are strongly shaped by their personal experience of being taught or of teaching. Researchers may refer to this as *craft knowledge* or *practical knowledge* to distinguish it from what others have referred to as *didactical knowledge'* or *mathematics education knowledge*, in particular, knowledge derived from research reported in the field. Knowledge derived from research (in short, research-based knowledge) is considered as a further source of teachers' professionalisation. In particular, research-based knowledge on: (a) students' ways of understanding and thinking; (b) ways of learning mathematics; and (c) ways of teaching particular mathematical concepts are viewed as providing a rich source for teachers' doing their work. Teachers need to engage with research, in the sense of keeping up to date with the latest developments and findings in research on students' ways of thinking, understanding, and learning, and on effective instructional techniques to inform their pedagogical content knowledge. In addition to the latest research findings, teachers should become familiar with the implications of this research for their day-to-day practice, and for education policy and practice more broadly. With this perspective, research is viewed as a key source of teachers' broader professional identity, one that reinforces other pillars of teacher quality: notably teachers' knowledge base and teaching experience.

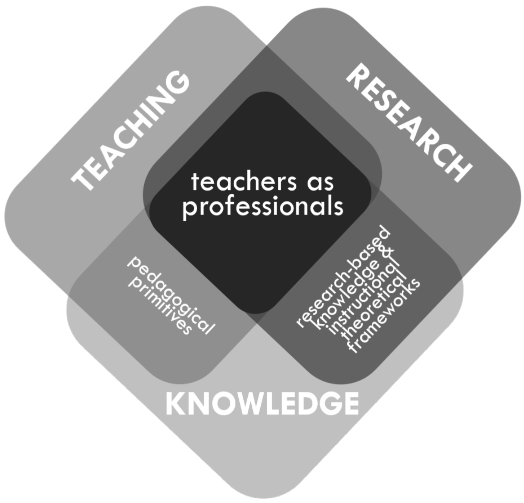


Figure 1: Sources of teachers' professionalisation

It is this conceptualisation of sources of teachers' professionalisation that enables an elaboration of knowledge resources for teaching mathematics. Consequentially, in contrast to any narrow or simplified view, the idea of teachers' professional knowledge essentially conveys the need to *integrate* knowledge from *various* sources including knowledge derived through teaching experience/practice (pedagogical primitives) and research (research-based knowledge and instructional theoretical frameworks).

Lessons from Past Approaches Conceptualising Mathematics Teachers' Knowledge

Over the past decades, a range of research work on conceptualising teachers' knowledge has been developed often taking Shulman's initial work as a point of departure, a considerable number of which has been located in mathematics education research (e.g., Adler & Davis, 2006; Ball, Thames, & Phelps, 2008; Blömeke, Kaiser, & Lehman, 2010; Even, 1990; Ma, 1999; Fennema & Franke, 1992; Kilpatrick et al., 2006; Rowland et al., 2005; Schoenfeld & Kilpatrick, 2008), and how such knowledge can be operationalised and measured (Baumert et al., 2010; Blömeke et al., 2014; Hill et al., 2007; Schilling et al., 2007; Tatto et al., 2008, 2012). Crucial lessons we have learned from these and related work on conceptualising mathematics teachers' knowledge have been identified and described elsewhere (Scheiner, 2015). In short, Shulman's (1986, 1987) conceptualisation of domains of teachers' knowledge, in particular, subject matter knowledge (SMK) and pedagogical content knowledge (PCK), has been made specific to teaching mathematics. The distinction between SMK and PCK, although being ambitious in empirical investigations, continue to be widely used, in particular since it is considered as a useful tool in describing teachers' knowledge for research purposes and in devising pre-service teachers' and professional development programs. The multidimensional nature of mathematics teachers' knowledge has been demonstrated by further refining the categories SMK and PCK and accentuating sub-dimensions that are specific for the purposes of teaching mathematics, such as describing and conceptualising a particular kind of mathematical content knowledge considered as unique for teaching mathematics.

In this work, the author wants to point to a further aspect that is about the dominating and guiding idea of most of the approaches on conceptualising mathematics teachers' knowledge developed in the past, namely the idea about teachers' *unpacking of mathematics content* in ways accessible for their students. In doing so, past approaches have centred their focus on the mathematics content; making the mathematics content a point of departure. Approaches guided by this *philosophy* often use the notion of *mathematical knowledge for teaching* in describing the teachers' knowledge base. From the author's perspective, the use of the notion of *mathematical knowledge for teaching* is insufficient since it seems not to capture other dimensions besides the subject content. Thus, this work calls for using the notion of *knowledge for teaching mathematics* including an epistemological, a cognitive, and a didactical dimension in addition to the subject content dimension. In doing so, it is intended to extend the current perspective on teachers' knowledge in the sense of going beyond a more or less purely content perspective by taking into account several other perspectives important in in this issue.

Conceptualising Mathematics Teachers' Knowledge: Past Trends and New Perspectives

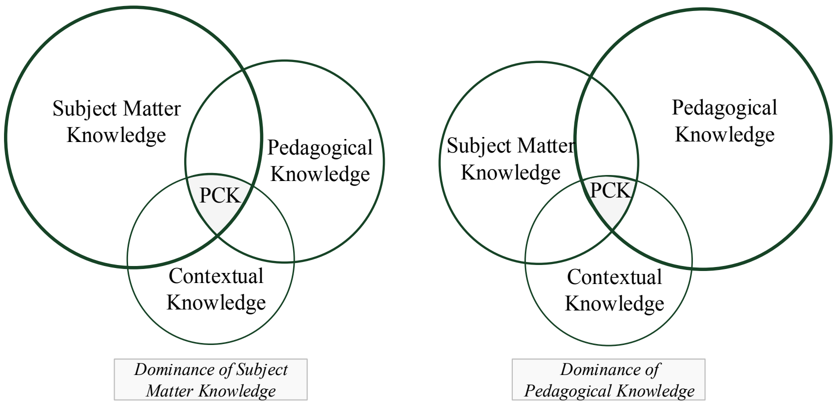
In the past, the literature concentrated its focus on what the teachers' knowledge is about. In doing so, the literature limited its focus on the *content* teachers do or should possess. Research has made progress in identifying various facets of mathematics teachers' knowledge arguing that teachers' subject matter knowledge is about substantive and syntactic structures of the discipline (Schwab, 1978); and mathematics teachers' content knowledge, in particular, seems to be about ways of understanding and ways of thinking (Harel, 2008), or school mathematical knowledge and academic content knowledge (Bromme, 1994), among others. Mathematics teachers' knowledge, as argued in the literature, is about the epistemological foundations of mathematics and mathematics learning (see, Bromme, 1994), students' cognitions (Fennema & Franke, 1992), in particular, knowledge of students' common conceptions (see Shulman & Sykes, 1986), knowledge of students' cognitive difficulties involved in concept construction (Harel, 2008), and the interpretation of students' emerging thinking (Ball et al., 2008), as well as "the most useful ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9), including teachers' illustrations and alternative ways of representing concepts (and the awareness of the relative cognitive demands of different topics) (Rowland et al., 2005) and knowledge of the design of instruction (Ball et al., 2008), among others.

However, what seems to be missing in the current landscape on various approaches of conceptualising mathematics teachers' knowledge are efforts in going beyond what the knowledge for teaching mathematics is about by taking into account: (1) how the knowledge is structured and organised; (2) on which sources teachers' draw on their knowledge; and (3) whether the knowledge is stable and coherent or contextually-sensitive and fluid. In short, the major issues that need better resolution if we are to understand teachers' acquisition of an integrated knowledge base are questions concerning: (1) the *form*; (2) the *source*; and (3) the *nature* of mathematics teachers' knowledge.

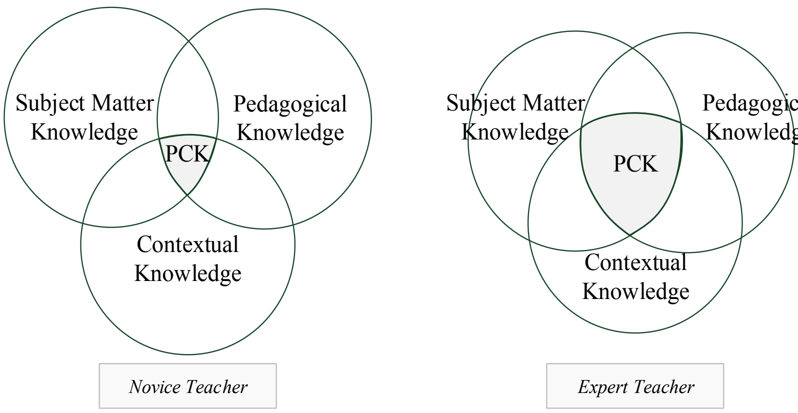
The Form of Teacher Knowledge

The initial point in this issue is the assumption that examining teacher expertise may help to advance our understanding of what makes the knowledge for teaching specialised since expert teachers are considered as focal elements in the movement towards excellence in education (Sternberg & Horvath, 1995). Findings in research on expert teachers, and, in particular, on expert teachers' knowledge show that the concept of domain-specific knowledge structures is vital. Among various differences, Sternberg and Horvath (1995) consider knowledge as "perhaps the most fundamental difference between experts and novices" (p. 10). The same authors conclude that research findings indicate that an expert in the domain of teaching differs from a novice not only in the amount of subject matter knowledge and pedagogical knowledge but also in the organisation of their domain-relevant knowledge.

Magnusson, Krajcik, and Borko (1999) illustrate one way (among several possible other ways) to think about the interaction of the domains of knowledge in the development of pedagogical content knowledge. They suppose that the knowledge bases (subject matter, pedagogical, and contextual knowledge) may unequally influence the development of pedagogical content knowledge due to differences in the amount of knowledge in each domain. However, taking the research findings on expert teachers' knowledge into account, it may be suggested that after a certain amount of subject matter knowledge, pedagogical knowledge, or contextual knowledge these knowledge bases do not have a higher relative influence on PCK. Rather, as shown in Figure 2a, it is not merely the amount of knowledge in each knowledge domain (subject matter knowledge, pedagogical knowledge, or contextual knowledge) that matters most but the degree of integration of the knowledge bases. Expert teachers, from this point of view, would show a greater overlap, symbolising increased integration of the three knowledge bases, than novice teachers (see, Figure 2b).



(a) The potential (impact) of the dominance of a particular knowledge base on PCK



(b) The (potential) impact of the degree of integration of knowledge bases on PCK

Figure 2: The (potential) impact of the dominance of a particular knowledge base and the degree of integration of knowledge bases on PCK

The Source of Teacher Knowledge

A further aspect in conceptualising the knowledge specialised for the purposes of teaching mathematics is to examine the constituent knowledge bases that influence this particular kind of knowledge. In the past, Shulman's pedagogical content knowledge was considered almost always as the only form of knowledge unique for the purposes of teaching. In Shulman (1987), pedagogical content knowledge was defined as "that special amalgam of content and pedagogy … It represents the blending of content and pedagogy " (Shulman, 1987, p. 8, italics added). However, this perspective is problematic for many reasons, including the fact that the amalgamation of content and pedagogy leads not only to a too broad category but lacks in both subject- and context-specificity. Still, the mathematics education research community has identified specific dimensions built upon Shulman's initial work on PCK. The various refinements of PCK seem to converge in three dimensions, namely: (1) knowledge of students' mathematical understandings (KSU); (2) knowledge of learning mathematics (KLM); and (3) knowledge of teaching mathematics (KTM). The former two refer to a cognitive and an epistemological perspective, while the latter refers to a didactical perspective on this issue. In this work, knowledge of students' mathematical understanding (KSU), knowledge of learning mathematics (KLM), and knowledge of teaching mathematics (KTM), together with mathematical content knowledge per se (MCK per se) and mathematical content knowledge for teaching (MCK for teaching) build the knowledge bases that constitute the particular kind of knowledge that is considered as specialised for the purposes of teaching mathematics. In doing so, past and current approaches in research on mathematics teachers' knowledge are turned on their heads in the sense of taking the identified (and refined) knowledge dimensions as building blocks for the construct of *knowledge for teaching mathematics*.

The Nature of Mathematics Teacher Knowledge

Certainly, approaches mentioned above do not converge on a clear conceptualisation of PCK. Indeed they portray differences of opinion and a lack of clarity about the nature of PCK and its development. Research approaches consider PCK as a knowledge dimension on either: (1) a cross-subject level; (2) a discipline-specific level; (3) a domain-specific level; or (4) a topic-specific level. Some researchers also hold the view that PCK can be considered as a knowledge dimension regarding several levels. In recent studies, PCK seems more often to refer to a broad and general form of knowledge, sometimes even losing its discipline-specificity. Fernández-Balboa and Stiehl (1995), for instance, analyse PCK in professors across several fields, including biology, business, and education, among others. However, in line with Hashew (2005), the author argues that PCK seems to have lost one of its most important characteristics, namely its topic-specificity. The work by Smith, diSessa, and Roschelle (1993), for instance, reminds us that knowledge is concept-specific and highly context-sensitive. For instance, the *knowledge in pieces* framework developed by diSessa calls for viewing knowledge as microstructures coming in a loose structure of quasi-independent, atomistic knowledge pieces.

Final Remarks: Future Directions

Although the various frameworks and models on the construct of mathematics teachers' knowledge have provided crucial insights on what mathematics teachers' knowledge is about, several of the discipline-specific frameworks represent conceptualisations of mathematics teachers' knowledge by a very general approach that seem ad hoc. The author, by contrast, does not believe in the existence of a general framework on teachers' knowledge but rather thinks that in investigating the form and nature of teachers' knowledge various frameworks may be discovered, which will be quite specific to particular mathematical concepts and individuals.

The author calls for paying attention to investigating what in this paper is called *knowledge for teaching mathematics* considered as a pool of personal and private constructed pieces of knowledge that have been transformed along a variety of knowledge bases identified by previous research investigating the multidimensionality of teachers' knowledge. In more detail, this work emphasises the view that teachers' professional knowledge specialised for teaching mathematics is the repertoire of *knowledge atoms* that have been transformed along: (1) knowledge of students' mathematical understanding (KSU); (2) knowledge of learning mathematics (KLM); and (3) knowledge of teaching mathematics (KTM), taking (4) mathematical content knowledge per se (MCK per se) and

(5) mathematical content knowledge for teaching (MCK for teaching) as the cornerstones (see, Figure 3). Notice that: (i) the notion of *transformation* implies that the constituent knowledge bases are inextricably combined into a new form of knowledge that is more powerful than the sum of its parts (*form*); (ii) in contrast to Shulman and his proponents' work, it is KSU, KLM, and KTM, together with MCK per se and MCK for teaching that build the knowledge dimensions that serve as the constituent knowledge bases for teaching mathematics (*source*); (iii) the notion of *knowledge atom* indicates that knowledge is of a microstructure, highly context-sensitive, and concept-specific and has to be considered as of a fine-grained size (*nature*); and (iv) The notion of *repertoire* indicates that knowledge is personal and private and that teacher education programs can only provide (as good as possible) rich resources for building up a fruitful repertoire of knowledge atoms.

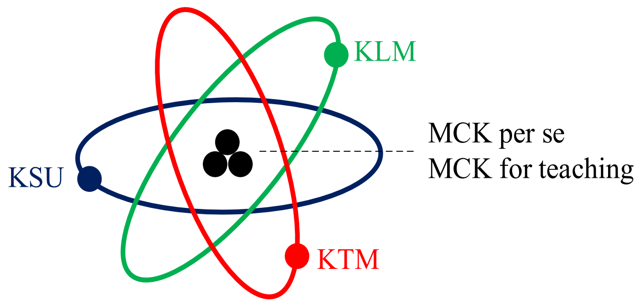


Figure 3: The *knowledge atom*

Acknowledgements

I wish to thank Gabriele Kaiser, Chris Rasmussen, and Alan H. Schoenfeld for their helpful comments in discussions on several key ideas put forward in this paper. The opinions, findings, and conclusions expressed in this paper are those of the author and do not necessarily reflect the views of the researchers mentioned.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Baumert, J., Kunter, M., Blum, W., Brunner, M, Voss, T., Jordan, A., Klusmann, U., & Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180.

Blömeke, S., Hsieh, F.-J., Kaiser, G., & Schmidt, W. H. (Eds.). (2014). *International perspectives on teacher knowledge, beliefs and opportunities to learn. TEDS-M results*. Heidelberg: Springer.

Blömeke, S., Kaiser, G., & Lehmann, R. (Eds.). (2010). *TEDS-M 2008: Professionelle Kompetenz und Lerngelegenheiten angehender Mathematiklehrkräfte für die Sekundarstufe I im internationalen Vergleich* [Cross-national comparison of the professional competency of and learning opportunities for future secondary school teachers of mathematics]. Münster: Waxmann.

Bromme, R. (1994). Beyond subject matter: A psychological topology of teachers' professional knowledge. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winkelmann (Eds.). *Mathematics didactics as a scientific discipline: The state of the art* (pp. 73-88). Dordrecht: Kluwer.

diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition and Instruction, 10*(2-3), 105-225.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics, 21*(6), 521-544.

Fennema, E., & Franke, K. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-163). New York: Macmillan.

Fernández-Balboa, J. M., & Stiehl, J. (1995). The generic nature of pedagogical content knowledge among college professors. *Teaching and Teacher Education, 11*(3), 293-306.
Harel, G. (2008). A DNR perspective on mathematics curriculum and instruction. Part II: With reference to teacher's knowledge base. *ZDM – The International Journal on Mathematics Education, 40*, 893-907.
Hashweh, M. Z. (2005). Teacher pedagogical constructions: a reconfiguration of pedagogical content knowledge. *Teachers and Teaching, 11*(3), 273-292.
Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111-156). Charlotte: NCTM/Information Age Publishing.
Kilpatrick, J., Blume, G., & Allen, B. (2006). *Theoretical framework for secondary mathematical knowledge for teaching.* University of Georgia & Pennsylvania State University.
Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Assoc.
Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Ledermann (Eds), *Examining pedagogical content knowledge* (pp. 95-132). Dordrecht, The Netherlands: Kluwer.
Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.
Rowland, T., & Turner, F. (2008). How shall we talk about 'subject knowledge' for mathematics teaching? *Proceedings of the British Society for Research into Learning Mathematics, 28*(2), 91-96.
Scheiner, T. (2015). Lessons we have (not) learned from past and current conceptualizations of mathematics teachers' knowledge. To appear in *Proceedings of the 9th Congress of European Research in Mathematics Education*. Prague, Czech Republic: CERME.
Schilling, S. G., Blunk, M., & Hill, H. C. (2007). Test validation and the MKT measures: generalizations and conclusions. *Measurement: Interdisciplinary Research and Perspectives, 5*(2–3), 118–127.
Schoenfeld, A. H. (2010). *How we think: A theory of goal-oriented decision making and its educational applications*. New York: Routledge.
Schoenfeld, A. H., & Kilpatrick, J. (2008). Toward a theory of proficiency in teaching mathematics. In D. Tirosh & T. Wood (Eds.), *International handbook of mathematics teacher education* (Vol. 2, pp. 321-354). Rotterdam, The Netherlands: Sense Publishers.
Schwab, J. J. (1978). Education and the structure of the disciplines. In I. Westbury & N. J. Wilkof (Eds.), *Science, curriculum, and liberal education* (pp. 229-272). Chicago: University of Chicago Press. (Original work published 1961).
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.
Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1-22.
Shulman, L. S. & Sykes, G. (1986). *A national board for teaching? In search of a bold standard: A report for the task force on teaching as a profession*. New York: Carnegie Corporation.
Smith, J. P., III, diSessa, A. A,, & Roschelle, J. (1993). Misconceptions reconceived: a constructivist analysis of knowledge in transition. *Journal of the Learning Sciences, 3*(2), 115-163.
Sternberg, R. J., Horvath, J. A. (1995). A prototype view of expert teaching. *Educational Researcher, 24*(6), 9-17.
Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework*. East Lansing, MI: Teacher Education and Development International Study Center, College of Education, Michigan State University.
Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, R., et al. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)*. Amsterdam: IEA.

Understanding Geometric Ideas: Pre-service Primary Teachers' Knowledge as a Basis for Teaching

Rebecca Seah
RMIT University
<rebecca.seah@rmit.edu.au>

This paper reports part of an ongoing investigation into aspects of pre-service teachers' geometric knowledge. One hundred and fifty-two Australian pre-service teachers responded to a series of questions that reflect the type of knowledge teachers are expected to know and teach. Analysis of their responses shows that teacher knowledge can be understood through the interplay between individual teachers' formal figural concepts and personal figural concepts. Errors and misconceptions of geometric properties can be addressed by strengthening the link between formal and personal knowledge through visualisation.

Introduction

As one of the oldest disciplines, the learning of geometry is an important aspect of developing intuition in mathematics, spatial reasoning and visualising skills, deductive reasoning, logical argument, and proof (Jones, 2002). Spatial reasoning, the capacity 'to see, inspect and reflect on spatial objects, images, relationships, and transformations' (Battista, 2007, p. 843) are linked to many technological advances and scientific discoveries. It consistently plays a critical role in influencing educational and occupational outcomes of individuals who go on to achieve advanced education credentials in science, technology, engineering, and mathematics (Graham & Pegg, 2011). Despite its importance, there has been scant attention given to research in geometry when comparing to content such as number, algebra and measurement (MacDonald, Davies, Dockett & Perry, 2012). The declining research emphasis has direct and significant impact on the teaching of geometry at all school levels.

To begin, the introduction of new topics in mathematics such as probability, statistics, and computer science has resulted in a reduction of time devoted to the study of geometry in many mathematics classrooms (Mammana & Villani, 1998). Beginning teachers taught under curricula that neglected geometry are likely to overlook the importance of visual and spatial reasoning, as seen in the absence of visual and spatial reasoning mentioned in the Australian Curriculum: Mathematics (Lowrie, Logan, & Scriven, 2012). Indeed, geometry learning today is characterised by memorising the vocabulary and applying formulae in routine arithmetic calculations (Barrantes & Blanco, 2006). There is also a lack of theories to support instructional design efforts. Much of the research into the development of geometric thinking is largely framed within the van Hiele levels (Owen & Outhred, 2006). These studies reported that many students struggle with recognizing geometrical shapes in non-standard orientation, perceiving class inclusions of shapes, visualising geometrical solids in 2D format, and solving problems that require spatial reasoning (Elia & Gagatsis, 2003; Shaughnessy, 1986). Many pre-service and experienced teachers share the same misconceptions about geometry as the students whom they will eventually teach (Fujita & Jones, 2007; Wang & Kinzel, 2014). While the van Hiele levels have provided a general description of the geometric development, they lack the depth needed to inform instructional design (Battista, 2007). Specifically, van Hiele's labelling of 'visual' to the

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins*
(Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 571–578. Sunshine Coast: MERGA.

lowest level is problematic because visualisation is needed at all levels of development (Jones, 2002).

While the key to supporting meaningful student learning lies in teachers possessing a number of identifiable and differentiable knowledge bases termed mathematical knowledge for teaching (Ball & Hill, 2008), codifying the type of geometric knowledge teachers need is difficult. As a discipline, geometry has grown to include more than 50 different aspects and theories (Graham, Bellert, & Pegg, 2007). Disagreements abound in the aims, content and methods of teaching from primary years to higher education level. As such, 'there has not yet been found – and perhaps there does not exist at all – a simple, clean, linear, "hierarchical' path from elementary to the more advanced achievements of geometry' (Mammana & Villani, 1998, p. 337). All types of geometric concepts appear to develop over time, becoming increasingly integrated and synthesised (Jones, 2002, p. 130). Secure knowledge of two and three dimensional shapes then acts as a conceptual glue that provides coherence and relevance to the learning of more advance geometry (Usiskin, 2012).

Much of the difficulties involved in learning two and three dimensional shapes are caused by a disjuncture between personal geometric knowledge derived from experience and formal geometric knowledge deriving from axioms, definitions, theorems, and proofs. Not so well known is the construct of visualisation and its role in bridging this gap to support learning. Available research suggests that learners tend to be better at drawing a correct image of a shape than providing a definition (Fujita & Jones, 2007). Many learners also have a tendency to make decisions based on figural constraints rather than on formal geometric knowledge (Fischbein, 1993).

This paper reports part of an ongoing investigation into teacher geometric knowledge. The larger study focuses on developing frameworks that can contribute to the design of instructional sequences. The responses of 152 pre-service teachers are considered in order to: (a) determine the gaps between Australian pre-service teachers' personal and formal geometric knowledge; and (b) the role of visualisation in the construction of geometric ideas.

Theoretical Framework

Geometry deals with mental entities constructed through the use of geometrical representations. In the form of points, lines, angles, and shapes, these are not simply representations of actual objects experienced in the world. Rather, they are used in an attempt to take an abstract concept and make it concrete (Phillips, Norris & Macnab, 2010). Geometric representations encompass both figural and conceptual characters (Fischbein, 1993). Figural characters depict properties that represent a certain shape and can be classified as external (embodied materially on paper or other support) or iconical (centred on visual images) (Mesquita, 1998). According to Mesquita, figures can also be determined in terms of 'finiteness' (referring to specific forms) and 'ideal objectiveness' with no reference made to specify its forms. For example, the image ◇ may be considered as a square with the unit of 3 (finiteness) or a quadrilateral with no reference made to its form (idea objectiveness). On the other hand, conceptual characters are *concept image* - the collective mental pictures, their corresponding properties and processes that are associated with the concept (Vinner, 1991). Such an image represents an ideal phenomenon, bound by its *formal concept definition* - a form of words used to specify that concept (Tall & Vinner, 1981, p. 152), and developed through the process of visualising.

Phillips, Norris, and Macnab (2010) found 23 definitions and explicit statements relating to visualisation. They point to a three-fold distinction between physical objects serving as: visualisations; mental objects pictured in the mind; and cognitive processing in which objects are interpreted within the person's existing network of beliefs, experiences, and understanding. Individuals develop their own personal *concept images* and *concept definitions* through experience. They may be referred to as *personal figural concepts* whereas '*formal figural concepts*' refer to *concept image* and *concept definitions* that are based on the axiomatic system (Fujita & Jones, 2007). Problems with visualisation may create disjuncture between *personal figural concepts* and *formal figural concepts.* A learner's first encounter with any geometric ideas is often through the use of objects or geometrical figures. Definitions are used to help form a concept image. Once the image is formed, the definition becomes dispensable or even forgotten (Vinner, 1991). From a didactical point of view, the role visualisation play in the interaction between *personal figural concepts* and *formal figural concepts* may help to understand how geometric knowledge is constructed and thereby inform pedagogical and curricular decisions.

Method

A total of 152 Australian primary pre-service trainee teachers in the third year of a four-year primary teacher education course participated in this study. The participants had undertaken two method courses on number, measurement, geometry, probability, and statistics and were reminded of the geometry topics they have studied prior to the study. Five multiple choice and two short answer questions were presented to the participants and relate to pi, angle, and properties of two and three dimensional shapes. They represent a sample of concepts participants are expected to know and teach. Details of the questions together with the analysis of the data are presented below.

Results and Discussions

The results of participants' correct responses on five multiple choice questions are summarised in Figure 1. No questions obtained 100% accuracy. The best performance was question 2 while the poorest score was question 3.

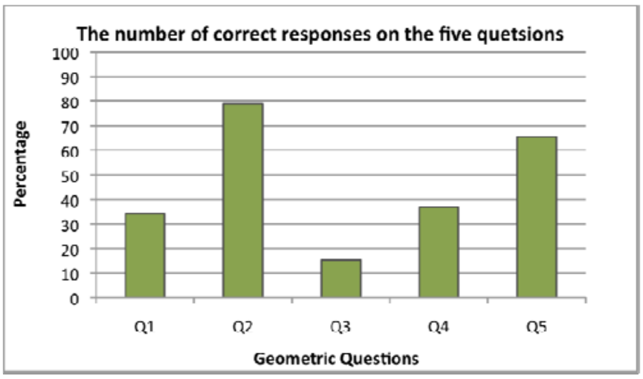


Figure 1. The amount of correct responses on the five questions

The first question asked the participants to select the most appropriate statement about π. It then asked participants to describe an activity that develops an understanding of this relationship. Knowledge of ratio written as a fraction and the relationship between circumference and diameter, through visualisation, can help participants to deduce the correct answer. The results spread across the four options (Table 1). Two thirds of the participants understood the relationship between circumference and diameter whereas 50 participants taught it is related to circumference and radius. Some participants scribbled down $\pi r^2 = C$, $d = r^2$, $2\pi r$, $\pi = r \times 3.14$, $d = 3 \times C$ or πr^2 to help them determine the right answer. Others drew diagrams (Figure 2). None of these participants answered correctly.

Table 1
Breakdown of Responses for Question 1

Questions:	Responses	
1. Select the correct statement about pi	(No.)	
a. Pi is the ratio of circumference to radius in a circle	16	11%
b. Pi is the ratio of circumference to diameter in a circle	52	34%
c. Pi is the ratio of radius to circumference in a circle	34	22%
d. Pi is the ratio of diameter to circumference in a circle	50	33%

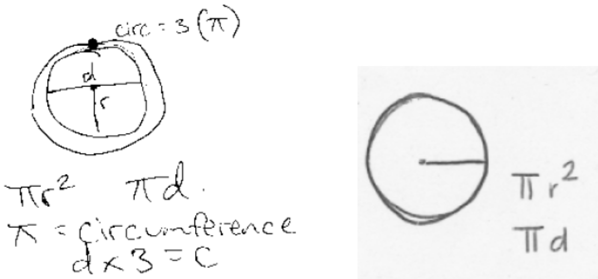


Figure 2. Drawings of two participants.

When asked to describe an activity to develop an understanding of this relationship, many participants mentioned measuring round objects of different sizes and then compare the results to establish the connection. However, a large number of descriptions, as shown below, lacked clarity and showed a lack of *formal figural concept* for the circle.

> Participant A: Pi is the relationship between the radius and circumference. Measure the diameter, half it and use the pi formula to work out the circumference.
>
> Participant B: The diameter is two times the radius, which is half the circumference. Students can measure different bottles and then half the total of the object measured.
>
> Participant C: Circumference is half double the diameter or diameter is half the circumference.
>
> Participant D: Circumference being the distance from one side to the other within the shape, and the diameter being the distance around the shape. I would draw a large circle on the ground and have students use formal and informal or standard and non-standard units to measure the two and see the difference.

During the method course, many pre-service teachers were intrigued by the history of π and methods used by mathematicians to determine the ratio. While almost all could recite π as equal to 'three point one four', few understood that it is an expression of a relationship between the circumference of a circle and its diameter. It would appear that despite the course work, many participants continued to demonstrate a lack of *formal figural concep*t for pi. They did not understand the relationships the formulas they have written sought to express. They also could not infer from the diagrams that since the circumference of any circle is about three times larger than its diameter (based on visualising), the correct answer will have to be 'b' - pi is the ratio of circumference to diameter (based on number understanding). Several participants viewed the diameter as half the size of circumference, albeit confusing both terms.

Question two assessed participants' knowledge of two dimensional shapes. It received the highest correct response (Table 2). Among the 120 correct responses, 77 participants drew figures to obtain the answer. One participant drew and wrote 'equal opposite sides that never meet'. Although the definition is not entirely correct, it showed her attempt to use both her personal *concept definition* and *concept figure* to obtain the answer (Figure 3).

Table 2
Breakdown of Responses for Question 2

Questions:	Responses	
2. David thinks of a regular 2D shape. It has only 3 pairs of parallel sides. The shape could be	(No.)	
a. A parallelogram	17	11%
b. A pentagon	10	7%
c. An octagon	5	3%
d. A hexagon	120	79%

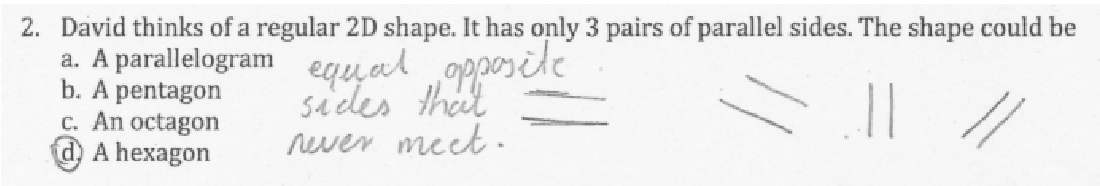


Figure 3. Using definition and figures to deduce the right answer.

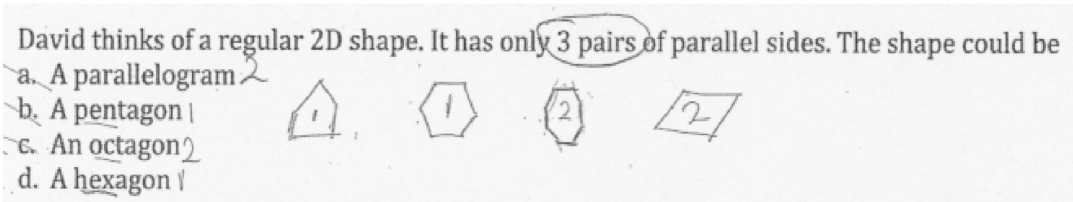


Figure 4. A participant's attempt to draw shapes to solve the problem.

Seventeen (11%) participants answered parallelogram when asked to determine a shape with only 3 pairs of parallel sides. They could have assumed 'parallel sides' as synonym to 'parallelogram'. Those (10 participants, 7%) who chose pentagon could have confused the Greek prefixes of 'penta' and 'hexa' whereas it is unclear how five (3%) participants chose octagon. Among them, one participant drew the four options and attempted to identify the parallel lines (Figure 4). His drawing indicated that he understood parallel lines could be represented vertically (| |), horizontally (=), or diagonally (/ /). However, he could only identify one pair of parallel lines for hexagon base on his diagram. Since the other pair of lines (/ \ and / \) did not rest on the same plane, he concluded that they are not parallel. Because the question asks for 3 pairs, he chose 'octagon' as it has more than six sides. In this case, his visual interpretation of the diagram was incorrect and he did not have sufficient *formal figural concept* for the regular hexagon.

The participants' knowledge of solids was weak and appears to be restricted to prism. Question 3 received the lowest score with 23 (15%) students responding correctly (Table 3). Forty percent of the participants inferred that 'deca' means 10, ignoring the 'do', deduced that a dodecahedron must have 10 faces. Fifty participants may have thought that tetrahedron is made up of triangles with three vertices and so gave the response ‘d’. These participants did not comprehend the Greek origin of these terms. Unlike the hexagonal prism, whose image is easier to be formed in the mind, the participants may not have had sufficient experience with the solids listed in question 3. As such, they were unable to represent three dimensional shapes using two dimensional diagrams. Their responses also suggested a lack of *concept definition* for three dimensional shapes. For question 5, 99 (66%) participants comprehended that a hexagonal prism has 8 faces, 18 edges and 12 vertices. Although 13 participants also knew that a hexagonal prism has 8 faces, they assumed that it has 16 edges instead of 18. This could be due to a counting error or that they were engrossed in the term hexagonal to mean ‘6’. Thirty-nine participants presumed that ‘hexagonal’ meant ‘six’ and chose either ‘b’ or ‘c’.

Table 3
Breakdown of Responses for Question 3 and 5

Questions:	Responses	
3. Select the correct statement about 3D shapes.	(No.)	
a. A dodecahedron has 10 faces	61	40%
b. An octahedron has 6 vertices	23	15%
c. A cube has as many faces as vertices	17	11%
d. A tetrahedron has twice as many edges as vertices	50	33%
5. A hexagonal prism has		
a. 8 faces, 18 edges and 12 vertices	99	66%
b. 6 faces, 16 edges and 10 vertices	10	7%
c. 6 faces, 12 edges and 10 vertices	29	19%
d. 8 faces, 16 edges and 12 vertices	13	9%

An angle is a form of measurement that calculates the amount of turn from one direction to another. Knowing that polygons can be viewed as containing triangles helps understand the patterns for finding the sum of the internal angles for a polygon. Few participants comprehend this idea. When asked to determine the internal angles of regular polygons (Question 4, Table 4), only 55 participants (36%) gave the correct answer.

Table 4
Breakdown of Responses for Question 4

Questions:	Responses	
4. The internal angle of a regular pentagon is		
a. 120°	56	37%
b. 108°	55	36%
c. 110°	35	23%
d. 102°	5	3%

When asked how they would define an angle and demonstrate to a child that the sum of the interior angles of a triangle is always 180 degrees, a number of participants attempted to describe how an angle looks like rather than stating the nature and scope of an angle and its relation to measurement (Figure 5). Also, they could not provide an activity to help a child construct this idea.

> Participant A: An angle is a particular line in which something can be of varying degrees for example the angle could be 90° or 180°.
>
> Participant B: An angle is the degree of two points of radius from a center (starting) point.

Others simply ignored the definition and described how they would get children to draw and measure angles. One participant understood the sum of the interior angles of a triangle is always 180° but drew a triangle with three 90° (Figure 5), demonstrating a lack of *concept image* for triangles.

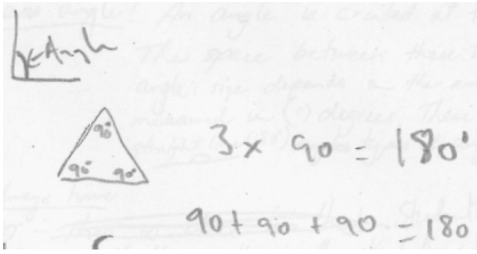


Figure 5. An attempt to define 'angle' by one participant

Conclusion

Teaching for geometric and spatial reasoning requires teachers to have a conceptual understanding of the structures and properties of shapes and solids, their positions in space, and the connectedness between them in the formation of theorems and the learning of other mathematical concepts. The findings indicate that only a small group of pre-service teachers demonstrated sufficient *formal figural concept* knowledge relating to the topics addressed in this study. Many were not ready to teach geometry at the level required of them. While the van Hiele model suggests that these participants are still at level 2 or below and show a lack of geometrical reasoning ability, the constructs of *personal figural concept* and *formal figural concept* provides greater insights into individuals' understanding of geometric ideas.

Individuals' *personal figural concepts* are constructed through experience with various geometric figures and the definitions attributed to these representations. The conceptual characteristics of a figure ares governed by its definition, which in turn is a statement that describes the nature, scope and meaning of a particular concept. For *personal figural concept* to be aligned with *formal figural concept*, well-developed *concept image* and *concept definitions* through visualisation are needed. The findings reveal that many participants' mental images of geometric shapes showed a lack of conceptual understanding. For example, they were able to draw and identify circumference and diameter but did not have the *concept definition* needed to comprehend the relationship between them. They could describe how an angle looks like but were unable to reason using properties of triangles. Many also could not accurately visualise and interpret the figures they had drawn, assuming that diameter was half the length of circumference. They also lacked a *personal figural concept* for three dimensional shapes, suggesting the lack of knowledge to this topic.

Similar to Fujita and Jones' (Fujita & Jones, 2006) findings, regression has happened after participants have completed the method course. One explanation could be that the geometric ideas presented were new to them and have not influenced their underlying beliefs and cognitive processes. This, coupled with their school experience may be the reason why participants can recite formulas but cannot provide the *concept definition*. This study only addresses a limited range of geometric ideas. Further research is needed to investigate the extent of teacher geometric knowledge and classroom practices, and how tasks can be designed to challenge and promote visualisation in the construction of *formal figural concepts*.

Reference:

Ball, D. L., & Hill, H. C. (2008). Measuring teacher quality in practice. In D. H. Gitomer (Ed.), *Measurment issues and assessment for teaching quality* (pp. 80-98). Thousand Oaks, CA: Sage.

Barrantes, M., & Blanco, L. J. (2006). A study of prospective primary teachers' conceptions of teaching and learning school geometry. *Journal of Mathematics Teacher Education, 9*, 411-436.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, North Carolina: Information Age Publishing.

Elia, I., & Gagatsis, A. (2003). Young children's understanding of geometric shapes: The role of geometric models. *European Early Childhood Education Research, 11*(2), 43-61.

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics, 24*(2), 139-162. doi: 10.1007/BF01273689

Fujita, T., & Jones, K. (2006). Primary trainee teachers' knowledge of parallelograms. *Proceedings of the Bristish Society for Research into Learning Mathematics, 26*(2), 25-30.

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education, 9*(1), 3-20.

Graham, L., Bellert, A., & Pegg, J. (2007). Supporting students in the middle school years with learning difficulties in mathematics: Research into classroom practice. *Australasian Journal of Special Education, 31*(2), 171-182.

Graham, L., & Pegg, J. (2011). Evaluating the QuickSmart numeracy program: An effective Australian intervention that improves student achievement, responds to special education needs, and fosters teacher collaboration. *The (Korean) Journal of Educational Administration, 29*(2), 87-102.

Jones, K. (2002). Issues in the teaching and learning of geometry. In L. Haggarty (Ed.), *Aspects of teaching secondary mathematics: Perspectives on practice* (pp. 121-139). London: RoutledgeFalmer.

Lowrie, T., Logan, T., & Scriven, B. (2012). Perspectives on geometry and measurement in the Australian Curriculum: Mathematics. In B. Atweh, M. Goos, R. Jorgensen, & D. Siemon (Eds.), *Engaging the Australian National Curriculum: Mathematics - Perspectives from the field* (pp. 71-88). Online Publication: Mathematics Education Research Group of Australasia.

MacDonald, A., Davies, N., Dockett, S., & Perry, B. (2012). Early childhood mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Research in mathematics education in Australasia 2008-2011* (pp. 169-192). Rotterdam: Sense Publishers.

Mammana, C., & Villani, V. (1998). *Perspectives on the teaching of geometry for the 21st century*. Dordrecht: Kluwer Academic Publishers.

Mesquita, A. L. (1998). On conceptual obstacles linked with external representations in geometry. *Journal of Mathematical Behavior, 17*(2), 183-196.

Owen, K., & Outhred, L. (2006). The complexity of learning geometry and measurement. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education. PME (1976-2006). Past, Present and future* (pp. 83-115). Rotterdam: Sense Publishers.

Phillips, L. M., Norris, S. P., & Macnab, J. S. (2010). *Visualization in matheamtics, reading and science education*. Dordrecht: Springer.

Shaughnessy, B. (1986). Characterising the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31-48.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics, with special reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151-169.

Usiskin, Z. (2012). *The shapes of geometry and their implications for the school geometry curriculum*. Paper presented at the 12th International Congress on Mathematics Education, COEX, Seoul, Korea.

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 65-81). Dordrecht: Kluwer Academic Publishers.

Wang, S., & Kinzel, M. (2014). How do they know it is a parallelogram? Analysing geometric discourse at van Hield level 3. *Research in Mathematics Education*. doi:10.1080/14794802.2014.933711

Mathematical Language Development and Talk Types in Computer Supported Collaborative Learning Environments

Duncan Symons
The University of Melbourne
<duncan.symons@unimelb.edu.au>

Robyn Pierce
The University of Melbourne
<r.pierce@unimelb.edu.au>

In this study we examine the use of cumulative and exploratory talk types in a year 5 computer supported collaborative learning environment. The focus for students in this environment was to participate in mathematical problem solving, with the intention of developing the proficiencies of problem solving and reasoning. Findings suggest that students engaged in exploratory talk may more regularly attempt the use of technical (tier 3) mathematical vocabulary.

Introduction

The development of mathematical language is essential to student understanding and growth in mathematics; see for example (Austin & Howson, 1979; Morgan, Craig, Schuette, & Wagner, 2014). In this paper we will examine the use of mathematical language by Year 5 students, in the context of a Computer Supported Collaborative Learning (CSCL) environment.

We were interested in the 'talk types' (Mercer & Wegerif, 1999) that would become evident during student online discussion. The three types we looked for in discussion were Mercer and Wegerif's 'disputational talk', 'cumulative talk', and 'exploratory talk'. Our primary intention in this paper is to answer the question; 'will the density of use of Beck, McKeown and Kucan's (2002) tier-three vocabulary (see below) be greater in identified examples of 'exploratory talk' compared with the other two talk types?' A secondary question that we also aim to explore is, 'Is there a relationship between students' (teacher identified) mathematical ability and their use of tier-three mathematical vocabulary?' Finally, we aim to investigate whether the density of tier-three mathematical vocabulary use changed throughout the intervention and also if there were any ability groups (below level, at level or above level) where changes were more obvious.

Talk Types

Mercer and Wegerif (1999) identified three broad talk types when they analysed many hours of videotaped discussion amongst British primary age students: 'disputational talk', 'cumulative talk' and 'exploratory talk'. Designating and analysing talk types in this way allowed the authors to consider the ways that students use language to collaboratively construct knowledge and problem solve. his approach to the analysis of student discussion since it is prominent within CSCL literature. We rely on Mercer and Wegerif's (1999) definitions for the three talk types in this study:

> **Disputational talk**, which is characterised by disagreement and individualised decision making. There are few attempts to pool resources, or to offer constructive criticism of suggestions. Disputational talk also has some characteristic discourse features – short exchanges consisting of assertions and challenges or counter-assertions.

> **Cumulative talk**, in which speakers build positively but uncritically on what the other has said. Partners use talk to construct a 'common knowledge' by accumulation. Cumulative discourse is characterised by repetitions, confirmations and elaborations.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 579–586. Sunshine Coast: MERGA.

> **Exploratory talk**, in which partners engage critically but constructively with each other's ideas. Statements and suggestions are offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. Compared with the other two types, in exploratory talk knowledge is made more publicly accountable and reasoning is more visible in the talk (p. 85).

The discourse analysed in this paper occurred in the online environment. That is, it occurred asynchronously in a discussion board. (However we use the term 'utterances' when referring to students' statements.) Like Mercer and Wegerif, we were interested in gaining an understanding of how students jointly construct knowledge. Our interest in identifying and analysing the three talk types stemmed from the author's speculation that CSCL environments designed to foster greater levels of exploratory talk are more likely to result in higher levels of higher order and critical thinking. We hypothesised that given that 'exploratory talk' is represented by talk where public accountability is evident, in addition to reasoning being visible, a greater density of technical mathematical vocabulary may be present when students engage in this talk type.

Vocabulary - The Three-Tier Framework

Beck, McKeown, and Kucan (2002) established a basic system for the classification of vocabulary. In their system vocabulary is classified as tier-one, tier-two and tier-three. They established these terms as a means to frame teaching and learning in the area of vocabulary development. Their framework has since been appropriated by various researchers for the purposes of understanding aspects of mathematical language development, see for example (Marzano & Simms, 2013).

Tier-one vocabulary encompasses everyday language. These words are the most basic and are used with a high degree of frequency, particularly in spoken language. Tier-one vocabulary includes such words as 'warm', 'cold', 'talk', 'cat', 'dog' etc.

Tier-two vocabulary represents words that are primarily used in written language. They are words with a very high degree of utility. These words are generally utilised by more mature users of language. As a result of their usage primarily in written language, they can be more difficult for students to learn independently. Examples of tier-two vocabulary include, 'proceed', 'following', 'retrospect', 'contradictory' etc.

Tier-three vocabulary includes words with a technical or domain-specific usage. Generally, these words are of a very limited usage, however in the case of this study we see them occurring more frequently because of the mathematical context of the study. They are generally the most difficult words for students to acquire because of the very limited opportunities students have to experiment with them. In the context of mathematics, Tier – three vocabulary would include, for example: 'formula', 'equation', 'symmetry', 'median'.

Method

The present study took place as part of a project in an Australian suburban primary school over a ten-week period. The first author had previously taught at this school and so was familiar with their curriculum and the students' computer skills.

Participants in the project were 54 Grade 5 students (ranging in age between 10 – 12 years old). There were 26 boys and 28 girls between two classes. Thirty-two percent of the student participants are from a language background other than English (ACARA, 2015). This had implications for this study as working within a CSCL environment places significant demands on students' general literacy abilities. The 54 students were placed in 10 mixed ability groups within the online space. These groups were created on the basis of teacher judgement (students were classified as either below level, at level or above level in mathematics). Teachers classified students on the basis of a series of tests they had conducted, assessing the students' level of procedural and algorithmic fluency and general understanding across key areas of mathematics.

Over the ten weeks in which the unit was delivered students collaboratively solved and/or investigated nine mathematical problems incorporating aspects of each of the content strands of the Australian Curriculum; namely, Number and Algebra, Statistics and Probability and Measurement and Geometry (ACARA, 2014).

Through a one-to-one netbook program every student had their own access to Microsoft Windows so online collaboration generally took place at the students' homes. This required an internet connection so if students did not have internet access at home, they were given the opportunity at lunch times to access the internet in their classroom.

Students were expected to engage in iterative online discourse where they would build on each other's ideas. This is a principal goal of collaborative mathematical problem solving. No online facilitator took part in the CSCL. This decision was taken in order to avoid discussion and communication between students being stifled by an 'expert'.

However the participants did receive support. Each week for the first 7 weeks, prior to the students commencing work on each online problem, an hour of standard classroom discussion was facilitated by the first author of this paper. This time was spent with the class performing three basic tasks: discussing expectations of behaviour, and appropriate approaches to collaboration within the online space; reviewing the previous week's solutions and discussing challenges and successes that students perceived; explaining, reading through and discussing the following week's problem. In weeks 8 and 9 a different pedagogical approach was taken. The level of support was greatly reduced, no discussion of the problems took place and students were asked to solve the problems in their class time but only through working in the CSCL environment.

Analysis of data for this paper was undertaken using qualitative data analysis software NVivo (2014) and was based on two forms of coding. Firstly, all online discussion within each of the ten small groups was coded in terms of talk type. For this analysis one of the three talk types (cumulative, exploratory or disputational) was identified for each discussion, for each group, for each problem. As indicated by Mercer and Wegerif (1999), often this meant that whilst a predominant talk-type was identifiable aspects of the others were also present. In these cases we coded according to the one we believed was *most* in evidence. When disagreement within a group occurred in a manner that moved the group forward in their thinking we chose to classify these episodes as 'exploratory' talk rather than 'disputational' talk. We believe that the lack of disputational talk may be a result of regular teacher led classroom discussions about constructive modes of online communication. Secondly, all online discussion was coded for examples of tier-three mathematical vocabulary. This coding is undertaken at the word level. After coding the two respective approaches were cross-tabulated to detect patterns and associations.

Results and Discussion

Figure 1 shows an example illustrative of discussion from the online message board coded as exploratory talk with tier-three mathematical vocabulary bolded. The discussion in Figure 1 is provided verbatim (with pseudonyms) from the online space. In this example students worked on a problem where they were required to make a conjecture about whether cats' names or dogs' names are generally longer. The students researched a number of the most common cats' names and dogs' names; calculated the mean, median and mode of these data, graphed results using Microsoft Excel and discussed their results.

Interestingly, the only two talk types that we detected were 'cumulative' and 'exploratory'. In this example of exploratory talk we see the students attempting to decide upon appropriate mathematical vocabulary to describe the three common measures of central tendency. One student offers the word 'maintain' as a possibility. Eventually though, they are able to arrive at the conclusion that the words 'mean', 'median' and 'mode' are the words that they have been seeking to find. This suggests that students' vocabulary may benefit from the co-negotiation of definitions, trialling and experimentation with new terms that the context of this setting allows.

Sunny	I think dogs and cats are the same number of letters because in my **graph** it came up with 8 fives and eight fives each. So that my Information Please reply Thanks guys
Sienna	Hi Sunny, where is your **graph**?
Sienna	Hi everyone, i have done the exel spread sheet and the names that i have got are feamale and male. i am neally compleated.
Holly	Hey guys what do you do after you have writen down all the names and numbers?
Sienna	hi everyone. What are the three words that we have to do. They are the M words. What are they?
Sunny	Same I forgot about those m words I think one was maintain. I am not sure about if it is right. Please reply under Thanks guys
Sunny	Guys I know the m words they are **mean**, **mode** and **median**. I just remembered today. I hope this helps you in you **bar graph**, **column graph** and lastly **line graph** etc. Please reply if you are on edmodo. Thanks guys And see you tommorow

Figure 1. Example of discussion coded as exploratory talk.

Figure 2 shows an example from the online discussion where cumulative talk is apparent. This discussion is again taken from a small group of students attempting to collaboratively solve the previously described 'Pet Names' problem. In this example we see Annie positioning herself as 'leader' within the group. She repeatedly rephrases her desire for suggestions or agreements related to whether she should provide information about the various pet names. No constructive criticism is present, however eventually we see some 'common knowledge' emerging. In this excerpt of discussion we see no tier-three mathematical vocabulary. The group did not present any analysis of their pet names.

Annie	hey guys do we need to do male and female cats names if you do please post
Kevin	do you what me to rshoq
Annie	what is rshoq?
Kevin	that mans resuch
Annie	no i was thinking i have already done the female and the male cat names and they are french names is that alright with you guys
Annie	and you spell research like this.
Sheldon	hi
Annie	hi just tell me if you guys want to know the names and i will tell you
Annie	i will tell you anyway the female names are:
	Sassy Misty Princess Samantha Kitty Puss Fluffy Molly Daisy Ginger
	Midnight Precious Maggie Lucy Cleo Whiskers Chloe Sophie Lily Coco
Annie	And my male names are:
	Max Sam Tigger Tiger Sooty Smokey Lucky Patch Simba Smudge Oreo Milo Oscar Oliver Buddy Boots Harley Gizmo Charlie Toby

Figure 2. Example of discussion coded as cumulative talk.

Table 1 shows the number of examples of talk types identified and the number of examples of tier-three mathematical vocabulary. The dominant talk type throughout all discussion during the ten weeks of data collection was cumulative talk. Forty-nine examples of cumulative talk were identified, whilst only 27 examples of exploratory talk were identified. Across the data we see an average of between 7 and 8 mathematical tier-3 words used during examples of cumulative discussions, whilst we see between 10 and 11 examples of this type of vocabulary used in examples of exploratory talk. This indicates that students engaged in exploratory talk were more likely to use tier-three vocabulary than when they are engaged in cumulative talk.

Table 1
Tier 3 Vocabulary use in Cumulative and Exploratory talk

Talk Type	Tier-three Vocabulary Used within Talk Types	Identified examples of Talk Type	Average No. Tier-three Vocabulary per Example
Cumulative Talk	361	49	7.4
Exploratory Talk	284	27	10.5

Table 2 shows the density of tier-three mathematical vocabulary use by differing ability levels of students. With the exception of the ‘at level’ boys a possible association can be seen between the density of mathematical tier-three vocabulary use and the student ability level. One possible explanation for the lower than expected use of tier-three vocabulary use in this group is that 7 of the 11 students in this group had a Language Background Other than English compared with 17 out of 54 overall.

Table 2
Density of Tier three Vocabulary use in Student Utterances

	No. Of Students	Tier-three Vocabulary Use	Total No. of Utterances	Tier-three Vocabulary Use per Utterance
Above Level Boys	5	129	198	0.65
Above Level Girls	3	78	96	0.81
At Level Boys	11	77	301	0.26
At Level Girls	12	183	350	0.52
Below Level Boys	7	35	96	0.42
Below level Girls	9	128	308	0.42

Table 3 shows the density of mathematical tier-three vocabulary use throughout the study. There does not appear to be any clear evidence of progressive growth in students' use of mathematical tier-three vocabulary throughout the period. However each problem offered different opportunities. We have also indicated rates of online participation of the ten small groups throughout the period. We see that in weeks 2 and weeks 7 the fewest number of groups participated in online discussion. These weeks also correspond with the lowest number of tier-three mathematical terms used. It is possible to conjecture, that in these two weeks students found it more difficult to engage with the tasks. Even though there was a classroom introduction, including explicit discussion of the required mathematical language the mathematical content required was new and also difficult for some students. For example, in week 7, when they undertook the Pet Names problem, students were required to calculate a central measure (mean, median and mode). The development of skills and understanding in this area of statistics does not appear in the Australian Curriculum (ACARA, 2014) until year 7.

It is also worth considering the change in pedagogical approach that took place in the final 2 weeks of the intervention. The classroom based support and facilitation that the students had benefited from, for the previous 7 weeks was withdrawn in the final two weeks for the purpose of gaining some understanding of whether students could transfer any of the learning that had occurred in the previous weeks without the same high level of support. Taking this into account, the average number of tier-three mathematical terms used per group (with weeks 2 and 7 removed) in the period of high support was 9.9 and in the final weeks without support it was 7.4. Our hypothesis that students should be able to transfer their learning after having received sustained support for the previous 7 weeks appears to be invalid. It is important to consider though the particular area of learning that we are assessing in this context. For each of the weeks before the final 2, the classroom facilitator (the first author of this paper), would introduce the new vocabulary and facilitate an extensive discussion and co-negotiation of these terms with students. Students were being 'pre-loaded' with the tier-three mathematical vocabulary required for the problem they would be discussing in the online environment before they were asked to collaborate. Naturally, they were able to better utilise this vocabulary, having been extensively prepared. As there was no specific mathematical content focus over the period of the intervention, each week a new and different set of vocabulary was required of the students. When the classroom support was taken away, so was the students' opportunity to familiarise themselves and become somewhat comfortable with vocabulary that would be of high utility to them in the online space in that week.

Table 3
Density of Tier three Vocabulary use throughout Intervention

Week	Examples of Tier-Three Vocabulary	Number of Groups Participating in Online Discussion	Average Number of Tier-Three words per (participating) group
Week 1	68	9	7.6
Week 2	19	6	3.2
Week 3	93	9	10.3
Week 4	129	9	14.3
Week 5	108	10	10.8
Week 6	64	10	6.4
Week 7	25	7	3.6
Week 8	85	10	8.5
Week 9	57	9	6.3

Our final analysis allowed us some understanding of if there was any significant difference in the growth in density of tier-three mathematical vocabulary use amongst the ability groups. Weeks 2 and 7 were removed from calculations.

Again, we do not believe that any the three groups showed clear progression in density of use of tier-three mathematical vocabulary over the period. However, some observations are possible. Firstly, all three groups on average used fewer tier-three mathematical words without classroom support than with support. The 'above level' group used 3.6 (per student, per problem) with support and 3.0 without. The 'at level' students used 1.6 with support and 1.2 without support and the 'below level' students used 1.5 with support and 1.1 without support. It appears that the 'above level' students made the greatest gains when provided support and equally their rate of use of these mathematical terms decreased the most of all three groups (whilst still using a greater number of these words than the remaining groups) when support was removed. The average density of tier-three vocabulary use between the 'at level' and 'below level' students appears very similar both with and without support (in fact the change of 0.4 that was evident without support was identical). This however, must be considered alongside the marked difference in density of tier-three mathematical vocabulary detected between 'at level' boys and girls, which we have attributed partially to the high level of LBOTE students in the boys group. Implications include the importance of deliberately teaching mathematical vocabulary and providing opportunities for students to see the value of its use.

Implications

These observations lead to a number of implications for teaching and further research. We hypothesise that if an intervention was replicated over the same period where a single mathematical content area remained the focus, a theorised growth in the density of student tier-three mathematical vocabulary may occur. We also believe that whilst no clear evidence of growth in student use of tier-three mathematical vocabulary is present, other areas of learning may have made clearer growth and student learning may have transferred from the period in which they received great support to the period of support being removed. These areas include the mathematical proficiencies of problem solving and reasoning, critical thinking and the level to which students engage in genuinely collaborative learning. Applying a more fine-grained approach to the coding and analysis of the talk-types, whereby each individual student utterance in the online space becomes the unit of analysis may prove to help investigate these matters.**Error! Reference source not found.**

The data indicate that students will use tier-three mathematical vocabulary more regularly when engaged in exploratory talk than when engaged in cumulative talk. We have also shown that cumulative talk is likely to be the dominant talk type, given the conditions described. We suggest that it may be beneficial to specifically encourage the engagement of students in exploratory talk in order to prompt them to more regularly experiment with newly acquired vocabulary. Explicitly teaching students about the three talk types and discussing their various attributes and characteristics, including why exploratory talk might be the most productive talk type, may promote this. Such teaching would include an explanation of the importance of building a repertoire of technical mathematical vocabulary. It is envisaged that this may result in groups 'self-regulating' their discussion and being aware of when talk had become less productive.

Additionally, data in this study suggests that a relationship exists between student levels of procedural mathematical achievement (as classified by their teacher) and the density of tier-three mathematical vocabulary use. Our data shows, that students classified, as 'below level' less regularly attempted the use of this type of vocabulary than their peers classified as 'above level'. Furthermore, data suggests that LBOTE students are less likely to attempt this high level vocabulary. Further research would be required to test the hypothesis that a targeted approach to the teaching of tier-three mathematical vocabulary may lead to improved results in procedural assessments of mathematical ability.

References

ACARA. (2014). *The Australian Curriculum V5.2*. Australian Curriculum and Reporting Authority Retrieved from http://www.australiancurriculum.edu.au/.

ACARA. (2015). My School. from http://www.myschool.edu.au

Austin, J. L., & Howson, A. G. (1979). Language and Mathematical Education. *Educational studies in Mathematics, 10*(2), 161-197. doi: 10.2307/3482013

Beck, I. L., McKeown, M. G., & Kucan, L. (2002). *Bringing words to life: Robust vocabulary instruction*. New York: Guilford Press.

Marzano, R. J., & Simms, J. A. (2013). *Vocabulary for the common core*. IN: Marzano Research Laboratory.

Mercer, N., & Wegerif, R. (1999). Is exploratory talk productive talk? *Learning with computers: Analysing productive interaction* (pp. 79). London: Routledge.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. *ZDM, 46*(6), 843-853. doi: 10.1007/s11858-014-0624-9

NVIVO. (2014). NVIVO [Qualitative Data Analysis]: QSR International.

The Individual Basic Facts Assessment Tool

Sandi Tait-McCutcheon
Victoria University of Wellington
<sandi.mccutcheon@vuw.ac.nz>

Michael Drake
Victoria University of Wellington
<michael.drake@vuw.ac.nz>

There is an identified and growing need for a levelled diagnostic basic facts assessment tool that provides teachers with formative information about students' mastery of a broad range of basic fact sets. The Individual Basic Facts Assessment tool has been iteratively and cumulatively developed, trialled, and refined with input from teachers and students to meet that need. The development of the tool, results from test trials, and our next steps are described in this article.

The importance of students knowing their mathematics basic facts is not a new phenomenon. The instant recall of basic fact knowledge is acknowledged as an important goal for mathematics education and an essential precursor for students' success in mathematics (Boaler, 2012; Ministry of Education [MoE], 2007a; Tait-McCutcheon, Drake, & Sherley, 2011; van de Walle, 2009; van de Walle, Karp, & Bay-Williams, 2013). Students' fluency with sophisticated tasks such as problem solving and higher-order processing is enhanced by their ability to instantly recall basic facts (Kilpatrick, Swafford, & Findell, 2001). Their short-term memory is freed-up and they are better positioned to focus on the more challenging strategic aspects of the task (Kling & Bay-Williams, 2014; Neill, 2008). Research has also acknowledged a strong correlation between basic facts fluency and mathematics achievement (Kilpatrick et al., 2001).

While the importance of being able to recall basic facts is well established, how this is best achieved has not been well defined. Traditional testing of basic facts has occurred through timed tests of short duration whereby students attempt to solve a specific number of addition, subtraction, multiplication, and/or division problems, randomly written in terms of difficulty (Kling & Bay-Williams, 2014). For example 50 facts within three minutes (Clarke & Holmes, 2011). Such an approach to testing is problematic and unlikely to elicit a true picture of student achievement (Crooks, 1988).

Evidence from previous research into the teaching, learning, and assessing of basic facts in New Zealand (Sherley & Tait-McCutcheon, 2008; Tait-McCutcheon et al., 2011), indicated that practice in schools, while changing, too often had the limited notion that learning basic facts meant learning the multiplication facts. Internationally, assessment tools and teaching interventions have focussed predominantly on multiplication facts (Clarke & Holmes, 2011; Kling & Bay-Williams, 2014; Skarr et al., 2014). As such, there is a strong possibility of a disconnect between the test content and student's class work. Some students could be repeatedly tested on facts they already know whilst others could be tested on facts related to operations they have very little understanding of, or use for.

Our stance is that basic facts are basic because they are fundamental and underpin the student's next learning steps, not because they are easy. They are facts in that they are pieces of mathematical information that are committed to and can be retrieved from long-term memory. The definition of basic facts in this research comes from Neill (2008) "any number or mathematical fact (or idea) that can be instantly recalled without having to resort to a strategy to derive it (p. 19). One implication of this definition is that the notion of *basic facts* is not a stable, fixed body of knowledge but is contextual as the facts being learned change with age and developing mathematical concepts. A second implication is

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 587–594. Sunshine Coast: MERGA.

that because students need to continually increase their fact mastery, all teachers need a robust basic facts programme as an integral part of their mathematics curriculum.

In relation to the issue of timed basic facts tests, McCloskey (2014) asked, “[C]ould the timed test be changed into a different form of performance with more meaningful assessment purposes and yet maintain the traditionalised purpose that teachers and parents seem to value?” (p. 35). This paper is a response to McCloskey’s question. It outlines the development and use of the Individual Basic Facts Assessment (IBFA), a tool for identifying a students’ current level of basic fact knowledge and fluency, an approach to basic fact testing referred to in Tait-McCutcheon et al. (2011).

Uses, Utility, And Apprehension

The authors concerns regarding the questionability of timed tests eliciting a true picture of student achievement have been documented in the literature. Three themes identified from the literature include: assessment measures and uses, what is valued, and the relationship between timed tests and math anxiety.

One assumption commonly made about timed basic facts tests is that correct answers are derived from knowledge. However, because the time given is to complete the whole test students could immediately recall the answers they know and then use a mix of efficient or inefficient strategies to determine other answers (Tait-McCutcheon et al., 2011). For example, Clarke and Holmes (2011) gave students three minutes to complete the test to ensure “knowledge rather than strategisation of solutions” (p. 205), but, this approach assumes that students used the same amount of time to solve each problem. As Kling and Bay-Williams (2014), contended timed testing “offers little insight about how flexible students are in their use of strategies or even which strategies a student selects” (p. 490).

The dilemma of speed versus accuracy and what gets valued is the second theme identified from literature. Popham (2008) suggested there was no value in “pressuring kids to be math perfect in minutes” (p. 87). Seeley (2009) warned against “overemphasizing fast fact recall at the expense of problem solving and conceptual experiences” because such emphasis can give students “a distorted idea of the nature of mathematics and of their ability to do mathematics” (p. 2). The danger being that the speed in which the test was completed could be valued more than the accuracy of the answers, speed could be erroneously equated with mathematical ability or fluency, and students could interpret their responsibility as having to be quick (Boaler, 2012; Kling & Bay-Williams, 2014).

The third theme is the relationship between timed tests and math anxiety. Boaler (2012) claimed a “direct link between timed tests and the development of math anxiety” (p. 2). Timed tests have been shown to trigger math anxiety in all students and the claim from Kling and Bay-Williams (2014) is that “some of our best mathematical thinkers are often those most negatively influenced by timed testing” (p. 490). Stress can block students working memory, causing symptoms similar to stage fright and making even familiar facts unretrievable (Beilock, 2011). The more aware students became of their inability to recall known facts the more apprehensive they became about their performance and results. The stress or anxiety caused by the timed test conditions may pressure students to revert to less efficient strategies such as finger counting, head bobbing, or touch point counting (van der Walle, 2009). For some students the prospect of doing a timed test could be enough to elicit a negative emotional response, with many disliking “not only tests, but also math” (Popham, 2008, p. 87).

Despite the noted disadvantages of traditional timed tests, the assessment of student’s basic facts knowledge remains a requirement and expectation for many teachers and parents (McCloskey, 2014; Seeley, 2009). Our aim was to develop and trial an assessment tool that more accurately measured basic fact recall, provided cleaner data, identified the use of knowledge or strategy, and reduced anxiety.

Method

Design-based research (D-BR) was the most appropriate methodological frame for this research for the following reasons: the iterative, cumulative, and cyclic nature of the research and theory development, the positioning of the research within the naturally occurring phenomena of classrooms, and the flexibility of the research design (Gravemeijer & van Eerde, 2009; Kennedy-Clark, 2013). Gravemeijer and Cobb (2006) used the following adage to explore the underlying philosophy of design research: “if you want to change something, you have to understand it, and it you want to understand something, you have to change it” (p. 45). The researchers of this study determined they wanted to change the current tools for testing students’ basic fact knowledge recall. Once the affordances and limitations of current tools were better understood, we set about designing, trialling, understanding, and improving the IBFA tool.

The theoretical rationale in this study was to understand the teaching, learning, and assessing of basic facts, the applied rationale was to use our empirically supported theories to influence how basic facts are taught, learned, and assessed in New Zealand schools. Context theory related to the challenges and opportunities presented in designing an alternative IBFA tool and outcomes theory related to the outcomes associated with the intervention to improve the teaching, learning, and assessment of basic facts.

Research Settings and Participants

Four schools participated in phases one or two of the IBFA tool design and development. Table 1 provides relevant data of the schools, teacher participants, and students.

Table 1
The research settings and participants

School Name (Pseudonym)	Decile	Category	Teachers	Students	Year Group
Ponga Primary	8	Full Primary	1	23	5 - 8
Nikau Intermediate	4	Intermediate	4	96	7 - 8
Nikau Secondary	6	Secondary	3	63	9
Rimu Intermediate	8	Full Primary	3	81	7 - 8

Data collection and analysis

A mixed methods data collection occurred to allow for a more robust understanding of the learning environment (The Design-Based Research Collective, 2003). Forms of data included observations from researchers, teachers, and students, student test papers, and interviews between researchers, teachers, and students. Data were analysed immediately, continuously and retrospectively alongside literature reviews coupled with the systematic and purposeful implementation of research methods (Wang & Hannafin, 2005).

Patterns, thoughts, and items of interest were noted during the analysis phase. Data were eyeballed “to see what jumps out” (Miles, Huberman, & Saldaňa, 2014, p. 117). For example, each set of stage results were considered for unusual results and explored in relation to the Number Framework (MoE, 2007a), and items located in nearby stages. This process could lead to an item being moved between stages. The design and revision of the IBFA questions were based on the researchers’ anticipations of which stages to place each problem and in what order. As such, “each cycle in the study is a piece of research in itself” (Plomp, 2007, p. 25) that contributed to the growing body of knowledge.

Formative evaluation of both quantitative and qualitative data informed the improvement and refinement of the IBFA tool and guidelines (Kennedy-Clark, 2013). This allowed us to measure the effects of the test and to develop richer pictures of teacher and student knowledge acquisition. A mixed methods approach increased the credibility and adaptability of the research allowing for retrospective analysis and formative evaluation. The positionings of the researchers and teachers within the study also ensured adaptability (Plomp, 2007). Researchers took on the roles of designer, advisor and facilitator without losing sight of being a researcher, and teachers took on the role of researcher, designer, and advisor without losing sight of being a teacher. The inclusion of different expert groups within the study provided an extended degree of rigor and mitigated issues of accessibility (Wang & Hannafin, 2005).

IBFA Tool Design and Development

The IBFA tool was designed, elaborated, trialed, and revised in an attempt to further understand and improve the educational processes of assessing basic facts. The guidelines for understanding and administering the tool are as follows:

> The response time for each item was aligned with the NDP expectations of what knowing means and allowed students 4 seconds to answer one question rather than 5 minutes to answer 100. The assessment includes basic-facts questions written as both number problems (e.g., 9+9 =, which is read as “nine plus nine equals”) and problems written in words (e.g. double what is ten?). To alleviate any prerequisite literacy requirement that could adversely affect students’ mathematical proficiency each item is read aloud to the class as well as displayed visually on a timed slideshow. As it is possible for students to strategise within the four seconds allocated for each item students are asked to annotate their answers with a “k” if they know the answer instantly or with an “s” if they strategise. (Tait-McCutcheon et al., 2011, p. 336)

The first version was designed to meet the following criteria. First to assess facts derived from the Number Framework (MoE, 2007a) and related to The New Zealand Curriculum (MoE, 2007b). Secondly, to provide a visual and aural, readily administered and easily marked test, useable with a whole class that offered reliable, relevant data that could be interpreted and actioned by students, teachers, and parents. Thirdly, to position students as active constructors of meaning by focussing them on individual facts, rather than a collection of facts, addressing the *amount of time* issue for an individual fact, and determining if an answer resulted from knowledge, strategy, or a combination. Fourthly, to address commonly recognised problems that students have when learning a particular set of facts and to include facts commonly found to be problematic and be sufficient in number to identify whether or not a student knows a particular set of facts. Fifthly, to provide results that identify students’ next learning steps, teachers next teaching steps, and a clean stage descriptor for reporting purposes.

IBFA Version One (V1)

IBFA V1 was trialled at Ponga Primary School as part of the research described in Tait-McCutcheon et al. (2011). Given the sample size the tool was found to be fit for purpose, however, the design process identified issues that would need to be addressed in larger scale trialling. For example, the Number Framework (MoE, 2007a, pp. 21 & 22) identifies that at Stage 6 students should know their multiplication facts and some corresponding division facts but not know all of their division facts until Stage 7, so it was unclear which division facts should be located at which stage.

The second trialling of V1 occurred at Nikau Intermediate and Secondary Schools. Teachers at both schools indicated the format was suitable for a range of ages and student abilities. However, the time set for the items (4 seconds) was an issue at the higher stages. While the time allocation was considered the maximum that should be allowed for knowledge recall, a review of the items indicated that concepts such as common factors needed to be found using a mix of knowledge recall and strategising. Such items were either simplified to retain the skill but better target knowledge recall and take less time, or were replaced with items from a different fact set. Also identified was a trend relating to start and change unknown formats (Sarama & Clements 2009). Students tended to find these harder than result unknown, but it was unclear whether this was a teaching issue or due to item difficulty. Clusters of items were explored to identify the appropriate location of particular sets of facts. For example, a cluster of items relating to the subtraction facts to 10 was spread over Stages 4 and 5. Results suggested that these were better located at Stage 5. For the multiplication and division facts over Stages 6 and 7, the numbers in the Stage 6 items were simplified and result unknown format applied to determine if this gave better discrimination.

Matters outside the initial scope of the research were also identified. For example, some students noted their correct answers as a total out of 60 rather than identifying what stage, which sets of facts they had mastered, and what their next learning steps were. Other classes had not used the ‘K’ or ‘S’ notations to indicate if they knew the answer or needed to work it out. The instructions for teachers were revised, as was the answer sheet, to ensure students and teachers better understood the purpose behind the test’s structure and to ensure data from different classes and schools were of a similar standard.

Researchers and teachers made significant contributions to further developing the content of the IBFA V1 and the theories underpinning the use of it (Gravemeijer & van Eerde, 2009). It was important to have synchronicity between both groups as to what the data was telling us and what we were identifying as next teaching and learning steps. As Mason (2002) suggested, this “process of refinement” (p. 181) was also part of the research as teachers reported back what they noticed and provided both practical and scientific ways to enhance the next research phase and their teaching (Gravemeijer & van Eerde, 2009). As such, the creation of V2 started to move the IBFA forward from the Number Framework to include lessons from trialling.

IBFA Version 2 (V2)

Version 2 was trialled at Rimu School. Students marked their own papers with later re-marking by the researchers. Twenty-six papers (32%) were found to have errors. One particular problem with student marking occurred when students ‘lost track’ of where they were up to in the test and produced a set of answers that were misplaced by one or two.

Another was when students put an unusual format for an answer. Teachers can adapt for such issues but students struggled to do so.

The changes introduced to Version 1 created *cleaner* results – in that there were fewer papers with *odd men out* (single items that many students answered incorrectly at a stage or single items correct at a stage). Fewer *random* results were found (individual correct items beyond the previous pattern of correct items). These processes suggested that the changes more correctly positioned sets of facts at a developmental stage and that the items were better targeting likely problems when learning a set of facts. Clearer trends were also evident. For example, at Stage 6, it was common to find, as Neill (2008) reflected, students who knew their multiplication and division facts but did not know their addition and subtraction facts to and from 20 – with particular classes tending to have this problem.

To identify items that were easier or harder than the rest at a stage, papers with 1 to 3 or 7 to 9 correct at a stage, and paired items (such as, "19 – □ = 8" and "8 + what equals 19?") were also analysed. For example, the cluster of items on the multiplication and division facts over stages 6 and 7 still tended to be answered consistently – all correct or incorrect, so these were moved to Stage 6 for V3. In V2, students again found start and change unknown formats slightly harder than result unknown, but not to the point where such items warranted location at a higher developmental stage.

Finally, the sets of facts assessed in V2 were mapped back to items in the IBFA, the Number Framework, and other fact frameworks (see NZCER, 2015; Van de Walle et al., 2013). This resulted in alterations to several items and the development of additional items at the higher stages. The sets of facts addressed in V3, and their related test question numbers can be found in Appendix A.

Conclusion

IBFA is a basic facts test designed to be used concurrently with other forms of assessment to support the ongoing learning of basic facts. Developmentally, the IBFA is progressing towards meeting its initial aims of providing teachers with a reliable assessment capable of providing information about students' mastery of a broad range of basic fact sets. Using a PowerPoint that only allows 4 seconds per question has ensured that students are not able to strategise across a collection of items or take a long time over any one item. Having the teacher read out the question alongside the visual prompt has made the assessment accessible to a broad range of students. With both V1 and V2, teachers report they were able to quickly gather information from students and that the data collected was easy to mark, interpret, and use to support their teaching of basic facts. V2 also allows cleaner stage decisions to be made as there are fewer *odd men out* – individual questions at a stage which are the only item that a number of students get right or wrong.

Our next phase of development is to trial Version 3 (V3) across a wide range of ages, including students in rural and low decile schools. One purpose is to evaluate the changes made to V2, in particular whether improved instructions and information about using the 'K' and 'S' codes along with better placed items show a stronger transition from knowledge recall to strategising. A second purpose is to move the research forward to include a teaching intervention based on Tait-McCutcheon et al. (2011), for which a supporting resource booklet has been written (Drake, 2014). We welcome teachers and researchers who are interested in trialling the IBFA or developing a basic facts programme based on these materials to contact either author.

References

Beilock, S. (2011). *Choke: What the secrets of the brain reveal about getting it right when you have to*. New York: Simon & Schuster, Free Press.

Boaler, J. (2012). Timed tests and the development of math anxiety. *Education Week*. Retrieved http://www.edweek.org/ew/articles/2012/07/03/36boaler.h31.html

Clarke, S. & Holmes, M. (2011). Mastering basic facts? I don't need to learn them because I can work them out. J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices* (Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia, Alice Springs). Adelaide: MERGA.

Crooks, T. J. (1988). The impact of classroom evaluation practices on students. *Review of Educational Research, 6*, 101–116.

Drake, M. (Unpublished). *Learning basic number facts: A DIY guide for parents, learners, and teachers.*

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 45-85). New York, NY: Routledge.

Gravemeijer, K. & van Eerde, H. H. H. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *The Elementary School Journal, 109*(5), 510-524.

Kennedy-Clark, S. (2013). Research by design: design-based research and the higher degree research student. *Journal of Learning Design, 6*(2), 26-32.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington: National Academy Press.

Kling, G. & Bay-Williams, J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics, 20*(8), 489-497.

McCloskey, A. (2013). The promise of ritual: A lens for understanding persistent practices in mathematics classrooms. *Educational Studies in Mathematics, 86*, 19-38. doi: 10.1007/s10649-013-9520-4

Mason, J. (2002). *Researching your own practice: the discipline of noticing*. London: Routledge Falmer.

Miles, M. B., Huberman, A. M., & Saldaňa, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd Edn), Los Angeles: Sage Publications

Ministry of Education. (2007a). *Book 1: The number framework.* Wellington: Author.

Ministry of Education. (2007b). *Book 2: The diagnostic interview*. New Zealand: Author.

New Zealand Council for Education Research (NZCER). (2015) Basic facts concept map. Retrieved April 10, 2010, from http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_basic_facts.php.

Neill, A. (2008). Basic facts: Start with strategies, move onto memorisation. *Set 3*, 19-24.

Plomp, T. (2007). Educational design-based research: An introduction. In T. Plomp & N. Nieveen (Eds.), *An Introduction to Educational Design-based research*. pp. 9-33. SLO: Netherlands ICD

Popham, W. J. (2008). Timed tests for tykes. *Educational Leadership, 65*(8), 86-87.

Ramirez, G., Gunderson, E., Levine, S., & Beilock, S. (2013): Math Anxiety, Working Memory, and Math Achievement in Early Elementary School, *Journal of Cognition and Development, 14*:2, 187-202. doi.org/10.1080/15248372.2012.664593

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.

Seeley, C. (2009). Faster isn't smarter: The trap of timed tests. *Maths Solutions Message 18*. Retrieved http://www.mctm.org/mespa/FasterIsntSmarter.pdf

Sherley, B., & Tait-McCutcheon, S. L. (2008). *Practice +assess ≠ knowledge: Basic facts and spelling lists.* Retrieved http://www.nzmaths.co.nz/findings-nz-numeracy-developmentprojects- 2007?parent_node.

Skarr, A., Zielinski, K., Ruwe, K., Sharp, H., Williams, R. L., & McLaughlin, T. F. (2014). The effects of direct instruction flashcard and math racetrack procedures on mastery of basic multiplication facts by three elementary school students. *Education and Treatment of Children, 37*(1), 77-93. doi: 10.1353/etc.2014.0007

Tait-McCutcheon, S. L., Drake, M. & Sherley, B. (2011). From direct instruction to active construction: Teaching and learning basic facts. *Mathematics Education Research Journal, 23*, 321–345. doi: 10.1007/s13394-011-0018-z

The Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5-8.

Van de Walle, J. (2009). *Reform mathematics vs. the basics: Understanding the conflict and dealing with it.* Retrieved http://www.mathematicallysane.com/analysis/reformvsbasics.asp.

Van de Walle, J., Karp, K., & Bay-Williams, J. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). New Jersey: Pearson.
Wang, F., & Hannafin, M. J. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology, Research and Development, 53*(4), 5-23.

Appendix A IBFA Fact Sets

Curriculum Level	Numeracy Stage	Fact sets assessed (Item numbers in brackets)
One	Three: Counting	Addition & subtraction facts to five (1 - 4) Zero facts (5, 6) Doubles to 10 (7, 8, 10) Plus one facts (Number sequence) (9)
	Four: Advanced Counting	Addition and subtraction facts to 10 (1, 2) Doubles to, and halves from 20 (3 - 5) Ten and facts (teen facts) (6, 7) Multiples of 10 that add to 100 (8 - 10)
Two	Five: Early Additive	Addition facts to 20 (1, 2) Subtraction facts from 10 (3, 4) Multiplication facts for the 0, 1, 2, 5, and 10 times tables (5 - 10) Multiples of 100 that add to 1000 (11)
Three	Six: Advanced Additive	Addition and subtraction facts to 20 (1 – 5) Multiplication facts to 100 and corresponding division facts (6 – 12) Square numbers (13) Compatible numbers to 100 (14)
Four	Seven: Advanced Multiplicative	Multiplication & division facts beyond 10 × 10, facts with tens, hundreds and thousands (1 – 3) Division with remainder (4) Fraction↔ decimal↔ percentage conversions for 1/2 - 1/5, 1/10 (5 – 8) Square roots of numbers to 100 (9) Quantities of an amount (10, 11) Factors and multiples (12) Factors (including primes) to 100 (13) Compatible numbers to 1 (14)
Five	Eight: Advanced Proportional	Integer facts for +/-/×/÷ (1 – 4) Fraction↔decimal↔percentage conversions (5, 6) Simple powers of numbers to 10 (7, 8) Common multiples & lowest common multiples to 10 (9 & 14) Divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10 times tables (10, 11) Common factors and highest common factor to 100 (12, 13)

Affording and Constraining Local Moral Orders in Teacher-Led Ability-Based Mathematics Groups

Sandi Tait-McCutcheon
Victoria University of Wellington
<sandi.mccutcheon@vuw.ac.nz>

Joanna Higgins
Victoria University of Wellington
<joanna.higgins@vuw.ac.nz>

Mary Jane Shuker
Victoria University of Wellington
<mary-jane.shuker@vuw.ac.nz>

Judith Loveridge
Victoria University of Wellington
<judith.loveridge@vuw.ac.nz>

How teachers position themselves and their students can influence the development of afforded or constrained local moral orders in ability-based teacher-led mathematics lessons. Local moral orders are the negotiated discursive practices and interactions of participants in the group. In this article, the developing local moral orders of 12 teachers and their highest and lowest mathematics groups are examined with particular attention paid to teacher positioning and the patterns of differentiated positioning between the groups.

Two teachers at the same primary school in New Zealand were teaching their lowest ability-based group of year five students how to apply a compensation strategy to solve multiplication problems. For example: 6 x 9 as (6 x 10) – 6 and 6 x 11 as (6 x 10) + 6. Two students in the first class were arguing about the amount to compensate. One student claimed:

> It's the number at the front.

and another claimed:

> No, it's the number that stays the same.

Other students were following the argument and written recordings and asking questions. At no point did any group member look to the teacher to settle the disagreement. The teacher directed the disagreeing students to use their talk, text, and actions to explain and justify their claims. They were reminded they needed to ensure they were being understood by their group. Others in the group were required to demonstrate their understanding by applying both strategies to solve a different problem and determine which claim was correct. Through the discussion the misconstrued 'front number' strategy was sorted out and the group moved to solving more complex problems using the now named 'same number' compensation strategy. In this example students were expected to explain and justify their thinking, ask and answer questions, settle their own disagreements, understand, and be understood, apply new learning, and remedy their own and others' misconceptions. Disagreeing was a legitimate part of mathematical discussions and the teacher was not the fountain of all knowledge.

Students in the second group were also arguing. In this group the argument was about having to record the equation using a specific strategy when the answer was already known. One student asked:

> Why do I need to write (6 x 10) + 6 = 66 when I just know that 6 x 11 is 66?

The teacher reiterated their expectation:

> I want you to use the compensation strategy to solve 6 x 9 and 6 x 11.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 595–602. Sunshine Coast: MERGA.

The first student quietly said:

> 60 – 6 is 54 and 6 x 11 is 66.

Overhearing this, a second student told the first their strategy was wrong:

> No that's not right, 6 x 11 = 66 isn't compensation, you have to use the compensation strategy, aye Miss, you have to use compensation aye?

The teacher provided the required confirmation:

> Yes I want you to use the compensation strategy.

Even more quietly the first student complained:

> That's just dumb, I know 6 x 11 is 66 why do I have to write it down that way? I know 6 x 11 = 66.

A third student tried to explain:

> You are right but being right isn't enough you had to use the right strategy too.

In this second example the expectations appeared to be that specific strategies must be applied, different thinking was not required, and existing knowledge was not valued.

These two examples illustrate how teacher positioning can influence the development of diverse, and potentially detrimental or beneficial, local moral orders. Local moral orders are a construct of positioning theory and are the agreed to patterns of interaction created and developed between participants, in this case teachers and students (Davies & Harré, 1990; Harré & van Langenhove, 1991). They develop from the ways participants view themselves and others, the way they act and interact, how they may feel obliged to act and interact, what can be said or done, who can action the saying or doing, when it can be said or done, and what the reactions to the words and actions can be (Harré & van Langenhove, 1999). There can be more than one developing local moral order within an interaction but all are contextualised to these participants, at this time, telling particular stories, from particular points of view (Harré, 2012).

Local moral orders are similar to Yackel and Cobb's (1996) social and socio-mathematical norms. How things are done become taken-as-shared by the group and what is taken-as-shared has a sense of oughtness (Linehan & McCarthy, 2000). However, there are two key differences. Moral, in the context of norms, refers to moral accountability or honourable behaviour such as in an expected code of conduct. Local moral orders in positioning theory have a moral quality because they are associated with the rights and duties of participants (Harré, 2012). The second difference pertains to how participants are located in the interaction. Local moral orders locate participants in positions whereas norms locate participants in roles. Positions were posited as a more dynamic and fluid notion than role which was perceived to be more static and symbolic (van Langenhove & Harré, 1999).

This article draws on the findings from a larger study (Tait-McCutcheon, 2014) where the key research question addressed was: How do teachers in New Zealand primary schools position themselves and students in their lowest and highest mathematics strategy groups so that mathematical know-how could be shared?

Mathematical know-how was defined according to Pólya (1965) as independence, judgement, and creativity. The focus of this article is the developing local moral orders that afforded or constrained the sharing of teacher and student mathematical know-how. The local moral orders were identified and explained by examining three discursive practices of positioning theory: positions, storylines, and social acts (Harré & van Langenhove, 1999).

Positions, Storylines and Social Acts

Positioning theory, proposed that when people participate in genuine, sequential, naturally occurring talk, text, and actions with others they do so from a position (Davies & Harré, 1990; Harré and van Langenhove, 1991). From a position, participants give and attempt to give, meaning to their own and others' talk, text, and actions by establishing, and attempting to establish their own and others' rights and duties (Harré & Moghaddam, 2003). The rights and duties any person has within a position are influenced by past, present, and future interactions of the group and influence the developing local moral orders of that group (Harré & van Langenhove, 1999). For example, teachers and students have individual and collective rights and duties, but their rights and duties are "interlaced with the expectations and history of the community" (Linehan & McCarthy, 2000, p. 442). Qualitatively different or fixed rights and duties can result in some students having substantially different opportunities to participate (Anderson, 2009; Barnes, 2003; Yamakawa, Forman, & Ansell, 2005).

Storylines make participants past or projected future words and actions meaningful to themselves and others "by telling a kind of story about them" (Slocum-Bradley, 2010, p. 93). The stories participants tell about themselves and others, and how those stories are accepted or rebutted help to define the local moral orders. There are numerous contextualised commentaries, interpretations, and relationships in play as the storyline advances and the exact same words and actions can convey a different storyline to different participants (van Langenhove & Harré, 1999). Storylines are neither complete nor correct because perspectives within any storyline may differ, participants may choose to be complicit or resistant, and the presence or absence of certain positions may alter the storyline. However, storylines do tend to follow already established patterns of development within a cluster of narrative principles and practices (Harré & Moghaddam, 2003). As such, the creation and survival of any storyline is contingent on it being jointly constructed and sustained. Social acts are the talk, text, and actions of participants that become significant to the interaction when they are appropriated by others and given increased, reduced, new, or different meaning (Davies & Harré, 1990). The social force participants have, and their social acts that are appropriated affect the existing and developing local moral order (van Langenhove & Harré, 1999). The relationship between positions, storylines, and social acts and local moral orders is mutually determining. The positions, storylines, and social acts of the group create the local moral orders and the local moral orders shape the positions, storylines, and social acts. Therefore, within any local moral order participants, conversations, expectations, and behaviours are susceptible to change (Harré, 2012).

Method

This study was underpinned by a social constructivist theoretical perspective whereby knowledge was considered from the personal view of an individual and the collective view of a group (Bobis, Mulligan, & Lowrie, 2004). A qualitative research paradigm was used to examine teachers' acts of positioning, to reason about those positionings, and to interpret relationships and consequences between positioning and each groups developing local moral order. The bounded and socially situated nature of this research within the highly subjective social phenomenon of teaching and learning meant a qualitative case study was an appropriate methodological choice. Case study research is exploratory and resonates with the reader's own experiences and existing understandings, provides insights into how things become the way they are, and generates discoveries of new learning. The end product of a qualitative case study is a “rich, thick description of the phenomenon under study” (Merriam, 2009, p. 43).

Two schools, Pacific and Tasman, were purposefully recruited to participate in the larger study because of their commonalities (professional development, lesson organisation, and ability grouping) and differences (static/changing staff, decile rating, and ethnic diversity). Twelve teachers of students in years zero to six (aged 5-11) participated. Years of teaching experience ranged from one to 24 and nine of the 12 teachers had participated in the New Zealand Numeracy Development Project (NDP), (Ministry of Education, 2007) professional development in the past three years either as pre-service or in-service teachers.

Three data sources were used extensively in this research: video and audio recordings, transcriptions, and observations. Each teacher was video and audio recorded for three consecutive lessons teaching their lowest and highest mathematics group, resulting in 72 lessons observed and transcribed. Written observations that included field and personal notes where undertaken for the duration of each lesson and theoretical notes were added after the observations. Qualitative data analysis required a fluid, evolving, dynamic approach that included contrasting, comparing, replicating, cataloguing, and classifying from concrete data toward more conceptual levels (Denzin & Lincoln, 2011). A constant comparative method (Corbin & Strauss, 2008) was chosen as the most appropriate method for data analysis. The analytic approach taken was look, think, look again, think again, through-out the following five phases of analysis:

1. Identify examples of teacher positioning and code as talk, text, or action. Note relationships between codes and group as themes. Develop tentative concepts from themes. Build categories through which theory was being created.
2. Identify mathematical contexts in which the teacher positionings occurred.
3. Plot teachers’ positioning acts according to codes and contexts.
4. Identify potential negative instances and conflicts within the data.
5. Establish themes to describe the positioning pattern of each teacher with their lowest and highest strategy group.

The trustworthiness of this research was tested and affirmed by considering the reliability, credibility, transferability, dependability, and confirmability of the qualitative research methods (Lincoln & Guba, 1985). Triangulation of participant sources, data sources, and data analysis confirmed emerging findings and the reliability of conclusions (Merriam, 2009). Credibility was enhanced through the processes of member checking and peer debriefing (Cohen, Manion, & Morrison, 2007). The thick descriptions used to tell the story of teacher positioning provided transferability for the reader and “accurate explanations and interpretation of the events” to a different setting (Cohen et al., 2007, p. 405). Dependability and confirmability were achieved through the rigour of the data collection, data analysis, and theory generation processes, documenting procedures for checking and rechecking the data, including negative instances, and conducting a data audit trail.

Findings and Discussion

This study identified three key findings where the developing local moral orders afforded the sharing of mathematical know-how from teachers and students and three key findings where the sharing of mathematical know-how was afforded for teachers but constrained for students. These findings are discussed in relation to the literature.

The local moral orders that afforded the sharing of mathematical know-how for both teachers and students emphasised the visibility, fluidity, and contestability of the mathematics; the importance of teachers and students contributions to the teaching and learning; and the expectation teachers and students would take a mathematical stand by agreeing with or challenging the shared know-how. Teachers and students ensured and enabled the visibility, fluidity, and contestability of the mathematics through their suggestions, observations, explanations, questions, and reflections. Teachers further ensured visibility by providing time and space within the lessons for suggestions, observations, explanations, questions, and reflections to be shared and responded to. Teachers and students were able to maintain the discussions around, and the complexity of, the task and the mathematical interest and depth of teachers and students understanding simultaneously developed (Chapin, O'Connor, & Anderson, 2009).

Teachers and students had important know-how to share, observations to make, and questions to ask that benefitted and progressed the teaching and learning (Hunter, 2007). Both were expected to take a mathematical stand and have a mathematical opinion that could be understood by others. They were expected to analyse their thinking, know when they were correct or mistaken, understand why, and know how they could prove they were correct, or fix their error (Chapin & O'Connor, 2007). They also had a duty to know when another group member was correct or incorrect and again know why. Correct and incorrect answers, misconceptions, disagreements, and questions from teachers and students provided resources for targeted teaching and learning (Anthony & Walshaw, 2009). When teachers purposefully listened to students' mathematics they gained knowledge about what students knew and how they were constructing new knowledge. This better positioned teachers to "generate interpretations of what they noticed and to generate conjectures about student thinking that would support the development of their ability to teach for understanding" (Choppin, 2011, p. 195). Teachers positioned themselves as active listeners, observers, and responders who had something mathematically important to learn from students. They then formatively applied what they had learned to question students in ways that shaped and further developed the mathematical talk, text, and actions.

Teacher and student know-how were predominant social acts because both had a voice within the mathematical discussions and both were responsible for the groups progress. Know-how acknowledged as valuable raised the individual and collective status of group members and the intellectual value of their reasoning (Hunter, 2007). The more the group experienced agency within their own and others learning, the more they learned they had control over their own and others successes and failures. Teachers and students had personal latitude within the teaching and learning because both had authority and were considered competent contributors to the mathematics (Wagner & Herbel-Eisenmann, 2013). The local moral order of the teachers whose positioning afforded the sharing of mathematical know-how and their students was collectively and collaboratively evolving.

The local moral orders that afforded the sharing of teacher know-how but constrained the sharing of student know-how emphasised the predominant positioning of the teacher; the teacher as gatekeeper; and the hurried pace of the lessons. The mathematics within predominantly belonged to the teacher and as such, the teacher had a considerably higher profile than students. Teachers were more significant within the group because they positioned themselves to do most of the mathematical talk and tasks within the lesson, often before the students had the same opportunity. They asked and answered questions, modelled and explained correct and incorrect answers, summarised learning for students, and dismissed opportunities to explore incorrect answers or different or advanced explanations. Instead of modelling, reasoning, and reflecting, these teachers tended to make authoritative statements and decisions and give directions that were quick, correct, and one-dimensional. Whilst students were receiving clear messages about "what they need to know and learn" (Ewing, 2011, p. 68), they were positioned as passive recipients of knowledge who had a duty to listen to the teacher and repeat answers and explanation. The request for repetition did not seem to be to be a means for ensuring students were paying attention to what was being said but rather to ensure they had heard correctly (Chapin, et al., 2009). There was limited time or space for students to make decisions or express their own thoughts. The fewer opportunities students had to share their mathematical know-how the fewer opportunities they had to experience reasoning and act purposefully and reflectively with others (Choppin, 2011).

When the sharing of mathematical know-how was constrained for students the teacher was positioned as the gatekeeper of the mathematics (Wagner & Herbel-Eisenmann, 2013). Teachers led, students followed, and there was little demarcation between these positions. Teacher knowledge and authority limited positions made available to students and teacher's personal mathematical beliefs and values were dominant within the discussions and developing mathematics (Davies & Hunt, 1994). The know-how shared, by whom, and when was determined by teachers. They gave themselves the right to provide correct answers and explanations, target specific strategy use, and ask closed and known answer questions. Teacher know-how was shared as self-enclosed messages to be understood. Steering students toward particular solutions and strategies and smoothing that path for them did not enable know-how to be experienced or grappled with (Chapin & O'Connor, 2007). Students were positioned by teachers as passive onlookers whose duty it was to behave appropriately, watch, listen, and mimic. These duties appeared to take precedence over mathematical thinking. The know-how of the teacher became the predominant social act because theirs was the voice most heard. Other significant social acts were the words and actions of students who endorsed the teacher positioning, provided correct answers, and applied designated strategies.

Conclusions and Implications

This article contributes new knowledge to understanding the teaching and learning of mathematics by employing the lens of local moral orders and the discursive practices of positions, storylines, and social acts for analysis. The mathematical opportunities of the 24 groups of students in this study were qualitatively different because of the developing local moral orders in which their learning occurred. The positions of teachers and students, the storylines being told, and the social-acts being valued reiterated and reinforced that qualitative difference. When teachers or students limited themselves or were limited by others to constrained positions, their rights and duties within that position become restricted (Davis & Hunt, 1994; Yamakawa et al., 2005). The longer the teacher or student is constrained by the positioning, the less likely the positioning could be altered or disrupted (Anderson, 2009; Barnes, 2003). The danger for some teachers and students is that they may become entrenched in an exponential pattern of constrained teaching and learning.

One such pattern was identified across the afforded and constrained local moral orders. This pattern was that more teachers afforded the sharing of mathematical know-how with their highest group than their lowest. Ten of the twelve teachers created local moral orders with their highest group that promoted and expected active participation, authentic involvement, and reflection from themselves and their students. Six of the 12 teachers created similar local moral orders with their lowest group. Therefore eight groups of students did not have the same opportunities to engage with their own and peers' know-how. They did participate in their teacher's know-how but access was narrow and restrictive. Students in these eight groups were marginalised from mathematical engagement because of their corresponding imitative, procedural, and simplified duties. Interactions occurred mainly between the teacher and an individual student and the goal appeared to be to follow specific strategies and determine correct answers. By positioning themselves as the dominant participant in the mathematical discussions, these teachers were limiting their opportunities for their students' to connect in mathematically meaningful ways and for them to connect in mathematically meaningful ways with their students (Boaler, 2014). Teachers and students opportunities for successful mathematics teaching and learning were marginalised and unlikely to alter levels of achievement.

It is important to note that situating the study within the NDP mathematics programme and numeracy strand may have predetermined the mathematical pedagogies teachers selected and simultaneously predetermined the positionings they would take and give. The NDP could be considered a more structuralist approach to teaching and learning mathematics and as such teachers could have promoted the "direct instruction of explicit mathematical representations and procedures" (Murphy, 2013, p. 108). When teachers' positionings constrained the sharing of mathematical know-how the goal appeared to be to push students toward the recommended strategy and correct answer. An adherence to the NDP teaching materials may have substantiated or exacerbated that goal. However, the evidence remains that for eight groups of students the developing local moral orders in which their mathematical learning occurred constrained their opportunities to share their mathematical know-how. These students mathematical know-how was not positioned as a valuable teaching and learning tool. It would be of value to these findings and to the international mathematics community to extend this research to include mathematics programmes less structured than the NDP. Increased understanding of the affording local moral orders in particular could assist all teachers to further define and explore effective teaching positions.

References

Anderson, K. T. (2009). Applying positioning theory to the analysis of classroom interactions: Mediating micro-identities, macro-kinds, and ideologies of knowing. *Linguistics and Education, 20*, 291-310.

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics: Educational practices series, 19*. International Academy of Education, International Bureau of Education & UNESCO.

Barnes, M. (2003). Patterns of participation in small-group collaborative work. Paper presented as part of the symposium *Patterns of participation in the classroom* at the Annual Meeting of the American Educational Research Association, Chicago.

Boaler, J. (2014). Ability Grouping in Mathematics Classrooms. *Encyclopaedia of Mathematics Education, 1-5.*

Bobis, J., Mulligan, J. T., & Lowrie, T. (2004). *Mathematics for children: Challenging children to think mathematically* (2nd ed.). Frenchs Forest, NSW: Pearson Education Australia.

Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W. G. Martin & M. E. Strutchens (Eds.), *The learning of mathematics* (pp. 113-128). Reston, VA: The National Council of Teachers of Mathematics.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Sausalito, CA: Math Solutions.

Choppin, J. (2011). The impact of professional noticing on teachers' adaptations of challenging tasks. *Mathematical Thinking and Learning, 13*(3), 175-197.
Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). London: Routledge.
Corbin, J., & Strauss, A. (2008). *The basics of qualitative research* (3rd ed.). Los Angeles, CA: Sage.
Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour, 20*(1), 43–63.
Davies, B., & Hunt, R. (1994). Classroom competencies and marginal positioning. *British Journal of Sociology of Education, 15*(3), 389-408.
Denzin, N. K., & Lincoln, Y. S. (Eds.). (2011). *The sage handbook of qualitative research* (4th ed.). Thousand Oaks: Sage.
Ewing, B. (2011). Direct instruction in mathematics: Issues for schools with high indigenous enrolments: A literature review. *Australian Journal of Teacher Education, 36*(5), 63-91.
Harré, R. (2012). Positioning theory: Moral dimensions of social-cultural psychology. In J. Valsiner (Ed.), *The Oxford handbook of culture and psychology* (pp. 191–206). NY: Oxford University Press.
Harré, R., & Moghaddam, F. M. (2003). Introduction: The self and others in traditional psychology and in positioning theory. In R. Harré & F. M. Moghaddam (Eds.), *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (pp. 65-78). Westport: Praeger.
Harré, R., & van Langenhove, L. (1991). Varieties of positioning. *Journal for the Theory of Social Behaviour, 21*(4), 393-407.
Harré, R., & van Langenhove, L. (1999). The dynamics of social episodes. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 1-14). Oxford: Blackwell.
Hunter, R. (2007). *Teachers developing communities of mathematical inquiry.* (PhD Thesis). Massey University, Auckland, New Zealand.
Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.
Linehan, C., & McCarthy, J. (2000). Positioning in practice: Understanding participation in the social world. *Journal for the Theory of Social Behaviour, 30*, 435-453.
Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation* (2nd ed.). San Francisco, CA: Jossey-Bass.
Ministry of Education. (2007). *Numeracy Professional Development Projects Book 1: The number framework*. Wellington, New Zealand: Author.
Murphy, C. (2013). Making space for mathematics learning to happen in group work: Is this really possible? *Teachers and Curriculum, 13*, 108-111.
Pólya, G. (1965). *Mathematical discovery: On understanding, learning, and teaching problem solving* (Volume II). New York: John Wiley.
Slocum-Bradley, N. (2010). The positioning diamond: A trans-disciplinary framework for discourse analysis. *Journal for the Theory of Social Behaviour, 40*(1), 79-107.
Tait-McCutcheon, S. L. (2014). *Teacher practice in primary mathematics classrooms: A story of positioning*. Unpublished doctoral thesis retrieved http://researcharchive.vuw.ac.nz
van Langenhove, L., & Harré, R. (1999). Introducing positioning theory. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 14-31). Oxford: Blackwell.
Wagner, D., & Herbel-Eisenmann, B. (2013). Disbursing authority among mathematics students. *Proceedings of the Seventh International Mathematics Education and Society Conference* (pp. 483-491), Cape Town, South Africa.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.
Yamakawa, Y., Forman, E., & Ansell, E. (2005). Role of positioning in constructing an identity in a third grade mathematics classroom. In K. Kumpulainen, C. E. Hmelo-Silver, & M. Cesar (Eds.), *Investigating classroom interaction: Methodologies in action* (pp. 179-202). Rotterdam: Sense.

Exploring relationship between scientific reasoning skills and mathematics problem solving

Nor'ain Mohd Tajudin
Sultan Idris Education University, Malaysia
<norain@fsmt.upsi.edu.my>

Mohan Chinnappan
University of South Australia
<mohan.chinnappan@unisa.edu.au>

Reasoning is considered to be an important proficiency in national mathematics curricula both in Australia (ACARA, 2014) and Malaysia (MOE, 2013). However, the nature of reasoning that supports learning and problem solving in mathematics is an area that requires further study (Schoenfeld, 2013). In this study we explored the link between *Scientific Reasoning Skills* (SRS) and mathematics problem solving performance among a cohort of Malaysian students. As expected, there was a positive relationship but the level of correlation between these two variables was moderate. Although the High-Achievement group performed significantly better than their peers in the Low-Achievement group in their solution outcomes, overall, all students exhibited low-levels of SRS. These findings suggest that while SRS could play a role in problem solving, components of SRS need further analysis in order to better explain how reasoning in science could facilitate problem solving processes.

Introduction

With increasing attention to logical arguments and justifications in mathematics, the study of reasoning that underpins these processes constitutes an important area of research (Schoenfeld, 2013; Santos-Trigo & Moreno-Armella, 2013). In this study, we focussed on reasoning that is associated with the development of arguments and justifications in the context of problem solving. Data on reasoning and how that reasoning is used by students during the course of their solution search was expected to deepen current understandings about the construction of arguments and generation of justifications by learners. In this study, we generate data relevant to this issue by analysing scientific reasoning processes that students could activate during their mathematical problem-solving attempts.

Literature review

Reasoning in Mathematics

Current research emphasises the importance of students engaging in reasoning in all strands of school mathematics (National Council of Teachers of Mathematics, 2009; Bieda *et al*., 2013; Santos-Trigo & Moreno-Armella, 2013; Stylianides *et al*., 2008). Ball and Bass (2003, p. 42) argued that 'mathematical reasoning is inseparable from knowing mathematics with understanding.' Several scholars have elaborated on the connection between learning mathematics with understanding and reasoning. Lakatos (1976) noted that complete mathematical understanding includes the engaging processes of thinking, in essence doing what makers and users of mathematics do: framing and solving problems, patterns recognition, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on. This observation about understanding and reasoning was recently reaffirmed by Schoenfeld (2013) when he commented that one variable seemed to have strongest impact on students learning - the amount of time students spent in explaining and justifying their ideas. While definitions used in the context reasoning differ, they deal in one way or another with a broad range of

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 603–610. Sunshine Coast: MERGA.

thinking skills involving arguments that drive the evolution of solutions to mathematical problems. In this sense, there are aspects of mathematical problem solving that could draw on scientific reasoning (Hand, Prain & Yore, 2001).

Scientific Reasoning Skills (SRS)

The range of SRS that students bring to learning and problem solving can be expected to assist them in making progress in multiple ways. SRS ‘encompasses the reasoning and problem-solving skills involved in generating, testing and revising hypotheses or theories, and in the case of fully developed skills, reflecting on the process of knowledge acquisition and knowledge change that result from such inquiry activities’ (Morris, Croker, Masnick & Zimmerman, 2012, p. 65). Scientific reasoning differs from other skills in that it requires additional cognitive resources as well as an integration of cultural tools. Further, scientific reasoning emerges from the interaction between internal factors (e.g., cognitive and metacognitive development) and cultural and contextual factors. According to Lawson (2004), scientific reasoning pattern is defined as a mental strategy, plan, or rule used to process information and derive conclusions that go beyond direct experience. In a similar vein, Hand, Prain and Yore (2001) argued that scientific reasoning involves the ability to construct powerful arguments for learners’ actions. Thus, SRS is related to cognitive abilities such as critical thinking and reasoning that assist students in producing knowledge during problem solving through evidence–based reasoning. Given the connectedness between knowledge generation via arguments and reasoning that gird these arguments, students with higher levels SRS could be expected to be superior problem solvers.

Conceptual framework

Lawson’s (2000) Classroom Test of Scientific Reasoning provided the framework to guide the analysis and interpretation of data in the present study.

Lawson’s Classroom Test of Scientific Reasoning

In this study, we focus on a set of domain-general reasoning skills that are commonly needed for students to make progress with mathematical problems which includes exploring a problem, formulating arguments, manipulating and isolating variables, and observing and evaluating the production of new information. Lawson’s Classroom Test of Scientific Reasoning (LCTSR) provides a theoretical lens for assessing a range of SRSs. The test was designed to examine a set of general reasoning ability dimensions which are crucial for the solution of problems in STEM including conservation of matter and volume, proportional reasoning, control of variables, probability reasoning, correlation reasoning, and hypothetical-deductive reasoning. The validity of the LCTSR had been established by several studies (e.g. Lawson, Bank & Lovgin, 2007).

LCTSR allows for the observation of three levels of reasoning: *concrete, transitional* and *formal* operational reasoning. The *concrete* operational reasoning refers to thinking pattern that enable one to understand concepts and statements that make a direct reference to familiar actions and observable objects, and can be explained in terms of simple associations (for example, all squares are rectangles but not all rectangles are squares). Students, at this level of reasoning, are also able to follow step-by-step instructions provided each step is completely specified (for example, solving two linear equations). Students are also able to relate his/her viewpoint to that of another in familiar situations (for example, students respond to difficult mathematical problems by applying a related correct rule). At this stage, students are unconscious of his/her own reasoning patterns, inconsistencies among various statements he/she makes, or contradictions with other known facts.

In contrast to *concrete* reasoning, *formal* reasoning patterns enable students to construct possible explanations as a starting point for reasoning about a causal situation. They can reason in a deductive manner to test their hypotheses. In other words, they can postulate causal factors, deduce the consequences of these possibilities and then empirically verify which of those consequences, in fact, occurs. Lawson (1978) categorised students at this stage as ‘reflective thinkers’. For example, in solving mathematical problem, students’ reasoning can be initiated with development of representations, use of symbols and planning a course of action.

The *transitional* operational stage is where students remain confined to *concrete* thinking or are only capable of partial *formal* reasoning. For example, proportional reasoning is the ability to compare ratios or develop arguments about the equality between two ratios. At *concrete* operational stage, students are not aware of ratio dependence and seek solutions by guessing. At the *transitional* stage, students are aware of objective dependence and seek solutions by estimation and later calculation, but assume that the change in one quantity produces the same change in the other quantity. In the *formal* stage, proportionality is discovered and applied to obtain correct solutions. Clearly, in all three levels of reasoning, students generate qualitatively different types of information that are driven by arguments and justifications.

Purpose of the Study

The review of literature indicates that reasoning skills are transferable across science and mathematics (Hand, Prain & Yore, 2001), and that we could learn a great deal about the role of reasoning in mathematical problem solving by examining potential links between the two (Lehrer & Schauble, 2000). The purpose of the study was to identify the levels of SRS attained by a cohort of upper Malaysian secondary school students (16-17-year-olds) and examine the impact of SRS on their mathematical problem-solving performance. We sought data relevant to the following three research questions:

1. What are the levels of SRS among upper secondary school students?
2. Is there a relationship between SRS and mathematics problem-solving performance?
3. Does achievement level of students affect their SRS and mathematics problem-solving performance?

Methodology

Design

This study employed a blend of descriptive and correlational research design as our interest was to generate information about the relationship between one independent (Achievement level) and two dependant variables (SRS and Mathematics Problem Solving).

Participants

A total of 351 students from 14 Malaysian secondary schools participated in the present study. Participants in this study were upper secondary school students or Year 11 students (16-17-year-olds). Participants were assigned to High or Low achievement groups on the basis of their performance in a centralised Malaysian examination, Lower Secondary Evaluation Examination (LSEE). The High-achievement group (Grades A or B in LSEE) comprised Science stream students (n = 98) and the Low–achievement group (Grades C or D in LSEE) were Non-Science stream students (n = 253). This is based on the Malaysian Education Evaluation System in placing students into Science and Non-Science streams. Under this system, students needed to score high marks in mathematics in order to go into the Science Stream in comparison to their peers in the Non-Science stream.

Tasks

This section provides details of two tasks that were used in this study. As discussed earlier, there were two tests used in order to generate scores for two dependent variables: Test 1 - Scientific Reasoning Test (SRT); Test 2 - Mathematics Problem-solving Test (MPST).

Task 1 - Scientific Reasoning Test (SRT)

The SRT was used to measure the students' level of SRS. It has been adapted from LCTSR (Lawson, 2000). We wanted to determine the internal consistency of the items which involved the generation of inter-item correlation matrix and computing of Kuder-Richardson 20 internal-consistency reliability coefficient. The final test had a KR-20 coefficient of 0.856. The test consisted of 12 paired items and was designed in a 'two-stage' multiple-choice format to illustrate problem scenarios. With each scenario, the first question focuses on the scenario content, while the second question asks for reason as to why the first answer is correct. Each answer for the first question has a corresponding reason in the second question.

For example, in one of the tasks, students' reasoning about conservation of volume was evaluated. Firstly, students have to think based on their experience or previous knowledge about where the water will rise to when the glass marble is put into cylinder. Then, students have to justify as to why the water rose at that level. This involves students applying the conservation reasoning to perceptible objects and properties. Making prediction and giving explicit explanation are important to successful completion of this item. Prediction, explanation and the generation of relevant new information are important processes of mathematical problem solving. Thus, we argue that these reasoning skills would contribute to the solution outcome of mathematical problems.

Scoring Rubric for SRT

The range of scores of SRS level is 0-12 which decomposes into three levels as suggested by Lawson: 0-4 (Concrete); 5-8 (Transitional); 9-12 (Formal).

Task 2 - Mathematics Problem Solving Test (MPST)

The MPST was designed to measure students' mathematics problem-solving performance by drawing on SRSs. The test was prepared by a panel of experienced mathematics educators, experienced teachers and mathematics curriculum experts. We were concerned to ensure that the solution of the problems necessitated the activation of one or more levels reasoning that was postulated in the framework of SRS. The items for the test were selected from a pool of resources such as textbooks, reference books and examination papers. The test consisted of 40 items that covered all core mathematical strands in the Malaysian Mathematics Syllabus (Year 8 – Year 11). The reliability of the test was established by following a process that was similar to SRT. The reliability index for MPST was 0.895.

For example, Item 18 required students to work out the perimeter of an irregular shape that was located within a rectangle. The solution required students to generate arguments about different ways to determine the perimeter and test their hypothesis. Students were categorised as having concrete reasoning level if they needed reference to familiar actions, objects, and descriptive properties. At this level, their reasoning was initiated with observations and step-by-step moves. For Item 18, students may only show a superficial understanding of concepts of perimeter, area of a rectangle and a circle without any way linking these to solving the problem. Students were categorised as having formal reasoning level if they can be initiated with possibilities, used symbols to express ideas, planned a lengthy procedure given the overall goal while being critical of his/her own reasoning patterns. In this case, students may systematically plan to find perimeter of the irregular shaded region. This will involve finding the curve length of a semicircle and a quadrant using the formulae for the area and using the given information of the radius length. Students were categorised as having a *transitional* reasoning level if they remained confined to concrete thinking or are only capable of partial formal reasoning such as they only understood and applied concepts of perimeter and area of a rectangle and area of a circle in a new context. Students responses for MPST were scored as 1- correct response; 0- incorrect response.

Procedures

There were three phases in the study. The first phase was concerned with the development and fine-tuning of MPST. The details are explained in the MPST task section. During the second phase, we pilot tested both the tests to allow for familiarisation of the data collection processes, to validate the instruments and to establish their reliability. The third phase involved the administration of the two tests. Both tests were administered during regular mathematics classes. Researchers and classroom teachers assisted with the administration of the tests. Students were invited to complete the SRT in the first week of their regular mathematics lesson. They were allowed a maximum of 40 minutes for SRT. The MPST, a one-hour paper and pencil test, was administered in the second week, again, during their regular mathematics lesson.

Results and Analysis

Three research questions were of interest to the present study. Data relevant to these research questions are presented below.

Research Question 1: What are the levels of SRS among Upper Secondary School Students?

Table 1 shows the overall level of SRS exhibited by the participating Year 11 students. The findings showed that 330 (94%) of the students achieved Level 1 (*concrete*) of SRS, 20 (5.7%) Level 2 (*transitional*) and 1 (0.3%) Level 3 (*formal*). The overall mean level of the SRS was 1.76. This indicates that majority of the participating students were functioning at the *concrete* level of SRS.

Table 1
Level of SRS

SRS Level	N	Percentage(%)	SRS Mean Score	Standard Deviation
Concrete	330	94.0	1.50	1.18
Transitional	20	5.70	5.65	0.81
Formal	1	0.30	9.00	-
Total	351	100.00	1.76	1.55

Research Question 2: Is there a Relationship between the SRS and Mathematics Problem Solving Performance?

Overall, the correlation between the SRS and mathematics problem solving performance was significant indicating a positive relationship between the two variables [($r = 0.593$), $p<0.05$]. The coefficient of correlation ($r = 0.593$) indicating that there was a moderate positive relationship between the SRS and mathematics problem solving performance .This suggests that if a student had a high score in SRS, he/she are expected to achieve a high score in MPST.

Research Question 3: Does Achievement Level of Students Effect their SRS and Mathematics Problem-Solving Performance?

A t-test analysis was conducted to compare the mean scores of the overall level of SRS for the two Achievement levels (High/Low). Analysis as presented in Table 2 showed there were differences in mean overall SRS between High and Low-Achievement groups [$t (349) = 9.260$, $p < 0.05$]. The mean SRS level for High-Achievement group (mean = 2.99) was better than the corresponding score for peers in the Low-Achievement group (mean = 1.28). However, SRS score for both groups of students was 1.76 (Table 1) suggesting the students had acquired concrete reasoning level.

The total scores for MPST were converted to percentages. Mean percentages for the High-Achievement group and the Low Achievement group were 81.02 and 46.86 respectively (Table 2). The results also showed there were differences in mean MPST percentages between the High and Low-Achievement groups [$t(349) = 16.789$, $p < 0.05$]. The mean MPST percentage score for the High-Achievement group was higher than the Low-Achievement group. Taken together, students in the High-Achievement group produced higher scores for SRS and MPST than their peers in the Low-Achievement group.

Table 2
SRS score and MT score Vs Achievement Group

Dependent Variable		Achievement Group High (n= 98)	Low (n=253)	t-value	p-value
SRT score	Mean	2.99	1.28	9.260	**
	SD	1.66	1.21		
MPST score	Mean	81.02	46.86	16.789	**
	SD	13.42	18.32		

**p<0.01

Discussion

The study was designed to generate data relevant to issues about relation between the level of SRSs and mathematics problem solving among a cohort of Malaysian students. The first research question addressed the level of SRS among upper secondary school students. We found participants to have acquired moderate levels of SRS. Almost all the students (94.0%) were in the concrete reasoning level and others were in the transitional (5.7%) and formal (0.3%) reasoning levels. The second research question addressed the relationship between SRS and mathematics problem solving. The results indicate that there was a moderate positive correlation between the SRS and mathematics problem solving ability as measured by the MPST. Data analysis relevant to Research Question 3 showed that students in the High-Achievement group performed significantly better than their Low-Achievement peers in both the MPST and SRS. Given the positive correlation between SRS and MPST, it was expected that the higher MPST scores of particularly the High-Achievement group can be attributed to their superior SRS. However, the SRS scores for all the students including the High-Achievement group was relatively low suggesting they were operating at concrete level.

Interestingly, the higher SRS scores for the High-Achievement group (in comparison to the Low-Achieving Group) is still low in terms of the overall SRS level that they had achieved. The mean SRS score for this group was 2.99 which falls well into the concrete reasoning level. However, as shown in Table 2, despite relatively low SRT scores for the High-Achievement group, the score on MPST for this group was significantly higher (mean = 81.02) in comparison to the Low-Achievement group.

We offer two possible explanations for the above pattern of results. Firstly, it might be that students in High-Achievement group were using reasoning and information generating strategies that do not involve the use of SRSs, a claim that needs further investigation. A second possibility is that concrete level SRS may be sufficient for the solution of the type problems that were provided in our MPST. If the latter is indeed the case, the suggestion is that we may have to develop more complex and sensitive problems in order to constrain students to activate transitional and formal levels of scientific reasoning during their solution attempts. In our next phase of this study, we are planning to pursue this hypothesis.

The scores for SRS and problem-solving for students in the Low-Achievement group were low in comparison to their high-achieving peers. In the absence of further data about how SRSs could foster problem solving in mathematics, it is too early to argue that low-achieving students could benefit from instructions to improve their SRSs. We also suggest that the scoring of SRS and MPST needs fine-tuning in order to make the comparison more sensitive to the skills underpinning the two variables.

In our analysis of level of SRS and mathematical problem solving, we did not consider the cultural context of the participating students. The students in this study had three types of linguistic backgrounds - Malay, Mandarin and Tamil. It would be interesting to explore the link between students' linguistic background, scientific reasoning skills and mathematical problem solving outcomes. In the present study, we drew on Lawson's work concerning the three levels of scientific reasoning skills on the assumption that these levels would be sufficient in order to capture the multitude of reasoning that could be activated during novel mathematical problem solving. As mentioned above, although all students were operating at Level 1 of SRS, their performance in MPST, particularly for the High-Achievement group, was high. It would seem that students were indeed engaging in substantial reasoning when they completed the MPST tasks.

References

ACARA (Australian Curriculum Assessment and Reporting Authority) 2009. Shape of the Australian Curriculum: Mathematics. Retrieved 1 December 2014 from http://www.acara.edu.au/verve/_resources/australian_curriculum_-_maths.pdf.

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics*, (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics.

Bieda, K. N., Ji, X., & Drwencke, J. & Picard, A. (2013). Reasoning-and-proving opportunities in elementary mathematics textbooks. *Journal for Research in Mathematics Education*. Retrieved 5 January 2014 from http://dx.doi.org/10.1016/j.ijer.2013.06.005.

Hand, B. M, Prain, V., & Yore, L. (2001). Sequential writing tasks influence on science learning. In P. Tynjala, L. Mason & K. Lonka (Eds.). *Writing as a Learning Tool* (pp. 105–129). Boston, MA: Kluwer Academic Press.

Lakatos, I. (1976). *Proofs and refutation*, Cambridge, UK: Cambridge University Press.

Lawson, A. E. (2004). The nature and development of scientific reasoning: A synthetic View. *International Journal of Science and Mathematics Education*, 2, 307–338.

Lawson, A. E. (1978). The development and validation of a classroom test of formal reasoning. *Journal of Research in Science Teaching, 15*(1), 11–24.

Lawson, A. E. (2000). *Classroom test of scientific reasoning (revised)*. Retrieved 10 September 2013, from http://www.ncsu.edu/per/TestInfo.html.

Lawson, A. E., Banks, D. L. & Logvin, M. (2007). Self-efficacy, reasoning ability, and achievement in college biology. *Journal of Research in Science Teaching, 44*(5), 706–724.

Lehrer, R. & Schauble, L. (2000). Developing model-based reasoning in mathematics and science. *Journal of Applied Developmental Psychology, 21*(1). 39-48.

MOE (Ministry of Education) (2013). *Malaysia Education Blueprint 2013-2025 (Preschoool to Post-Secondary Education)*, Putrajaya, Malaysia: Kementerian Pendidikan Malaysia.

Morris, B. J., Croker, S., Masnick, A. M. & Zimmerman, C. (2012).The emergence of scientific reasoning. In H. Kloos, B. J. Morris and J. L. Amaral (Eds.), *Current Topics in Children's Learning and Cognition* (pp. 61–82). Rijeka, Croatia: Tech - Open Access Publisher. Retrieved 2 January 2014 from http://creativecommons.org/licenses/by/3.0.

National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: reasoning and sense making.* Reston, VA: NCTM.

Santos-Trigo, L. & Moreno-Armella, L. (2013). *International perspectives on problem solving research in mathematics education, a special issue. The Mathematics Enthusiast*, *10*(1&2), 3–8.

Schoenfeld, A. H. (2013). Reflections on problem solving theory and practice. In L. Santos-Trigo & L. Moreno-Armella, *International Perspectives on Problem Solving Research in Mathematics Education, a special issue. The Mathematics Enthusiast, 10*(1&2), 9–34.

Stylianides, G. J. (2008). An analytic framework of reasoning-and-proving. *For the Learning of Mathematics*, *28*(1), 9–16.

Developing Adaptive Expertise with Pasifika Learners in an Inquiry Classroom

Zain Thompson
Massey University
<z.thompson@massey.ac.nz>

Jodie Hunter
Massey University
<j.hunter1@massey.ac.nz>

In the current reform of mathematics classrooms teachers are charged with the role of facilitating collaborative groups during problem-solving activity. The challenge is for teachers to engage students in making mathematical meaning during collaborative group discussions. In this paper we draw on the concept of adaptive expertise to report on teachers' actions to engage students in co-constructing collective knowledge. We address how teachers attended to students' cultural values and socio-mathematical norms to promote and cultivate adaptive expertise.

Introduction

The current educational climate emphasises children in the 21st century becoming literate and numerate (Ministry of Education, 2010). Despite this aspiration, within New Zealand the National Standards data highlights the significant underachievement of many Pasifika students (Ministry of Education, 2012). Many researchers (e.g., Berryman, Bishop, Cavanagh, & Teddy, 2009; Clark, 2001; Spiller, 2012) note that Pasifika students along with indigenous Maori students are marginalised in the schooling system due to inequitable practices. This includes deficit theorising by teachers, identity issues, lack of effective pedagogical actions, and cultural misunderstandings. In order to alleviate these practices and increase the achievement of Pasifika learners, New Zealand's national Pasifika education strategy – the *Pasifika Education Plan (PEP) 2013 – 2017* emphasises the need to increase achievement by responding to the identities, languages, and cultures of different Pasifika groups (Ministry of Education, 2013).

The PEP policy is prominent at a government level; however, there are limited guidelines for schools and teachers on how to implement the goals of the strategy. Furthermore, few studies specifically focus on culturally responsive practices for Pasifika learners in mathematics. Research studies (e.g., Averill & Clark, 2012; Hunter & Anthony, 2011) which do address this area indicate that cultural values can work against Pasifika students as they are not accustomed to questioning or arguing during mathematics - practices integral to success in mathematics. Despite these challenges, a current initiative recognised by New Zealand's Ministry of Education as having a significant positive impact on student learning is the *Pasifika Success Project in Mathematics*. This initiative draws on the PEP's strategy and aims to develop teachers' pedagogical actions and culturally responsive practices while supporting them to construct an inquiry-based mathematics classroom. The purpose of this paper is to outline how two teachers from the Pasifika Success Project drew on the cultural backgrounds of their students to enhance their mathematical practices. The focus of the paper is on the teachers' actions that led to their students developing adaptive skills during collaborative problem-solving activity.

Background Research

Creating high quality learning environments enables students to engage in mathematical learning. Guiding students to develop adaptive expertise in such

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 611–618. Sunshine Coast: MERGA.

environments is essential to give students the opportunity to construct mathematical knowledge. The construct of *adaptive expertise* was introduced by Hatano (1982) where he initially related the notion to adaptive adults in the workplace. However, more recent research involves student learning in mathematics education. Hatano (2003) argues that a pressing issue in mathematics education is how students can learn so they develop adaptive expertise. He describes adaptive expertise as the ability to flexibly and creatively apply meaningfully learned procedures where learners explore a range of possibilities and try to make sense of their actions. Hatano (2003) compares this term to routine expertise, which he defines as being able to carry-out mathematics exercises quickly and accurately with limited understanding. He argues that if an educational environment is oriented toward solving a set of problems, students will become experts defined in relation to accuracy and speed (routine experts). In contrast, when learning environments meet varied demands, students are able to develop adaptive and flexible skills therefore developing adaptive expertise.

Similarities can be drawn from Hatano's (2003) routine and adaptive expertise constructs to Boaler's (2006) notion of multi-dimensional and uni-dimensional classrooms. Boaler's (2006) four-year study of an equitable approach to mathematics teaching and learning illustrated how the multi-dimensionality of classrooms contributed to high student achievement. Boaler (2006) describes a uni-dimensional classroom as a classroom where only one dimension of mathematical work is valued and to be a successful mathematician students have to execute procedures correctly. Boaler (2006) advocates for heterogeneous groupings in multi-dimensional classrooms where many dimensions of mathematical work are valued such as asking good questions, mathematically explaining ideas, and justifying solution strategies. Hatano (2003) shares a similar view. He argues that through questioning, conceptual knowledge that is related to a procedure can be developed. This can occur through discourse and students asking why each step is needed during its application. Hatano (2003) notes that this process is similar to Schoen's (1983) notion of *reflection in action*. The similarity being that both researchers argue that knowledge is constructed through the process of solving problems.

Developing an understanding of mathematical concepts through exploration is supported by research on developing adaptive expertise in mathematics. For example, Markovits and Sowder (1994) designed a three-month unit that focused on providing opportunities for students to explore the relationships between numbers and a range of operations. Rather than introducing new procedures, this was aimed at encouraging the development of a deeper conceptual understanding of the content they had already acquired. Following the intervention, students from the experimental group were compared to students taught with a traditional curriculum. The results indicated that the students in the experimental group had greater number-sense. These researchers concluded that the exploration of the relationships between numbers and differing operations aided the students in solving novel problems. Similarly, Mercier and Higgins (2013) associate the development of adaptive expertise with the opportunity for students to be innovative and exploratory with mathematical concepts. This includes exposure to multiple solution strategies. By allowing students to explore and reflect upon the different solution strategies, these researchers contend that each student will choose a strategy that is "personal and insightful" supporting them to become more flexible and adaptive.

Participation in collaborative discussions can be a powerful way for students to develop adaptive expertise within mathematics. Staples and Colonis (2007) differentiate between two types of discussions: sharing and collaborative discussions. In sharing discussions, they note that students are urged to understand others' ideas however they maintain a connection to their own ideas. In contrast, these researchers define collaborative discussions as extending beyond understanding others' ideas to responding to, extending, or connecting to the ideas to form a new perspective. Similarly, Chapin and O'Connor (2007) advocate the use of talk moves by teachers to engage students in academically productive talk. These talk moves include *revoicing*, *repeating*, *reasoning*, *adding on*, and *teacher wait time.* Use of these can promote collaborative and equitable discourse hereby contributing to the development of adaptive expertise.

The development of productive discourse during collaborative discussion requires a suitable classroom environment. To achieve this, Yackel and Cobb (1996) propose the use of socio-mathematical norms: social norms that are unique to mathematics. Such norms describe appropriate mathematical discourse and engage students in mathematical practices such as mathematical explanations and argumentation. In Yackel and Cobb's (1996) study, they explored the development of norms in inquiry-based mathematics classrooms. A key finding of this study was that when teachers set up socio-mathematical norms, students developed flexibility and sophistication in their use of mathematical constructs, key aspects of adaptive expertise.

Also of importance to this study is the notion of culturally responsive practice. Drawing on students' cultural backgrounds facilitates student engagement in learning mathematics. Hawk, Cowley, Hill, and Sutherland (2005) urge educators to attend to the cultural well being of Pasifika students by building on their cultural capital. An example of the importance of this is found in the study by Averill and Clark (2012). This study focused on high school students' perceptions towards the cultural value of respect. These researchers found that students believed teachers were respectful if they held high expectations and believed in their students' abilities. This included giving students time to think and problem-solve during mathematics rather than explaining a solution directly to students. Similarly, Hunter and Anthony (2011) found in their study that the case teacher drew on his students' concepts of respect and reciprocity to encourage students to actively listen, question, and support each other during the learning of mathematics. This led to positive outcomes for the Pasifika students as they were positioned to engage in inquiry discourse and develop collective mathematical practices.

The theoretical framework of this study draws on a socio-cultural perspective. This perspective views students' mathematical activity as a social process that develops as students participate in mathematical practices (Yackel & Cobb, 1996). Hatano and Oura (2003) observed in their studies that gaining adaptive expertise occurred in socio-cultural contexts. These contexts are related to student interests, values, and identity where learning is socially significant, such as solving real-world problems. Extending this socio-cultural perspective is Lave's (1996) emphasis on the community as opposed to the individual. Lave (1996) states that mathematics should not be viewed as an abstract task or individual practice but an activity that is deeply bound in social activities within a community. Learning can therefore be viewed as occurring through participation in practices and the gradual attainment of expertise which contributes to the development of children's mathematical identities.

Methodology

This paper reports on a case study of two classrooms drawn from the wider Pasifika Success Project. The case study design was used to gain an in-depth understanding of factors that contributed to Pasifika learners developing adaptive expertise in inquiry-based mathematics classrooms. The research took place in November towards the end of the school year, which coincided with the near completion of the PLD project for teachers. Participants were students from two New Zealand urban primary schools, aged ten to thirteen years old, of Pacific Nations ethnicity. These students came from low socio-economic home environments and many spoke English as their second language.

Data collection involved semi-structured focus group interviews and video-recorded footage of mathematics lessons. Field notes were used to support the classroom observations. The interview and video-recordings were wholly transcribed and, through an iterative process using a grounded approach, patterns and themes were identified. Five key themes related to cultural values emerged from the analysis of the data: respect, reciprocity, service, inclusion, and leadership.

Results and Discussion

In this section the five key themes that were identified from the data analysis will be discussed in reference to the teachers' actions that drew on their students' cultural backgrounds. These actions that presented students with the opportunity to develop adaptive expertise involved developing collaboration within groups; promoting collaborative discourse; and fostering inclusion and adaptive skills through using mathematical practices.

Developing Collaboration within Groups

Both teachers deliberately set up their classroom learning environments with a focus on an exploratory approach to mathematics learning: students were encouraged to explore and be innovative with mathematics concepts. To support this, students were presented with cognitively demanding group-worthy problems and the expectation was that they would collaboratively explore the problem and engage in sense making. These actions drew on the cultural value of respect: teachers showing respect for their students' abilities to construct knowledge while problem solving (Averill & Clark, 2012). Within their planning the teachers identified key mathematical ideas to which student learning was connected to during the conclusion of the lesson. The structure of this learning environment ensured opportunities for students to think creatively and collaboratively generate multiple solution strategies to mathematical tasks.

Purposeful grouping of students was used to set up a learning environment that promoted mathematical practices and drew on cultural values. The social and grouping arrangements of the students consisted of a heterogeneous make-up (varying attainment levels) with groupings of three or four students. The teachers purposefully assigned students their groups based on social and mathematical skills and changed these groups on a daily basis. This required students to develop their ability to work with different students. When questioned about this grouping arrangement, students showed a consensus in favour of this approach and responded with the following comments:

Laisa: We work with different people all the time, doing different roles. We now like working with anyone in our class.

Mere: People can have different skills and think differently from you. That's a good thing. You can learn new things from them.

Laisa: In same ability groups you don't learn much because you are all the same.

As evidenced in the comments from the students, they valued the opportunity to work in heterogeneous groupings with different members of the classroom community.

In order to promote equitable participation the teachers refrained from assigning students to roles within the group. Instead students were given the flexibility to take turns at different roles at their own discretion. This also countered academic and status differences within the groups (Boaler, 2006). However, working in this way necessitated that students adapted to carrying out a multitude of roles within a group. This was recognised by the students:

Tania: We have learnt more skills by doing different roles.

Laisa: Everyone has something to contribute to the group - strategies, questions, explaining differently.

Mere: We don't have one leader - leaders always change. We see ourselves as learning from each other.

The classroom environments drew on the cultural backgrounds of the students. Within the comments from the students links were made to the cultural values of inclusion, leadership, and reciprocity. For example, inclusion is depicted by the need for equitable participation as noted by the students. They emphasised that all group members have skills to contribute to group work and that students carry out different roles. Mere made reference to leadership as a shared role where students take turns leading their group; this may be enacted by constructing and explaining part of a solution to group members. Reciprocity was illustrated by the students' view of themselves as learning from one another and Laisa noting that every group member has important mathematical skills to contribute during group work.

Promoting Collaborative Discussions

The development of collaborative discussions was a key element of these classrooms. The case teachers used specific talk moves to promote collaborative discourse among group members. During the small group phase of the lesson, the teachers monitored the group activity and the students' participatory actions. When the teachers noticed limited use of mathematical practices, they responded by asking certain students to explain their reasoning and for other group members to ask questions, repeat an explanation in their own words, agree or disagree with a reason, or add on to the group's idea.

The collaborative grouping structures described in the earlier section and development of specific norms enabled students to develop their own discourse to support one another during collaboration. For example, in one lesson the different group members engaged in collaborative discourse to contribute towards the development of the group's solution strategy.

Sefu: So it's twelve point six five kilometres plus five point seven eight kilometres equals … what's a strategy we can use? We can do different strategies.

Lenni: What about partition a number?

David: Twelve point six o plus zero point zero five.

Lenni: Then that one is five point seven o plus

Sefu: Zero point zero eight.

Lenni: So what do we do now? Add the tenths then hundredths numbers together.

David: 60 plus 70?

Lenni: 130. But remember it will actually be one point three zero not one hundred and thirty because there's a decimal point in the middle.

In this discussion the students collaboratively developed a solution strategy along with constructing place value knowledge as Lenni reasons with his group members about the correct terminology associated with the concept.

Students also identified the importance of collaboration while engaging in discourse to develop solution strategies during the focus group interviews:

Laisa: When people feed off each other's ideas it becomes deeper thinking. We find that we come up with new things that we didn't think about.

Tina: It's important to be ready for mind change - when you are used to doing one strategy and you see another person using a new strategy you can connect to that and learn it.

These responses draw on the value of co-constructing solution strategies together by building on and extending one another's ideas (Staples & Colonis, 2007). This involves making sense of different ideas and synthesising the ideas to develop new knowledge.

Furthermore, value was also placed on inclusion of others during collaborative activity in mathematics lessons:

Sally: It's about sharing your knowledge with your group members.

Kali: If someone hasn't got it we spend time practising and going over a problem, helping each other and our solutions before presenting.

Fia: We don't just think about ourselves - we help others to get on-track.

Sally: So no one is left out. So we know that everyone is learning.

Tini: We feel more successful if our whole group gets it.

However the students also recognised that they needed to use specific strategies to ensure that they supported each other within their group discourse. Interestingly, these paralleled some of the talk moves outlined by Chapin & O'Connor (2007) that were used by their teachers:

Kali: We say can you add on? Can you paraphrase, to see if the audience is still following us?

Tini: Do you agree? Do you disagree? Can you explain and justify your strategy? Does anyone have another strategy?

It is evident that Kali and Tini developed adaptive expertise from participating in collaborative activity. These students adopted the talk moves used by their teacher and adapted these to support each other during collaborative discussions. The talk moves supported students in exploring various concepts in mathematics and encouraged them to make sense of their actions: this is viewed by Hatano (2003) as adaptive expertise.

Again links can be made to Pasifika values when examining both the students' comments and interaction from the classroom. In particular, reference was made to the value of service in relation to the importance of serving group members so everyone in the group experiences success (Boaler, 2006). Additionally other comments referred to by these students strongly value inclusion to ensure no one is left out and all group members learn.

Fostering Inclusion and Adaptive Skills through using Mathematical Practices

In both classrooms, the teachers led the development of socio-mathematical norms during mathematics. These included providing mathematical explanations, using different representations, and justifying solution strategies using mathematical reasoning (Yackel & Cobb, 1996). Through emphasising the social and socio-mathematical elements of the classrooms, learning environments which valued many dimensions of mathematical work were developed (Boaler, 2006). This was evident when students were asked to describe how they worked during mathematics.

Laisa: We ask a lot of questions - about what we don't understand about the problem. We paraphrase - people have to explain in their own words.

Tania: We stop and check on each other to see if we understand or agree. Do they agree with the answer or the strategy we used?

Mere: They have to justify their answer and why they disagree. They have to try and convince us.

Tina: We make sure everyone's on-board. We paraphrase and add on. If we have different answers we justify until we come to an agreement. We have to make sure everyone has got it.

Laisa: We use pen and paper and write or draw whatever we like to help each other.

Mere: If you are asking questions you are getting a better understanding of what you're doing. Also, if you paraphrase you are getting a better understanding of what the problem is about so you're building your knowledge and get deeper thinking.

Again, within the descriptions from the students, the links to Pasifika values such as service and inclusion are evident. The students emphasise the importance of ensuring all group members understand by enacting many important dimensions of mathematical work, including questioning, using different representations and justification (Boaler, 2000). Engaging in these practices while solving problems enables students to construct their own knowledge and develop adaptive expertise (Hatano, 2003).

Conclusion and Implications

The findings of this study indicate that students are able to develop adaptive and flexible skills when teachers set up appropriate learning environments that promote adaptive expertise. Key to this is the use of an exploratory approach to problem solving that gives students the opportunities to create multiple strategies and engage in mathematical practices (Mercier & Higgins, 2013). Pasifika students were given the opportunity to think creatively and construct their own knowledge that exemplifies the value of respect. Similar to what Averill and Clark (2012) described, the teachers showed respect for their students' abilities.

Heterogeneous grouping was also used to promote adaptive expertise. The regular mixing of groups contributed to the students' developing adaptive skills to be able to work with different students and carry out different group roles. Also linked to this were connections to cultural values such as reciprocity, inclusion, and leadership. This is similar to Boaler's (2006) findings that highlighted the value of reciprocal learning when students were placed in heterogeneous grouping.

By carefully structuring collaborative discourse, teachers were able to promote the values of reciprocity and service. Collaborative discussions enabled students to build on to one another's ideas when co-constructing a solution strategy (Colonis & Staples, 2007). Students demonstrated service by supporting each other to understand group solutions while engaging in mathematical practices. During small group discussions, students also displayed adaptive expertise by generating their own talk moves. Placing an emphasis on socio-mathematical norms encouraged students to use and value different dimensions of mathematical work; this included mathematical explanations and justification (Yackel & Cobb, 1996). Lastly, the findings reflect Hatano's (2003) perspective of knowledge construction, when students are given the opportunity to solve problems in a learning environment with varied demands, students are able to construct knowledge and develop adaptive expertise.

This study presents a culturally responsive approach to the teaching of mathematics that produces positive outcomes for Pasifika learners. In this study, students were positioned as adaptive co-constructors of knowledge. If educators are able to view students as knowledge creators who develop adaptive skills during mathematics learning, this may counter deficit beliefs towards students' abilities. A key finding is that when learning environments and teachers' pedagogical actions draw on students' cultural backgrounds, learners can develop adaptive skills that support them in using mathematical practices. This study adds to the existing research base on culturally responsive teaching for Pasifika learners by analysing Pasifika values and how they can be related to mathematics learning in specifically designed classrooms.

References

Averill, R., & Clark, M. (2012). Respect in teaching and learning mathematics: Professionals who know, listen to and work with students. *Set: Research Information for Teachers*, *3*, 50.

Bishop, R., Berryman, M., Cavanagh, T., & Teddy, L. (2009). Te kotahitanga: Addressing educational disparities facing Māori students in New Zealand. *Teaching and Teacher Education*, *25*(5), 734-742.

Boaler, J. (2006). Urban success: A multidimensional mathematics approach with equitable outcomes. *Phi Delta Kappan*, *87*(5), 364.

Chapin, S.H. & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W.G. Martin, M. Strutchens, & P. Elliot (Eds.), *The learning of mathematics* (pp. 113-139). Reston, VA: NCTM.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, *31*(3-4), 175-190.

Hatano, G. (1982). Cognitive consequences of practice in culture specific procedural skills. *Quarterly Newsletter of the Laboratory of Comparative Human Cognition*, *4*(1), 15-18.

Hatano, G. (2003). Foreword. In A.J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. xi-xiii). Mahwah, NJ: Lawrence Erlbaum Associates.

Hatano, G., & Oura, Y. (2003). Commentary: Reconceptualizing school learning using insight from expertise research. *Educational Researcher*, *32*(8), 26-29.

Hawk, K., Tumama Cowley, E., Hill, J., & Sutherland, S. (2005). The importance of the teacher/student relationship for Maori and Pasifika students. *Set: Research information for Teachers, 3*, 44–50.

Hunter, R., & Anthony, G. (2011). Forging mathematical relationships in inquiry-based classrooms with Pasifika students. *Journal of Urban Mathematics Education*, *4*(1), 98-119.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, culture, and activity*, *3*(3), 149-164.

Markovits, Z., & Sowder, J. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education*, 4-29.

Mercier, E. M., & Higgins, S. E. (2013). Collaborative learning with multi-touch technology: Developing adaptive expertise. *Learning and Instruction*, *25*, 13-23.

Ministry of Education. (2010). *Statement of Intent 2010-2015.* Wellington: Author.

Ministry of Education. (2012). *Pasifika Education Plan monitoring report 2010.* Wellington: Author.

Ministry of Education. (2013). *Pasifika Education Plan 2013-2017.* Wellington: Author.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action* (Vol. 5126). Basic books.

Spiller, L. (2012). How can we teach them when they won't listen? *How teacher beliefs about Pasifika values and Pasifika ways of learning affect student behaviour. SET: Research Information for Teachers*, *3*, 58-67.

Staples, M., & Colonis, M. (2007). Making the most of mathematical discussions. *The Mathematics Teacher, 101*(4)*,* 257-261.

Getting out of Bed: Students' Beliefs

Jane Watson
University of Tasmania
<Jane.Watson@utas.edu.au>

Rosemary Callingham
University of Tasmania
<Rosemary.Callingham@utas.edu.au>

Responses of 223 students in grades 6 to 11 to questions related to beliefs about getting out of bed on the left side are analysed from two perspectives. On one hand the items explore subjective beliefs about chance. On the other hand the different wording and context of the items provide opportunity to show different levels of understanding of students' explanations. Rasch analysis is used to place the items on a scale with other statistical literacy items in order to suggest potential levels of difficulty.

Background

Over the years there appears to have been a rise and fall in interest related to students' subjective beliefs about probability. In 1972 Kahneman and Tversky (1972) introduced the representativeness heuristic to explain people's subjective solutions to probability problems based on an event's similarity to its parent population or the way in which it was produced. This was followed by Tversky and Kahneman's (1973) availability heuristic related to subjective decisions on probability based on remembered incidents or scenarios. Fischbein (1975) suggested that preconceived ideas and superstitions could influence children's probabilistic decisions before the age when they reached the stage of formal operations. Fischbein and Gazit (1984) followed this work in a study of teaching intended to change such beliefs.

Subjective factors specifically affecting students' explanations of outcomes from trials in contexts where equally likely random outcomes should be expected, such as coins, dice, and urns, were studied by J. Truran (1985) and K. Truran (1995). The beliefs they uncovered included the use of mental powers, the use of physical manipulation, the need to change outcomes on multiple trials, the intervention of outside forces (such as God), the need to achieve a specific outcome for a game, the attribution of luck (or lack of luck), the kind of material out of which a device is constructed, and the greater difficulty of getting outcomes associated with higher numbers.

In a 1999 review of cultural influences on subjective beliefs about probability, Amir and Williams concluded, "it is widely believed and accepted that the children bring informal knowledge acquired in daily life from their culture which might interfere with their learning of probability" (p. 85). Their study in England sought to characterise these influences. On one hand the cultural influences included superstitiousness, religiousness, personal experience with games, and interpretation of language used to describe probabilistic occurrences. These influences were similar to those described by Truran (1995). On the other hand they also identified biases identified earlier as representativeness, equiprobability, and availability, as well as the outcome approach of Konold (1989).

Recently Sharman (2014) again considered the influence of culture on probabilistic thinking, using examples from research carried out in Fiji. From a very different cultural setting than Amir and Williams (1999), she reported similar attributions to their research, calling for more research from this perspective.

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 619–626. Sunshine Coast: MERGA.

The current study

The current study arose following the previous use of a survey item some years earlier (Watson, Collis, & Moritz, 1995). The item, called *James* here was adapted from an item used by Fischbein and Gazit (1984): “Joseph endeavours to enter the classroom, each day, by putting the right foot first. He claims that this increases his chances of getting good marks” (p. 5). Fischbein and Gazit did not ask for explanations, only an opinion of “Yes” or “No”, where “No” was correct. There was no reward for suggesting “Yes” that the belief might help Joseph and the overall results were inconsistent across grades and teaching conditions (p. 14-15). Joseph became James, and his action to increase his chances of getting good marks was to get out of bed on the left side.

The original James item was used in surveys with 1014 students in Years 3, 6, and 9 (Watson et al., 1995) and assessed using the SOLO model (Biggs & Collis, 1982). Overall, 58% of students (rising from 43% in Year 3 to 68% in Year 9) dismissed the claim, whereas between 5% of Year 3 and 30% of Year 9 could offer more sophisticated reasoning about James’ beliefs. Later the item was included with two other belief items in a longitudinal study (Watson, Caney, & Kelly, 2004) that looked at change over two and four years and compared beliefs about chance with chance measurement questions. For the James item the change over four years for 148 students starting in Year 3 was from 63% to 73% for dismissing the claim and from 4% to 16% for providing more sophisticated reasoning. For 117 students initially in Year 6 the change to Year 10 was an increase from 64% to 68% for dismissing the claim and from 14% to 23% for giving more sophisticated reasoning. The positive change in performance for the combined belief item scores was significant in each 2-year period.

The James item might not have been used again except for a media item that addressed the specific issue of getting out of bed on the left side. The article from Reuters news agency is reproduced in Figure 1 (Majendie, 2008). Because of the existence of *expert* opinion, it was decided to include the supporting evidence and see how it influenced students’ opinions about James’ belief. The amended items are in Figure 2, with items 1 (JMES), 2 (FENG), and 3 (PSYC) included in one survey of statistical literacy and item 4 (BED) included in a parallel survey. Hence in one survey, students could make three responses in relation to getting out of bed on the left side, whereas in the other only one item was used combining the opinions of the experts.

Getting out of bed on the left side is the right side

Sleep scientists, feng shui experts and psychologists put their heads together to analyse the best way to get up in the morning.

Left is best, they decreed in a study undertaken by the hotel chain Premier Inn.

Feng shui expert Jan Cisek said getting out of the bed on the left is associated with all that people hold dear – family and health, money and power.

Psychology and motivation expert Pete Cohen said the left side helps us all to think rationally about the day ahead.

“The right side of the brain is responsible for emotions like fear and stress which only dilute your potential for having a positive experience,” he said.

Figure 1. Getting out of bed on the left side is the right side (Majendie, 2008).

Item 1 – JMES

Every morning James gets out of the left side of the bed. He says that this increases his chance of getting good marks.
Explain what you think of this claim.

Item 2 – FENG

Now consider a newspaper headline:

> **Left is the right way to exit bed**
> Feng shui expert Jan Cisek said getting out of the bed on the left was associated with all that people held dear – family and health, money and power…

Explain what you think of this claim.

Item 3 – PSYC

Also in the same article:

> **Left is the right way to exit bed**
> …Psychologist Pete Cohen said that getting out of bed on the left side helped us to think rationally about the day ahead.

Explain what you think of this claim.

Item 4 – BED

The following extract is from a newspaper.

> **Left is the right way to exit bed**
> Feng shui expert Jan Cisek said getting out of the bed on the left was associated with all that people held dear – family and health, money and power. Psychologist Pete Cohen said the left side helped us to think rationally about the day ahead.

Explain what you think of these claims?

Figure 2. The four probabilistic reasoning items used.

The introduction of the authentic media extracts places the items in Figure 2 in a cultural context not considered in previous research. The purpose of this analysis, hence, is to explore what difference the form of question makes in eliciting student responses to the belief about getting out of bed in a particular fashion and its effect on life situations, especially James' chances of getting good marks. Do students respond differently to the different stimuli and how does the sophistication compare with other statistical literacy questions?

Method

Sample

All participants were students who were part of the StatSmart project and who had already completed at least three StatSmart assessments (see Callingham & Watson, 2007 for details of the research design). These students were all in classes with teachers who were part of the project at a point during the study when they did not have to undertake an assessment but where their peers were undertaking one of the StatSmart tests. Teachers requested another test to occupy the small numbers of students in their classes who fell into this category, and this situation provided the opportunity to trial new items, including the four of interest: BED (Test Form X), JMES, FENG, and PSYC (Test Form Y). Of the 248 students (M, n=132, 53.2%; F, n=116, 46.8%) from three different states (South Australia, Tasmania, and Victoria) who undertook the two test forms, 229 provided valid answers to one or more of the four target items. The distribution of valid responses to these items across grades is shown in Table 1, broken down by test form.

Table 1
Number of students in each grade for each survey

Grade	6	7	8	9	10	11	Total
Form X	5	27	11	31	25	4	103
Form Y	8	23	23	24	48		126
Total	13	49	33	54	70	4	229

Instruments

The two test forms consisted of 23 (Test X) and 25 (Test Y) items of which 20 were common to both forms. The common items were taken from an item bank of statistical questions that had been used in a number of past studies to measure statistical literacy (e.g., Callingham & Watson, 2005), including some also included in the StatSmart tests. Because the two test forms were linked by common items they could be placed together on the same measurement scale using Rasch analysis, and ultimately linked to the larger StatSmart data set. Both tests and the items within them, through a consideration of fit to the Rasch model, met the standards required to allow valid inferences to be made from the data (Bond & Fox, 2007).

Analysis

All items were coded using rubrics developed on the basis of the complexity of response. The specific rubric used for all four target items is shown in Table 2.

Table 2
Rubrics used to code Getting-out-of-Bed items

Code	Description
0	No response
1	Agreement with James or Feng shui expert or Psychologist
2	Rejection of claim; simple disagreement with no justification
3	Presentation of one argument, either based on a lack of evidence, physical conditions, or based on a psychological belief that may assist performance
4	Combination of more than one argument, based on lack of evidence, physical conditions, and/or based on a psychological belief that may assist performance

Coded responses were analysed using Winsteps 3.80.1 Rasch measurement software (Linacre, 2013). In addition, examples of the text responses were collected as exemplars of different levels of response. A map of all the items, showing the relative difficulty of the items with their different coding levels, was produced by the software. This was examined to determine how the items behaved relative to each other, as well as in relation to other items. In addition, the map was compared qualitatively with Callingham and Watson's (2005) Statistical Literacy Hierarchy, using items from the earlier study to suggest possible levels on the hierarchy for the new items.

Results

Figure 3 shows the item map produced by the software showing all of the items in both X and Y surveys. The items of interest are shaded. The number attached refers to the code as shown in Table 2. Each # represents 3 students and the scale is shown in mean logits (mean of the log of the odds of response), the units of Rasch measurement. Items shown at the top of the scale are the hardest.

<table>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td>FENG .4</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td><td>+</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td>JMES .4</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td>+</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td>TATS .5</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td>PSYC .4</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td>HOSP .3</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td>BED .4</td><td>RUTH .4</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td>TED .3</td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td>+</td><td></td><td></td><td>HSE3 .2</td><td></td><td>SKINR.4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>.</td><td></td><td>|</td><td></td><td></td><td></td><td></td><td>TATS .4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>.#</td><td></td><td>|</td><td></td><td></td><td>BT1 .2</td><td>RAND .3</td><td>HSE1 .2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>.</td><td>T</td><td>|</td><td>T</td><td></td><td></td><td></td><td>T2X2 .4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>##</td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>.###</td><td></td><td>|</td><td></td><td></td><td></td><td>TEMP .3</td><td>HGT3 .4</td><td>TGPH .4</td><td></td><td></td></tr>
<tr><td>1</td><td>.###</td><td></td><td>+</td><td></td><td>HSE3 .1</td><td>HSE2 .2</td><td>RUTH .3</td><td>STOMR.4</td><td>RUTH .2</td><td>TATS .3</td><td>TEMP .2</td></tr>
<tr><td></td><td>.#####</td><td>S</td><td>|</td><td></td><td></td><td>SPN1 .3</td><td>CAR .3</td><td>SPN2 .4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>#########</td><td></td><td>|</td><td>S</td><td>FENG .3</td><td>F2.3 .2</td><td>F2.3 .3</td><td>MV10 .3</td><td>PSYC .3</td><td>SPN2 .3</td><td></td></tr>
<tr><td></td><td>.########</td><td></td><td>|</td><td></td><td></td><td>SPN1 .2</td><td>BED .3</td><td>HGT3 .3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>######</td><td></td><td>|</td><td></td><td>F2.3 .1</td><td>TEMP .1</td><td>TGPH .3</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>.######</td><td>M</td><td>|</td><td></td><td></td><td></td><td>JMES .3</td><td></td><td></td><td></td><td></td></tr>
<tr><td>0</td><td>.#######</td><td></td><td>+</td><td>M</td><td></td><td>T2X2 .3</td><td>STOMR.3</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>#######</td><td></td><td>|</td><td></td><td>HSE2 .1</td><td>HGT3 .2</td><td>SKINR.3</td><td>RAND .2</td><td>T2X2 .2</td><td>TED .2</td><td>TGPH .2</td></tr>
<tr><td></td><td>.#######</td><td></td><td>|</td><td></td><td>SKINR.2</td><td>HOSP .2</td><td>STOMR.2</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>.#####</td><td>S</td><td>|</td><td></td><td>HSE1 .1</td><td>HGT2 .2</td><td>BOX9 .3</td><td>TGPH .1</td><td>MV10 .2</td><td>PSYC .2</td><td></td></tr>
<tr><td></td><td>.##</td><td></td><td>|</td><td>S</td><td>TATS .2</td><td>BED .2</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>#</td><td></td><td>|</td><td></td><td></td><td>FENG .2</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>-1</td><td>##</td><td></td><td>+</td><td></td><td>FENG .1</td><td>PSYC .1</td><td>STOMR.1</td><td>BED .1</td><td></td><td></td><td></td></tr>
<tr><td></td><td>.</td><td>T</td><td>|</td><td></td><td>HGT3 .1</td><td>BOX9 .2</td><td>RAND .1</td><td>HGT1 .2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>#</td><td></td><td>|</td><td></td><td>BT1 .1</td><td>JMES .2</td><td>TATS .1</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>.</td><td></td><td>|</td><td>T</td><td>BOX9 .1</td><td>SPN2 .2</td><td>RUTH .1</td><td>SPN2 .1</td><td></td><td></td><td></td></tr>
<tr><td></td><td>.</td><td></td><td>|</td><td></td><td>HOSP .1</td><td>JMES .1</td><td>SKINR.1</td><td>T2X2 .1</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td>HGT1 .1</td><td>CAR .2</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>-2</td><td></td><td></td><td>+</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>.</td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td>CAR .1</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td>HGT2 .1</td><td>MV10 .1</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td>TED .1</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>|</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>-3</td><td></td><td></td><td>+</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 3. Item map of all items in X and Y surveys including coding levels.

All of the target items showed good spread along the scale, with the highest code for each item being among the hardest items in the surveys. Agreement and simple rejection of JMES appeared easy for students, but all codes 1 and 2 appeared on the bottom one-third of the scale. Examples of code 1 responses—agreement with the claims—included the following:

JMES: It is his lucky side.

FENG: I think that Jan Cisek thinks getting out of bed on the left side is lucky.

PSYC: Because your brain will work better.

BED: That it better to get out left side of bed.

Of interest is that for the PSYC item, using the psychologist's claim, students seemed somewhat more likely to use answers based on the workings of the brain, whereas with other items they tended to attribute responses to luck. Code 2 responses—simple rejection—included:

JMES: I think it makes no sense.

FENG: It's a superstition.

PSYC: I don't really get this claim, I really don't see how getting out of bed on the left side helps this.

BED: I think that these claims explain what experts think, however, I would not agree with them.

Students could recognise that the claims were made by *experts* and did not necessarily agree with the claims, but were unable to articulate a rationale for their disagreement.

There was a jump in difficulty from code 2 to code 3, shown by the relatively large gaps on the map. It seemed that for students to articulate their rejection of the claim required a greater level of understanding of the context and validity of the claims. Code 3 responses included:

JMES: It can help because it put [*sic*] him in a good mood.

FENG: Feng shui relies on supernatural that is unmeasurable elements so it would be unwise to think anything about it other that it is improbable.

PSYC: No proof. Unless there has been a scientific study, there is no way of knowing.

BED: It doesn't matter really. How can getting up a different way effect [*sic*] your thinking? If you believe it, it might happen.

FENG was more likely to be rejected on the grounds of supernatural or spiritual beliefs, whereas the effects of a belief about getting out of bed on a particular side were often cited as possibly being positive for the person concerned for the other items. Physical conditions, such as the bed being against the wall on the left side were also used as justification for rejecting the claim.

Code 4 responses were very difficult for students to achieve, particularly for JMES and FENG. Examples of code 4 responses included:

JMES: I think that this claim is irrational as it does nothing to change his chance of getting good marks, it's a superstition. But, if he believes it works, this could influence his marks either in a good way, because he is positive about it, or in a bad way, because he believes that it is all he has to do to achieve good marks.

FENG: Left is where the heart is, so spiritually people might think strongly about this, but there is no physical evidence.

PSYC: This is just a claim. There is no particular evidence of it and for some people their beds may be against the left side of the wall so how do they get off the bed on the left. Does that mean they think less? Again, I think there is no science or proof, only superstitious beliefs.

BED: I think that while they're interesting, there is no proof given in the article to actually say that the left side is better. These are only opinions, yet the title seems to state that it is better to get out of bed on the left side. Besides, what if the left side of your bed was up against a wall?!

Students responding at this level were able to conjecture about different conditions or ways in which the person concerned might be affected by their superstition or belief. They sought evidence or scientific proof. The relative difficulty of making a code 4 response to JMES and FENG may be because students could not see any possible merit in the justifications provided for the claim, and hence were unable to discern any plausible arguments that could be used. The PSYC and BED items, on the other hand, tended to attract responses that alluded to scientific or experimental evidence.

The informal comparison of the item map with the Statistical Literacy Hierarchy suggests that all code 4 responses were at the highest level of the hierarchy — Critical Mathematical. The descriptor for this level states that respondents demonstrate a "Critical, questioning engagement with context, using proportional reasoning particularly in media or chance contexts, showing appreciation of the need for uncertainty in making predictions, and interpreting subtle aspects of language" (Callingham & Watson, 2005, p. 3). At the other end of the scale, most code 1 responses appeared to be at the Informal level where responses show "Only colloquial or informal engagement with context often reflecting intuitive non-statistical beliefs, single elements of complex terminology and settings, and basic one-step straightforward table, graph, and chance calculations." The descriptors for these levels seem to be appropriate to the nature of the responses to the four items provided by participants in this study. Further work is needed to place the intermediate codes accurately in levels of the hierarchy.

Discussion

In this study four items addressing students' subjective beliefs about probability were used with other items that targeted more traditional statistical content, including central tendency, graph reading, and numerical probability. All items worked together to provide a single measurement scale, and the four focus items showed a progression in difficulty that was well spread out along this scale.

It could be considered that these items were not mathematical in their nature. No quantification of probability was given and they required no calculation. Responding to these items, however, did demand statistical reasoning especially at the high levels of response. The language demands of statistics can make it challenging for teachers, but the need to be able to *tell the story* (Pfannkuch, Regan, Wild, & Horton, 2010) is exemplified by the demands of context such as shown in the items used in this study.

The nature of the subjective beliefs about probability demonstrated by students in this study was similar to those shown in earlier research (e.g., Amir & Williams, 1999;

Fischbein, 1975; J. Truran, 1985; K. Truran, 1995; Tversky & Kahneman, 1973). The beliefs shown by students in this study included the supernatural, the importance of physical conditions, psychological beliefs, and trust in the scientific method. Students drew on their sometimes limited understanding of context to reason about the situations presented, such as knowledge about the right and left brain.

Being able to present a coherent, critical argument about a situation, referring to the evidence provided, is an important component of statistical reasoning. The students reported here had already been part of a large-scale study, StatSmart, for at least a year. The relative difficulty that they demonstrated in making high-level responses indicates that more work is needed to help students develop the language of statistics. Providing opportunities for students to reason about probabilistic contexts beyond the classroom activities of tossing coins and dice is an important step to developing the critical statistical reasoning skills needed in the complex society in which they live.

References

Amir, G.S., & Williams, J.S. (1999). Cultural influences on children's probabilistic thinking. *Journal of Mathematical Behavior, 18*, 85-107.

Biggs, J.B., & Collis, K.F. (1982). *Evaluating the quality of learning: The SOLO taxonomy*. New York: Academic Press.

Bond, T. G., & Fox, C. M. (2007). *Applying the Rasch model: Fundamental measurement in the human sciences* (2nd ed.*)*. Mahwah, NJ: Lawrence Erlbaum.

Callingham, R., & Watson, J. M. (2005). Measuring statistical literacy. *Journal of Applied Measurement, 6 (1)*, 29, 19-47.

Callingham, R. & Watson J.M. (2007) Overcoming research design issues using Rasch measurement: The StatSmart project. In P. Jeffery (Ed.) *Proceedings of the AARE annual conference, Fremantle, 2007*. Retrieved from http://www.aare.edu.au/07pap/cal07042.pdf

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children.* Dordrecht: D. Reidel.

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? An exploratory research study. *Educational Studies in Mathematics, 15*, 1-24.

Kahneman, D., & Tversky, A. (1972). Subjective Probability: A judgement of representativeness. *Cognitive Psychology*, *3*, 430-454.

Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction, 6,* 59-98.

Linacre, J.M. (2013). *Winsteps 3.80.1* [Computer Software]. Chicago: Winsteps.com.

Majendie, P. (2008). *Getting out of bed on the left side is the right side*. Reuters. Retrieved from: http://www.reuters.com/article/2008/01/14/life-bed-idUSL1350332320080114

Pfannkuch, M., Regan, M., Wild, C. J., & Horton, N. J. (2010). Telling data stories: Essential dialogues for comparative reasoning. *Journal of Statistics Education, 18*(1), 1-38. Retrieved from www.amstat.org/publications/jse/v18n31/pfannkuch.pdf.

Sharma, S. (2014). Cultural influences in probabilistic thinking. In E. Chernoff & B. Sriraman (Eds.), *Probabilistic thinking: Presenting plural perspectives* (pp. 657-681). Amsterdam: Dordrecht.

Truran, J. (1985). Children's understanding of symmetry. *Teaching Statistics, 7*(3), 69-74.

Truran, K. (1995). Animism: A view of probability behaviour. In B. Atweh & S. Flavel (Eds.), *Galtha* (Proceedings of the 18th annual conference of the Mathematics Education Research Group of Australasia, pp. 537-542). Darwin, NT: MERGA.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive Psychology, 5*, 207-232.

Watson, J.M., Caney, A., & Kelly, B.A. (2004). Beliefs about chance in the middle years: Longitudinal change. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics Education for the third millennium: Towards 2010* (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia, Townsville, pp. 581-588). Sydney, NSW: MERGA.

Watson, J.M., Collis, K.F., & Moritz, J.B. (1995). Children's understanding of luck. In B. Atweh & S. Flavel (Eds.), *Galtha* (Proceedings of the 18th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 550-556). Darwin, NT: MERGA.

Improving Student Motivation and Engagement in Mathematics Through One-to-one Interactions

Jennifer Way
University of Sydney
<jennifer.way@sydney.edu.au>

Amelia Reece
University of Sydney
<Amelia.reece@sydney.edu.au>

Janette Bobis
University of Sydney
<janette.bobis@sydney.edu.au>

Judy Anderson
University of Sydney
<judy.anderson@sydney.edu.au>

Andrew Martin
University of NSW
<andrew.martin@unsw.edu.au>

The phenomenon of the 'middle-years dip' in mathematics engagement and achievement has been a cause of concern for over a decade. This paper presents an example of one upper-primary classroom identified as having higher than average levels of student engagement, with the purpose of documenting specific teaching strategies that align with known key elements of motivation and engagement. Drawing on evidence from teacher interviews, observation notes and lesson video with recorded dialogue, we argue that particular types of one-to-one interactions between teacher and student can have a powerful influence on student engagement.

Introduction

In Australia, the under-participation of middle-years (late primary - early secondary) students in mathematics has been widely reported (Sullivan & McDonough, 2007). For example, national reporting of numeracy benchmarks (MCEETYA, 2005) highlight the drop in numeracy results experienced by New South Wales students during the vital transition period of primary to secondary school (years 5/6 to 7/8). This decline has resulted in fewer students continuing with further mathematics study in senior school and beyond, causing a shortage of suitable employees for mathematics-related occupations (DEEWR, 2008). The dual issues of under-participation and under-achievement in mathematics are often described in terms of declining motivation and engagement, and a substantial body of research has found that motivation and engagement are positively associated with student academic achievement (Martin, 2007; Stipek, Salmon, Givvin, Kazemi, Saxe, & MacGyvers, 1998). However, this relationship is not necessarily causal, at least in the short-term. High levels of motivation and engagement do not ensure high levels of achievement and vice versa. There are many mathematically capable students who opt out of mathematics study as soon as it becomes an option. Yet there has been much difficulty in clearly identifying the actual causes of declining motivation and disengagement during this crucial time for students. A number of factors are at play, including social influences, curriculum, pedagogy and personal changes in students relating to early adolescence. In recent years researchers have achieved two significant advances towards a solution to the problem: a) coherent models that account for the multifaceted nature of engagement, drawing together the various definitions and theories; and b) reliable instruments for measuring the facets of engagement exhibited by individual students and monitoring changes over time (Martin, 2007, 2010).

Educators and researchers have long believed that the teacher is the key to determining the quality of learning in a classroom, but when looking for reasons behind the decline of

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 627–634. Sunshine Coast: MERGA.

engagement and performance in middle-years mathematics, educators have tended to be distracted by other factors such as the physical and social development of adolescents and societal influences. However, recent research utilising a multi-faceted model of engagement and an associated measurement instrument has identified the fact that it is not necessarily transition and personal development that causes engagement declines; rather, student, home, classroom, and school factors explain the bulk of such variance – and that amongst these factors, it is the variation in individual students that is the strongest (Martin, Anderson, Bobis, Way, & Vellar, 2012). This means that, potentially, the teacher can overcome the broader influences of developmental change, school and home by focusing on specific characteristics of individual students (Martin, Way, Bobis, & Anderson, 2015). The 'middle-years dip' in mathematics is not inevitable. The research reported in this paper extends this important finding by identifying specific strategies that one teacher uses to promote higher levels of student engagement in mathematics learning via her interactions with individual students.

Motivation and Engagement

'Engagement' is now generally accepted to be a multi-faceted construct that can be broadly described as three (interrelated) categories of engagement – behavioural, emotional and cognitive (Fredricks, Blumenfeld, & Paris, 2004). In general, 'motivation' can be described as a set of interrelated beliefs and emotions that influence and direct behaviour (Wentzel, 1999). However there have been numerous theories developed to explain the processes at work in both engagement and motivation - including attribution, expectancy-value, goal theory, self-determination, self-efficacy, and self-worth motivation theory. Such fragmentation has highlighted the need for a model that encompasses the strengths of the various theories and enables practitioners, such as teachers, to employ a framework that is easily translated into teaching strategies and communicated to students. The research reported here makes use of such a model, depicted diagrammatically as the student Motivation and Engagement Wheel (Figure 1); and represented in the associated Motivation and Engagement Scale, in the form of a validated questionnaire (Martin, 2007).

The student Motivation and Engagement Wheel (Hereafter referred to as 'M&E Wheel') identifies the thoughts, emotions and behaviours that enhance or impede motivation and engagement (Martin, 2007, 2010). The *Adaptive Cognition* section reflects the thoughts that boost motivation. These thoughts consist of *self-belief* (the student's belief and confidence in their ability to understand their schoolwork); *mastery orientation* (a learning focus, whereby the student is interested in developing new skills and understanding); and *valuing school* (the student's belief that the learning is useful and relevant). The *Adaptive Behaviours* section identifies behaviours that enhance motivation and is comprised of *persistence* (how the student perseveres with schoolwork); *planning* (the student's planning and monitoring of their progress); and *task management* (the student's study organisation, including time management). Adoption of these thoughts and behaviours results in increased motivation and engagement (Martin, 2007, 2010).

Thoughts and behaviours that reduce motivation and engagement are reflected in the *Impeding/Maladaptive Cognitions* and *Maladaptive Behaviour* dimensions (Martin, 2007, 2010). Negative thoughts include *anxiety* (feeling nervous about school work); *failure avoidance* (the student feels that if they do not complete their schoolwork they will be seen as a failure); and *uncertain control* (students feel unsure of how to do well and believe that their success is out of their control). Behaviours that hinder motivation and engagement are: *self-handicapping* (adoption of strategies that reduce chances of success, such as procrastination); and *disengagement* (giving up, withdrawing or accepting failure). For a fuller description of the M&E Wheel in relation to mathematics see Bobis, Anderson, Martin and Way (2011).

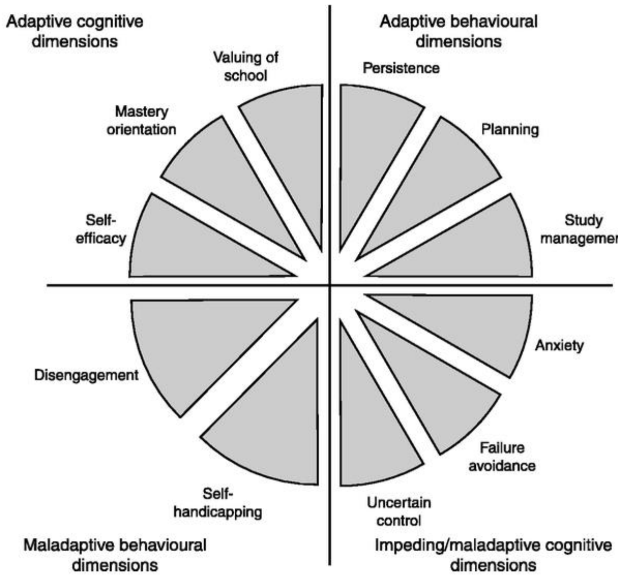


Figure 1. Motivation and Engagement Wheel (reproduced with permission from Lifelong Achievement Group (www.lifelongachievement.com) and Martin, 2010, p. 9.

Teacher-Student Interactions

As previously mentioned, the relationship that exists between student engagement and student achievement is not necessarily causal. This signifies that there may be highly motivated students demonstrating low levels of achievement and, conversely, students with low (or falling) levels of motivation achieving relatively highly. This situation suggests that, although teacher practices that enhance student engagement and those that improve student learning-outcomes may overlap, these practices are not necessarily indistinguishable. There is a growing body of research that asserts that positive interpersonal relationships between the teacher and student support both engagement and academic performance (E.g., Attard, 2013; Clarke et.al, 2002). The complementary nature of the pedagogy to support engagement and pedagogy to support learning is not surprising considering one of the three inter-related types of engagement is ‘cognitive’ engagement, that focuses on learning. For example, Hackenberg (2010) proposes that to build teacher-student relationships aimed at mathematical learning, teachers must assess and monitor the student’s mathematical thinking, attempt to view the mathematics from the student’s perspective and interpret the student’s feelings about the mathematics. She also highlights the reciprocal nature of these relations, in that the teacher needs to receive some positive responses or feedback from the students in order to build the relationship. If these relationships are built successfully, the student is likely to learn the mathematical content and, in turn, develop increased self-belief in their mathematical ability (Hackenberg, 2010). Increased self-belief and a focus on learning the mathematics content (mastery orientation) are positively associated with motivation and engagement (Martin, 2010).

In particular, one-to-one interactions between teacher and student may have significant value in building supportive relationships (Frymier & Houser, 2000), and promoting mathematics learning (Cheeseman, 2009). However, the specific nature of such individual interactions in mathematics classrooms remains under-researched, and little attention appears to be given to the specifics of these pedagogical relationships in teacher education and professional development (Sullivan, Mousley, & Zevenbergen, 2006).

Theoretical Perspective of the Study

The relevance of studying teacher-student interactions is supported by theories of social constructivism, which focus on the learner's construction of knowledge in a social context, including support from the teacher (Cobb, 1994). More specifically, the theory of *symbolic interactionism* has been used in mathematics education to explain how meaning is made through social interactions (Yackel & Cobb, 1996). Symbolic interactionism asserts that mathematical meaning is negotiated, and the theory can be used to explain how the teacher and students co-construct the social norms of the classroom related specifically to mathematics. These norms maintain established patterns of classroom interaction, regulate mathematical argumentation and influence learning opportunities for both the students and teacher (Yackel & Cobb, 1996). It follows that an appropriate approach to investigating teacher-student interactions is to closely observe particular established classrooms, with the understanding that each may be a unique situation.

Methods

This study's focus is expressed by the following research question: What interactions with students does one teacher use in a mathematics lesson, and how do these interactions relate to aspects of motivation and engagement?

Participants

This study was nested within a large mixed-methods project designed to research the phenomenon of the 'middle-years dip' in mathematics engagement and achievement. Longitudinal data from the Motivation and Engagement Scale questionnaire (Martin, 2007) identified six primary and secondary classrooms (from 200) in which student motivation and engagement was found at higher than expected levels, to become case studies. The set of six cases is currently undergoing systematic cross-case analysis, but data from one of the primary classrooms was analysed immediately via an Honours research project and is the subject of this report.

The Year 6 class was in a co-educational Catholic primary school in a large metropolitan area, with students from a wide range of cultural and socioeconomic backgrounds. The teacher, 'Kate', was female, aged 41-50, with over 21 years teaching experience and over seven years experience teaching upper primary. Kate collaborated with another Year 6 teacher in a team-teaching approach with 57 students. The two teachers planned the mathematics program together, with a unique structure that consisted of three groupings of students per lesson: *workshop group* (students having the most difficulty), *core group* (students needing consolidation and practice) and *enrichment group* (high achieving students needing further challenge). These groups were fluid, in that students chose which group to attend each lesson, based on their self-assessed ability after completing the whole-class, introductory task. In the observed lesson on fractions, Kate worked exclusively with the workshop group – the under-achieving students, who she referred to as "the little ones". It is this subgroup of students and this phase of the whole lesson that defined the boundaries of this particular case study (Yin, 2009).

Data Collection and Analysis

Kate's work with her group was video-audio recorded and field-notes were taken by the researcher. Semi-structured interviews were conducted with the teacher before and after the lesson to discuss student engagement, learning and pedagogy, with some specific questions about teacher-student interactions. For example, "Have you planned one-to-one interactions? If so, what kind of interactions? With whom?" and, "When you were walking around talking to the individual students, what kind of strategies did you use?"

Data analysis took place in three phases. The first phase used inductive (open-ended) analysis to identify and document all instances of interactions between Kate and her students through repeated viewings of the videoed lessons. These instances were then grouped into themes, then tentatively categorised according to commonalities. The second phase also used an inductive approach to identify themes in the interview transcripts and field notes. These themes were then applied in refining the researcher's interpretation of the themes and categories of teacher practice derived from lesson-video. The final phase involved looking for alignment between the identified teacher strategies and the elements of the M&E Wheel (Figure 1)

Findings

Kate declared that her major aim when working with the 'underachievers' group was to promote active participation and student understanding, saying, "the whole reason is getting them to understand why, rather than being told 'this is why'". To achieve this she deliberately interacted with individual students throughout the majority of the lesson, because "I know that I get better results with the one-on-one". Most of these interactions took place privately rather than in front of the group. Kate explained,

> For the little ones who couldn't answer, they wouldn't answer so why would you if you have got the other children there? ...You won't play tennis against someone who is McEnroe if you can't hit the ball. Why would you do that with maths?

Analysis of the interviews and the lesson video, interviews and field notes revealed three major categories of practice: pedagogical practices, practices contributing to a quality learning environment, and nonverbal practices - with strong alignment between the three data sources. Although there are some interrelationships between these categories, Kate's own explanations of what she was doing and why, provided further differentiation. Direct quotations from the teacher (Kate), from interview transcriptions and lesson dialogue, are included in the following descriptions of the categories. The lesson was dominated by Category 1 practices and therefore these are more fully explained in this short paper than the other two categories.

Category 1: Pedagogical Practices

Pedagogical practices refer to the teacher's practices that were chiefly concerned with the mathematical content and the students' learning of this content. Most of the one-to-one interactions that took place during the lesson involved these types of practices. Within this category, the following themes were identified:

Promote mastery orientation. Kate explained that maintaining an emphasis on student understanding requires the teacher to be flexible and adapt lessons to appropriately match the students' abilities. The emphasis on student understanding promotes *mastery orientation* (Adaptive Cognition quadrant of the M&E Wheel – Figure 1), which is associated with intrinsic motivation and is therefore a critical element of student engagement. There is also a link between mastery orientation and self-belief, suggesting that student success, achieved by mastering the content, increases their self-belief (Martin, 2007).

Encourage student self-regulation - Kate said that she assists students to "claim ownership of their learning" by allowing them to choose the 'ability' group they work with each lesson, and by encouraging students to reflect on their learning through questions such as "What did you learn?", and by pressing students to identify their preferred learning style. Kate gave the example of "We discuss in class...what kind of a learner are you? Do you need pictures? ...Are you good at listening to people?" Self-regulatory behaviours (planning, task management and persistence) comprise the adaptive behaviours of the M&E Wheel. They correlate to a mastery orientation and have been found to be conducive to both motivation and achievement (Martin, 2007).

Assess student understanding. To keep track of student understanding, Kate discussed the importance of monitoring, questioning and one-to-one conversations, stating in the pre-observation interview "It's a good way to pinpoint children where they are" and that she asks students having difficulties, "What can't you do?" since "If you don't ask them, you don't know". This was reflected throughout the lesson where Kate spent much of her time moving between students to monitor their progress and frequently assessed their understanding through comments such as "Ok. Show me what you've done", "How'd you go?" and "So did you do the figuring out...in your head? Or did you work it out on paper?" Monitoring student understanding in a manner that does not diminish a student's self-regulation corresponds with behaviours of planning, task management and persistence (M&E Wheel) that are positively associated with motivation and engagement.

Support students experiencing difficulty through prompting. Assessing student understanding throughout the lesson allowed Kate to support individual students experiencing difficulties with the task by providing prompts. Some of these prompts encouraged students to reflect on their thinking and included questions such as "Has that shown me that it's 4 lots of 3?" and "Is there a better way of showing...? Another way of showing?" Others provided clues about how to solve the answer, such as "But how many pieces do I need to cut it into?". Such prompts are intended to re-engage students in the task and allow them to experience a sense of accomplishment. Kate explained this was important "… otherwise they just find avoidance techniques. They go looking for other things to do. They are not feeling it's something they are comfortable with or capable of doing." Martin (2007) affirms that success-oriented students exhibit high self-belief and control, both of which are positively associated with motivation and engagement. Such students are also less likely to participate out of fear of failure – an impeding cognition (M&E Wheel).

Extend students when ready. This was evident in Kate's responses to students who correctly completed the task, such as "Can you draw it another way?", "Try another number where the top number is bigger than the bottom one" and "Can you do...this one's a double digit number, 14 over 12." Maintaining an appropriate level of challenge supports mastery orientation and therefore is positively associated with engagement.

Encourage student reasoning. Another feature of Kate's interactions with individual students was her press for students to justify their mathematical thinking through reasoning. "I want them to look at what they do and prove it, like tell me why...Getting them to see and compare and to make a judgment about why and give a reason". During the lesson, this was demonstrated through questions such as "Why is that one different?"

and "How did you know to split that into four bits?" Through challenging students to develop meaningful understandings, reasoning can be linked to mastery orientation and, thus, may contribute to student motivation and engagement.

Category 2: Practices Contributing to a Quality Learning Environment

This category is comprised of the teacher practices that contributed to setting the social and emotional tone of the classroom. These practices helped to create a learning environment where students felt safe and supported and contributed to building positive teacher-student relationships. Kate explained, "It gives the children who are part of that group the sense that someone is listening to me, someone is addressing me". Three themes emerged from the data: a) *Building positive teacher-student relationships* – Kate showed that she cared about the students' feelings, respected and valued them, by being attentive, polite and asking how they felt; b) *Providing encouragement* - illustrated through a variety of verbal and non-verbal communications, nodding and smiling and the frequent use of positive reinforcement; c) *Managing the learning environment* – through brief individual verbal interactions, repositioning students in the classroom, saying student names, raised eyebrows or a light touch on the student's hand or shoulder.

Category 3: Nonverbal Practices

Throughout the lesson, Kate exhibited a range of nonverbal practices when interacting one-to-one with students. These practices concerned her use of gaze, facial expression, gesture, proximity and touch. These included maintained eye contact in conversation, smiling, attentive listening, and pointing. Kate spent much of the lesson moving between students to monitor their work, standing close or even kneeling so that the interaction took place at eye level.

Discussion and Conclusion

In Kate's class, with its emphasis on interactions as the basis for building understanding in mathematics, we see an example of *symbolic interactionism* in action. With this group of 'underachievers', Kate had established socio-mathematical norms with a pattern of one-to-one interactions, which has a strong influence on the learning opportunities for both the students and teacher (Yackel & Cobb, 1996). As the observations were confined to this particular group of students it would be interesting to see whether the same interaction patterns were present when she taught the other two 'more advanced' groups of students.

The findings of this study resonate well with other research that has shown that effective teacher interactions focus largely on mathematical thinking (Cheeseman, 2009), and that monitoring student progress and providing prompts or extension is effective for supporting student motivation and engagement (Clarke et.al., 2002; Hackenberg, 2010; Sullivan et al., 2006). There was clear evidence that Kate deliberately attended to all three types of engagement, that is, behavioural, emotional, cognitive (Fredricks et.al, 2004), but placed particular emphasis on cognitive engagement. Many of Kate's practices aligned well to facets of the M&E Wheel (Martin, 2010). As such, it is possible that the high levels of motivation and engagement previously identified in this class were a result of the teacher's practices that encouraged students to adopt thoughts and behaviours known to increase student motivation and engagement.

Acknowledgements

The authors are grateful for the support for this research from the Australian Research Council, the Catholic Education Office (Sydney) and participating schools.

References

Attard, C. (2013). "If I had to pick any subject, it wouldn't be maths": Foundations for engagement with mathematics during the middle years. *Mathematics Education Research Journal, 25,* 569-587.

Bobis, J., Anderson, J., Martin, A., & Way, J. (2011). A model for mathematics instruction to enhance student motivation and engagement. In D. Brahier (Ed.), *Motivation and Disposition: Pathways to Learning Mathematics, National Council of Teachers of Mathematics Seventy-third Yearbook (2011)* (pp. 31–42), Reston,Va: NCTM.

Cheeseman, J. (2009). Challenging Children to Think: Teacher behaviours that stimulate children to examine their mathematical thinking. In J. Novotna & H. Moraova (Eds.), *Proceedings of the International Symposium Elementary Maths Teaching: The Development of Mathematical Understanding,* 11-23. Prague: Charles University.

Clarke, D.M., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., et al. (2002). *Summary of the Early Numeracy Research Project Final Report*. Fitzroy, Victoria: Mathematics Teaching and Learning Centre, Australian Catholic University.

Cobb, P. (1994). Where is the Mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13-20.

DEEWR. (2008). *Maths? Why Not?* Final Report Prepared for the Department of Education, Employment and Workplace Relations (DEEWR). Canberra: Author

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: potential of the concept, state of the evidence. *Review of Educational Research, 74,* 59-109.

Frymier, A. B., & Houser, M. L. (2000). The teacher-student relationship as an interpersonal relationship. *Communication Education, 49*(3), 207-219.

Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education, 41*(3), 236-273.

Martin, A. J. (2007). Examining a multidimensional model of student motivation and engagement using a construct validation approach. *British Journal of Educational Psychology, 77*(2), 413-440.

Martin, A.J. (2010). *Building classroom success: Eliminating academic fear and failure*. London: Continuum.

Martin, A.J., Anderson, J., Bobis, J., Way, J., & Vellar, R. (2012). Switching on and switching off in mathematics: An ecological study of future intent and disengagement amongst middle school students. *Journal of Educational Psychology, 104*(1), pp.1-18.

Martin, A.J., Way J., Bobis, J., & Anderson, J. (2015). Exploring the ups and downs of mathematics engagement in the middle years of school. *Journal of Early Adolescence, 35* (2), 199-244.

MCEETYA. (2005). *National Reporting on Schooling in Australia: Preliminary Paper.* Canberra: MCEETYA.

Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., & MacGyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education, 29*(4), 465-488.

Sullivan, P., & McDonough, A. (2007). Eliciting Positive Student Motivation for Learning Mathematics. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential Research, Essential Practice. Vol 2: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia*, 698-707. Hobart: MERGA.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006c). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms. *International Journal of Science and Mathematics Education, 4*(1), 496-503.

Wentzel, K. R. (1999). Social-Motivational processes and interpersonal relationships: Implications for understanding motivation at school. *Journal of Educational Psychology, 91*(1), 76-97.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Yin, R. K. (2009). *Case study research: Design and methods*, 4th ed. California: SAGE.

A Cross-cultural Comparison of Parental Expectations for the Mathematics Achievement of their Secondary School Students

Daya Weerasinghe
Monash University, Australia
<daya.weerasinghe@monash.edu>

Debra Panizzon
Monash University, Australia
<debra.panizzon@monash.edu>

This paper presents results from a survey of 80 parents and 120 secondary school students in Australia. Many parents report that their children put in all their effort into mathematics education but they believe that their children can do better if they try harder. This paradox is more evident among parents from Asian-Australian backgrounds compared to parents from other backgrounds who also report having high expectations in mathematics education, which is not the common perception in Australian media and society.

Introduction

Evidence around the high achievement of Asian students is available from comparisons of international studies such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). These studies indicate that students in many Asian countries perform better in mathematics than students in most European countries (Leung, 2012; Thomson et al., 2012). This same difference appears to occur between Asian-Australian students and European-Australian background students and there is therefore interest in factors contributing to those differences.

One relevant factor is arguably the involvement of parents in their children's education. Such involvement has captivated the attention of the world for some time. Chinese parents, for example, are often reported to spend time each day in monitoring the academic activities of their children (Chua, 2011). The term "Tiger mom" is sometimes used to describe an authoritarian parenting style in which parents give their children few choices, and seldom ask children for opinions (Baumrind, 1967; Maccoby & Martin, 1983). It is not only Chinese mothers who act as "Tiger moms". Chua (2011), for example, argued that non-Chinese parents from Korea, India and Pakistan have similar mindsets. The well-prepared offspring of these "Tiger moms" seem to be outperforming non-Asian counterparts at schools where both Asian and non-Asian ethnic background students study together (Chua, 2011).

In order to explore further the influences of parents on their children's education, and especially to explore differences between particular groups, the paper presents findings from a recent survey that seeks to identify the influences of parental involvement in the mathematics education of secondary students.

The Research Framework and Associated Literature

The research is informed by the theory of relative functionalism (Sue & Okazaki, 1990), which has been used to describe achievements of Asian-American students. Functionalism emphasises the adaptiveness of the mental or behavioural processes. In fact, migrants experience difficulties in upward mobility and issues with status in society if they belong to minority ethnic groups in their new country. It is likely that the recency of migration is a salient factor in influencing attitudes of migrant parents. Such parents are likely to be more involved in their children's education than other parents. The theory of relative functionalism explores the extent to which migrants adopt the cultural traits or

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 635–642. Sunshine Coast: MERGA.

social patterns of another country. Sue and Okazaki (1990) argued that education is increasingly functional as a means for mobility when other avenues such as sports, politics, entertainment, and so forth, are blocked. They also argued that the academic achievement of children of Asian-American migrants cannot be solely attributed to Asian cultural values but also to their migrant status. Similarly, in another study in an Australian context with primary school students, Dandy and Nettelbeck (2002) explained this theory as "immigrants attempt to exploit opportunities not available in their homelands, with the ultimate goal of upward social mobility by way of education" (p. 621). In explaining the outperformance of Asian students in countries such as Australia and America, Dandy and Nettelbeck (2002) and Sue and Okazaki (1990) considered those Asian background students as immigrants.

Of course it is not just migrants who take an interest in their children's education. Various studies have suggested that there is a significant relationship between parental involvement and the academic achievement of their children (e.g., Dandy & Nettelbeck, 2002; Fan, 2001; Hong & Ho, 2005) although it seems that the construct of parental involvement is multidimensional and complex. As Hornby and Lafaele (2011) described, the way that parents view their role in their children's education and the belief that parents have in their ability to help their children succeed at school were critical aspects in the study of parental involvement and their attitudes in children's mathematics education. However, Hoover-Dempsey and Sandler (1997) described a lack of confidence of parents in thinking that they may not have academic competence to help their children. Further, Hoover-Dempsey and Sandler argued that it is also critical what views parents hold about children's intelligence as well as how they learn and develop their abilities.

Importantly, as Leung (2012) highlighted, there are many variables within a country or culture that impact student achievement. Many of these variables are interrelated so it is difficult to isolate the effect of individual factors. Considering just one of these Ma (1999) argued that attitudes are important in mathematics participation, suggesting that efforts around improving cognitive skills alone may not necessarily lead to increased mathematics participation. The implication is that if parents spend more time on improving their children's attitudes towards mathematics, then this is likely to have an impact on their achievement.

Various studies have identified a focus on parental encouragement by ethnically Asian parents. In a study on parental roles and culture, Cai, Moyer, and Wang (1997) argued that Asian parents consistently motivate their children to achieve academic success and this encouragement may significantly contribute to the success of Asian students. Interestingly, in a comparison study of students in China and Australia, Cao, Bishop, and Forgasz (2007) found that the students in China had stronger perceived parental encouragement and higher perceived parental expectations than ethnically Chinese students in Australia. The authors also found that parents of Chinese speaking students and other non-European students in Australia have similar levels of parental encouragement but significantly higher levels of parental encouragement than English speaking students in Australia. This connects to their migrant status.

Some studies have found cultural differences in parental expectations for their children. In a survey of 239 Chinese, Vietnamese, and Anglo-Celtic Australian parents of primary school children aged 6 to 14 years in South Australia, Dandy and Nettelbeck (2002) found most parents had high expectations of their children's academic performance. They also found that Anglo-Celtic Australian parents seem to put less emphasis on academic achievement while having more flexible expectations when compared to Chinese- or

Vietnamese-Australian parents. However, Dandy and Nettelbeck (2002) stated that it is impossible to conclude that these factors are solely responsible for ethnic group differences in academic achievement. In a study of direct and indirect longitudinal effects of parental involvement on student achievement using a nationally representative sample of 24,599 eighth graders from 1,052 schools in USA, Hong and Ho (2005) randomly selected a sample of 1,500 students from Asian-American, African-American, Hispanic, and White groups with a total of 6,000 students for their analyses. They concluded that across all ethnic groups the higher the hopes and expectations of parents with respect to the educational attainment of their child, the higher the expectations of the child and greater their academic achievement. In another study based on cross-cultural comparison with 158 parents of students from two Chinese primary schools and one Anglo-Celtic primary school in Hong Kong, Phillipson and Phillipson (2007) argued that parents of different cultures have different intervention strategies and values in bringing up and educating their children.

The current study applies these perspectives in the Australian context with secondary school students using the following research questions:

> How do the expectations for their children in mathematics education vary between Asian-Australian and European-Australian background parents? Does either group have higher expectations than the other? What are the children's perceptions of the expectations of their parents?

Research Method

The data presented here are part of a larger study, which was planned primarily around surveys on parental involvement in mathematics education of their children, using two questionnaires one each for parents and children. In addition to parental expectations for their children, this study focussed on children's perspective about the expectations of their parents. Therefore, two separate instruments on mathematics education were developed with similar but different questions for parents and students. The instructions provided on the instruments informed participants that the responses should be in relation to mathematics education. Surveys were followed by semi-structured interviews for a parent and a child from purposively selected families, although these data are not presented here.

As this study involved participants from Asian and European backgrounds, it was required to invite multicultural schools to participate in the surveys. With the permission of the Department of Early Childhood and Education (DEECD), four multicultural schools with Asian and European background students in metropolitan Melbourne were invited to participate. Two of those schools are select-entry schools and the other two are public schools. Only three principals from the four schools agreed to participate in the study. Hence, the information about the student questionnaire was provided to secondary school students in one select-entry school and two public schools in the city of Melbourne. Next, the information about the parental questionnaire was given to families of those children who were interested in participating without being selective of their ethnic background or culture. The questionnaires were available online, and students and parents were able to respond whenever they wanted. For those who wanted to fill in the questionnaire on paper, a copy was provided.

A total of 200 volunteer participants from European-Australian and Asian-Australian backgrounds including 80 parents (28 European-Australian and 52 Asian-Australian parents) and 120 children (33 European-Australian and 87 Asian-Australian children) responded to the survey. The ethnic background of each participant was recorded. In addition to Australians of Anglo-Celtic heritage, the European group included participants living in Australia who were originally from other European countries including Russia, Italy, Greece, and Turkey. The Asian group consisted of ethnically Sri Lankan, Indian, Chinese, Vietnamese, Malaysian, Singaporean, and Bangladesh participants who also live in Australia. A four-point Likert scale was used to record the responses in the questionnaires of this study (1 = Strongly agree, 2 = Agree, 3 = Disagree and 4 = Strongly disagree). No neutral option was provided thereby forcing specific choices.

Firstly, a table of summarised data was used in data analysis. Secondly, the Mann-Whitney U-Test, which is the non-parametric version of the independent samples t-Test was performed on the ranked data. The Mann-Whitney U-test compares medians of the groups involved. While parametric tests often include assumptions about the shape of the population distribution (e.g., normally distributed), non-parametric techniques do not have such stringent requirements. As the data collected were measured only at the ordinal level (ranked), a non-parametric technique is suitable for data analysis (Pallant, 2013). This study satisfies other requirements for non-parametric tests, which require random samples and independent observations where each person can be counted only once. The dependent variable of the data gathered in response to the following four statements of interest in the study is ordinal and were coded using a discrete number from 1 to 4. Finally, cross-tabulation was used to explore the data and identify relationships further.

Results

While observing the responses to questionnaires on mathematics education, the following items from the parents' questionnaire were of particular interest because responses to those items apparently led to a contradiction. Hence, the responses were analysed to provide particular insights into differences between European-Australian and Asian-Australian background participants.

My child puts all his/her effort into school related tasks. (statement 1)

My child can get better marks if he/she tries harder. (statement 2)

The following items relevant to the above statements were selected from the students' questionnaire.

My parents believe that I put all my effort into school related tasks. (statement 3)

My parents believe that I can get better marks if I try harder. (statement 4)

Table 1 presents responses of the various groups. According to the results, the various patterns of responses between the two groups appear similar. The majority of parents and students agree with the four statements in Table 1. If a parent agrees that his/her child puts all his/her effort into school related tasks, one may question why a parent thinks his/her child can get better marks if the child tries harder. In this paradox, students demonstrate the same attitude about their parents' thoughts highlighting the importance of elucidating parental expectations further in order to identify any similarities and differences between cultures.

Table 1
Summary of responses of parents and students

Statement	Cultural background[1]	Strongly agree	Agree	Disagree	Strongly disagree	Total agree	Total disagree
1. My child puts all his/her effort into school related tasks.	E-A	9 (32.1%)	13 (46.5%)	6 (21.4%)	0 (0%)	78.6%	21.4%
	A-A	13 (25.0%)	30 (57.7%)	9 (17.3%)	0 (0%)	82.7%	17.3%
2. My child can get better marks if he/she tries harder.	E-A	8 (28.6%)	16 (57.2%)	2 (7.1%)	2 (7.1%)	85.8%	14.2%
	A-A	32 (61.6%)	18 (34.6%)	2 (3.8%)	0 (0%)	96.2%	3.8%
3. My parents believe that I put all my effort into school related tasks.	E-A	9 (27.3%)	18 (54.5%)	5 (15.2%)	1 (3.0%)	81.8%	18.2%
	A-A	27 (31.0%)	43 (49.5%)	16 (18.4%)	1 (1.1%)	80.5%	19.5%
4. My parents believe I can get better marks if I try harder.	E-A	15 (45.5%)	11 (33.3%)	6 (18.2%)	1 (3.0%)	78.8%	21.2%
	A-A	64 (73.6%)	22 (25.3%)	1 (1.1%)	0 (0%)	98.9%	1.1%

[1]E-A (European-Australian), A-A (Asian-Australian)

Although both European-Australian and Asian-Australian parents (85.8% and 96.2% respectively) think that their children can get better marks if they try harder, only 28.6% of European-Australian parents strongly agree with statement 2 while 61.5% Asian-Australian parents strongly agree with the statement. This difference between the two groups is further explored below.

Similarly, student responses for statement 4 align with the differences in parental expectations for statement 2 as discussed above. Students from both European-Australian and Asian-Australian backgrounds (78.8% and 98.9% respectively) report that their parents believe that they can get better marks if they try harder. Moreover, from the two groups 45.5% of European-Australian students strongly agree with statement 4 while 73.6% of Asian-Australian students strongly agree with the statement. Both these percentages of the two groups are the highest out of the four options of statement 4. However, it is worth exploring further the parental influence on Asian-Australian students because almost all of them (86 out of 87) agree with statement 4.

Comparing parents' and students' responses, 28.6% European-Australian parents strongly agree with statement 2 whereas 45.5% of their children strongly with statement 4. Also, 61.6% Asian-Australian parents strongly agree with statement 2 whereas 73.6% of their children strongly agree with statement 4. Even though strongly agreeing to these statements is a pressure on children, percentages of children's responses are higher than the parents' responses of similar items. Irrespective of culture, this shows beliefs of some children, which may improve their academic achievement.

Second level analysis using the Mann-Whitney U-Test confirmed that most of the European-Australian and Asian-Australian parents consider that their children put all their effort into school-related tasks (statement 1). This is implied by the median value of 2 (= Agree) for both cultural groups. Most of the parents from both European-Australian and Asian-Australian backgrounds think that their children can get better marks if they try harder (statement 2). However, according to the results, European-Australian parents agree with statement 2 with a median score of 2 (= Agree) while Asian-Australian parents strongly agree with statement 2 with a median score of 1 (= Strongly agree). Although parents from both cultural groups have high expectations for their children, according to median values it is evident that Asian-Australian parents have higher expectations than European-Australian parents.

Students responded in a similar manner. From the responses of children from both cultural backgrounds, the above findings about parental expectations are supported by statement 3 with a median score of 2 (= Agree). This implies that most of the offspring from both European-Australian and Asian-Australian backgrounds think that their parents believe that they put all their effort into school related tasks. The above findings are further supported by the same median values of 2 (= Agree) and 1 (= Strongly agree) for European-Australian and Asian-Australian students respectively for statement 4 as shown in the results. This suggests that the parents from both cultural backgrounds not only have high expectations for their children but also they have successfully conveyed the message to their children.

The results of the Mann-Whitney U test provide probability values (p-values) for the four statements. There is no statistically significant difference between the groups as the p-value for statement 1 is 0.819. Although parents from both cultural backgrounds demonstrate a similar view to statement 1, there is a statistically significant difference between the two groups for statement 2 as shown by $p = 0.005$. Similarly, with the responses of children there is no statistically significant difference between the groups as the p-value for statement 3 is 0.820. However, the difference between groups for statement 4 is statistically significant with $p = 0.001$. This means, even though the majority of parents agree with statement 2 and the majority of students agree with statement 4, the responses which are skewed towards “strongly agree” and “agree” have a significant difference between the two cultural groups.

Thirdly, cross-tabulation is used to further analyse these skewed data to investigate cultural differences (see Table 2).

Table 2
Cross-tabulation of responses of parents and children for the four statements

	Statement 1									Statement 3							
	European-Australian				Asian-Australian					European-Australian				Asian-Australian			
	SA	A	D	SD[2]	SA	A	D	SD		SA	A	D	SD	SA	A	D	SD
Statement 2 SA	**2**	**3**	3	0	**7**	**17**	8	0	Statement 4 SA	**1**	**10**	3	1	**22**	**29**	12	1
Statement 2 A	**4**	**9**	3	0	**5**	**12**	1	0	Statement 4 A	**4**	**5**	2	0	**4**	**14**	4	0
Statement 2 D	2	0	0	0	1	1	0	0	Statement 4 D	3	3	0	0	1	0	0	0
Statement 2 SD	1	1	0	0	0	0	0	0	Statement 4 SD	1	0	0	0	0	0	0	0

[2] Note: SA= Strongly agree, A= Agree, D= Disagree, SD= Strongly disagree

Cross-tabulation results demonstrate a variety of parental expectations and parenting styles. The majority of parents (i.e., 18/28 European-Australians and 41/52 Asian-Australians – shown bold in Table 2) either agree or strongly agree with both statements 1 and 2. This implies high expectations of parents that encourage high achievement of children in both cultural backgrounds. Considering the above fractions as percentages of 64.3% and 78.8% respectively, it is observed that parental expectations are relatively higher among Asian-Australians than European-Australians. Further, strongly agreeing with both statements 1 and 2 may create extreme pressure on children. Sometimes parents' beliefs and attitudes about their children's education act as barriers and prevent effective parental involvement. Literally, it does not make sense that students can put more effort into their work if they are already putting all their effort into their school work. However, this seems to be a technique used by parents to motivate their children. Some parents (6/28 European-Australians and 9/52 Asian-Australians) disagree with statement 1 but agreed or strongly agreed with statement 2. This seems to be because these parents are not satisfied with their children's effort and they expect more from them. Few parents (4/28 European-Australians and 2/52 Asian-Australians) agreed or strongly agreed with statement 1 and disagreed or strongly disagree with statement 2. These parents seem supportive of their children's effort. In this case the ratio is higher for European-Australian parents showing some flexibility in their expectations.

Offspring responses are similar to the results of parents. The figures show that the majority of children in both groups (i.e., 20/33 European-Australians and 69/87 Asian-Australians – shown bold in Table 2) have pressure from their parents and this pressure is higher among Asian-Australian children than European-Australian children. Further, there is extreme pressure felt by some children with 1/33 European-Australian and 22/87 Asian-Australian children strongly agreeing with both statements 3 and 4.

According to these results, even though acknowledging that their children do their best, Asian-Australian parents appear to be less satisfied with the effort of their children and have significantly higher expectations than European-Australian parents. However, it is impossible to underestimate the academic interaction of European-Australian parents with their children because results show that both Asian-Australian and European-Australian groups have high expectations in education of their children.

Conclusions

Although Chinese mothers are well known for putting pressure on their children's education it was found that parents from other Asian backgrounds also put pressure on their children. The expectations are higher among Asian-Australian parents and the results support the theory of relative functionalism about immigrant Asians (Sue & Okazaki, 1990). Therefore, in addition to Asian cultural values, beliefs, and practices, high expectations of Asian background parents may be explained by their migrant status too. Further, it is important to recognise that European-Australian parents too have high expectations for their children in mathematics education, even though it is not as significant as that of Asian-Australian parents. Overall, it appears that irrespective of culture, many parents have high expectations for their children. Interestingly, parent and student data provide similar results regarding parental expectations. Moreover, results found from students' data imply that the students are well aware of the expectations of their parents.

Opposed to the general perception, European-Australian parents also believe that their children can do better in mathematics if they try harder. One might argue that there has been an influence on European-Australian parents by the recently migrated Asian population. However, there is no substantive data to support this view from this study. Parental high expectations may have positive consequences for children resulting in improved performances in mathematics because parents' attitudes are naturally communicated to their children. Therefore, all parents should monitor their children's study habits regularly to improve their skills, attitudes, and confidence in mathematics learning. They should also motivate, encourage, and support their children to work diligently in order to enhance academic achievement.

While the results from the sample of participants used in this study provide some interesting insights, it must be acknowledged that the sample does not represent the country as a whole as the participants belong to three public schools in metropolitan Victoria. If there were a larger number of participants the responses could be analysed according to year levels, which might provide more interesting results.

References

Baumrind, D. (1967). Child care practices anteceding three patterns of preschool behaviour. *Genetic Psychology Monographs, 75*(1), 43-88.

Cai, J., Moyer, J. C., & Wang, N. (1997). *Parental roles in students' learning of mathematics: An exploratory study*. Chicago: American Educational Research Association.

Cao, Z., Bishop, A., & Forgasz, H. (2007). Perceived parental influence on mathematics learning: A comparison among students in China and Australia. *Educational Studies in Mathematics, 64*(1), 85-106.

Chua, A. (2011). *Battle hymn of the tiger mother*. Great Britain: Bloomsbury publishing.

Dandy, J., & Nettelbeck, T. (2002). A cross-cultural study of parents' academic standards and educational aspirations for their children. *Educational Psychology: An International Journal of Experimental and Educational Psychology*, *22*(5), 621-627.

Fan, X. (2001). Parental involvement and students' academic achievement: A growth modelling analysis. *The Journal of Experimental Education*, *70*(1), 27-61.

Hong, S., & Ho, H. (2005). Direct and indirect longitudinal effects of parental involvement on student achievement: Second-order latent growth modelling across ethnic groups. *Journal of Educational Psychology*, *97*(1), 32-42. doi: 10.1037/0032-0663.97.1.32.

Hoover-Dempsey, K. V., & Sandler, H. M. (1997). Why do parents become involved in their children's education? *Review of Educational Research*, *67*(1), 3-42. doi: 10.3102/00346543067001003.

Hornby, G., & Lafaele, R. (2011). Barriers to parental involvement in education: an explanatory model. *Educational Review*, 63(1), 37-52. doi: 10.1080/00131911.2010.488049.

Leung, F.K.S. (2012). What can and should we learn from international studies of mathematics achievement? In Dindyal, J., Cheng, L. P., & Ng, S. F. (Eds.), *35th Annual Conference of the Mathematics Education Research Group of Australasia*, 34-60. Adelaide: Mathematics Education Research Group of Australasia.

Ma, X. (1999). Dropping out of advanced mathematics: The effects of parental involvement. *Teachers College Record*, *101*(1), 60-81.

Maccoby, E. E., & Martin, J. A. (1983). Socialization in the context of the family: parent-child interaction. In P. H. Mussen (Ed.), *Handbook of child psychology* (Vol. 4, pp. 1-101). New York: Wiley.

Pallant, J. (2013). *SPSS survival manual* (5th ed.). Australia: Allen & Unwin.

Phillipson, S., & Phillipson, S.N. (2007). Academic expectations, belief of ability, and involvement by parents as predictors of child achievement: A cross-cultural comparison. *International Journal of Experimental and Educational Psychology, 27*(3), 329-348. doi: 10.1080/01443410601104130.

Sue, S., & Okazaki, S. (1990). Asian-American educational experience. *American Psychologist, 45*(8), 913-920.

Sullivan, P. (2011). Teaching Mathematics: Using research-informed strategies. *Australian Education Review*; no.59, ACER.

Thomson, S., Hillman, K., Werner, N., Schmidt, M., Buckley, S. & Munene, A. (2012). *Highlights from TIMSS & PIRLS 2011 from Australia's perspective*. Melbourne, Australia: ACER.

"I was in year 5 and I failed maths": Identifying the Range and Causes of Maths Anxiety in first year Pre-service Teachers.

Sue Wilson
Australian Catholic University
Sue.wilson@acu.edu.au

Mathematics anxiety affects primary pre-service teachers' engagement with and future teaching of mathematics. The study aimed to assess the level and range of mathematics anxiety in first year pre-service teachers entering their teacher education course, and to investigate the sources of this anxiety as perceived and identified by them. Data collection methods included the RMARS survey, and Critical Incident Technique. The results indicate that the most common negative impacts on pre-service teacher mathematical self-concept involved experiences with teachers. However, their current mathematics anxiety is most commonly aroused under testing or evaluation situations.

Introduction and Context

Anxiety towards mathematics has been identified as an issue nationally and internationally (OECD, 2015). Students compete globally in a world that is strongly based on using mathematics confidently. Successfully engaging with mathematics has social, economic and political implications. Mathematical know-how is widely regarded as essential not only to the life chances of individuals, but also to the health of communities and the economic well-being of nations (Walls, 2009).

This paper is part of a study that investigates primary (elementary) pre-service teachers' (PSTs') mathematics anxiety (maths anxiety), how it impacts upon their engagement with their teacher education course, and how it might be addressed. This is important, with increased scrutiny of teacher education courses, for example, the Australian Institute for Teaching and School Leadership (AITSL) stated that universities need to establish strategies to ensure PSTs have the required standard of numeracy to engage effectively in mathematics units in a rigorous program, (AITSL, 2011).

This paper examines the level and range of first year primary PSTs' maths anxiety at the beginning of their course. The research questions addressed by this study are:

1. With what range and extent of maths anxiety do first year PST present?
2. Is there any indication in the critical incident written responses as to what has stimulated this anxiety?

Theoretical Framework

The interpretive tradition is characterised by prioritising lived experiences, with a focus on meaning of interactions and events. The study aimed to access the narrative or storied nature of PSTs' experiences. The self-analysis of an emotionally-charged experience is an opportunity to analyse past actions and emotions; and the process of writing can be used to reflect on responses and decisions. The 'transactional model of emotion' (Lazarus, 1991) links motivational, social and cognitive dimensions. According to Lazarus, a lived experience consists of contextual and personal factors, which determine whether the event will be appraised: firstly as harmful or threatening (negative emotion), or challenging or beneficial (positive emotion); and secondly, for likely future outcomes, and their potential

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 643–650. Sunshine Coast: MERGA.

coping strategies. The appraisal can be analysed using binary or thematic analysis of written responses.

Literature Review

Two bodies of research informed this study. The first concerns maths anxiety in PSTs, and the second the use of reflective strategies, especially critical incident technique (CIT) in teacher education. Researchers of primary PSTs report high levels of mathematics anxiety, low confidence levels to teach elementary mathematics, and low mathematics teacher efficacy (Swars, Daane, & Giesen, 2006); and that high levels of teacher mathematics anxiety impact on student achievement (Beilock, Gunderson, Ramirez, & Levine, 2009) and can be perpetuated in classrooms (see Wilson, 2012). This transfer of mathematics anxiety from teacher to student has long-term educational implications.

The first year of study at university is particularly important (Krause, 2005), Recent research (e.g. Martin, 2012) reported success with strategies to increase engagement and reduce of anxiety in a first year education unit that linked practical activities with theory, and more studies of first year pre-service teachers are needed.

Surveys investigate the sources of maths anxiety. They measuring the existing level of maths anxiety by asking participants to rate the level of anxiety induced by different situations. Studies on gender differences in maths anxiety vary, with a number of studies reporting that females have higher levels than males. Age is another factor where contradictory findings are reported in the literature (see discussion in Wilson 2012).

Mathematics anxiety and its impact on students have been identified for many years. "Impoverished school mathematics experiences have left many pre-service teachers with strong *negative* affective responses about mathematics" (Namukasa, Gadanidis, & Cordy, 2009, p. 46 - 47). Previous researchers have investigated causes of maths anxiety, using a range of methods. Reflective thinking is important for professional practice to identify the assumptions that underlie thoughts and actions.

> During mathematics methods courses, it is important to give preservice teachers tools to deal with their recollections and experiences: If students reflect on occasions in their mathematical autobiography and discover that the interpretations of events can be changed, it can free them to search for new perspectives on their mathematical past and future (Kaasila, Hannul, & Laine, 2012, p. 991).

A number of researchers have used PSTs' mathematics autobiographies (Ellsworth & Buss, 2000; Sliva & Roddick, 2001; Lutovac & Kaasila, 2009). They identified the powerful effects of teachers. Teachers who are hostile, hold gender biases, or embarrass students in front of peers play a powerful role in maths anxiety (Vukovic, Keiffer, Bailey, & Harari, 2013). The perceptual changes that occur as a result of mathematics classroom experiences are persistent and enduring.

> People who claim that they were born without mathematical ability will often admit that they were good at the subject until a certain grade, as though the gene for mathematics carried a definite expiry date. Most people will also recall an unusual coincidence: that the year their ability disappeared, they had a particularly bad teacher (Mighton, 2004, p. 20).

Critical incidents have been used to foster reflection in teaching. Lerman (1994) developed "the idea of reflective mathematics teaching, offering the 'critical incident' as a device to stimulate reflection on teaching" (p. 52). The critical incident technique (CIT) focuses on real-life incidents. The advantages of using critical incidents come from their focus on observable behaviours (Pedersen, 1995) and participants' lived experience.

> "When analysing a critical incident, reflective individuals ask: Why did I view the original situation in that way? What assumptions about it did I make? How else could I have interpreted it? What other action(s) might I have taken that could have been more helpful? What will I do if I am faced again with a similar situation?" (Serratt, 2010, p. 379)

These incidents are descriptions of vivid events that people remember as being meaningful in their experience, and often can be identified, upon looking back, as a crisis or tipping point (Wilson, 2014). This study used CIT to investigate how PST feel about themselves as learners and future teachers of mathematics, by asking them to recall a critical incident which impacted on the way they feel. The critical incident may not have happened as they remembered. The aim of this writing is not to determine whether that event actually happened as remembered, but to help PST reflect on their perception of that event and its impact on their construction of what it means to learn mathematics and on themselves as a learner of mathematics.

> Like all data, critical incidents are created. Incidents happen, but critical incidents are produced by the way we look at a situation: a critical incident is an interpretation of the significance of an event. To take something as a critical incident is a value judgement we make, and the basis of that judgement is the significance we attach to the meaning of the incident (Tripp, 1993, p 8).

A benefit of CIT compared to mathematics autobiographies, is that instead of researchers selecting which parts to analyse for themes, in CIT the participant chose the experience and identifies the impact. Participants were not guided towards the selection of a negative experience, so their choice provides data on the proportions of PSTs' positive and negative responses.

Methodology

The study used two methods. A survey of level of anxiety responses to various situations was used to determine the range and type of maths anxiety. Ethics approval, based on accepted informed consent procedures, was received from the university's ethics committee, and agreement to use the RMARS survey was received from the author.

Given the complex nature of the phenomenon, and the aim of the study to access the narrative or storied nature of experience, a qualitative approach was appropriate to investigate the causes of this anxiety. This study is based in the interpretive paradigm. People create and associate their own meanings of their interactions with the world. PSTs' current experiences are filtered through their perceptions, reinforcing their attitudes.

The research study population consisted of two cohorts of students undertaking their first year mathematics unit on a major metropolitan campus and a smaller regional campus of an Australian university, in two successive years - a total of approximately 450 level 1 students from the Bachelor of Education (Primary) course. The data were collected in the participants' setting.

Methods

The RMARS (Alexander & Martray, 1989) was chosen for the survey because of its length, fit with the research question, appropriateness for group and strong psychometric information. It has been widely used in academic research, rigorously tested, and found to be psychometrically sound (Baloglu & Kocak, 2006; Dunkle, 2010). The RMARS is a 25-item, five-point (1 = not at all, to 5 = very much) Likert-type instrument. Thus, potential Total Anxiety scores range from "not at all" = 25, to "very much" = 125.

The RMARS assumes the multidimensionality of the construct, (Alexander & Martray, 1989, Baloglu, 2002), and has three subscales, for mathematics test anxiety (MTA, items 1-15), numerical task anxiety (NTA, items 16-20), and mathematics course anxiety (MCA, items 21-25). Possible scores for MTA could range from 5-45, and for NTA and MCA could range from 5-25.

The RMARS was used with minor modifications for the Australian context. A set of demographic questions was also used in the study. These asked for information such as age, gender, mathematics courses studied in high school, and the number of years/months since their last mathematics course. Data were coded onto an excel spreadsheet and analysed with the Statistical Package for Social Sciences (SPSS) 20.0. Means and standard deviations for the total scale scores on the RMARS were computed. Gender and age differences were examined for the total scale scores on the RMARS as well as the three subscales.

A critical incident approach was selected as the underpinning qualitative method as the study aims to access the narrative or storied nature of experience. In tutorials, PSTs were asked to write a written description of a critical incident (positive or negative) from their own school mathematics education that impacted on their image of themselves as learners of mathematics. PSTs were identified only by a code used to match CIT reflections with other data. Reflections were sealed in envelopes immediately and sent to the researcher. The data were not merged, as the use of the survey was pragmatic to answer the research question concerning the levels of anxiety. The qualitative data was used to explore the meaning individual PSTs ascribe to the problem of maths anxiety. Some initial results from the preliminary binary analysis (Lazarus, 1991) are presented. The binary analysis will be completed and followed by a more extensive thematic analysis.

Results and Discussion

Surveys from 219 PSTs were collected at the beginning of Semester 1, 2012. Sample 1 (57 PSTs: 45 female, 12 male) came from a city in a regional area and Sample 2 (162 PSTs: 140 female, 21 male, 1 not specified) was from a campus in a major metropolitan city. Response rates were 98% (Sample 1) and 70% (Sample 2). Surveys from 208 PSTs from the same two campuses were collected at the beginning of Semester 1, 2013. Means and standard deviations for the total scale scores on the RMARS were computed, and are shown in Tables 1 and 2.

Table 1
Total Anxiety Scores as measured by the RMARS, Semester 1, 2012

PST samples	n	range	mean	S. D.
Total PST	219	31-116	63.32	16.74
Campus 1	57	31-104	66.02	19.19
Campus 2	162	34-116	62.78	17.86
Females	185	31-116	64.01	18.44
Males	33	35-108	62.24	17.90
Less than 25 years	192	31-116	62.44	17.73
25 years and over	26	35-112	73.58	19.75

The PSTs exhibited a broad range of anxiety levels, ranging from almost no maths anxiety to very high levels of anxiety. An independent-samples t-test was conducted to compare campus differences in maths anxiety. In both years, there was a wide range within the cohorts ranging from very little maths anxiety to very high levels of anxiety, with half of the participants showing at least a fair amount, and 2% high to very high levels, of anxiety.

Table 2
Total Anxiety Scores as measured by the RMARS, Semester 1, 2013

PST samples	n	range	mean	S. D.
Total PST	208	30-116	64.74	18.39
Campus 1	63	30-110	64.05	18.07
Campus 2	145	31-116	65.03	18.58
Females	177	30-116	65.97	18.52
Males	31	33-89	57.71	16.19
Less than 25 years	192	30-116	64.43	18.42
25 years and over	16	32-103	68.44	18.75

No significant differences in Total Anxiety were found between the cohorts from the two campuses in either year. They were statistically equivalent on the total RMARS scores, as well as the three subscales (MTA, NTA, and MCA) (shown in Table 3).

Gender and age differences were examined for the total scale scores on the RMARS as well as the three subscales. In the first year, no significant differences were found between females and males on the total RMARS scores, or on the three subscales. However, in the 2013 cohort, female students had significantly higher levels, consistent with previous findings of gender differences in the RMARS scores (Alexander & Martray, 1989; Brady & Bowd, 2005; Baloglu & Kocak, 2006). In addition they had a significantly higher MTA component of their maths anxiety.

In the first year, significant differences were identified between age cohorts. The older group demonstrated higher levels of mathematics anxiety and larger standard deviations. Statistically significant differences were found between the scores of the younger and mature-age PSTs on the total RMARS scores, ($t(217) = 2.97$, $p < 0.005$); and on the three subscales (MTA, $t(217) = 2.12$, $p < 0.05$; NTA, $t(217) = 3.47$, $p = 0.001$; and MCA, $t(217) = 3.09$, $p < 0.05$), with mature-age PSTs receiving higher scores. This supported the findings of Baloglu and Kocak (2006) that older college students show higher levels of mathematics anxiety than younger ones. However, in 2013, there were no significant differences between age groups. This indicates that, although there were no significant differences between campuses, the level and distribution of maths anxiety in groups of incoming PSTs may vary from year to year.

Table 3 shows the factor analysis for the three contributing factors (MTA, NTA and MCA) for each of the two years. The score for each of the factors depends on the number of questions that contribute to that factor. In order to compare the levels of the anxiety components, each is presented as a score out of 5. The analysis shows that for both years, the mathematics test anxiety factor is much higher than the other two factors. This indicates that the primary factor that arouses PSTs' maths anxiety is testing or evaluation.

In the RMARS survey, participants rated their emotional responses to certain mathematical experiences in their lives, whereas the CIT identified past incidents that impacted on their feelings about themselves. Thus, both research methods focus on aspects of emotional responses to lived experiences (Lazarus, 1991), although the CIT involved open-ended responses, and the survey involved reducing the emotions to five levels.

Table 3
Means and Standard Deviations of the Sub-scales of the Revised Mathematics Rating Scale*

Factors	Years	
	2012 (n=219)	2013 (n=208)
Mathematics Test Anxiety (MTA) /5	2.98 (0.82)	3.06 (0.83)
Numerical Task Anxiety (NTA)/5	1.77 (0.81)	1.85 (0.84)
Mathematics Course Anxiety (MCA)/5	1.80 (0. 81)	1.90 (0.85)

* Standard deviations are reported within parentheses.

Preliminary binary analysis (Lazarus, 1991) has been completed on critical incident reflections from the 2012 Campus 2 cohort of PSTs. The participants chose the salient experiences and identified their impact. The initial analysis divided incidents into positive and negative experiences, based on Lazurus' (1991) characterisation of appraisal as harmful (negative emotion) or beneficial (positive emotion). Of the 236 descriptions of critical incidents, 102 (39%) were positive, 157 (61%) were negative and 2 (1%) described a neutral incident. These figures support findings by previous researchers (Namukasa et al., 2009).

The researcher then analysed the accounts for the most common factor. This was the teacher. Comments were coded as "teacher", only if they included the word "teacher". If a comment mentioned two teachers, in separate years, both were counted separately. Of the 236 PSTs, 135 (57%) wrote about the teacher. Analysis of the 140 comments about the teacher, found 46 (33%) were positive and 94 (67%) were negative. To illustrate the preliminary findings, the following examples of positive experiences show the impact of teachers who provided safe and supportive learning environments:

> Year 8 – my teacher made me comfortable and helped me understand the task in a way that was not uncomfortable.
>
> Year 11 to 12. Previously I had never been very good at maths. My teacher found ways to connect maths in ways I could relate to, making it fun. This developed my maths skills and attitude towards maths.

However, some PSTs retained intense memories of their experiences with disabling teachers, and these ranged from primary school to senior secondary. The following are examples of critical incidents that were coded as negative:

> In year 3, I didn't understand. The teacher gave up, gave me 'colouring in' while other students learned maths.
>
> Year 6 – I couldn't understand the concept of long division so the teacher gave up on me and said don't worry about it. Looking back it makes me feel like a failure.
>
> In primary school I had one teacher who would always put you on the spot in front of a class and he would read out everyone's results in front of everyone too. This always made me anxious and from then on I aimed to avoid maths.
>
> In year 9, my teacher would make us answer questions on the board and if we got it wrong, he would say "Poor _____. What can we do with you?"

> Year 11 … My teacher made it hard for me to learn and understand because he would give the class a time to finish answering a question and if I didn't know an answer, he would look at me and say "You should know this".

These comments reflected findings from other researchers (Ellsworth & Buss, 2000; Sliva & Roddick, 2001; Wilson & Thornton, 2008; Lutovac & Kaasila, 2009) on the important impact of individual teachers.

In addition, in some critical incident descriptions, PST identified that failure in tests had implications for their self-concept as learners of mathematics:

> I was in year 5 and I failed maths and since that day I hate maths. This experience makes me feel that I don't know anything about maths.

Combined with the survey findings of the significant contribution of MTA to high levels of maths anxiety, this has important implications for teacher education. The survey showed that maths anxiety may present differently in different situations, but evaluation and testing were identified as the most common source of maths anxiety. The connections between experiences identified in critical incidents as causes of maths anxiety and current sources of maths anxiety will be explored by further analysis of the data.

These results indicate that beginning teacher education students vary in affective responses towards learning mathematics. Teacher educators should be aware of the extent of range of anxiety that PST may present with at the beginning of their teacher education course, and hence that the needs of students coming to their teacher education mathematics units may vary considerably.

Conclusions

This paper demonstrates that PST come to their teacher education courses with a range of existing maths anxiety, identified through the RMARS survey. The initial findings of the binary analysis of the critical incidents indicate teachers and testing as important factors to be investigated. Further analysis is need to explore the connections between experiences identified by the initial analysis of the critical incidents as potential causes of maths anxiety, and current sources of maths anxiety identified by the survey. To achieve this, the qualitative data will be further analysed in terms of themes.

Participation of PST in writing critical incidents and reflections, provides insights into their views of themselves as future teachers of mathematics, and potentially impacts on their future teaching of mathematics and hence the achievement of their future students. The larger study will also investigate the impact of using bibliotherapy to address maths anxiety on engagement of PSTs in their mathematics units. This research has the potential to make an important contribution to the strategies available in teacher education courses to address maths anxiety.

References

Alexander, L., & Martray, C. (1989). The development of an abbreviated version of the mathematics anxiety rating scale. Measurement and Evaluation in Counselling and Development, 22, 143–150.

Australian Institute for Teaching and School Leadership. [AITSL]. (2011). Accreditation of initial teacher education programs in Australia: Standards and procedures http://www.aitsl.edu.au/verve/_resources/Accreditation_of_initial_teacher_education.pdf

Baloglu, M. (2002). Construct and current validity and internal consistency, split-half and parallel-model reliability of the revised mathematics anxiety rating scale. Doctoral thesis. Faculty of the Graduate School of Texas A & M University.

Baloglu, M. & Kocak, R. (2006). A multivariate investigation of the differences in mathematics anxiety. Personality and Individual Differences, 40, 1325–1335.

Beilock, S., Gunderson, E., Ramirez, G. & Levine, S. (2009) Female teachers' math anxiety affects girls' math achievement. Proceedings of the national Academy of Sciences of the United States of America. 107 (5) 1860-1863.

Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students, Teachers and Teaching: Theory and Practice, 11(1), 37-46.

Dunkle, S. (2010). Remediation of math anxiety in preservice elementary school teachers. Unpublished doctoral dissertation, Faculty of D'Youville College Division of Academic Affairs.

Ellsworth, J., & Buss, A. (2000). Autobiographical stories from preservice elementary mathematics and science students: Implications for K-16 teaching. School Science and Mathematics, 100(7), 355-364.

Gibbs, G. (1988). Learning by doing: A guide to teaching and learning methods. Oxford: Further Education Unit, Oxford Brookes University.

Kaasila, R., Hannula, M., & Laine, A. (2012). "My personal relationship towards mathematics has necessarily not changed but ..." analyzing preservice teachers' mathematical identity talk. International Journal of Science and Mathematics Education, 10: 975-995.

Krause, K. (2005). Serious thoughts about dropping out in first year: Trends, patterns and implications for higher education. Studies in Learning, Evaluation, Innovation and Development 2(3), 55-68.

Lazarus, R. (1991). Emotion and adaptation. New York, NY: Oxford University Press.

Lerman, S. (1994). Reflective Practice. in B. Jaworski & A. Watson (Eds) Mentoring in Mathematics Teaching (Ch 5) New York, NY: Routledge Falmer.

Lutovac, S. & Kaasila, R. (2009). Using narratives as innovative tools in Finnish teacher education. Proceedings of the Conference Development of Competencies in the World of Work and Education. University of Ljubljana , Slovenia . http://www.decowe.com/static/uploaded/htmlarea/files/Using_Narratives_as_Innovative_Tools_in_Mathematics_Education_Course_.pdf

Martin, D. (2012). Rich assessment in a first-year, teacher education (primary) mathematics education subject, International Journal of Pedagogies and Learning, 7(1), 62-72, DOI: 10.5172/ijpl.2012.7.1.62

Mighton, J. (2004). The myth of ability: Nurturing mathematical talent in every child. Toronto: The Text Publishing Company.

Namukasa, I., Gadanidis, G., & Cordy, M. (2009). How to feel about and learn mathematics: Therapeutic intervention and attentiveness. Mathematics Teacher Education and Development, 10, 46-63.

OECD. (2015). Does math make you anxious? PISA in Focus. http://www.oecd-ilibrary.org/education/does-math-make-you-anxious_5js6b2579tnx-en;jsessionid=cb6yjkqyloth.x-oecd-live-03

Pedersen, P. (1995). *The five stages of culture shock.* Westport, CN: Greenwood Press.

Serratt, O. (2010). *Knowledge solutions: Tools, methods and approaches to drive development forward and enhance its effects.* Mandalayong City, Philippines: Asian Development Bank.

Sliva, J., & Roddick, C. (2001). Mathematics autobiographies: A window into beliefs, values, and past mathematics experiences of preservice teachers. *Academic Exchange Quarterly, 5*(2), 101.

Swars, S., Daane, C., & Giesen, J. (2006). Mathematics anxiety and mathematics teacher efficacy: What is the relationship in elementary preservice teachers? *School Science and Mathematics, 106*(7), 306-315.

Tripp, D. (2012). *Critical incidents in teaching: Developing professional judgement.* Routledge Education Classic Edition. Milton Park, England: Routledge.

Vukovic. R., Keiffer, M., Bailey, S., & Harari, R. (2013). Mathematics anxiety in young children: Concurrent and longitudinal associations with mathematical performance. *Contemporary Educational Psychology, 38* (1) 1-10. Retrieved from http://www.sciencedirect.com/science/journal/0361476X/38/1.

Walls, F. (2009). *Mathematical subjects: Children talk about their mathematical lives*. London: Springer.

Wilson, S. (2012). Investigating pre-service teachers' mathematics anxiety using the revised mathematics anxiety scale (RMARS). In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), *Mathematics education: Expanding horizons* (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, pp. 777-784). Singapore: MERGA, Inc.

Wilson, S. (2014). "Fail at maths and you fail at life": Learned barriers to equal opportunities - mathematics anxiety and quality of life. In R. Brown & R. Faragher (Eds.), *Quality of Life and Intellectual Disability: Knowledge application to other social and educational challenges***.** Hauppauge, NY: Nova Science Publishers, Inc.

Wilson, S. & Thornton, S. (2008). "The factor that makes us more effective teachers": Two pre-service primary teachers' experience of bibliotherapy. *Mathematics Teacher Education and Development, 9,* 22–35.

Enhancing Mathematics (STEM) Teacher Education in Regional Australia: Pedagogical Interactions and Affect

Geoff Woolcott
Southern Cross University
<Geoff.Woolcott@scu.edu.au>

Tony Yeigh
Southern Cross University
<Tony.Yeigh@scu.edu.au >

This article reports on initial findings, including the mathematics components, of a multi-institutional Science, Technology, Engineering, and Mathematics (STEM) project, *It's part of my life: Engaging university and community to enhance science and mathematics education*. This project is focussed on improving the scientific and mathematical thinking of pre-service teachers (PSTs) by aligning their pedagogy with the scientific and mathematical thinking that occurs in authentic, real-world contexts. This article discusses emotional literacy and emotional regulation as aspects of self-reflective professional development and how these measures are conceptually related to improving competence and confidence for pre-service STEM teachers. This report details how emotional feedback was used in trials of a pilot program to enable PSTs to analyse, understand, and make use of emotional information to improve their teaching confidence, particularly in mathematics.

Introduction

This paper reports on the initial stages of an Office of Learning and Teaching (OLT) funded Science, Technology, Engineering, and Mathematics (STEM) project, *It's part of my life: Engaging university and community to enhance science and mathematics education*, that seeks to address a lack of confidence and competence in science and mathematics teaching in regional and rural Australian schools. The project addresses these issues through the development of interventions that focus on how mathematicians and scientists think and solve problems and how this may be linked to the ways that people solve problems in everyday life (Woolcott, 2015). In particular, this report focuses on the development and application of some of the affect measures used by the project to provide feedback in relation to pre-service teachers (PSTs) pedagogical self-reflections on their lesson preparation and lesson delivery in mathematics. Affect, as a measure of emotional experience and understanding, is viewed as fundamental to the professional development of confidence and competence in teacher training (Tobin & Ritchie, 2012), and the project's use of affective feedback thus represents an important aspect of achieving the larger project goals of improving these aspects of pre-service training. With this in mind, a brief framework to contextualise and position the project is presented, followed by a description of the affect-related measures being reported along with some key results of their use in initial project trials. Improvements to the measures; in particular, how to use these to better connect emotional literacy to appropriate research goals, are then recommended as a focus for ongoing research.

Background

Context and Theoretical Framework

There has been a steady reduction in the number of Australian students who are studying mathematics and science at both the secondary (high school) and tertiary levels of education (Lyons & Quinn, 2010). There is also a shortage of appropriately qualified mathematics and science teachers available to teach at the secondary school level,

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 651–658. Sunshine Coast: MERGA.

particularly in rural schools (e.g., Ainley, Kos, & Nicholas, 2008). For example, Thomson (2009), in a report based on data from the 2007 Trends in International Mathematics and Science Study (TIMSS), identified that many Year 4 teachers reported having little specific training or specialised education upon which to base their teaching of the TIMSS assessment topics. Similarly, Australia's Chief Scientist, Professor Ian Chubb and colleagues have repeatedly expressed concern in relation to the state of Australian STEM education (Chubb, Findlay, Du, Burmester, & Kusa, 2012). Importantly, an Australian Association of Mathematics Teachers' (AAMT, (2014) report on quantitative skills has proposed that one key step in developing mathematics literacy in schools was by "helping schools to teach STEM as it is practiced, in ways that engage students, encourage curiosity and reflection, and link classroom topics to the 'real world' ".

This project seeks to address such issues by clarifying links between content knowledge and confidence as related to contextualised or situated learning in Australian classrooms. In initial trials of the project, this was enacted by having PSTs work in groups to develop pedagogical contexts and scenarios, guided by expert mathematicians, scientists, and pedagogy mentors, to construct and optimise inter-dependent and collaborative scenario-based lessons that utilised local community contexts to increase the meaning of the lessons (e.g., Woolcott, 2015).

Sources of Feedback to Encourage Competence and Confidence

In terms of tracking the influences associated with STEM teaching, various sources of feedback were provided to encourage PSTs to analyse and reflect on their learning and teaching in a way that connected what they were teaching, and what their school students were learning, to the contextualised content of the lessons. It is important to note that these sources of feedback were incorporated into a series of iterated enhancement and feedback/reflection modules during initial trials of the project (Woolcott, 2015). Enhancement modules involved interactions between the PSTs and world-class science and mathematics researchers, and between PSTs and experienced educators who specialise in the area of classroom pedagogy. The feedback modules involved collaborative groups of PSTs analysing their teaching and how they had made use of the expert advice, as well as including input and guidance from their pedagogical mentors. As the PSTs developed experience across the modules, they then began mentoring less-experienced colleagues, providing yet another source of feedback for the project.

The Role of Affect in Teacher Confidence and Competence

An important part of the reflective processes for the project involved *affect* feedback, including emotion ratings, video recordings, and voice parameter analysis (Yeigh & Woolcott, 2014). Research by Tobin and Ritchie (2012) suggests that *emotional arousal* (positive or negative) is related to teaching competence and confidence in PSTs, and because of this the particular focus of this paper concerns how the project utilised some of these sources of feedback, determined as key sources, to assess and analyse PST affect in relation to the scenario-based lessons they developed in conjunction with the expert mathematicians, scientists, and pedagogy mentors. Emotional arousal was operationally defined as affect for the project because affect represents the external expression of emotion as attached to ideas or mental representations. Measures used, therefore, were concerned with how the PSTs were analysing and interpreting their emotions in relation to their teaching, and what impact this was having on their confidence and sense of competence about the teaching. In this respect, the project sought to measure the degree to which affect, and the corresponding ability to regulate emotions, moderated confidence in the PSTs, and how this may have influenced their competence.

Affect as a Basis for Critical-moment Reflection

Affect was measured from a variety of perspectives and using several different strategies, with an overall goal of measuring affect to have the PSTs learn how to identify and analyse their teaching-related affective states. This was done in order for the PSTs to assess their own emotions and motivations, and to ensure that the emotional and motivational climate of the classroom was optimally supportive for the learning of their students (Tobin & Ritchie, 2012). A discussion of the critical moments and the related emotion diaries follows. Other affect measures are reported elsewhere (e.g., Donnelly, Pfieffer, Woolcott, Yeigh, & Snow, 2014; Yeigh & Woolcott, 2014).

Methodology

Trial Structure

The methodology of the initial trials was developed around collaborative team discussions in order to produce a plan for a teaching lesson. The lesson was followed by self-reflection and a collaborative feedback/reflection session. In line with theory on the value of iteration in learning processes for PSTs (e.g., Davis & Dargusch, 2015), this sequence was repeated as iterations of the sequence: Enhancement Module; Teaching Lesson; and Feedback/Reflection Module (see Figure 1).

ENHANCEMENT - *to facilitate interactions with research scientists and mathematicians and university educators to improve competence in pedagogy and subject knowledge.*	TEACHING LESSON - *a topic nominated by the target school prior to Enhancement*	FEEDBACK/ REFLECTION - *to utilise self-evaluation to examine links between competence and confidence.*

Group A (PSTs) 3 iterations of enhancement/ feedback cycle	Group B (PSTs) Waiting list (no enhacement or feedback/reflection) – each PST teaches one lesson
Group A Control (no enhacement or feedback/reflection) – each PST teaches one lesson	Group B 3 iterations of enhancement/ feedback cycle

Figure 1. The iterated sequence of Enhancement Module, Teaching Lesson and Feedback/Reflection Module. The lower diagram shows the grouping of PSTs within these initial trials.

Each Module was here treated effectively as a discussion-based learning intervention for the PSTs. Each trial was preceded by a training session that explained the process to be undertaken and the rationale behind each Module. The initial mathematics trials were based around this process, with groups of 3-6 volunteer PSTs in each trial, with teaching done in schools local to the university.

Critical Moments

All teaching lessons included full audio/visual (video) recordings, and PSTs then used these to analyse and reflect on their teaching. In particular, they identified six *critical teaching moments* for each lesson, where each *moment* represented an important (positive or negative) emotional feeling or experience associated with the pedagogical process of instruction, that they felt influenced their competence and/or confidence in relation to the lesson. Instructions for providing this aspect of the affect data were for PSTs to record the start and finish times for six segments of the video identified as representing a critical moment for each lesson, and seeking to identify two segments from the first third of the lesson, two segments from the middle third of the lesson, and two from the final third of the lesson.

Critical moment data was also recorded in the same manner by observing PTSs, allowing comparisons to be made between experienced and observed affect for each teaching PST. These comparisons assisted in identifying affect-related issues for the PSTs, as well as highlighting affective trends in the overall iterations that took place during these trials.

Emotion Diary

PSTs were also asked to complete an emotion diary for the critical moment segments identified in relation to their teaching (see Figure 2). The emotion diaries used well-established affect icons and their meanings to represent the various emotional states PSTs might experience during teaching (or observe in another PST's teaching). To complete these diaries, PSTs were trained to recognise emotions in terms of observing changes in voice volume, pitch, tone, or other sound qualities when observing one another, and when analysing their own video recordings. They were also trained to notice how overall body language during teaching (e.g., facial expressions, breathing rate, sweating, vasodilation [blushing], posture, increased muscle tension, etc.) might indicate a particular feeling or bodily sensation.

Using this training to direct their diary recordings, both teaching and observing PSTs were instructed to complete an emotion diary for each critical moment segment identified in the video recording by the teaching PST. The diary was completed by selecting appropriate affect icons to represent the teaching PST's emotions during teaching, and then selecting from the scale a number that represented the intensity of the emotion next to the icon. As shown in Figure 2, the emotion diary also provided space to write open-ended comments about the selected emotions, and PSTs were encouraged to use this space to elaborate and explain their affective identifications in terms of what the teaching PST was doing at the time, what else might be going on in the classroom, and at whom the emotion seemed to be directed.

Emotion	Please describe briefly what you were doing, what was happening and at whom the emotion was directed.
Excitement/Enthusiasm	
Happiness	
Enjoyment	
Pride	
Anxiety/Worry	
Frustration	
Disgust/Contempt	
Annoyed/Irritated	
Disappointment	
Embarrassed	
Interested	
Confident	

1	2	3	4	5
Not at all	**A little**	**Moderately**	**Quite a bit**	**Extremely**

Figure 2. The emotion diary used to identify affect during teaching sessions and in relation to reflective lesson analysis (used with permission Tobin & Ritchie, courtesy of Henderson)

Results and Discussion

Early findings from these initial trials, which included both science and mathematics teaching, support the use of affective data to examine the thinking and behaviours that led to emotional states in pre-service teachers (PSTs). It was also felt that a need existed to report on the current findings promptly, as the purpose of these analyses was to assist PSTs improve their ongoing competence and confidence in STEM-related teaching, including the teaching of mathematics. These outcomes appear to support the efficacy of having PSTs learn how to identify and analyse their teaching-related affective states in order to assess their own emotions and motivations, and to ensure that they understand the relationship between emotional literacy and effective pedagogy.

Critical Moment Analysis and Emotion Diaries

Critical moment analysis involved both the teaching and observing PSTs using the video recordings to analyse and reflect on the affective states of the teaching PTSs during lesson delivery. For each lesson, the teaching PST initially identified and analysed six critical moments from the video, representing important points at which some form of affect had influenced their pedagogy. The non-teaching PSTs then also analysed the video according to the identified time signature for each moment, and provided feedback on the affect they observed in relation to each identified moment.

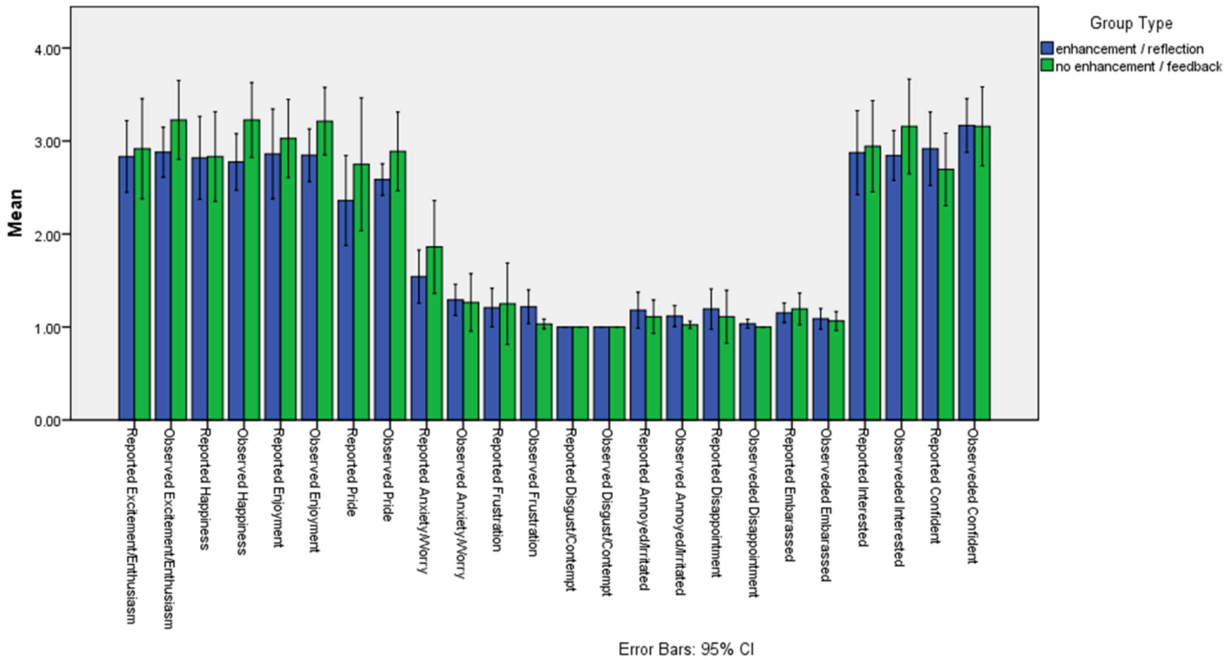


Figure 3. Critical moment data by group type (see also Donnelly et al., 2014)

Figure 3 shows a mean comparative overview of how these critical moments were analysed in terms of reported affect versus observed affect–for PSTs who had received enhancement for the lessons they delivered and for PSTs who had not received enhancement for the lessons they delivered. There were three significant differences in relation to these critical moment analyses, involving differences between reported and observed *anxiety/worry* ($t_{[17]}$= 2.62, p<0.02), between reported and observed *confidence* ($t_{[17]}$= -2.20, p<0.05), and between reported and observed *embarrassment* ($t_{[17]}$= 2.21, p<0.05). It should also to be noted that on average the *no enhancement* group tended to experience and report higher levels of positive emotion, and lower levels of negative emotion, than did the *enhancement* group.

With regard to the analysis of critical teaching moments, it is of interest that the *no enhancement* group tended to experience and report higher levels of positive emotion, and lower levels of negative emotion, than did the *enhancement* group. This was especially true for emotions relating to *Excitement/Enthusiasm*, *Happiness*, *Enjoyment*, *Pride*, and *Interested*, which all represent positive forms of affect. Note also, however, that the no enhancement group self-reported much greater *Anxiety/Worry* than the enhancement group, even though this was observed as lower than the enhancement group by others. Perhaps what was occurring here was that a greater sense of *pressure* took place for PSTs undergoing enhancement–a type of performance pressure–while a sense of *missing out* took place for PSTs when they were not receiving enhancement. In either case, the question again arises as to whether an intentional or unconscious emotion-regulation strategy may be occurring to control emotional display and, if so, how this might be operating.

A project survey was undertaken along with the affect analysis (Whannell, Woolcott, & Whannell, 2015) and, although it covers far wider ground than just the affective domains of the project, several findings from the factor analysis performed on the survey do appear relevant to the current paper, including the existence of a Teacher Reflection Scale (TRS) as a valid project construct (Cronbach's alpha 0.854). There is, in fact, a significant positive relationship between the TRS and mathematical thinking, being able to support school students, and pedagogical confidence. In addition, it is of particular interest that the correlation between the TRS and the number of mathematical curriculum units completed at university is negative. This suggests that the amount of experience that the respondents had in terms of formalised mathematical learning was inversely associated with their reflections on teaching practice or on the respondents' understanding of the impact of emotions on teaching. Considering that the identification of strategies to enhance PST confidence and competence through reflection is one of the primary aims of the project, these overall findings indicate that opportunity exists for the project to make a genuine contribution to the training of pre-service teachers of mathematics.

One of the clearest outcomes from this early analysis of the project affect data is that some sort of emotion-regulation strategy seems to be occurring in relation to emotional display. In this respect ongoing research will need to investigate the degree to which PSTs are aware of such strategies, why certain emotions seem to be controlled in a more strategic manner than others, and how emotional regulation takes place. Perhaps the use of a dedicated debriefing session, aimed at exploring these specific aspects of the reflective process, could be used to further train PSTs in this direction. Additionally, incorporating specific reflective prompts into the critical moment analysis strategy could also be used to elicit this sort of information. In both cases, the aim of improving PST emotional awareness, in terms of connecting the experience of distinct emotions to individual behavioural responses, would be further clarified.

Implications of Findings and Future Research Directions

It's part of my life is a multi-institution STEM project, designed to increase the competence and confidence of training mathematics and science teachers. This report has focused on initial analyses of how the project used affective measures as part of the iterative processes by which pre-service teachers (PSTs) explored and analysed the pedagogy connected to their teacher training. The findings in this paper reflect those found by Woolcott, Yeigh and colleagues (e.g., Donnelly et al., 2014; Yeigh & Woolcott, 2014) that the PSTs have exhibited a positive emotional bias overall, and also displayed greater changes in their negative versus positive emotions. These findings also suggest that when receiving enhancement for their lesson development (expert science or mathematics input, plus pedagogical guidance), the PSTs may feel pressure to perform, whereas when not receiving enhancement (developing their lesson in collaboration with other PSTs only) they may feel as though they are missing out on important information.

The analysis completed so far, however, (e.g., Donnelly et al., 2014; Woolcott, 2015; Whannell et al., 2015; Yeigh & Woolcott, 2014) supports the project's emphasis on reflective affect analysis to increase pedagogical confidence, and thus links this training strategy to the larger project goal of increasing competence through increasing pedagogical confidence. Importantly, differences between experienced (self-reported) affect and observed affect highlight the need to elaborate the reflective process in terms of consciously identifying the relationship between specific emotions and their behavioural correlates. Overall, these findings indicate that the project's use of affect analysis is appropriate as a means of addressing the lack of confidence and competence in science and mathematics teachers in Australian schools. Indeed, in this most essential criterion the project seems to be hitting the targets it has set for itself quite well. The findings also provide clear avenues for improvement with respect to some aspects of the reflective process, suggesting the need to forge clearer conscious correspondences between affect and behaviour on the part of training STEM teachers. In this respect the project will need to modify certain elements within the reflective process, and this is viewed as an important way forward for the ongoing project program. The effect of these modifications will be to better connect emotional literacy to the project research goals, in order to improve the overall project goal of developing quality teaching practices that are directed at the enhancement of science and mathematics teaching in Australia.

Acknowledgement

This paper reports on research associated with the project: *It's part of my life: Engaging university and community to enhance science and mathematics education*, a project supported by a three-year $1 million grant awarded to the Regional Universities Network (RUN) by the Australian Government Office for Learning and Teaching (OLT).

References

Australian Association of Mathematics Teachers [AAMT] (2014). *Identifying and supporting quantitative skills of 21st century workers*. Final Report. Adelaide: AAMT.

Ainley, J., Kos, J., & Nicholas, M. (2008). *Participation in science, mathematics and technology in Australian education*. Canberra: Australia Council for Educational Research.

Chubb, I., Findlay, C., Du, L., Burmester, B., & Kusa, L. (2012). *Mathematics, engineering and science in the national interest*. Canberra: Office of the Chief Scientist.

Davis, S.E., & Dargusch, J.M. (2015). Feedback, iterative processing and academic trust - teacher education students' perceptions of assessment feedback. *Australian Journal of Teacher Education*, *40*(1), 177-191.

Donnelly, J., Pfieffer, L., Woolcott, G., Yeigh, T., & Snow, M. (2014, October). *Emotion in pre-service teachers: Relations among self- and observer-reports on classroom videos and voice parameter analyses*. Paper Presented at the Australian Psychological Society Conference, Hobart.

Lyons, T., & Quinn, F. (2010). *Choosing Science: Understanding the declines in senior high school science enrolments*. Report from the National Centre of Science, ICT and Mathematics Education for Rural and Regional Australia (SiMERR Australia) Armidale, NSW: University of New England.

Tobin, K., & Ritchie, S.M., (2012). Multi-method, multi-theoretical, multi-level research in learning sciences. *The Asia-Pacific Education Researcher*, *21*(1), 117-129.

Thomson, S. (2009). International mathematics and science study shows mixed results for Australia. *Research Developments*, *20*, 1-4.

Whannell R., Woolcott, G., & Whannell, P. (2015, January). *Improving the confidence and competence of mathematics pre-service teachers at a regional university in Australia*. Paper presented at the Hawaii International Conference on Education, Honolulu.

Woolcott, G., (2015, February). *Enhancing science and mathematics teacher education in regional Australia: Evaluating an enhancement module for science pre-service teachers*. Paper presented at the Annual Conference of the National Association of Research in Science Teaching, Chicago. Published online.

Yeigh, T. & Woolcott, G. (2014, October). *Enhancing mathematics and science teacher education in regional Australia: Pedagogical interactions and their affective outcomes*. Paper presented at the Asian Conference on Education 2014 (ACE2014), Osaka.

Mathematics, Programming, and STEM

Andy Yeh
Queensland University of Technology
<a.yeh@qut.edu.au>

Vinesh Chandra
Queensland University of Technology
<v.chandra@qut.edu.au>

Learning mathematics is a complex and dynamic process. In this paper, the authors adopt a semiotic framework (Yeh & Nason, 2004) and highlight programming as one of the main aspects of the semiosis or meaning-making for the learning of mathematics. During a 10-week teaching experiment, mathematical meaning-making was enriched when primary students wrote Logo programs to create 3D virtual worlds. The analysis of results found deep learning in mathematics, as well as in technology and engineering areas. This prompted a rethinking about the nature of learning mathematics and a need to employ and examine a more holistic learning approach for the learning in science, technology, engineering, and mathematics (STEM) areas.

In the early 2000s when the personal computers were powerful enough to handle complex three-dimensional (3D) real time rendering, the author started developing an online 3D virtual reality-learning environment (VRLE) for learning mathematics. The focus of this VRLE (Figure 1) is on mathematical meaning-making of 3D geometry. This 3D VRLE was seen as an information and communication technology (ICT) tool, tutor, and/or tutee (Taylor, 1980) to facilitate the learning or knowledge construction of mathematics.

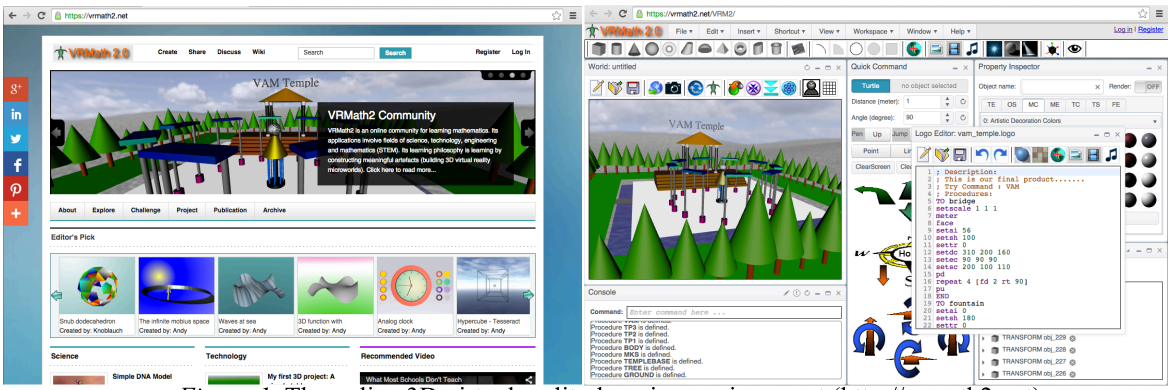


Figure 1: The online 3D virtual reality learning environment (http://vrmath2.net)

The design of the VRLE was informed by a semiotic framework (Yeh & Nason, 2004), which asserted the need of multiple semiotic resources for mathematical meaning-making. The implementation of the VRLE was elaborated elsewhere in Yeh (2007). The main components of the VRLE included an interactive 3D virtual space for visualising 3D objects, a customised Logo programming language for creating 3D virtual worlds, and an online forum for social discussion and presentation of the created 3D virtual worlds.

Among the semiotic resources, the programming language plays an imperative role of linguistic formalisation in aiding learners' mathematical expression. It serves as the symbolic representation of a mathematical function as well as the glue that binds all the representational modes together (Hoyles, Noss, & Adamson, 2002). Feurzeig, Papert and Lawler (2010) further confirmed the values of programming languages to the teaching and learning of mathematics. They argued that programming (in the example of Logo) not only

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 659–666. Sunshine Coast: MERGA.

enables pupils to access an accurate understanding of some key mathematical concepts, but also develops problem-solving skills and facilitates the expansion of mathematical culture to topics in biological and physical sciences, linguistics, etc.

In this paper, the authors report a review of a teaching experiment, in which the focus was originally on the learning of mathematics by primary students in the VRLE. In this review, we found that through programming, the primary students not only developed deeper understanding of mathematics, but also applied and practiced their problem solving skills in designing, creating, and engineering their 3D virtual worlds.

Theoretical and Conceptual Frameworks

The theoretical framework for this research is rooted in semiotics, from which all human cognition is viewed as meaning-making endeavour within systems of signs. Lemke (2001) proposed a mathematical account of semiotics and classified the mathematical signs or representations into three categories of semiotic resources, namely typological, topological, and social-actional resources. Typological resources are those signs that signify meaning by discrete means. Languages (including programming languages) and symbols are typical typological resources. Conversely, topological resources signify meaning by continuous means. They could be animations, change of colour spectrum or sound pitch, and continuous variation of viewpoints such as changing from the top view of a square to side view of a square to recognise the different shapes a square can transform into due to different 3D perspectives (Yeh & Hallam, 2011). The social-actional resources are non-exclusive to the above two. They are the means of meaning-making by cultural activities or gestures, or from discussion to negotiation of doing things together such as when building a house or designing a garden. Informed by this mathematical account of semiotics, the VRLE was designed and implemented. Initial research and evaluation (see Yeh, 2007, 2013; Yeh & Hallam, 2011) have reported deep learning of mathematics within rich typological (e.g., Logo programming), topological (e.g., interactive 3D virtual space), and social-actional (e.g., discussion forum and group project) resources.

Upon the review of the teaching experiment in Yeh (2007), the authors further confirmed that the Logo programming language (a typological resource) of the VRLE played a central role to connect other semiotic resources for mathematical meaning-making. Moreover, it was found that the social-actional resources (e.g., a building project in the teaching experiment) also contributed to the learning of, not only mathematics, but also other science, technology, engineering, and mathematics (STEM) areas including technology and engineering. A new conceptual framework is thus formed as shown in Figure 2 below.

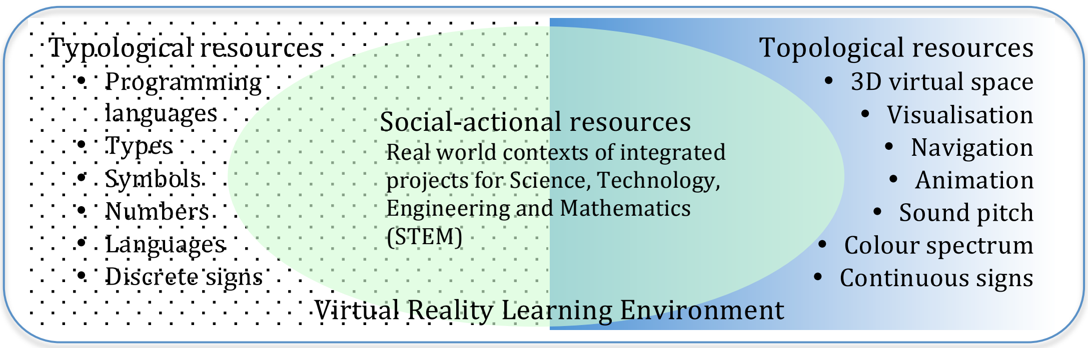


Figure 2: Semiotic framework for learning in STEM in VRLE

This new conceptual framework (Figure 2) is an initial attempt to explain what we found about the learning occurred in the VRLE. The framework postulates that the social-actional resources in the VRLE can provide the contexts of integrated projects in STEM areas. The rich semiotic resources in the VRLE thus enable learning beyond the field of mathematics, which in turn informs that the nature of learning (or meaning-making) about mathematics is not confined within mathematics itself, but in an interdisciplinary manner. This reasoning can also be applied to the learning in science, technology, and engineering, and even beyond STEM. In the next section, we will report on the reviewed teaching experiment to elaborate on this new conceptual framework. Due to the scope of this conference paper, the focus of this report is on how the programming connects all meaning-making resources to expand the learning from mathematics to STEM.

The Teaching Experiment

Three Year 5 students (Pseudo names: R2D2, Victor, Alekat20) participated in a 10-week (2 sessions per week, 1 hour per session) teaching experiment involving learning 3D geometry in the VRLE. In the first 8 weeks, the three participants were made familiar with the VRLE and were able to write Logo programs including 3D movement commands and procedures in the carefully designed learning activities by this teacher-researcher. In the last 2 weeks, the three participants had to work together as a team, choose a design project, and create the 3D virtual world for the project. This paper reports only on the participants' project in the last 2 weeks of the teaching experiment.

Data collected included video and audio recordings, participants' programming codes, 2D drawings, 3D virtual artefacts, and the teacher-researcher's field notes. For the purpose of this paper, we focus on the analysis of the programming process and report on the participants' learning in the STEM areas.

Results

The three participants decided to create a Temple, and started a generic problem solving cycle of think, plan, do, and check. They thought about what to have in the virtual Temple space, and then drew some initial plans (see Figure 3) in a collaborative and cooperative manner. The Temple was then divided into three parts and each participant designed and programed a part of the Temple.

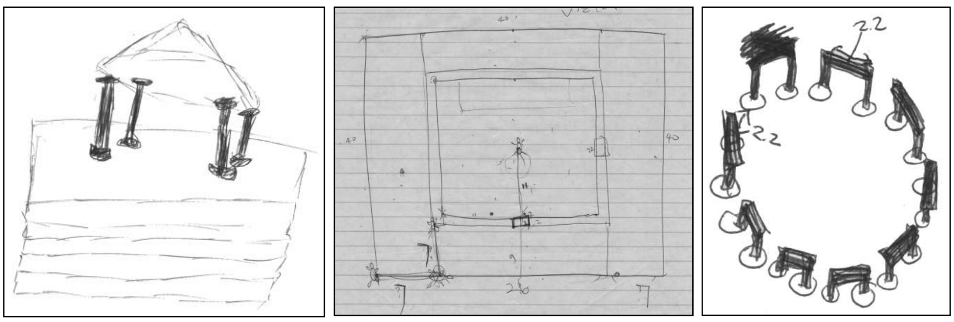


Figure 3: Three parts of the Temple plan

R2D2 was responsible for the stage with stairs and a kiosk. He observed his drawing design and thought that it could be achieved by scaling shapes of a cone, cylinders, and boxes. In doing the stairs, he experimented (i.e., trial and improvement) with different scales and started building the stair cases one by one:

up 0.4 fd 0.25 scaled 5.5 box

up 0.4 fd 0.25 scaled 5 box

up 0.4 fd 0.25 scaled 4.5 box …

Challenged by the teacher-researcher, he noticed a pattern firstly in the drawing of stairs then in the above commands as the scale of depth keeps decreasing by 0.5. After a few trials he came up with using a repeat to complete this stairs stage:

repeat 4 [up 0.4 fd 0.25 scaled 6-repcount*0.5 box]

The kiosk consisted a label, a cone roof, and four cylindrical posts, which were created in many cycles of think-plan-do-check, with calculations and trials of different sizes, locations, directions, and movements. R2D2 put every command in a procedure named *body* so the whole Temple stage could be created by just a simple command as *body*. In his spare time, he also wrote a *fountain* procedure with carefully chosen materials settings for running water (blue cylinders) and marble top (two overlapping spheres with different colours). He was very proud of his invention of the marble top because he found that although the two spheres were created at the same location, if they have different orientation then it will have the colour alternating effect. His creation of Temple part is shown in Figure 4 below.

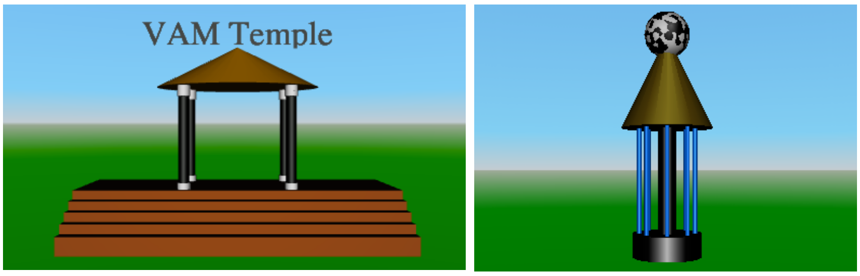
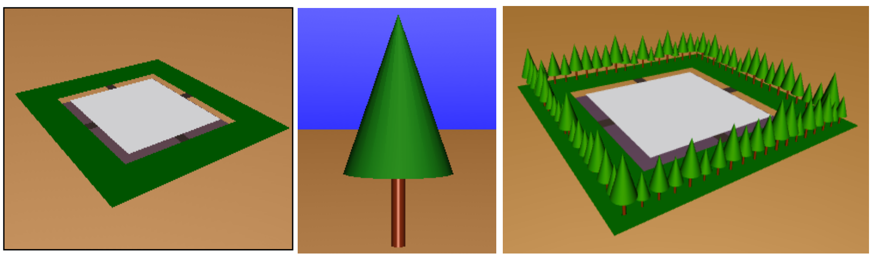


Figure 4: R2D2's Temple stage and fountain

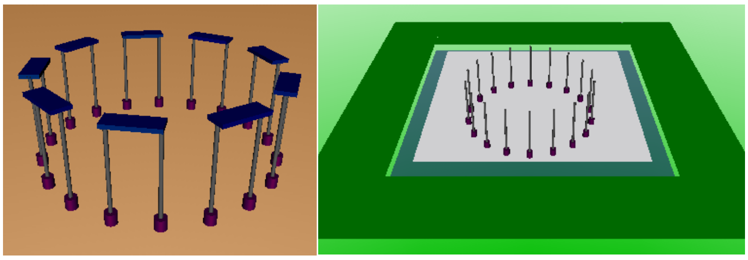
Victor was responsible for the Temple ground. He had more detailed plan with pre-calculated dimensions of the ground, centre court, and four bridges. Similar to R2D2, Victor decided to write a *ground* and a *bridge* procedure to create simple faces (planes) to achieve his design. He moved the turtle (i.e., the reference point in Turtle Geometry or Logo) in the 3D virtual space and recorded the track to create the 2D faces for the ground and bridges. After the construction of 2D faces, he wrote a *tree* procedure to create a simple 3D tree consisting of a cone and a cylinder. In order to generate different sizes of tree (a challenge by the teacher-researcher), the *tree* procedure was modified to take in an input. To create trees with random sizes (ranging from 1.1 to 2 times), Victor tried with brackets and eventually wrote: *tree (((random 10)+1)/10)+1*. The tree procedure was then repeated to create a tree fence surrounding the Temple ground (Figure 5).



```
TO tree :s
scale 0.2*:s 2*:s 0.2*:s
; some material settings here
up 1*:s cylinder
scale 1.5*:s 2.2*:s 1.5*:s
; some material settings here
up 1.1*:s cone
dn 2.1*:s
END
```

Figure 5: Victor's Temple ground and trees

Alekat20 took the design of part 3 of the Temple. She drew a circular structure to be placed at the centre of the Temple. Because in previous activities she had learnt about a polygon formula: *repeat :side [fd 1 rt 360/:side]*, she was able to apply the formula (challenged by the teacher-researcher) to create 18 cylinder posts but only 9 top slates (Figure 6) with many trials. To place this structure at the right place is another challenge. In Logo, a 2D circle or polygons can be easily created using the above polygon formula, which forwards same distance and turns same angle for many times (i.e., repeat). However, to find out the centre of a polygon or a circle is difficult for primary students. Eventually, Alekat20 solved this by a few trials of different starting locations and relying on the feedback she got between her Logo program (typological) and the navigation in 3D virtual space (topological) to place her *templebase* structure at a centre-south location.



```
TO templebase
home meter setscale 0.2 5 0.2
up 2.5 south 5 west 6.5
repeat 18 [cylinder fd 2.2 rt 360/18]
setscale 0.6 0.6 0.6 dn 2.2
repeat 18 [cylinder fd 2.2 rt 360/18]
setscale 1 0.2 2.5 up 4.7
repeat 9 [fd 1.1 box fd 1.1 rt 360/18 fd
2.2 rt 360/18]
END
```

Figure 6: Alekat20's Temple base on Temple ground

After all participants had completed their design of their Temple part, the final effort was to put everything together. Because there were many procedures created, the teacher-researcher suggested them to each create another procedure such as *tp1* (Temple Part 1) and include all their procedures in it. The team then quickly sorted out the sequence and merged all procedures and thus a final virtual Temple was created with a main procedure named *VAM* (an acronym related to their first names) (Figure 7).

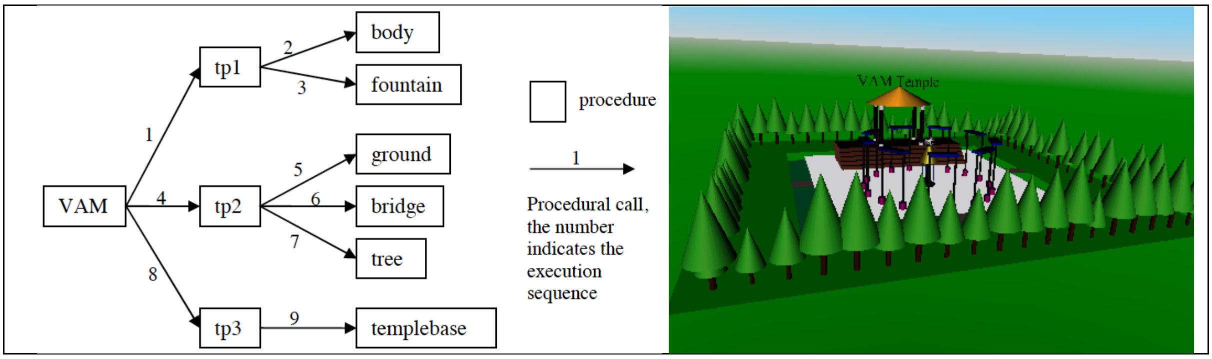


Figure 7: VAM Temple's procedural structure and virtual world

Discussion

From the results, we hope that we have shown deep mathematical learning among the three participants. Initially, the teaching experiment focused on the learning of geometry with 2D and 3D shapes, maps/plans, and location, direction, and movement. However, further analysis identified that learning in the areas of technology and engineering has also occurred. It can be sometimes difficult to separate the learning into individual disciplines. But here we will first try to discuss the learning we found according to disciplines, and then discuss the inter-relationships or integration of disciplines.

Learning in Mathematics

The mathematical concepts developed and applied in this students' final project not only included the intended 3D geometry, but also involved:

- Number and operations: This was evident where the participants designed and calculated the dimensions of their Temple parts. Whole numbers and decimal numbers were used throughout their programming. Operations were applied brilliantly to create intended results. For example, in R2D2's stairs stage, the use of *6-repcount*0.5* showed a decreasing mechanism by starting with a larger number 6.
- Measurement: This was evident from the plans they drew and the scales they wrote in the Logo program to change the sizes of 3D objects. The decimal scales such as 0.2 or 1.5 were not specifically discussed by the teacher-researcher with the participants. However, with the feedback from 3D virtual space and programming, the participants demonstrated good understanding and uses of decimal scales. It was usually a trial and improve process, in which they guessed a decimal scale in Logo, saw the 3D objects created, then made a sensible change of scales.
- Patterns: The teacher-researcher had this planned and thus always challenged the participants to use the *repeat* command to simplify the programming codes. By observing the geometrical patterns and the number patterns, participants developed ideas from describing (e.g., getting smaller), generalising (e.g., decreasing by 0.5), and then formalising in Logo programs.
- Algebra: The Logo programs created by the participants naturally contained many algebraic expressions. For example, the random sized tree *(tree (((random 10)+1)/10)+1)* involved variable (as an input of a procedure), order of operations, and functional thinking. In fact, a procedure in Logo is a sub-routine or a function. All participants were able to create and name their sub-routines and execute function calls.
- Chance: This was a contingency when the teacher-researcher challenged Victor to create different sizes of tree. The idea of random could be difficult to Victor. However, as demonstrated by his codes, he was able to utilise *random* command/function to generate a number range from 1.1 to 2.

The processes of learning mathematics in the VRLE are subtle and dynamic. We can confirm that the mathematical concepts and skills developed and applied in the VRLE are the results of the meaning-making (semiosis) within the multiple semiotic resources afforded in the VRLE. Further, what is common in the above discussion about the mathematical learning, is that it started and involved the Logo programming language. It is of course the nature of this VRLE, for its inclusion and design of Logo programming language as a core component. However, this reminds and informs that language, particularly a programming language can serve as a formalising agent for mathematical abstraction and logical thinking and reasoning.

Learning in Technology

Technology as framed in the Australian Curriculum, includes Design and Technologies, and Digital Technologies (ACARA, 2015). In this Temple project, the learning in technology was evident in:

- Design thinking and solutions (product): The whole process in this Temple project demonstrated a technological process as similar to the think-plan-do-check problem-solving cycle. The final VAM Temple was created through many refinements (prototyping) in designs of shapes, colours, and materials. There was a final product (solution) as a virtual Temple, albeit *virtual* but a different kind of *reality*. The virtual Temple is very tangible and real in a sense that the participants can see it, navigate, and walk in it.

- Computational and systematic thinking: As an ICT tool, the VRLE is a technology that engages the participants in designing and implementing digital solutions. The Logo programming language is a natural match for computational thinking, in which the participants practiced the problem solving, generated procedural and systematic codes to provide a solution (the virtual Temple), and evaluated the solution. The participants together managed the project and were able to create a systematic structure of the VAM Temple procedures (see Figure 7).

Learning in Engineering

The engineering design process is in a way similar to the generic problem solving cycle but can be broken down into more detailed cyclic steps: (1) identify the need or problem; (2) research the need or problem; (3) develop possible solution(s); (4) select the best possible solution(s); (5) construct a prototype; (6) test and evaluate the solution(s); (7) communicate the solution(s); and (8) redesign (Massachusetts Department of Education, 2006). In this Temple project, there was not enough time to go through the full cycle a few times. However, it was clear that the participants had gone through selecting materials and tools (programming commands and/or graphic user interface) to prototyping, testing, communicating, and redesigning their artefacts (e.g., the 3D stage, trees, centre court structures, and the programming codes etc.). The aspect of procedural thinking in the programming is also a key component of engineering design process. It involves flow-charting, data/variables, mathematical computations, and comparisons (e.g., greater and/or less than), logical operations (e.g., and, or) and controls (e.g., if, else). Some of them were not evident in this Temple project but they are certainly provided in the Logo programming language in this VRLE.

Learning in Mathematics, Programming and STEM

In the discussion above about the learning in individual disciplines, there are clearly some overlapping developments among disciplines. The meaning-making of mathematics is a dynamic and complex process among systems of signs in the VRLE. In this Temple project example, we found that the geometrical Logo programming language was the pivotal point linking all types of mathematical representations. When programming in the VRLE's Logo and virtual space, learners will mathematically analyse the real world context; generalise according to patterns and relationships; logically sequence the steps and commands; semantically and syntactically write in the programming language; execute the codes to create the virtual world; navigate in the 3D virtual space to examine and see the continuous visual feedback; rethink, recalculate, and repeat the earlier steps; and sometimes restart, redefine, and redesign their solutions.

From mathematics to programming, we also found that what were learnt was beyond merely mathematics as we originally focused. In building a virtual Temple, many of the technology and engineering concepts were learned and practiced by the participants. They collaboratively and cooperatively solved a problem (i.e., build a virtual Temple) by applying the design process (i.e., technology or engineering processes); selecting materials (i.e., material settings on virtual objects), programming with variable and procedures (e.g., the tree procedure), operating and communicating with computers (e.g., use of ICT tools and forums), thinking procedurally (e.g., sequence procedural calls), and creating systems and controls (e.g., combine functional calls). We can say that these participants have engineered a virtual Temple with technologies and mathematics through programming in the VRLE.

Conclusion

In our new conceptual framework (Figure 1), the social-actional semiotic resources such as designing and building structures can and will most certainly involve projects from science, technology, engineering, and mathematics. This semiotic framework thus has implications for future teaching and learning, not just for mathematics, but STEM as an integrated whole for a more holistic meaning-making approach. We would like to conclude that learning mathematics now encompasses other disciplines, particularly with areas in STEM. The nature of learning mathematics may be still within mathematics itself, but in the current technological world, at least in this VRLE, knowledge and skills of mathematics, technology, and engineering developed simultaneously. We need to rethink and consider how mathematics can be taught and learnt in an integrated way and utilise what current technologies such as this VRLE can offer. We also need to examine closely how programming can and should be included in the curriculum.

References

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2015). Australian Curriculum: Technologies. Retrieved 28 February 2015, from http://www.australiancurriculum.edu.au/technologies/rationale

Feurzeig, W., Papert, S., & Lawler, B. (2010). Programming-languages as a conceptual framework for teaching mathematics. *Interactive Learning Environments, 19*(5), 1-15. doi: 10.1080/10494820903520040

Hoyles, C., Noss, R., & Adamson, R. (2002). Rethinking the Microworld idea. *Journal of Educational Computing Research, 27*(1), 29-53.

Lemke, J. L. (2001). Mathematics in the middle: Measure, picture, gesture, sign, and word. Retrieved 6 February 2011, from http://academic.brooklyn.cuny.edu/education/jlemke/papers/myrdene.htm

Massachusetts Department of Education. (2006). Massachusetts Science and Technology/Engineering Curriculum Framework. http://www.doe.mass.edu/frameworks/scitech/1006.pdf

Taylor, R. P. (Ed.). (1980). *The computer in the school: Tutor, tool, tutee*. New York: Teachers College Press.

Yeh, A. (2007). *Knowledge Construction of 3D Geometry in Virtual Reality Microworlds.* (Doctoral dissertation, Queensland University of Technology, Brisbane, Australia). Retrieved from http://eprints.qut.edu.au/16648/

Yeh, A. (2013). Constructing a frame of cube: Connecting 3D shapes with direction, location and movement. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics educations: Testerday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia. Melbourne: MERGA.

Yeh, A., & Hallam, J. (2011). Young children's understandings about "square" in 3D virtual reality microworlds. In J. Clarrke, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), Mathematics : Traditions and [new] practices (Proceedings of of the 23rd biennial conference of the AAMT and the 34th Annual Conference of the Mathematics Education Research Group of Australasia) Alice Springs, NT: MERGA/AAMT.

Yeh, A., & Nason, R. (2004, December). *Toward a semiotic framework for using technology in mathematics education: The case of learning 3D geometry.* Paper presented at the International Conference on Computers in Education, Melbourne, Australia.

Research Presentation Abstracts

Laying the Foundation for Proportional Reasoning

Ann Downton
Australian Catholic University
<Ann.Downton@acu.edu.au>

Multiplicative thinking is required when engaging in proportional reasoning tasks. While proportional reasoning does not always develop naturally in students, providing students from a young age with tasks that require students to think multiplicatively may lay the foundation for them to do so. This paper reports the findings of Grade 3 students' performance on tasks relating to different multiplicative structures and the influence of each structure on students strategy choice.

The Development and Evaluation of an Individualised Learning Tool for Mathematics students with Intellectual Disability: IMPELS

Agbon Enoma
Curtin University
<a.enoma@postgrad.curtin.edu.au>

John Malone
Curtin University
<J.Malone@curtin.edu.au>

IMPELS is an Individualised Mathematics Planning and Evaluation of Learning Tool for Students with Intellectual Disability. IMPELS was evaluated against 3 number sense tools and subjected to standard validity and reliability assessments. Results obtained indicated that IMPELS correlated strongly with the tools, ranging from 0.70 to 0.91 and 0.45 to 0.70 for Pearson and Spearman's Rho correlation coefficients respectively. Cronbach's alpha and Spilt-Half Reliability (KR-20) was 0.96. IMPELS is useful for the collection of baseline data to inform the development of individual education plans (IEPs) and for monitoring the progress of learning of individual students.

Capturing Mathematical Learning in an Inquiry Context: There are Some Things Not Easily Measured

Kym Fry
University of Queensland
<k.fry1@uq.edu.au >

This paper presents the theoretical findings from a PhD study into assessing mathematical learning in an inquiry context. The pedagogy of inquiry will continue to struggle to prove its worth, while student improvement in mathematics continues to be measured in terms of data gained through assessment designed for more traditional pedagogies. Findings from this study revealed high levels of student thinking about mathematics in inquiry when teachers artfully engineered feedback gained through formative assessment into teaching and learning experiences. Learning mathematics in inquiry reflected a complex and highly interactive journey, not easily measured using traditional school assessment practices.

Teacher Professional Growth through using a Critical Mass Mentoring System: Effective Whole School Teacher Professional Development.

Judy Hartnett
Mathematics Education Consultant
<judyhartnett@bigpond.com>

Jim Midgley
Principal: St Eugene's Catholic College
<jmidgely@bne.catholic.edu.au>

Professional development for teachers utilises a significant portion of school budgets. Too often the impact on the performance of teachers, individually or collectively or on the learning outcomes of students is limited. One school principal devised a mentoring system that has been shown to bring about profound and sustainable cultural change, where all staff willingly take responsibility for their own professional learning and play a crucial role in the professional development of their peers.

Anatomy of a Mathscast

Carola Hobohm
University of the Sunshine Coast
<chobohm@usc.edu.au>

Linda Galligan
University of Southern Queensland
<Linda.Galligan@usq.edu.au>

This paper reports on continued research of student produced mathscasts to support learning. Teachers and pre-service teachers, enrolled in a university course, were asked to create and peer-critique mathscasts to explain concepts in middle school. This paper discusses results of students' use of a mathscast rubric that was developed by the authors to assist in the creation and evaluation of mathscasts. Surveys, practice mathscasts with informal feedback, and students' final mathscasts are analysed. The paper concludes with an outline of future directions.

An Exploration of Strategies That Teachers Use When Teaching Beginning Algebra

Christina Lee
Edith Cowan University
christina.lee@ecu.edu.au

Christine Ormond
Edith Cowan University
c.ormond@ecu.edu.au

Algebra is essential in higher mathematics and initiation into the language and conventions of algebra, and the development of algebraic thinking, are crucial in the earlier years of schooling. It is the teachers' beliefs about the nature of mathematics and teaching and learning which underpins their approach to teaching algebra. In this paper, we explore the strategies used by four teachers to teach beginning algebra.

Factors Influencing Social Process of Statistics Learning within an IT Environment

Ken W. Li
Hong Kong Institute of Vocational Education (Tsing Yi)
<kenli@vtc.edu.hk>

Merrilyn Goos
The University of Queensland
<m.goos@uq.edu.au>

Information Technology (IT) plays an educational role of organising the learning environment to promote social interaction among students as well as between students and a teacher but little has been known about what underlying factors influence such social interaction within the context of statistics learning. A questionnaire-based survey was therefore conducted to gather data relevant to this issue. The data were then summarised by using Factor Analysis into factors: co-learning, teacher's scaffolding assistance, positive working relationship linking with social interaction.

Identifying categories of Pre-service Teachers' Mathematical Content Knowledge

Sharyn Livy
Monash University
<sharyn.livy@monash.edu>

An important issue related to the current discussion about teachers' knowledge and qualifications is to improve and enhance their preparation of numeracy skills. Further studies, including longitudinal studies designed to identify mathematical content knowledge (MCK) pre-service teachers' gain during teacher education are important for course design and developing effective primary numeracy teachers. This paper reports on one pre-service teacher's development of MCK but was informed by an historical overview of theoretical frameworks and the findings of a four-year longitudinal study of 17 pre-service teachers' MCK. The results identified how and when different categories of MCK were developed and can be used to improve future course design.

Using Drawings and Discussion to Prompt Young Learners to Reflect Upon and Describe Their Mathematical Understandings

Andrea McDonough
Australian Catholic University
<andrea.mcdonough@acu.edu.au>

Jill Cheeseman
Monash University
<jill.cheeseman@monash.edu>

With an interest in children's learning of mathematics and ways to gain insights into this learning, we explore the possible value of an open-ended self-assessment task, *Impress Me*, and follow-up interview. Ten children in their first or second year of school recorded their understandings using drawing and/or writing during the period in which they were taught lessons on mass measurement and then met individually with an interviewer/researcher to discuss their portrayal and their learning. The *Impress Me* recording was found to be a useful initial prompt to stimulate discussion and other effective interview prompts are identified in this paper.

Language and Mathematics: Exploring a New Model to Teach in Bi/Multilingual Mathematics Classroom

Charly Muke
St Theresa's Abergowrie College, Townsville
<charly_muke@yahoo.com.au>

The Australian curriculum now recognises the importance of language in mathematics learning. However, little recognition is given to the fact that most urban schools have many students who are from families who speak a non-English first language (L1). Participants in this session will be introduced to a *language-use model* for teaching mathematics. This model can be used in planning mathematics lessons that will highlight important aspects of language, particularly for English as a later language ELL students' learning. Examples will come from research carried out in Papua New Guinea with multilingual teachers.

Exploring the Influence of Early Numeracy Understanding Prior to School on Mathematics Achievement at the End of Grade 2

Andrea Peter-Koop
Bielefeld University
<andrea.peter-koop@uni-bielefeld.de>

Sebastian Kollhoff
Bielefeld University
<sebastian.kollhoff@uni-bielefeld.de>

This paper reports first results of a 3-year longitudinal study that seeks to explore the impact of early number skills and knowledge as demonstrated prior to school on achievement in school mathematics at the end of junior primary school. The study investigates the development of early numeracy understanding of 334 children from one year prior to school entry until the end of grade 2. The study identifies second graders that are vulnerable in their mathematics learning and compares their performance with their achievements over the past three years.

An Irish Response to an International Concern: Challenges to Mathematics Teaching

Lisa O'Keeffe
University of South Australia
<lisa.okeeffe@unisa.edu.au>

Olivia Fitzmaurice
University of Limerick
<olivia.fitzmaurice@ul.ie>

Patrick Johnson
University of Limerick
<patrick.johnson@ul.ie>

The issue of *quality* teachers has been well debated internationally. Both Ireland and England have previously flagged their concern about the lack of qualified teachers in secondary mathematics and overreliance on traditional teaching methods. The UK response to *improving teaching standards* has included changes to the structure of teacher training and skills testing and currently in Australia the topical education issue is the introduction of similar skills test for all trainee teachers. Despite facing similar issues, Ireland didn't follow suit with skills testing. Instead the Irish government opted to implement a number of strategies which included the upskilling of practising teachers of mathematics. However, while this addressed one side of the issue a key aspect of the challenge still remained "challenging pre-service teachers to do more than talk the talk' (Prendergast et al., 2013). Hence, this paper aims to present the Mathematics Education team's (at University of Limerick) response – "Mathematical Thinking"

Reference

Prendergast, M., P. Johnson, O. Fitzmaurice, M. Liston, L. O'Keeffe, and O'Meara, N. (2014). Mathematical Thinking: Challenging Prospective Mathematics Teachers to Do More than 'Talk the Talk'. *International Journal of Mathematical Education in Science and Technology.* 45 (5), 635–647.

An Analysis of Modelling Process based on McLuhan's Media Theory: Focus on Constructions by Media in Cases of Using Geoboard

Hiro Ozasa
Saitama University
< s13ac202@mail.saitama-u.ac.jp>

The aim of this paper is to examine transitions between phases of a modelling process in cases of introduction of square root using paper and electric geoboard. The method is to analyse construction by media based on McLuhan's media theory (McLuhan, 1987; Tokitsu, 2012). As conclusions, the followings about the both cases are found; (1) transitions between phases of the modelling process are same *but* constructions by media are different, and (2) a new mathematical problem can be posed because of the constructions by media.

References

McLuhan, M. translated by Mori, T. (1986). *The Gutenberg Galaxy: The Making of Typographic Man*. Tokyo, Japan: Misuzu Shobo. (in Japanese)

Tokitsu, K. (2011). The communication model in M.McLuhan's theory of media and its implication in education, *Department*al *Bulletin Paper of Hiroshima Bunka Gakuen University, 17*, 25-34. (in Japanese)

The Knowledge Dimension of Revised Bloom's Taxonomy for Integration

Farzad Radmehr
Victoria University of Wellington
<Farzad.radmehr@vuw.ac.nz>

Robin Averill
Victoria University of Wellington
< Robin.averill@vuw.ac.nz>

Michael Drake
Victoria University of Wellington
< Michael.drake@vuw.ac.nz>

In this paper, the knowledge dimension of Revised Bloom's taxonomy (RBT) in the context of integration in stage 1 university calculus is presented. For this purpose, eleven subcategories of the knowledge dimension of RBT are introduced and through document analysis of chapter 4 of the handbook of RBT, subcategories are defined. Then, using materials frequently employed for teaching integration, the knowledge dimension of RBT in the context of integration is explored. The study findings may enable enhanced opportunities for dialogue between teachers, lecturers, and researchers about metacognitive knowledge in relation to teaching integration and to the development of tools for designing educational objectives, teaching activities, and assessments based on RBT.

Developing an analysing tool for dynamic mathematics-related student interaction regarding affect, cognition and participation

Laura Tuohilampi
University of Helsinki
<laura.tuohilampi@helsinki.fi>

In this study, a video excerpt of two boys working on a mathematical open-ended problem is discussed. In the video, affective and social factors overrule development of logical thinking. Analysing such an episode is challenging, as appropriate tools are few. This study elaborates the video excerpt to find out what affective, cognitive and social phenomena exist in the episode, aiming to develop an analysing tool for such purpose. In addition, a framework called Patterns of Participation will be adapted to test its purposefulness to the analysis. As a result, it was found out that most of the essential features of the episode were revealed. However, it is suggested including theories of emotions, student engagement and positioning would make the tool more profound.

Thinking Strategies Used by 7th-Grade Students in Solving Number Sense Problems

Palanisamy Veloo
SEGi University, Malaysia
<pkveloo@segi.edu.my>

Parmjit Singh
University of Technology MARA, Malaysia
<parmj378@salam.uitm.edu.my>

Forty-five Grade 7 students from a larger sample of 118 were interviewed on the thinking strategies they used in solving number sense problems. Students were categorised into high, middle and low ability groups. Students were asked a series of questions designed to assess the thinking strategies they used in solving number sense questions. The study also investigated the extent to which misconceptions and learned rules were sufficiently fixed that they continue to influence students' responses despite having been given lessons on number sense. Results show that many students continued to apply rule-based methods in attempting to solve number sense questions.

Round Table Discussion Abstracts

Working Across Disciplinary Boundaries in Pre-service Teacher Education

Merrilyn Goos
The University of Queensland
<m.goos@uq.edu.au>

Judy Anderson
The University of Sydney
<judy.anderson@sydney.edu.au>

Jo Balatti
James Cook University
<josephine.balatti@jcu.edu.au>

Kim Beswick
University of Tasmania
<kim.beswick@utas.edu.au>

Tricia Forrester
University of Wollongong
<tricia@uow.edu.au>

Jenni Way
The University of Sydney
<jennifer.way@sydney.edu.au>

In Australia, a suite of national projects has been funded by the Australian government to promote strategic change in mathematics and science pre-service teacher education. This round table session will share some of the interdisciplinary strategies being trialled in one project, *Inspiring Mathematics and Science in Teacher Education* (IMSITE), and invite feedback from participants on the transferability of strategies to other institutional contexts and the sustainability of these strategies over time.

The specific objectives of the IMSITE project are:

- to develop and validate a repertoire of strategies for combining knowledge of content and pedagogy in mathematics and science; and
- to connect academics from different communities of practice – mathematics, science, education – in order to collaboratively design and implement these new teacher education approaches.

Six universities and 23 investigators – mathematicians, scientists, and mathematics and science teacher educators – are the core participants in the project, with more universities to be added in 2015.

The first half of the round table session will showcase interdisciplinary strategies such as:

- Collaborative development and delivery of new content and pedagogy courses by mathematicians and mathematics educators;
- Reciprocal tutoring by mathematicians and mathematics educators into each other's courses;
- Peer observation by mathematicians and mathematics educators of each other's teaching;
- Development of a mathematics specialisation in primary pre-service programs.

The remainder of the session will invite discussion of challenges to interdisciplinary collaboration ("siloing" of disciplines, inflexible workload and course funding models, cultural differences between the disciplines) and ways to overcome these.

Promoting Positive Emotional Engagement in Mathematics of Prospective Primary Teachers

Joanna Higgins
Victoria University Wellington
<Joanna.Higgins@vuw.ac.nz>

Janette Bobis
University of Sydney
<janette bobis@sydney.edu.au>

Good teaching is described as that which is "charged with positive emotion" (Hargreaves, 1998, p.835). Yet, primary pre-service teacher education programs predominantly focus on the development of knowledge and pedagogy while affective aspects, including emotions, are only implicitly treated (Gootenboer, 2008). To date, research exploring the role emotions play in the process of learning to teach mathematics has received little attention (Hogden & Askew, 2007).

The round table will begin by outlining the rationale and theoretical underpinnings of a trans-Tasman research project that aims to deepen primary pre-service teachers' [PST] emotional and intellectual engagement in learning to teach mathematics. The Mathematics Emotional Engagement [MEE] project aims to develop and assess the effectiveness of an innovative teaching approach designed to promote positive emotional engagement in learning and teaching mathematics. The study explores the impact of a three-step interventional framework, referred to as 'AIR', that utilises a series of research-based instructional activities involving preservice primary teachers in: (1) **A**ttending to their existing emotional responses towards the learning and teaching of mathematics; (2) **I**nterpreting the causes and potential impact of existing emotional responses; and (3) **R**esponding to their emotions with strategies to ameliorate negative affects on their learning and teaching of mathematics. Data from the first stage of the project—developing and refining AIR instructional strategies—will provide the stimulus for discussion amongst participants.

References

Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 11*(6), 479-497.

Hargreaves, A. (1998). The emotional practice of teaching, *Teaching and Teacher Education, 14* (8), 835-845.

Hogden, J., & Askew, M. (2007). Emotion, identity and teacher learning, *Oxford Review of Education, 33*(4), 469-487.

Senior Secondary Students' Pre-calculus and Calculus Understanding

Michael Jennings
The University of Queensland
<msj@maths.uq.edu.au>

Peter Adams
The University of Queensland
<pa@maths.uq.edu.au>

There are substantial and ongoing concerns in the Australian and international secondary and tertiary education sectors about students' transition from secondary to tertiary mathematics. Declining enrolments in *advanced mathematics* in secondary schools and less stringent university entry requirements are seen as a major concern for the future of STEM education in Australia.

In this round table, I will present data collected from secondary school students on pre-calculus and calculus topics. These data were collected from two groups of students: those studying *intermediate mathematics* in the last two years of secondary school; and those studying both *intermediate* and *advanced mathematics.*

The results suggest that there are distinct differences in students' procedural and conceptual understanding depending on which mathematics they studied in the last two years of secondary school. Students who studied both *intermediate* and *advanced mathematics* performed considerably better in all questions, not only on the calculus questions but also on junior mathematics pre-calculus topics such as gradient of a straight line. The data also showed that both groups of students had difficulty identifying lines parallel to axes, as well as explaining the meaning of the definition of the derivative.

This presentation is part of a two-year state-wide longitudinal project that is investigating the transition from secondary to tertiary mathematics.

Investigating Mathematical Inquiry

Katie Makar
The University of Queensland
<k.makar@uq.edu.au>

Jill Fielding-Wells
University of Tasmania
<jill.wells@utas.edu.au>

Kym Fry
The University of Queensland
<k.fry1@uq.edu.au>

Sue Allmond
Jindalee State School
<sallm6@eq.edu.au>

Jude Hillman
Jindalee State School
<jhill98@eq.edu.au>

The aim of this Round Table is to bring together a community of researchers who focus on the teaching, learning, assessment, and research of a mathematical inquiry approach. We invite those interested in the study of mathematical inquiry to discuss their work or aspects of inquiry that are in need of research. A few questions are listed below to provoke conversation. Bring your own!

1. What shared and unshared perspectives do we have of mathematical inquiry?
2. What are purposes of mathematical inquiry?
3. How can mathematical inquiry be used to assess learning?
4. What signature practices characterise inquiry pedagogy in mathematics education?
5. How is mathematical inquiry similar to or different from inquiry in other content areas, such as science?
6. How does the teaching of mathematical inquiry fit into the broader repertoire of pedagogies used by teachers in the course of a year?
7. What challenges do teachers and students face in adopting mathematical inquiry?
8. Does an inquiry approach benefit children with different backgrounds differently?
9. What are key benefits and drawbacks of learning mathematics through inquiry?
10. Do particular strands of mathematics fit better with inquiry?
11. Does mathematical inquiry improve learning in mathematics?
12. Is mathematical inquiry scalable?
13. How can different paradigms contribute to a diversity of insights into mathematical inquiry?
14. What key research areas are strongly tied to mathematical inquiry (e.g., argumentation, socio-mathematical norms, collaboration)?
15. What are possible programs of research for mathematical inquiry?

Short Communication Abstracts

A Problem Solving Lesson: Pre-service Teachers Initiation to Lesson Study

Jaguthsing Dindyal
National Institute of Education
<jaguthsing.dindyal@nie.edu.sg>

Three pre-service teachers (PST) who had no prior experience with lesson study had to use a lesson study approach to plan and teach a problem solving lesson. This paper documents how the three PSTs were initiated into the Japanese style of lesson study and then how as a team they went about planning their research lesson on problem solving for a primary three class and then teaching it. The focus is on some of the issues that surfaced when preparing this problem solving lesson on *magic squares* and how they addressed them.

Teachers' Beliefs about Knowledge of Content and Students and its Effect on their Practice

Vesife Hatısaru
Middle East Technical University
<vhatisaru@hotmail.com>

This study investigated mathematics teachers' beliefs about teachers' knowledge of content and students (Ball, Thames, & Phelps, 2008) about particular mathematical content and its effect on teaching practice. Two teachers participated in the study. Data were collected through classroom observations and an interview. The interview was based on An, Kulm, Wu, Ma, and Wang (2002) and focused mainly on teachers' beliefs about knowledge of students' thinking, approach to planning the mathematics instruction, students' homework, and importance and approach to grading homework. The study indicated both teachers believed the importance of teachers' understanding the way students think about a certain mathematics subject or the difficulties they experience with it. Nevertheless, it is seemed the teachers' beliefs had no effect on their teaching practice. Moreover, they had limited awareness of how to identify students' difficulties.

References

An, S., Kulm, G., Wu, Z., Ma, F., & Wang, L. (2006). A comparative study on mathematics teachers' beliefs and their impact on the teaching practice between the U.S. and China. In F. K.S. Leung, K. D. Graf, & F. J. Lopez-Real (Ed.), *Mathematics education in different cultural traditions: A comparative study of East Asia and the West, The 13th ICMI Study,* (pp. 449-464). New York, NY: Springer.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Exploring Students' Views on using iPads in Mathematics.

Janelle Hill
Monash University
<Janelle.hill@monash.edu.au>

The use of iPads in education is increasing, with increasing numbers of studies focussing on teacher use of this tool in mathematics teaching and learning. As a stakeholder group, the views of students must also be investigated. As part of a larger case study, the views of Year 5 to Year 12 students from one Victorian school were sought about the use of iPads in mathematics. A number of concerns related to the perceived negative impact of iPad use in mathematics learning arose and will be further explored in the presentation.

Mapping school students' aspirations for STEM careers

Kathryn Holmes
University of Newcastle
<Kathryn.holmes@newcastle.edu.au>

Adam Lloyd
University of Newcastle
<adam.lloyd@newcastle.edu.au>

Jenny Gore
University of Newcastle
<jenny.gore@newcastle.edu.au>

Max Smith
University of Newcastle
<maxwell.smith@newcastle.edu.au>

Declining enrolments in STEM disciplines and a lack of interest in STEM careers is concerning at a time when society is becoming more reliant on complex technologies. We examine student aspirations for STEM careers by drawing on survey data from 8235 school students in Years 3 to 11 who were asked to indicate their occupational choices and give reasons for those choices. These data are also examined in relation to student SES, gender, prior achievement and educational aspirations. The analysis provides a strong empirical basis for understanding current student interest in STEM and exploring implications for educational policy and practice.

Breaking down Barriers

Peter Howley
The University of Newcastle
<peter.howley@newcastle.edu.au>

Engaging cohorts including less quantitatively-adept students and educating them about the value of Statistics has its challenges. This talk will outline two successes: the first resulted in a first-year Statistics for Business course increasing student satisfaction scores from under 3.5 out of 5 to 4.72 whilst maintaining 'challenge' scores and reducing Failure rates previously exceeding 25% to 7-12%; the second is a national project-based learning activity (piloted in the Hunter Region in 2014) which facilitates boundary encounters (between secondary, tertiary, and industry sectors and students having varied backgrounds and areas of interest) and develops key communication, research and quantitative skills.

Building upon the Language Model of Mathematics

Harry Kanasa
Griffith University
<h.kanasa@griffith.edu.au>

Kevin Larkin
Griffith University
<h.kanasa@griffith.edu.au>

The language model of mathematics is a useful framework to conceptualise the teaching and learning of mathematics from a constructivist perspective. The model currently proposes that students move along two dimensions (visual and verbal) towards increasing levels of mathematical abstraction. We present the case for theorising the existence of a third dimension, the gestural, by drawing upon established theories of learning within mathematics and also from brain based learning. Examples will be provided on how the addition of the gestural dimension can enhance mathematics education at all levels.

The Australian Mathematics Competition: What's the Score?

Andrew Kepert
The University of Newcastle
<Andrew.Kepert@newcastle.edu.au>

Mike Clapper
Australian Mathematics Trust
<Mike.Clapper @amt.edu.au>

The Australian Mathematics Competition (AMC) is a problem-solving competition for Primary and Secondary students. Each paper has 30 problems graded from routine to baffling, challenging and rewarding students of all abilities. The competition's quality depends on the collective effort of dozens of Mathematics Educators (Primary to University) who write and scrutinise the papers in several stages. Our current work is to ensure the AMC provides a reliable challenge for students. Tools for calibrating the performance of questions and papers across a range of question types are improving the competition, measured by the relative performance of each question, and by each paper's aggregate score.

A Focus Question Approach to the Teaching of Mathematics

John Ley
University of Western Sydney
<j.ley@uws.edu.au>

This is a presentation on a focus question approach to teaching mathematics (FFQA) which is the title of my thesis. It is proposed to investigate the impact of the FFQA at the commencement of each mathematics lesson on the learning and motivation of students. The style of five questions that I propose to investigate has the first four questions as instrumental style questions that focus on procedural knowledge, with the final question using a relational understanding approach with some of the questions being open ended investigational style questions focusing on conceptual knowledge. The research is ongoing.

Promoting the Development of Foundation Content Knowledge in *all* Primary Pre-service Teachers

Chris Linsell
University of Otago College of Education
<chris.linsell@otago.ac.nz>

Naomi Ingram
University of Otago College of Education
<naomi.ingram@otago.ac.nz>

Megan Anakin
University of Otago College of Education
<megan.anakin@otago.ac.nz>

A feature of Linsell and Anakin's (2013) concept of foundation content knowledge is that all pre-service teachers should have a growth oriented disposition and extend their knowledge, whether or not it is initially strong. This study reports on the use in mathematics pedagogy classes of introductory problems designed to encourage all first year primary pre-service teachers to become aware of the features of foundation content knowledge and to extend their own knowledge. Eighty-one percent of those pre-service teachers whose foundation content knowledge was not initially strong considered the introductory problems helpful, compared to 61% of those whose knowledge was strong.

References

Linsell, C., & Anakin, M. (2013). Foundation content knowledge: What do pre-service teachers need to know? In V. Steinle, L. Ball, & C. Bardini (Eds.), *Yesterday, Today and Tomorrow* (Proceedings of the 36th Mathematics annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 442-449). Adelaide: MERGA.

Paternal influence on school students' aspirations for STEM careers

Adam Lloyd
University of Newcastle
<adam.lloyd@newcastle.edu.au>

Jenny Gore
University of Newcastle
<jenny.gore@newcastle.edu.au>

Max Smith
University of Newcastle
<maxwell.smith@newcastle.edu.au>

There is a growing awareness of the important and differential influence fathers have on child lifestyle behaviours compared to mothers. This 'paternal' influence could potentially carry across to children's early career aspirations. A sample of n = 8235 school students in Years 3 to 11 were asked to indicate their occupational choices, give reasons for those choices and also provide information about their parents education and occupation. Using regression analysis, associations between paternal and maternal education levels and occupations with children's STEM career aspirations were modelled. The findings provide further evidence of the potential differential influence parents have on their child's aspirations.

Understanding Mathematics: Teacher Knowledge, Task Design and Evaluating Students' Mathematical Reasoning

Christine Mae
Catholic Education Office Sydney
<christine.mae@syd.catholic.edu.au>

Janette Bobis
University of Sydney
<janette.bobis@sydney.edu.au>

Jenni Way
University of Sydney
<jennifer.way@sydney.edu.au>

This presentation describes a research project designed to understand the relationship between teachers' conceptual understandings of mathematics, the tasks they design for their students and their evaluation of students' responses to tasks. Using Timperley's (2008) *Teacher Knowledge Building and Inquiry Cycle*, Year 5 and 6 primary teachers and leaders at a range of career stages engaged in tasks to highlight the connection between what students need to know, what teachers need to know and what teachers need to learn. The implications for developing teachers' understandings of mathematics will be discussed in terms of system-level professional learning.

References

Timperley, H. (2008). Teacher professional learning and development, *Educational Practices Series-18*, International Bureau of Education, UNESCO.

The Pattern and Structure of the *Australian Curriculum–Mathematics*

Catherine McCluskey
Macquarie University
<catherine.mccluskey@students.mq.edu.au >

Joanne Mulligan
Macquarie University
<joanne.mulligan@mq.edu.au >

Michael Mitchelmore
Macquarie University
<mike.mitchelmore@mq.edu.au >

The mathematical proficiencies in the *Australian Curriculum—Mathematics* describe the processes students are engaged in while developing mathematical concepts (ACARA, 2014). This presentation focuses on how the proficiencies: understanding, problem solving, reasoning and fluency, may work together to build patterns of thinking which can lead to generalised understandings of mathematical concepts. The authors connect the combined role of these proficiencies with a proposed Generalised Model of Patterning (McCluskey, Mitchelmore, & Mulligan, 2013), highlighting the role of patterning in the development of conceptual understandings within and beyond mathematics.

Mathematical Thinking in a Context of ‘General Thinking’: Implications for Mathematics Education

Corinne Miller
Southern Cross University
<Corinne.miller@scu.edu.au>

Geoff Woolcott
Southern Cross University
<Geoff.woolcott@scu.edu.au>

Christos Markopoulos
Southern Cross University
< christos.markopoulos@scu.edu.au >

This new project explores the similarities and differences of mathematical thinking and ‘general thinking’, as well as related motivational and emotional aspects, focusing on how these differ in educational contexts. It will examine assumptions of the underlying feature of mathematics curriculum design and pedagogy, for example, that linear structure is the most efficient means of building mathematical knowledge or that number-based knowledge is a reliable indicator of mathematical skill. Insights gained will be used to improve the current paradigms in course structure and pedagogy for classroom mathematics in order to develop a structure better aligned to student capabilities and potentials.

Conceptual Connectivity in Mathematics

Joanne Mulligan
Macquarie University
<joanne.mulligan@mq.edu.au>

Geoff Woolcott
Southern Cross University
<Geoff.woolcott@scu.edu.au>

Human environmental interactions involve general conceptual connectivity processes such as categorisation, abstraction and generalisation. These are linked to the development of mathematics concepts, but research in this area is relatively new in mathematics education. A conceptual connectivity lens, however, has been used in cases where there are difficulties in mathematics learning, such as developmental dyscalculia, as well as in studies of mathematical pattern and structure with young gifted children. This presentation suggests that such studies support the determination that individual differences in processing of environmental information are an important way forward in understanding what underpins mathematics conceptual development.

References

Australian Curriculum Assessment and Reporting Authority [ACARA] (2014). *Australian curriculum.* Retrieved 11 October, 2014, http://www.australiancurriculum.edu.au/

McCluskey, C., Mitchelmore, M. C., & Mulligan, J. T. (2103). Does an ability to pattern indicate that our thinking is mathematical? In V. Steinle, L. Ball, & C. Bandini (Eds.), *Mathematics education: Yesterday, today & tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 482-489). Adelaide: MERGA.

Primary-Middle Pre-Service Teachers reported use of the Mathematics Textbook

Lisa O'Keeffe
University of South Australia
<lisa.okeeffe@unisa.edu.au>

The 1999 TIMSS video study highlighted a heavy reliance on the mathematics textbook in Australian classrooms (Hiebert et al., 2003). This promoted further investigation by Vincent & Stacey (2008) who have documented the differences between mathematical textbooks and concerns with regard to problem solving. However, there is much anecdotal evidence to suggest that the role of the textbook may be changing and that the emergence of digital technologies may in fact replace the mathematics textbook (Hu, 2011). Hence, this exploratory study intends to a brief insight into the current status of the mathematics textbook and its use within Australian classrooms.

Examining a Students' Resource for Reconstructing the Limit Concept at Need: A Structural Abstraction Perspective

Thorsten Scheiner
University of Hamburg, Germany
<Thorsten.Scheiner@uni-hamburg.de>

Márcia M. F. Pinto
Federal University of Rio de Janeiro, Brazil
<marciafusaro@gmail.com>

This presentation examines a student's learning of the limit concept of a sequence compatible with his strategy of making sense, through which the structural abstraction framework evolves and is further refined. The attention is focused on a student's generic representation of the limit concept that allows him to generate meaningful components specific to particular contexts. Further, a sketch of the basic ideas of structural abstraction is given, and the use of the generic representation as a resource to reconstruct the meaning of the concept at need is discussed. Additionally, the importance of structural abstraction for learning mathematics is elaborated.

Pre-service Teachers' Views on Mathematics Homework Practices

Sven Trenholm
University of South Australia
<sven.trenholm@unisa.edu.au>

Mohan Chinnappan
University of South Australia
<mohan.chinnappan@unisa.edu.au>

Literature suggests that homework plays an important role in mathematics learning yet, in the Australian context, there is limited related research on this issue. This exploratory study sets out to better understand pre-service teachers' intentions and practices concerning mathematics homework. Using a survey design, we analysed data collected from a questionnaire administered to 98 (71% response rate) pre-service teachers (PSTs), all in the third year of their BEd program and completing a third course in mathematical methods as well as professional experience. Contrary to our expectation, the difference in perceptions among PSTs teaching upper and lower primary grades were not statistically significant.

Teaching out-of-field: Meanings, representations and silences

Colleen Vale
Deakin University
<colleen.vale@deakin.edu.au>

Linda Hobbs
Deakin University
< linda.hobbs@deakin.edu.au >

Christopher Speldewinde
Deakin University
<christopher.speldewinde@deakin.edu.au>

Zahra Parvanehnezhadshirazian
Deakin University
<zahra.p@deakin.edu.au>

Teaching out-of-field is a concern internationally, and in Australia, and is linked to social, economic and educational costs for students and teachers along with an ethical and social justice issue for the community. At the national level, out-of-field teaching is most often represented as a problem of teacher quality involving less qualified teachers. Using a critical lens, meanings and representations of government policy and stakeholder perspectives and practices are analysed. The findings show how teaching out-of-field occurs and is legitimated and reveal the opportunities for contesting these positions to improve the outcomes for students and out-of-field teachers.

Promoting Financial Literacy in Pre-service Teacher Education through On-line Modules

Leigh Wood
Macquarie University
<leigh.wood@mq.edu.au>

Carmel Coady
University of Western Sydney
<c.coady@uws.edu.au>

Joanne Mulligan
Macquarie University
<joanne.mulligan@mq.edu.au>

Michael Cavanagh
Macquarie University
<michael.cavanagh@mq.edu.au>

Damian Bridge
Macquarie University
<damian.bridge@mq.edu.au>

Opening Real Science (ORS) is 3-year Australian Government project led by Macquarie University supported by the Office for Learning and Teaching under the Enhancing the Training of Mathematics and Science Teachers Scheme (ETMST). ORS is developing a series of modules for implementation in teacher education programs, some of which focus on financial literacy: budgeting, investing and protecting, and modelling. The modules will be designed for active learning incorporating digital literacy themes to showcase implementation of technology integration into curriculum. Currently there are several trials in progress at three partner Australian universities. Evaluation data will inform the design-based approach to program re-development aimed at building the mathematical competence, and confidence of teachers.